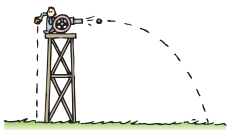
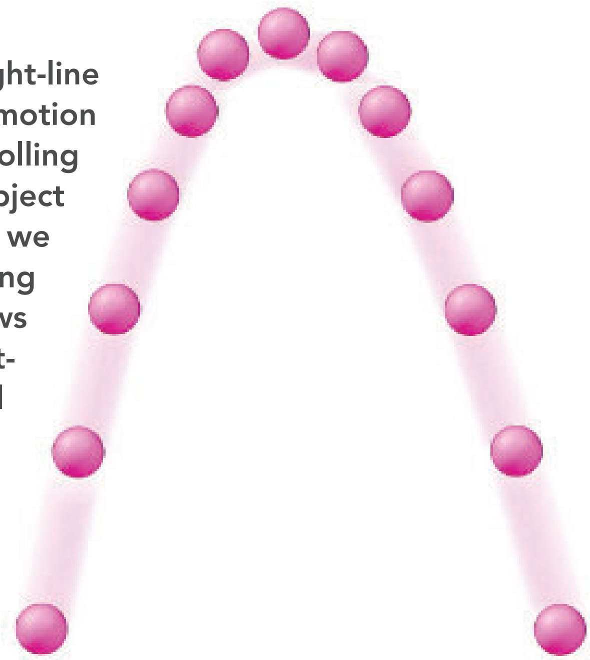
5 PROJECTILE MOTION



THE BIG IDEA Projectile motion can be described by the horizontal and vertical components of motion.

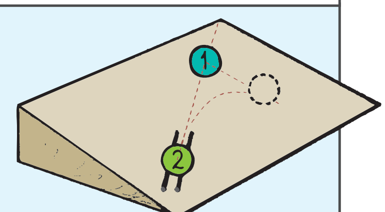


In the previous chapter, we studied simple straight-line motion—linear motion. We distinguished between motion with constant velocity, such as a bowling ball rolling horizontally, and accelerated motion, such as an object falling vertically under the influence of gravity. Now we extend these ideas to nonlinear motion—motion along a curved path. Throw a baseball and the path it follows is a curve. This curve is a combination of constant-velocity horizontal motion and accelerated vertical motion. We'll see that the velocity of a thrown ball at any instant has two independent "components" of motion—what happens horizontally is not affected by what happens vertically.

discover!

How Should You Aim to Hit a Falling Target?

1. On one corner of a rectangular piece of rigid cardboard, tape two 5-cm lengths of soda straws so they form a trough. Angle the straws to point toward the diagonal corner of the cardboard.
2. Draw a straight line passing through the center of the soda straws and extending to the top of the cardboard.
3. Tilt the cardboard so that Marble 1 will roll downhill. Hold Marble 1 in the upper right corner of the cardboard on the line you've drawn. Place a second marble in the trough formed by the two soda straws.
4. At the same time, release Marble 1 and launch Marble 2 by giving it a flick with your finger.



Analyze and Conclude

1. **Observing** Did you hit Marble 1? If so, what determined the point of collision?
2. **Predicting** What would happen if you used marbles with different masses?
3. **Making Generalizations** Why do the two marbles fall the same vertical distance from the line in the same amount of time?

5.1 Vector and Scalar Quantities

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It is often said that a picture is worth a thousand words. Sometimes a picture explains a physics concept better than an equation does. Physicists love sketching doodles and equations to explain ideas. Their doodles often include arrows, where each arrow represents the magnitude and the direction of a certain quantity. The quantity might be the tension in a stretched rope, the compressive force in a squeezed spring, or the change in velocity of an airplane flying in the wind.

A quantity that requires both magnitude and direction for a complete description is a vector quantity. Recall from Chapter 4 that velocity differs from speed in that velocity includes direction in its description. Velocity is a vector quantity, as is acceleration. In later chapters we'll see that other quantities, such as momentum, are also vector quantities. For now we'll focus on the vector nature of velocity.

Recall from Chapter 2 that a quantity that is completely described by magnitude only is a scalar quantity. Scalars can be added, subtracted, multiplied, and divided like ordinary numbers. ✓ **A vector quantity includes both magnitude and direction, but a scalar quantity includes only magnitude.** When 3 kg of sand is added to 1 kg of cement, the resulting mixture has a mass of 4 kg. When 5 liters of water are poured from a pail that has 8 liters of water in it, the resulting volume is 3 liters. If a scheduled 60-minute trip has a 15-minute delay, the trip takes 75 minutes. In each of these cases, no direction is involved. We see that descriptions such as 10 kilograms north, 5 liters east, or 15 minutes south have no meaning.

CONCEPT CHECK How does a scalar quantity differ from a vector quantity?

Physics on the Job

Air-Traffic Controller

Busy airports have many aircraft landing or taking off every minute. Air-traffic controllers are responsible for guiding pilots to their destinations. Using an understanding of vectors, air-traffic controllers determine the proper speed and direction of an aircraft by taking into account the velocity of the wind, path of the aircraft, and local air traffic. They use radar equipment as well as their view from the control tower to follow the motion of all aircraft flying near the airport.

FIGURE 5.1 ▲
This vector, scaled so that 1 cm = 20 km/h, represents 60 km/h to the right.

5.2 Velocity Vectors

The vector in Figure 5.1 is scaled so that 1 centimeter represents 20 kilometers per hour. It is 3 centimeters long and points to the right; therefore it represents a velocity of 60 kilometers per hour to the right, or 60 km/h east.

The velocity of something is often the result of combining two or more other velocities. For example, an airplane's velocity is a combination of the velocity of the airplane relative to the air and the velocity of the air relative to the ground, or the wind velocity. Consider a small airplane slowly flying north at 80 km/h relative to the surrounding air. Suppose there is a tailwind blowing north at a velocity of 20 km/h. This example is represented with vectors in Figure 5.2. Here the velocity vectors are scaled so that 1 cm represents 20 km/h. Thus, the 80-km/h velocity of the airplane is shown by the 4-cm vector, and the 20-km/h tailwind is shown by the 1-cm vector. With or without vectors we can see that the resulting velocity is going to be 100 km/h. Without the tailwind, the airplane travels 80 kilometers in one hour relative to the ground below. With the tailwind, it travels 100 kilometers in one hour relative to the ground below.

Now suppose the airplane makes a U-turn and flies *into* the wind. The velocity vectors are now in opposite directions. The resulting speed of the airplane is 80 km/h − 20 km/h = 60 km/h. Flying against a 20-km/h wind, the airplane travels only 60 kilometers in one hour relative to the ground.

We didn't have to use vectors to answer questions about tailwinds and headwinds, but we'll now see that vectors are useful for combining velocities that are not parallel.

Consider an 80-km/h airplane flying north caught in a strong crosswind of 60 km/h blowing from west to east. Figure 5.3 shows vectors for the airplane velocity and wind velocity. The scale is 1 cm = 20 km/h. The sum of these two vectors, called the *resultant*, is the diagonal of the rectangle described by the two vectors.

