Determine whether the events are independent or dependent. Then find the probability.

1. A red die and a blue die are rolled. What is the probability of getting the result shown?



SOLUTION:

Since the outcome of tossing the red die does not affect the outcome of rolling the blue die, these events are independent.

P(3 and 5)

 $=P(3)\cdot P(5)$ Probability of independent events

$$=\frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36}$$
 $P(3) = \frac{1}{6} \text{ and } P(5) = \frac{1}{6}$

The probability is $\frac{1}{36}$.

2. Yana has 4 black socks, 6 blue socks, and 8 white socks in his drawer. If he selects three socks at random with no replacement, what is the probability that he will first select a blue sock, then a black sock, and then another blue sock?

SOLUTION:

Since the socks are being selected with out replacement, the events are dependent.

$$P(blue) = \frac{6}{18} \text{ or } \frac{1}{3}$$

$$P(black|blue) = \frac{4}{17}$$

$$P(\text{bluel(blue and black)}) = \frac{5}{16}$$

P(blue and black and blue)

$$=P(black) \cdot P(blue|black) \cdot P(blue|(blue and black))$$

$$=\frac{1}{3} \cdot \frac{4}{17} \cdot \frac{5}{16} \text{ or } \frac{5}{204}$$

The probability is $\frac{5}{204}$ or about 0.025.

A die is rolled twice. Find each probability.

3. P(2 and 3)

SOLUTION:

P(2 and 3)

 $=P(2)\cdot P(3)$ Probability of independent events.

$$=\frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36}$$
 $P(2) = \frac{1}{6} \text{ and } P(3) = \frac{1}{6}$

The probability is $\frac{1}{36}$.

4. P(two 4s)

SOLUTION:

P(4 and 4)

 $=P(4)\cdot P(4)$ Probability of independent events.

$$=\frac{1}{6} \cdot \frac{1}{6} \text{ or } \frac{1}{36}$$
 $P(4) = \frac{1}{6} \text{ and } P(4) = \frac{1}{6}$

The probability is $\frac{1}{36}$.

5. *P*(no 6s)

SOLUTION:

P(no 6 and no 6)

 $=P(no 6) \cdot P(no 6)$ Probability of independent events

$$=\frac{5}{6} \cdot \frac{5}{6} \text{ or } \frac{25}{36}$$
 $P(\text{no } 6) = \frac{5}{6}$

The probability is $\frac{25}{36}$.

6. *P*(two of the same number)

SOLUTION:

P(two of the same numbers)

$$= P(2 \text{ 1s}) + P(2 \text{ 2s}) + P(2 \text{ 3s}) + P(2 \text{ 4s}) + P(2 \text{ 5s}) + P(2 \text{ 6s})$$

$$= \frac{1}{6} \cdot \frac{1}{6} + \frac{1}{6} \cdot \frac{1}{6}$$

$$= 6\left(\frac{1}{36}\right) = \frac{1}{6}$$
Simplify.

The probability is $\frac{1}{6}$.

A bag contains 8 blue marbles, 6 red marbles, and 5 green marbles. Three marbles are drawn one at a time. Find each probability.

7. The second marble is green, given that the first marble is blue and not replaced.

SOLUTION:

P(green|blue)

$$= \frac{P(\text{green and blue})}{P(\text{blue})} \quad \text{Conditional probability}$$

$$= \frac{\frac{40}{342}}{\frac{8}{19}} \qquad \qquad P(\text{blue}) = \frac{8}{19} \text{ and } P(\text{green and blue}) = \frac{5}{19} \cdot \frac{8}{18}$$

$$= \frac{40}{342} \cdot \frac{19}{8} = \frac{5}{18} \qquad \text{Simplify.}$$

The probability is $\frac{5}{18}$.

8. The second marble is red, given that the first marble is green and is replaced.

SOLUTION:

P(red|green)

$$= \frac{P(\text{red and green})}{P(\text{green})} \quad \text{Conditional probability}$$

$$= \frac{\frac{30}{61}}{\frac{5}{19}} \qquad P(\text{green}) = \frac{5}{19} \text{ and } P(\text{red and green}) = \frac{6}{19} \cdot \frac{5}{19}$$

$$= \frac{30}{361} \cdot \frac{19}{5} = \frac{6}{19} \qquad \text{Simplify.}$$

The probability is $\frac{6}{19}$.