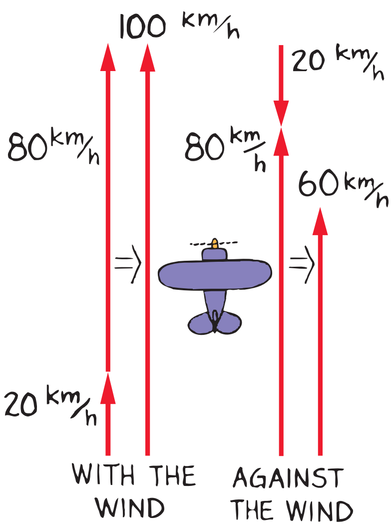


FIGURE 5.2 ▲
The airplane's velocity relative to the ground depends on the airplane's velocity relative to the air and on the wind's velocity.

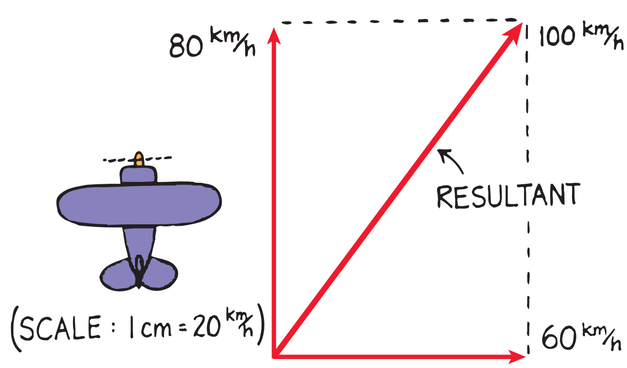
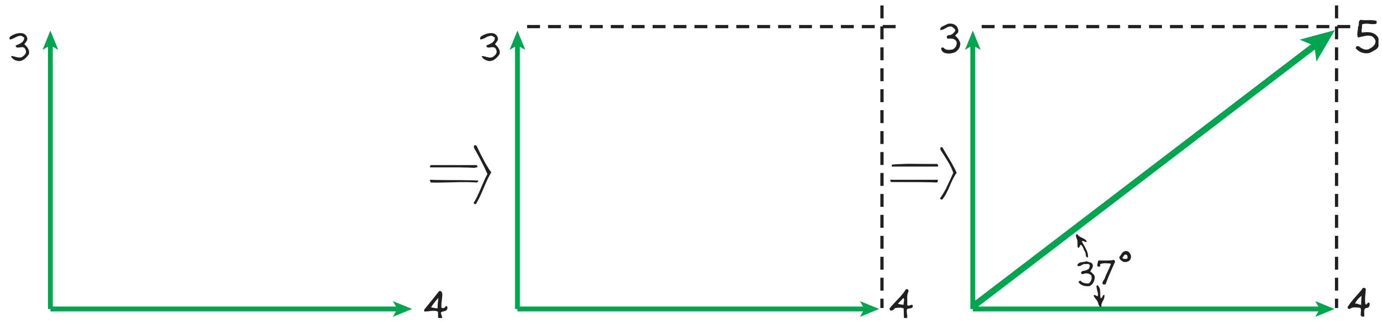


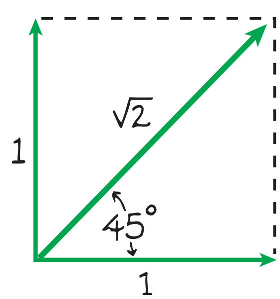
FIGURE 5.3 ▶
An 80-km/h airplane flying in a 60-km/h crosswind has a resultant speed of 100 km/h relative to the ground.

FIGURE 5.4 ▼
The 3-unit and 4-unit vectors add to produce a resultant vector of 5 units, at 37° from the horizontal.



The resultant of two perpendicular vectors is the diagonal of a rectangle constructed with the two vectors as sides. We learned this in Chapter 2. Here, the diagonal of the constructed rectangle measures 5 cm, which represents 100 km/h. So relative to the ground, the airplane moves 100 km/h northeasterly.[5.2.1]

In Figure 5.4 we see a 3-unit vector at right angles to a 4-unit vector. Can you see that they make up the sides of a rectangle, and when added vectorially they produce a resultant of magnitude of 5? (Note that $5^2 = 3^2 + 4^2$.)



◀ **FIGURE 5.5**
The diagonal of a square is $\sqrt{2}$, or 1.414, times the length of one of its sides.

In the special case of adding a pair of equal-magnitude vectors that are at right angles to each other, we construct a square, as shown in Figure 5.5. For any square, the length of its diagonal is $\sqrt{2}$, or 1.414, times either of its sides. Thus, the resultant is $\sqrt{2}$ times either of the vectors. For instance, the resultant of two equal vectors of magnitude 100 acting at right angles to each other is 141.4.[5.2.2]

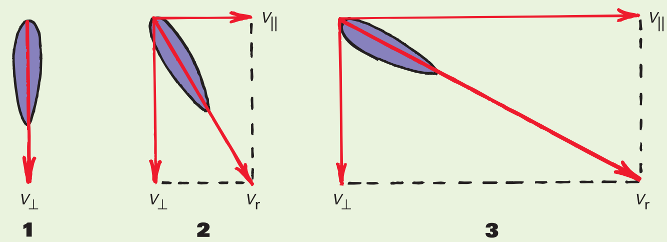
CONCEPT CHECK What is the resultant of two perpendicular vectors?

think!

Suppose that an airplane normally flying at 80 km/h encounters wind at a right angle to its forward motion—a crosswind. Will the airplane fly faster or slower than 80 km/h?
Answer: 5.2

Physics of Sports

Surfing Surfing nicely illustrates component and resultant vectors. (1) When surfing in the same direction as the wave, our velocity is the same as the wave's velocity, $v_\perp$. This velocity is called $v_\perp$ because we are moving perpendicular to the wave front. (2) To go faster, we surf at an angle to the wave front. Now we have a component of velocity parallel to the wave front, $v_\parallel$, as well as the perpendicular component $v_\perp$. We can vary $v_\parallel$, but $v_\perp$ stays relatively constant as long as we ride the wave. Adding components, we see that when surfing at an angle to the wave front, our resultant velocity, v_r, exceeds $v_\perp$. (3) As we increase our angle relative to the wave front, the resultant velocity also increases.



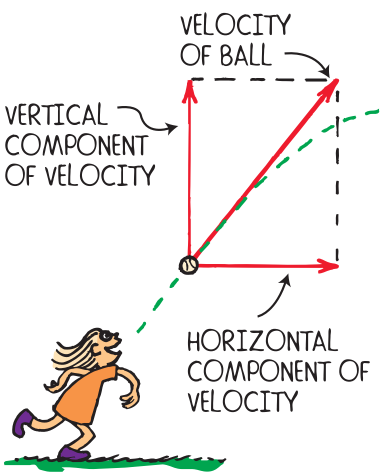


FIGURE 5.6 ▲
A ball's velocity can be resolved into horizontal and vertical components.

5.3 Components of Vectors

Often we will need to change a single vector into an equivalent set of two *component* vectors at right angles to each other. Any vector can be "resolved" into two component vectors at right angles to each other, as shown in Figure 5.6. Two vectors at right angles that add up to a given vector are known as the **components** of the given vector they replace. The process of determining the components of a vector is called **resolution.** Any vector drawn on a piece of paper can be resolved into vertical and horizontal components that are perpendicular. **The perpendicular components of a vector are independent of each other.**

Vector resolution is illustrated in Figure 5.7. Vector *V* represents a vector quantity. First, vertical and horizontal lines are drawn from the tail of the vector (top). Second, a rectangle is drawn that encloses the vector *V* as its diagonal (bottom). The sides of this rectangle are the desired components, vectors *X* and *Y*.

CONCEPT CHECK How do components of a vector affect each other?

FIGURE 5.7 ▶
Vectors *X* and *Y* are the horizontal and vertical components of a vector *V*.

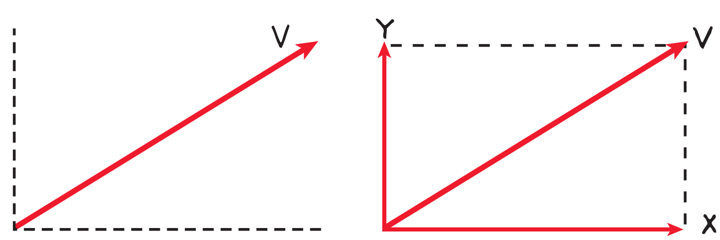
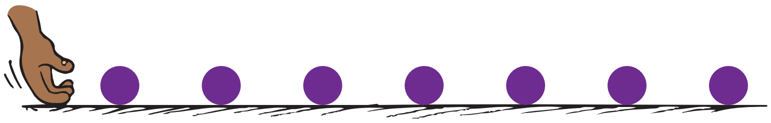
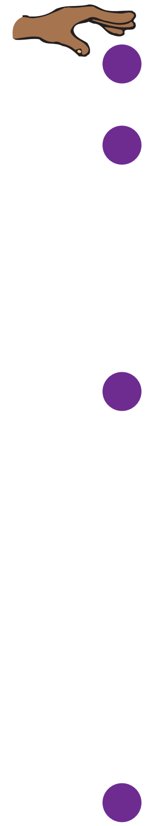


FIGURE 5.8
Projectile motion can be separated into components.

a. Roll a ball along a horizontal surface, and its velocity is constant because no component of gravitational force acts horizontally.



b. Drop it, and it accelerates downward and covers a greater vertical distance each second.



5.4 Projectile Motion

A cannonball shot from a cannon, a stone thrown into the air, a ball rolling off the edge of a table, a spacecraft circling Earth—all of these are examples of *projectiles*. A **projectile** is any object that moves through the air or space, acted on only by gravity (and air resistance, if any). Projectiles near the surface of Earth follow a curved path that at first seems rather complicated. However, these paths are surprisingly simple when we look at the horizontal and vertical components of motion separately.

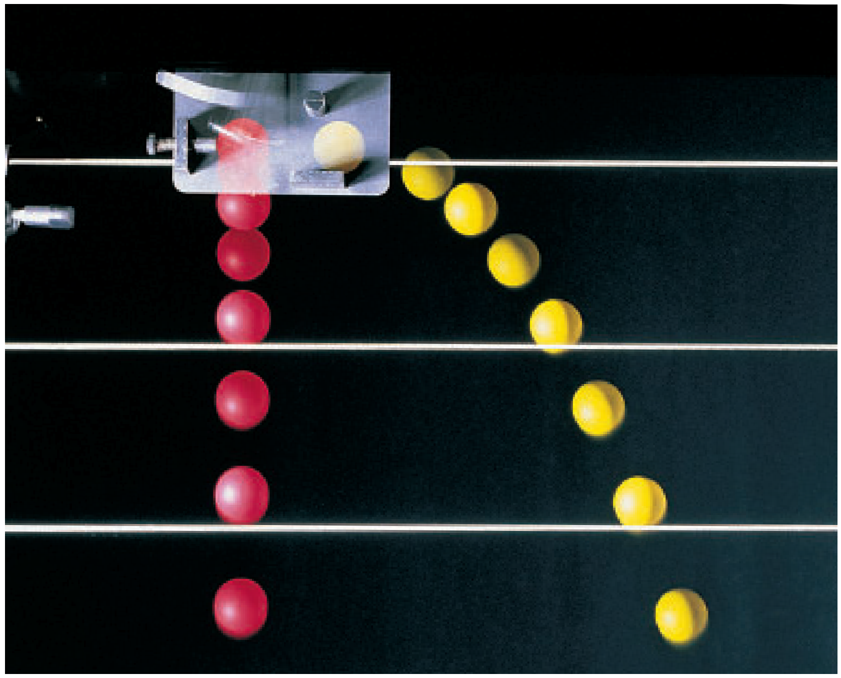
The horizontal component of motion for a projectile is just like the horizontal motion of a ball rolling freely along a level surface without friction. When friction is negligible, a rolling ball moves at constant velocity. The ball covers equal distances in equal intervals of time as shown in Figure 5.8a. With no horizontal force acting on the ball there is no horizontal acceleration. The same is true for the projectile—when no horizontal force acts on the projectile, the horizontal component of velocity remains constant.

The vertical component of a projectile's velocity is like the motion for a freely falling object. Gravity acts vertically downward. Like a ball dropped in midair, a projectile accelerates downward as shown on the right in Figure 5.8b. Its vertical component of velocity changes with time. The increasing speed in the vertical direction causes a greater distance to be covered in each successive equal time interval. Or, if the ball is projected upward, the vertical distances of travel decrease with time on the way up.

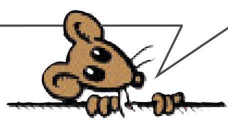
Most important, the horizontal component of motion for a projectile is completely independent of the vertical component of motion. Each component is independent of the other. Their combined effects produce the variety of curved paths that projectiles follow.

CONCEPT CHECK **Describe the components of projectile motion.**

FIGURE 5.9 ▶
A strobe-light photo of two balls released simultaneously from a mechanism that allows one ball to drop freely while the other is projected horizontally. Notice that in equal times both balls fall the same vertical distance.



The curved path shown in Figure 5.9 is the combination of constant horizontal motion and vertical motion that undergoes acceleration due to gravity.



5.5 Projectiles Launched Horizontally

Projectile motion is nicely analyzed in the multiple-flash exposure in Figure 5.9. The photo shows equally timed successive positions for two balls. One ball is projected horizontally while the other is simply dropped. Study the photo carefully, for there's a lot of good physics here. Analyze the curved path of the ball by considering the horizontal and vertical velocity components separately. There are two important things to notice. The first is that the ball's horizontal component of motion remains constant. The ball moves the same horizontal distance in the equal time intervals between each flash, because no horizontal component of force is acting on it. Gravity acts only downward, so the only acceleration of the ball is downward. The second thing to note is that both balls fall the same vertical distance in the same time. The vertical distance fallen has nothing to do with the horizontal component of motion. ✓ **The downward motion of a horizontally launched projectile is the same as that of free fall.**

The path traced by a projectile accelerating only in the vertical direction while moving at constant horizontal velocity is a *parabola.* When air resistance is small enough to neglect—usually for slow-moving or very heavy projectiles—the curved paths are parabolic.

Toss a stone from a cliff and its path curves as it accelerates toward the ground below. Figure 5.10a shows how the trajectory is a combination of constant horizontal motion and accelerated vertical motion.

think!

At the instant a horizontally pointed cannon is fired, a cannonball held at the cannon's side is released and drops to the ground. Which cannonball strikes the ground first, the one fired from the cannon or the one dropped?
Answer: 5.5

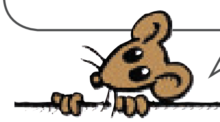
CONCEPT CHECK Describe the downward motion of a horizontally launched projectile.

discover!

Which Coin Hits the Ground First?