9. The third marble is red, given that the first two are red and blue and not replaced.

SOLUTION:

P(red)(red and blue))

$$= \frac{P(\text{red and (red and blue}))}{P((\text{red and blue}))} \quad \text{Conditional probability}$$

$$= \frac{\frac{240}{5814}}{\frac{48}{342}}$$
 P(red and blue)= $\frac{6}{19} \cdot \frac{8}{18}$ and

$$P(\text{red and (red and blue})) = \frac{6}{19} \cdot \frac{5}{18} \cdot \frac{8}{17}$$

$$=\frac{240}{5814} \cdot \frac{342}{48} = \frac{5}{17}$$
 Simplify.

The probability is $\frac{5}{17}$.

10. The third marble is green, given that the first two are red and are replaced.

SOLUTION:

P(green|(red and red))

$$= \frac{P(\text{green and (red and red)})}{P((\text{red and red}))} \quad \text{Conditional probability}$$

$$= \frac{\frac{180}{19^3}}{\frac{36}{10^2}}$$
 P(red and red)= $\frac{6}{19} \cdot \frac{6}{19}$ and

$$P(\text{green and (red and red})) = \frac{5}{19} \cdot \frac{6}{19} \cdot \frac{6}{19}$$

$$= \frac{180}{19^3} \cdot \frac{19^2}{36} = \frac{5}{19}$$
 Simplify.

The probability is $\frac{5}{19}$.

DVDS There are 8 action, 3 comedy, and 5 drama DVDs on a shelf. Suppose three DVDs are selected at random from the shelf. Find each probability.

11. P(3 action), with replacement

SOLUTION:

$$P(3 \text{ action}) = \frac{8}{16} \cdot \frac{8}{16} \cdot \frac{8}{16} = \frac{1}{8} P(\text{action}) = \frac{8}{16}$$

The probability is $\frac{1}{8}$.

12. P(2 action, then a comedy), without replacement

SOLUTION:

$$P(\text{action, action, and comedy}) = \frac{3}{16} \cdot \frac{8}{15} \cdot \frac{7}{14} = \frac{1}{20}$$

The probability is $\frac{1}{20}$.

13. **CARDS** You draw a card from a standard deck of cards and show it to a friend. The friend tells you that the card is red. What is the probability that you correctly guess that the card is the ace of diamonds?

SOLUTION:

Given that the card is red, the probability it is an ace is $\frac{1}{26}$. The probability is $\frac{1}{26}$.

14. **HONOR ROLL** Suppose the probability that a student takes AP Calculus and is on the honor roll is 0.0035, and the probability that a student is on the honor roll is 0.23. Find the probability that a student takes AP Calculus given that he or she is on the honor roll.

SOLUTION:

P(AP Calchonor roll)

$$= \frac{P(\text{AP Calc and honor roll})}{P(\text{honor roll})} \quad \text{Conditional probability}$$

$$= \frac{0.0035}{0.23} \qquad \qquad P(\text{honor roll}) = 0.23 \text{ and}$$

$$P(\text{AP Calc and honor roll}) = 0.0035$$

$$\approx 0.015 \qquad \text{Simplify.}$$

The probability is about 0.015.

15. **DRIVING TESTS** The table shows how students in Mr. Diaz's class fared on their first driving test. Some took a class to prepare, while others did not. Find each probability.

Status	Class	No Class	
passed	64	48	
failed	18	32	

- **a.** Paige passed, given that she took the class.
- **b.** Madison failed, given that she did not take the class.
- **c.** Jamal did not take the class, given that he passed.

SOLUTION:

Calculate the column and row totals.

Status	Class	No Class	Totals
passed	64	48	112
failed	18	32	50
Totals	82	80	162

P(passiclass)

$$= \frac{P(\text{pass and class})}{P(\text{class})} \quad \text{Conditional probability}$$

$$= \frac{\frac{64}{162}}{\frac{82}{162}} \qquad P(\text{class}) = \frac{82}{162} \text{ and } P(\text{pass and class}) = \frac{64}{162}$$

$$= \frac{64}{82} = \frac{21}{41} \qquad \text{Simplify.}$$

b.