1. Place a coin at the edge of a table so that it hangs over slightly. Place a second coin on the table some distance from the first coin.
2. Slide the second coin so it hits the first one and both coins fall to the floor below. Which coin hits the ground first?
3. **Think** Does your answer depend on the speed of the coin? Explain.

5.6 Projectiles Launched at an Angle



In Figure 5.10, we see the paths of stones thrown horizontally and at angles upward and downward. The dashed straight lines show the ideal trajectories of the stones if there were no gravity. Notice that the vertical distance that the stone falls beneath the idealized straight-line paths is the same for equal times. This vertical distance is independent of what's happening horizontally.

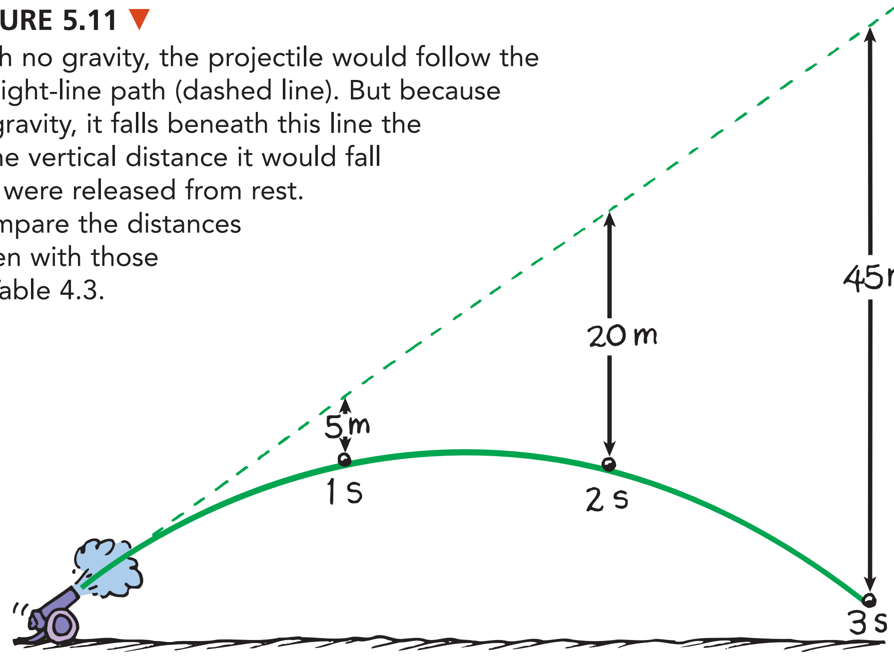
FIGURE 5.10 ▼
No matter the angle at which a projectile is launched, the vertical distance of fall beneath the idealized straight-line path is the same for equal times.

a. The trajectory of the stone combines horizontal motion with the pull of gravity.

b. The trajectory of the stone combines the upward motion with the pull of gravity.

c. The trajectory of the stone combines downward motion with the pull of gravity.

FIGURE 5.11 ▼
With no gravity, the projectile would follow the straight-line path (dashed line). But because of gravity, it falls beneath this line the same vertical distance it would fall if it were released from rest. Compare the distances fallen with those in Table 4.3.



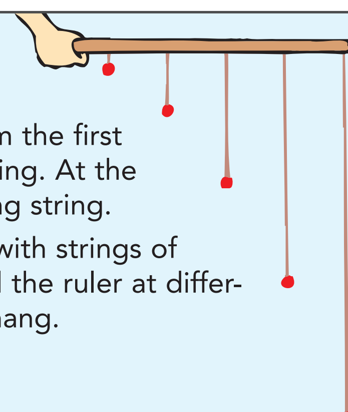
think!

A projectile is launched at an angle into the air. Neglecting air resistance, what is its vertical acceleration? Its horizontal acceleration?
Answer: 5.6.1

Figure 5.11 shows specific vertical distances for a cannonball shot at an upward angle. If there were no gravity, the cannonball would follow the straight-line path shown by the dashed line. But there is gravity, so this doesn't occur. What happens is that the cannonball continuously falls beneath the imaginary line until it finally strikes the ground. The vertical distance it falls *beneath any point on the dashed line* is the same vertical distance it would fall if it were dropped from rest and had been falling for the same amount of time. This distance, introduced in Chapter 4, is given by $d = \frac{1}{2}gt^2$, where t is the elapsed time. Using the value of 10 m/s^2 for g in the equation yields $d = 5t^2$ meters.

discover!

How Can You Model Projectile Motion?



1. Mark a ruler at five equal spaces. From the first mark, hang a bead on a 1-cm long string. At the next mark, hang a bead on a 4-cm long string.
2. Hang beads on the next three marks with strings of lengths 9 cm, 16 cm, and 25 cm. Hold the ruler at different angles and see where the beads hang.
3. **Think** Why is this model accurate?

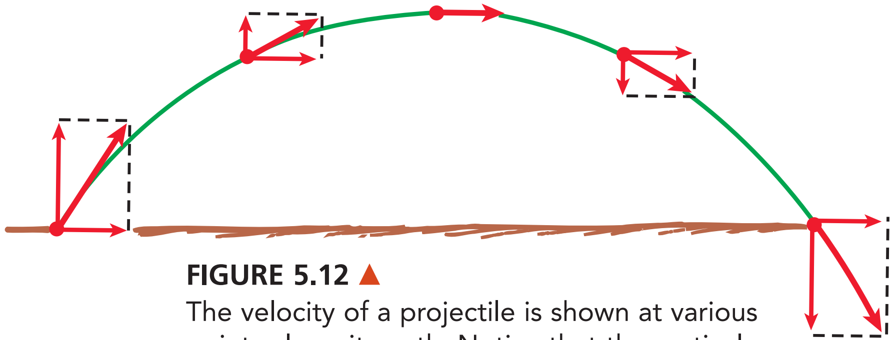


FIGURE 5.12 ▲
The velocity of a projectile is shown at various points along its path. Notice that the vertical component changes while the horizontal component does not. Air resistance is neglected.

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Height We can put this another way. Toss a projectile skyward at some angle and pretend there is no gravity. After so many seconds *t*, it should be at a certain point along a straight-line path. But due to gravity, it isn't. Where is it? The answer is, it's directly below that point. How far below? The answer is $5t^2$ meters below that point. How about that? **The vertical distance a projectile falls below an imaginary straight-line path increases continually with time and is equal to $5t^2$ meters.**

Note also from Figure 5.11 that since there is no horizontal acceleration, the cannonball moves equal horizontal distances in equal time intervals. That's because there is no horizontal acceleration. The only acceleration is vertical, in the direction of Earth's gravity.

Figure 5.12 shows vectors representing both the horizontal and vertical components of velocity for a projectile on a parabolic path. Notice that the horizontal component is always the same and that only the vertical component changes. Note also that the actual resultant velocity vector is represented by the diagonal of the rectangle formed by the vector components. At the top of the path the vertical component shrinks to zero, so the velocity there *is* the same as the horizontal component of velocity at all other points. Everywhere else the magnitude of velocity is greater, just as the diagonal of a rectangle is greater than either of its sides.

Range Figure 5.13 shows the path traced by a projectile with the same launching speed but at a steeper angle. Notice that the initial velocity vector has a greater vertical component than when the projection angle is less. This greater component results in a higher path. However, since the horizontal component is less, the range is less.

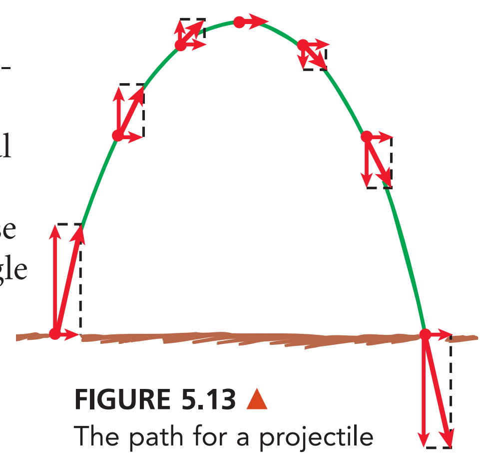


FIGURE 5.13 ▲
The path for a projectile fired at a steep angle. Again, air resistance is neglected.

FIGURE 5.14 ▶
The paths of projectiles launched at the same speed but at different angles. The paths neglect air resistance.

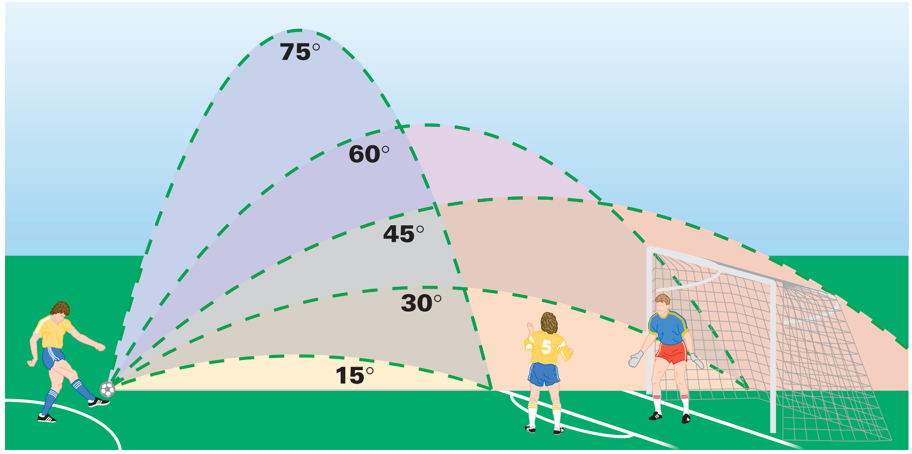


FIGURE 5.15 ▲
Maximum range is attained when the ball is batted at an angle of nearly 45°.

Horizontal Ranges Figure 5.14 shows the paths of several projectiles all having the same initial speed but different projection angles. The figure neglects the effects of air resistance, so the paths are all parabolas. Notice that these projectiles reach different heights (altitude) above the ground. They also travel different horizontal distances, that is, they have different *horizontal ranges.*

The remarkable thing to note from Figure 5.14 is that the same range is obtained for two different projection angles—angles that add up to 90 degrees! For example, an object thrown into the air at an angle of 60 degrees will have the same range as if it were thrown at 30 degrees with the same speed. Of course, for the smaller angle the object remains in the air for a shorter time. Maximum range is usually attained at an angle of 45°. For a thrown javelin, on the other hand, maximum range is achieved for an angle quite a bit less than 45°, because the force of gravity on the relatively heavy javelin is significant during launch. Just as you can't throw a heavy rock as fast upward as sideways, so it is that the javelin's launch speed is reduced when thrown upward.

Physics of Sports

Hang Time Revisited

Recall our discussion of hang time in Chapter 4. We stated that the time one is airborne during a jump is independent of horizontal speed. Now we see why this is so—horizontal and vertical components of motion are independent of each other. The rules of projectile motion apply to jumping. Once the feet are off the ground, if we neglect air resistance, the only force acting on the jumper is gravity. Hang time depends only on the vertical component of liftoff velocity. It turns out that jumping force can be somewhat increased by the action of running, so hang time for a running jump usually exceeds that for a standing jump. However, once the feet are off the ground, only the vertical component of liftoff velocity determines hang time.

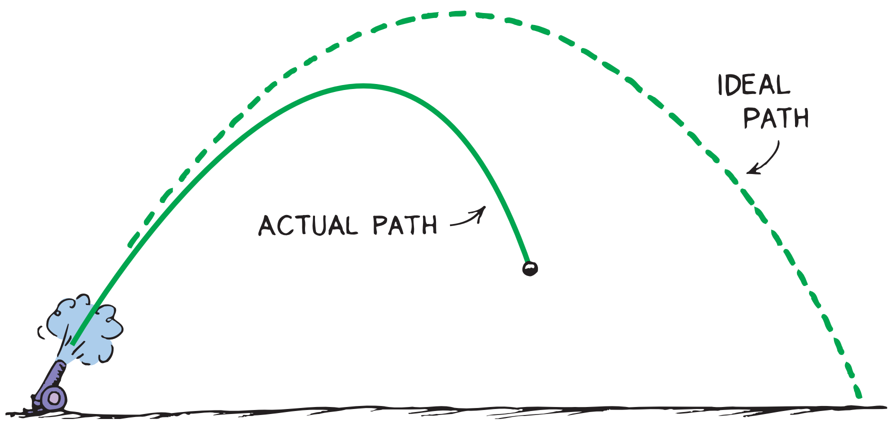


FIGURE 5.16
In the presence of air resistance, the path of a high-speed projectile falls below the idealized parabola and follows the solid curve.

Speed We have emphasized the special case of projectile motion for negligible air resistance. As we can see in Figure 5.16, when the effect of air resistance is significant, the range of a projectile is diminished and the path is not a true parabola.

If air resistance is negligible, a projectile will rise to its maximum height in the same time it takes to fall from that height to the ground. This is due to the constant effect of gravity. The deceleration due to gravity going up is the same as the acceleration due to gravity coming down. The speed it loses going up is therefore the same as the speed it gains coming down, as shown in Figure 5.17. So the projectile hits the ground with the same speed it had originally when it was projected upward from the ground.

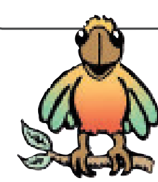
For short-range projectile motion such as a batted ball in a baseball game, we usually assume the ground is flat. However, for very long range projectiles the curvature of Earth's surface must be taken into account. We'll see that if an object is projected fast enough, it will fall all the way around Earth and become an Earth satellite! More about satellites in Chapter 14.

CONCEPT CHECK **Describe how far below an imaginary straight-line path a projectile falls.**

think!

At what point in its path does a projectile have minimum speed?
Answer: 5.6.2

The longest time a jumper is airborne for a standing jump (hang time) is 1 second, for a record 1.25 meters (4 ft) height. Can anyone in your school jump that high, raising their center of gravity 1.25 meters above the ground? Not likely!



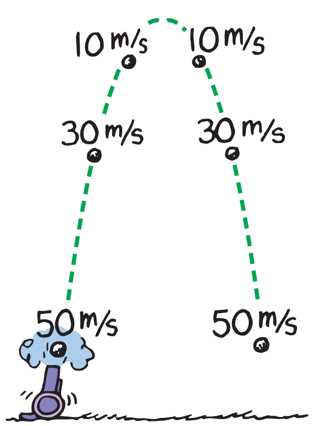


FIGURE 5.17
Without air resistance, the speed lost while the cannonball is going up equals the speed gained while it is coming down. The time to go up equals the time to come down.