P(fail|class)

$$= \frac{P(\text{fail and no class})}{P(\text{no class})} \quad \text{Conditional probability}$$

$$= \frac{\frac{32}{162}}{\frac{80}{162}} \qquad P(\text{no class}) = \frac{80}{162} \text{ and } P(\text{fail and no class}) = \frac{32}{162}$$

$$= \frac{32}{80} = \frac{2}{5} \quad \text{Simplify.}$$

c.

P(no class|pass)

$$= \frac{P(\text{no class and pass})}{P(\text{pass})} \quad \text{Conditional probability}$$

$$= \frac{\frac{48}{162}}{\frac{112}{162}} \qquad \qquad P(\text{plass}) = \frac{112}{162} \text{and} P(\text{no class and pass}) = \frac{48}{162}$$

$$= \frac{48}{112} = \frac{3}{4} \qquad \text{Simplify.}$$

16. **SCHOOL CLUBS** King High School tallied the number of males and females that were members of at least one afte each probability.

Gender	Clubs	No Clubs
male	156	242
female	312	108

- **a.** A student is a member of a club given that he is male.
- **b.** A student is not a member of a club given that she is female.
- **c.** A student is a male given that he is not a member of a club.

SOLUTION:

Calculate the column and row totals.

		Clubs	
male	156	242	398
female	312	108	420
Totals	468	350	818

P(club memberimale)

$$= \frac{P(\text{club member and male})}{P(\text{male})} \quad \text{Conditional probability}$$

$$= \frac{\frac{156}{818}}{\frac{398}{818}} \qquad \qquad P(\text{male}) = \frac{398}{818} \text{ and}$$

$$P(\text{club member and male}) = \frac{156}{818}$$

$$\frac{156}{398} = \frac{78}{199}$$
 Simplify.

$$P(\text{club member|male}) = \frac{P(\text{club member and male})}{P(\text{ male})} \quad \text{Conditional probability}$$

$$= \frac{\frac{156}{818}}{\frac{398}{818}} \qquad \qquad P(\text{male}) = \frac{398}{818} \text{ and } P(\text{club member and male})$$

$$= \frac{156}{398} = \frac{78}{199} \qquad \qquad \text{Simplify.}$$

P(non member|female)

$$= \frac{P(\text{non member and female})}{P(\text{ female})} \quad \text{Conditional probability}$$

$$= \frac{\frac{108}{815}}{\frac{420}{815}} \qquad \qquad P(\text{female}) = \frac{420}{818} \text{ and}$$

 $=\frac{108}{420}=\frac{9}{35}$

$$P(\text{non member and female}) = \frac{108}{818}$$

Simplify.

$$P(\text{non member|female}) = \frac{P(\text{non member and female})}{P(\text{ female})} \quad \text{Conditional probability}$$

$$= \frac{\frac{108}{815}}{\frac{420}{815}} \qquad \qquad P(\text{female}) = \frac{420}{818} \text{ and } P(\text{non member an member and female)}$$

Simplify.

 $\frac{108}{420} = \frac{9}{35}$

P(maleinon member)

$$= \frac{P(\text{male and non member})}{P(\text{non member})} \quad \text{Conditional probability}$$

$$= \frac{\frac{242}{818}}{\frac{350}{818}} \qquad P(\text{non member}) = \frac{350}{818} \text{ and}$$

$$P(\text{male and non member}) = \frac{242}{818}$$

$$= \frac{242}{350} = \frac{121}{175} \qquad \text{Simplify.}$$

17. FOOTBALL ATTENDANCE The number of students who have attended a football game at North Coast High School is shown. Find each probability.

Class	Freshman	Sophomore	Junior	Senior
attended	48	90	224	254
not attended	182	141	36	8

- a. A student is a freshman and has not attended a game.
- **b.** A student has attended a game and is an upperclassman (a junior or senior).

SOLUTION:

Calculate the column and row totals.

class	Freshman	Sophomore	Junior	Senior	Totals
attended	48	90	224	254	616
not	182	141	36	8	367
attended					
Totals	230	231	260	262	983

- **a.** Given that the student is a freshman, the $P(\text{not attended}) = \frac{182}{230} = \frac{91}{115}$ or about 79.1%. **b.** Given that the student attended the game, the $P(\text{upperclassman}) = \frac{478}{616} = \frac{239}{308}$ or about 77.6%.