5 REVIEW

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Concept Summary

- A vector quantity includes both magnitude and direction, but a scalar quantity includes only magnitude.
- The resultant of two perpendicular vectors is the diagonal of a rectangle constructed with the two vectors as sides.
- The perpendicular components of a vector are independent of each other.
- The horizontal component of motion for a projectile is just like the horizontal motion of a ball rolling freely along a level surface without friction. The vertical component of a projectile's velocity is like the motion for a freely falling object.
- The downward motion of a horizontally launched projectile is the same as that of free fall.
- The vertical distance a projectile falls below an imaginary straight-line path increases continually with time and is equal to $5t^2$ meters.

Key Terms

components *(p. 72)*
projectile *(p. 73)*
resolution *(p. 72)*

think! Answers

5.2 A crosswind would increase the speed of the airplane and blow it off course by a predictable amount.

5.5 Both cannonballs fall the same vertical distance with the same acceleration *g* and therefore strike the ground at the same time. Do you see that this is consistent with our analysis of Figure 5.9? Ask which cannonball strikes the ground first when the cannon is pointed at an upward angle. In this case, the cannonball that is simply dropped hits the ground first. Now consider the case when the cannon is pointed downward. The fired cannonball hits first. So upward, the dropped cannonball hits first; downward, the fired cannonball hits first. There must be some angle where both hit at the same time. Do you see it would be when the cannon is pointing neither upward nor downward, that is, when it is pointing horizontally?

5.6.1 Its vertical acceleration is *g* because the force of gravity is downward. Its horizontal acceleration is zero because no horizontal force acts on it.

5.6.2 The minimum speed of a projectile occurs at the top of its path. If it is launched vertically, its speed at the top is zero. If it is projected at an angle, the vertical component of velocity is still zero at the top, leaving only the horizontal component. So the speed at the top is equal to the horizontal component of the projectile's velocity at any point. How about that?

5 ASSESS

Check Concepts

Section 5.1

1. How does a vector quantity differ from a scalar quantity?
2. Why is speed classified as a scalar quantity and velocity classified as a vector quantity?

Section 5.2

3. If a vector that is 1 cm long represents a velocity of 10 km/h, what velocity does a vector 2 cm long drawn to the same scale represent?
4. When a rectangle is constructed in order to add perpendicular velocities, what part of the rectangle represents the resultant vector?

Section 5.3

5. Will a vector at 45° to the horizontal be larger or smaller than its horizontal and vertical components? By how much?

Section 5.4

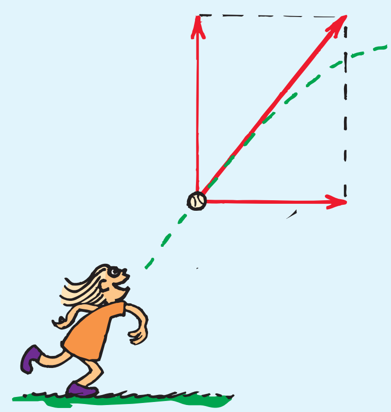
6. Why does a bowling ball move without acceleration when it rolls along a bowling alley?
7. In the absence of air resistance, why does the horizontal component of velocity for a projectile remain constant while the vertical component changes?
8. How does the downward component of the motion of a projectile compare with the motion of free fall?

Section 5.5

9. At the instant a ball is thrown horizontally over a level range, a ball held at the side of the first is released and drops to the ground. If air resistance is neglected, which ball strikes the ground first?

Section 5.6

10. a. How far below an initial straight-line path will a projectile fall in one second?
    b. Does your answer depend on the angle of launch or on the initial speed of the projectile? Defend your answer.



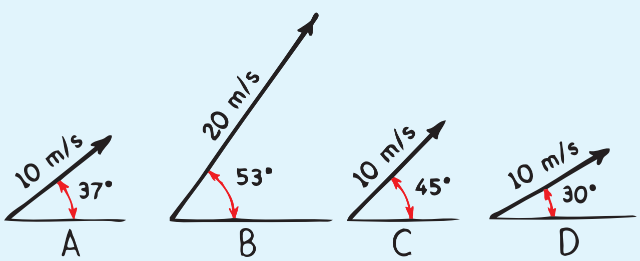
11. Neglecting air resistance, if you throw a ball straight up with a speed of 20 m/s, how fast will it be moving when you catch it?
12. a. Neglecting air resistance, if you throw a baseball at 20 m/s to your friend who is on first base, will the catching speed be greater than, equal to, or less than 20 m/s?
    b. Does the speed change if air resistance is a factor?
13. What do we call a projectile that continually "falls" around Earth?

5 ASSESS *(continued)*

Think and Rank

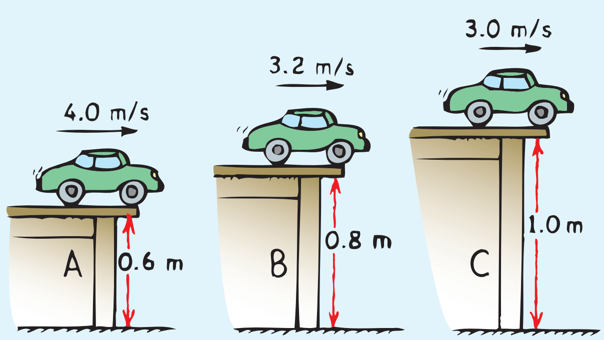
Rank each of the following sets of scenarios in order of the quantity or property involved. List them from left to right. If scenarios have equal rankings, then separate them with an equal sign. (e.g., A = B)

14. The vectors represent initial velocities of projectiles launched at ground level.



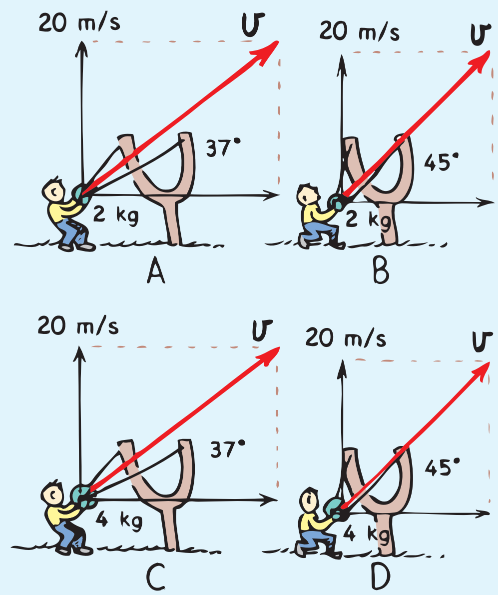
a. Rank them by their vertical components of velocity from greatest to least.
b. Rank them by their horizontal components of velocity from greatest to least.

15. A toy car rolls off tables of various heights at different speeds as shown.



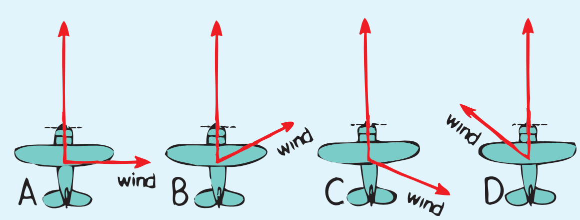
a. Rank them for the time in the air, from greatest to least.
b. Rank them for horizontal range, from greatest to least.

16. Water balloons of different masses are launched by slingshots at different launching velocities *v*. All have the same vertical component of launching velocities.



a. Rank by the *time* in the air, from longest to shortest.
b. Rank by the maximum *height* reached, from highest to lowest.
c. Rank by the maximum *range,* from greatest to least.

17. The airplane is blown off course by wind in the directions shown. Use the parallelogram rule and rank from highest to lowest the resulting speed across the ground.



Plug and Chug

For Questions 18–19, recall that when two vectors in the same or exactly opposite directions are added, the magnitude of their resultant is the sum or difference of their original magnitudes.

18. Calculate the resultant velocity of an airplane that normally flies at 200 km/h if it encounters a 50-km/h tailwind. If it encounters a 50-km/h headwind.

19. Calculate the magnitude of the resultant of a pair of 100-km/h velocity vectors that are at right angles to each other.

For questions 20–21, recall that the resultant V *of two vectors* A *and* B *at right angles to each other is found using the Pythagorean theorem:*

$$V = \sqrt{A^2 + B^2}$$

20. Calculate the resulting speed of an airplane with an airspeed of 120 km/h pointing due north when it encounters a wind of 90 km/h directed from the west. (Recall, speed is the magnitude of velocity.)

21. Calculate the speed of raindrops hitting your face when they fall vertically at 3 m/s while you're running horizontally at 4 m/s.

Think and Explain

22. Whenever you add 3 and 4, the result is 7. This is true if the quantities being added are scalar quantities. If 3 and 4 are the magnitudes of vector quantities, when will the magnitude of their sum be 5?

23. Christopher can paddle a canoe in still water at 8 km/h. How successful will he be at canoeing upstream in a river that flows at 8 km/h?

24. How does the vertical distance a projectile falls below an otherwise straight-line path compare with the vertical distance it would fall from rest in the same time?

25. The speed of falling rain is the same 10 m above ground as it is just before it hits the ground. What does this tell you about whether or not the rain encounters air resistance?

26. Marshall says that when a pair of vectors are at right angles to each other, the magnitude of their resultant is greater than the magnitude of either vector alone. Renee says he is speaking in generalities and that what he says isn't always true. With whom do you agree?

27. How is the horizontal component of velocity for a projectile affected by the vertical component?

28. Rain falling vertically will make vertical streaks on a car's side window. However, if the car is moving, the streaks are slanted. If the streaks from a vertically falling rain make 45° streaks, how fast is the car moving compared with the speed of the falling rain?

29. An airplane encounters a wind that blows in a perpendicular direction to the direction its nose is pointing. Does the effect of this wind increase or decrease speed across the ground below? Or does it have no effect on ground speed?

30. A projectile is launched vertically at 50 m/s. If air resistance can be neglected, at what speed will it return to its initial level? Where in its trajectory will it have minimum speed?

31. A batted baseball follows a parabolic path on a day when the sun is directly overhead. How does the speed of the ball's shadow across the field compare with the ball's horizontal component of velocity?

32. When air resistance acts on a projectile, does it affect the horizontal component of velocity, the vertical component of velocity, or both? Defend your answer.

5 ASSESS *(continued)*

33. You're driving behind a car and wish to pass, so you turn to the left and pull into the passing lane without changing speed. Why does the distance increase between you and the car you're following?

34. Brandon launches a projectile at an angle of 75° above the horizontal, which strikes the ground a certain distance down range. For what other angle of launch at the same speed would the projectile land just as far away?

35. When you jump up, your hang time is the time your feet are off the ground. Does hang time depend on your vertical component of velocity when you jump, your horizontal component of velocity, or both? Defend your answer.

36. The hang time of a basketball player who jumps a vertical distance of 2 feet (0.6 m) is 2/3 second. What will be the hang time if the player reaches the same height while jumping a horizontal distance of 4 feet (1.2 m)?

Think and Solve

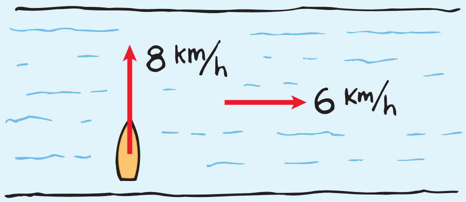
37. Sneezlee flies at a speed of 10 m/s in still air.

a. If he flies into a 2-m/s headwind, how fast will he be traveling relative to the ground below?

b. Relative to the ground below, how fast will he travel when he experiences a 2-m/s tailwind?

c. While flying at 10 m/s, suppose that he encounters a 10-m/s cross wind (coming at a right angle to his heading). What is his speed relative to the ground below?

38. A boat is rowed at 8 km/h directly across a river that flows at 6 km/h, as shown in the figure.



a. What is the resultant speed of the boat?

b. How fast and in what direction can the boat be rowed to reach a destination directly across the river?

39. If a 14-unit vector makes an angle of 45° with the horizontal, what are its horizontal and vertical components?

40. Harry accidentally falls out of a helicopter that is traveling at 15 m/s. He plunges into a swimming pool 2 seconds later. Assuming no air resistance, what was the horizontal distance between Harry and the swimming pool when he fell from the helicopter?

41. Refer to the previous problem.

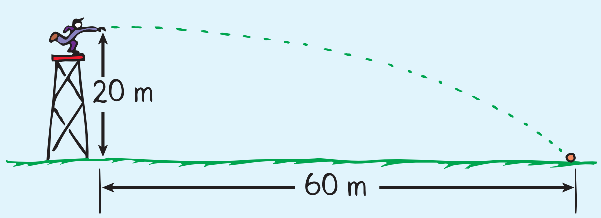
a. What are the horizontal and vertical components of Harry's velocity just as he hits the water?

b. Show that Harry hits the water at a speed of 25 m/s.

42. Harry and Angela look from their balcony to a swimming pool below that is 15 m from the bottom of their building. They estimate the balcony is 45 m high and wonder how fast they would have to jump horizontally to succeed in reaching the pool. What is your answer?

43. A girl throws a slingshot pellet directly at a target that is far enough away to take one-half second to reach. How far below the target does the pellet hit? How high above the target should she aim?

44. The boy on the tower in the figure below throws a ball a distance of 60 m, as shown. At what speed, in m/s, is the ball thrown?



45. A cannonball launched with an initial velocity of 141 m/s at an angle of 45° follows a parabolic path and hits a balloon at the top of its trajectory. Neglecting air resistance, how fast is it going when it hits the balloon? What is the acceleration of the cannonball just before it hits the balloon?

46. Joshua throws a stone horizontally from a cliff at a speed of 20 m/s, which strikes the ground 2.0 seconds later.

a. Use your knowledge of vectors and show that the stone strikes the ground at a speed of about 28 m/s.

b. At what angle does the ball strike the ground?

47. On a bowling alley, Isabella rolls a bowling ball that covers a distance of 10 meters in 1 second. The speed of the ball is 10 m/s. If the ball were instead dropped from rest off the edge of a building, what would be its speed at the end of 1 second?

48. A bowling ball is moving at 10 m/s when it rolls off the edge of a tall building. What is the ball's speed one second later? (*Hint:* think vectors!)

49. Calculate Hotshot Harry's hang time if he moves horizontally 3 m during a 1.25-m high jump. What is his hang time if he moves 6 m horizontally during this jump?

50. Megan rolls a ball across a lab bench y meters high and the ball rolls off the edge of the bench with horizontal speed v.

a. From the equation $y = \frac{1}{2}gt^2$, which gives the vertical distance y an object falls from rest, derive an equation that shows the time t taken for the ball to reach the floor.

b. Write an equation showing how far the ball will land from a point on the floor directly below the edge of the bench.

c. Calculate the time in the air and the landing location for $v = 1.5$ m/s and a bench height of 1.2 m.

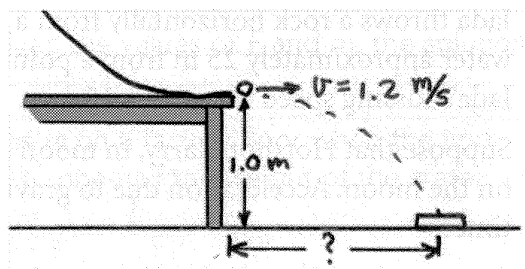
More Problem-Solving Practice
Appendix F

sample problem

In a physics lab, a 10-g marble, after rolling down a ramp, shoots off the edge of a table with a horizontal velocity of 1.2 m/s. The table is 1.0 m high. You want the marble to land in a shallow cup on the floor. How far from the edge of the table should you place the cup?

▶ **Step 1** Identify the underlying physics concept and write the equation that best expresses it:

The concept is *projectile motion* in the absence of air drag, where the horizontal component of motion remains constant while the vertical component has an acceleration of 10 m/s^2. You are looking for the horizontal distance the ball travels when it leaves the table. The main equation for this is $d = vt$.



▶ **Step 2** Rewrite the equation so that the term whose value you're looking for is isolated on the left side of the equation:

In this case, you are looking for *d* so there is no need to rewrite the equation.

▶ **Step 3** Substitute the values of the terms, with their units, for the variables in your equation. (This may involve repeating the process for unknown terms.)

$d = vt = (1.2 \text{ m/s})t$. You must find t (the time in the air) in order to solve for d. Find the time from the formula for the vertical distance $d = \frac{1}{2}gt^2$. Rearranging the equation to solve for $t = \sqrt{2\frac{d}{g}}$. Since you are concerned with two distances, horizontal and vertical, use x for horizontal distance, and y for vertical distance. Then $x = vt = v\sqrt{2\frac{y}{g}}$.

▶ **Step 4** Do the necessary calculations.

$$v\sqrt{\frac{2y}{g}} = 1.2 \text{ m/s} \sqrt{\frac{2(1.0 \text{ m})}{1.0 \text{ m/s}^2}} = 0.54 \text{ m}$$

▶ **Step 5** Evaluate your solution:

If the cup were taller, then you'd solve the problem by letting y be the vertical distance below the tabletop to the top of the cup. (You may have to take the cup's height into account if you do this in lab!) Note that the marble's mass doesn't matter.

1. An experimental jet plane, just after takeoff, climbs with a speed of 500 km/h at an angle of 37° with the horizontal. At what rate is the plane gaining altitude?

2. A plane is flying at an altitude of 8000 m at a speed of 250 m/s. At what horizontal distance ahead of its target must a water balloon be released to strike the target on the ground (neglecting air resistance)?

3. Kayla throws a rock horizontally at 12 m/s from the top of a 45-m high cliff. How far will the rock land from the base of the cliff?

4. Tyler accidentally falls out of an airplane that is traveling horizontally at 45 m/s. He plunges into the water below 3 seconds later. Assuming no air resistance, what was the horizontal distance between Tyler's falling point and the water splash?

5. Jada throws a rock horizontally from a bridge 32 m above the water, which hits the water approximately 25 m from a point immediately below the bridge. Show that Jada's tossing speed was about 10 m/s.

6. Suppose that Hotshot Harry, in moon attire, jumps a vertical distance of 8.0 m on the moon. Acceleration due to gravity there is 1.6 m/s^2. What is Harry's hang time?

7. Suppose that Harry makes the same jump from a horizontally-moving skateboard on the moon's surface. Is his hang time more, less, or the same?

Problems with Trigonometry

8. A soccer ball is kicked with an initial speed of 24 m/s at an angle of 30° to the ground. Neglect air resistance. (a) Calculate the vertical component of the initial velocity. (b) What is the vertical component of velocity at the top of its upward movement? (c) What is the acceleration of the ball on the way up? (d) Show that the ball spends 1.2 s in the air on the way up. (e) Show that the ball rises to a height of 7.2 m. (f) How much total time does the ball spend in the air?

9. Benjamin throws a small coconut at 18 m/s and at a projection angle of 55° above the horizontal to his friend Junior, who is up in a tree house. Junior catches the coconut right at the top of its trajectory. (a) Show that the speed of the coconut just as Junior catches it is about 10 m/s. (b) Show that Junior's tree house is almost 11 m above the ground.

10. Skyler kicks a football. Two seconds later it hits the ground 37 m away. What was the initial speed and direction of the football?

11. A stream of water leaves a high-pressure hose at 25 m/s and makes a 57° angle with the ground. Show that the water lands 57 m away.

12. Adam is skateboarding across the gym floor at 5.5 m/s when he throws his keys straight up (relative to himself) at 7 m/s. Show that Adam has moved 7.7 m across the floor when he catches his keys again.

13. A golf ball leaves the tee an angle of 30 degrees above the horizontal with a speed of 50 m/s. It lands at the same height from which it was hit. Ignore air resistance. (a) How long is it in the air? (b) What is the maximum height it reaches? (c) Show that the ball lands 217 m downrange.

Chapter 5

5.2.1 Whenever a pair of vectors are at right angles (90°), their resultant can be found by the Pythagorean Theorem, a well-known tool of geometry. It states that the square of the hypotenuse of a right-angle triangle is equal to the sum of the squares of the other two sides. Note that two right triangles are present in the rectangle in Figure 5.3. From either one of these triangles we get

$$\begin{aligned} \text{resultant}^2 &= (60\ \text{km/h})^2 + (80\ \text{km/h})^2 \\ &= 3600\ (\text{km/h})^2 + 6400\ (\text{km/h})^2 \\ &= 10{,}000\ (\text{km/h})^2 \end{aligned}$$

The square root of 10,000 $(\text{km/h})^2$ is 100 km/h, as expected.

5.2.2 An important property of vectors is that they can be moved around as long as their length and direction are not changed. Vectors can be rearranged into a chain, tail-to-head in any order. A vector drawn from the tail of the first vector to the head of the last vector represents the resultant of the entire chain of vectors.

Chapter 7

7.0 The terms *push* and *pull* usually invoke the idea of a living thing exerting a force. So, strictly speaking, to say "the wall pushes on you" is to say "the wall exerts a force as though it were pushing on you." As far as these mutual forces are concerned, there is no observable difference between the force exerted by you, a living being, and the force exerted by a wall, a nonliving object.

7.5 In both cases we consider the football to be the system. In the first case the foot (A) exerts a force on the ball (B). The force exerted by A is the net force on the system (the ball), so the ball accelerates. In the second case there are two forces on the ball—A on B, and C on B. Note two things: A doesn't interact with C and vice versa—so A and C do not make up an action-reaction pair of forces. Also note there are three objects involved, not just two. Read on to the horse-cart problem, which may make this clearer.

Chapter 8

8.2 This relationship is derived by rearranging Newton's second law to make the time factor more evident. If we equate the formula for acceleration, $a = F/m$, with what acceleration actually is, $a = \Delta v/\Delta t$, we get $F/m = \Delta v/\Delta t$. From this we derive $F\Delta t = \Delta(mv)$.

Chapter 9

9.1 For the more general case, work is the product of the *component of force* acting in the direction of motion and the distance moved. (No work is done when force and distance are perpendicular to each other.)