Chino Valley Unified School District
High School Course Description

CONTACTS	
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A. COVER PAGE - COURSE ID	
1. Course Title	Calculus AB
2. Transcript Title/Abbreviation	Calc AB
3. Transcript Course Code/Number	5128
4. Seeking Honors Distinction	Yes
5. Subject Area/Category	Meets the “c” mathematics UC/CSU requirement
6. Grade level(s)	11-12
7. Unit Value	5 units per semester / 10 total credits – math
8. Length of course	Two (2) semesters / one (1) year
9. Was this course previously approved by UC?	Yes
10.Is this course classified as a Career Technical Education course?	No
11.Is this course modeled after an UC approved course?	Yes
12. Repeatable for credit?	Yes
13. Date of Board Approval:	May 5, 2016

14. Brief Course Description:

This course is taught at the rigor of a college level Calculus course. The course covers functions, limits and continuity, differentiation and integration of functions of a single real variable. Applications from higher level sciences are addressed as well. The class is similar in rigor to the AP Calculus course, but is taught at a slower pace.

15. Prerequisites	Integrated Math 1, Integrated Math 2, Integrated Math 3

16. Context for Course:

In Calculus, we approach problems from each of the four major angles: graphically, numerically, algebraically, and verbally. Since calculus connects with so many other disciplines, especially physics, we take advantage of many opportunities to offer demonstrations and activities that allow the students to see and touch the calculus we learn.

Instructional Methods and/or Strategies are specifically geared to support the delivery of the curriculum and the course goals in a balanced fashion. Whole class instruction, small group instruction, discussion, think-pair-share, power point presentations, student demonstration, daily assignments, warm-up quizzes, and interactive instruction are all used on a regular basis for all portions of the course outline. Student presentations and projects are assigned and displayed for all units. Poster projects, Riemann Sum projects, Area and Volume projects are a few examples of larger student work than the typical daily assignment. Study guides are assigned for each unit. Students are encouraged to receive and/or provide peer tutoring to help reinforce their mathematical understanding. Instructional approaches support the eight Standards of Mathematical Practice in the Common Core State Standards as stated explicitly in items 1-8 below.

Make sense of problems and persevere in solving them

Students make sense of problems and persevere in solving them throughout all units of Calculus. They analyze the meaning of a problem for understanding, including givens, constraints, relationships, and goals of each problem. They make conjectures about the form and meaning of the solution and plan a solution pathway, looking for multiple entry points. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Calculus students use their graphing calculator to get the information they need to explore, analyze, graph, calculate, and solve problems. Calculus students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships. They also use graphs, diagrams, illustrations, and concrete objects or pictures to help conceptualize and solve a problem, specifically for the calculus of motion, related rates, and area and volume problems. Calculus students check their answers to problems using multiple methods (graphical, algebraic, numerical and verbal approaches), and continually explain and justify their answers. The eight common core state standards are embedded in assignments and assessments.

Reason abstractly and quantitatively

Calculus students make sense of quantities by indicating units of measure for distance, displacement, position, velocity, acceleration, average value, anti-derivatives, related rates, lengths of curves, area, and volume problems. Students reason abstractly and quantitatively throughout all units of calculus. They can represent a given situation symbolically and manipulate the representing symbols. They give meaning to the symbols in an applied context appropriate for rates of change, area, volume in physics, business, and a variety of applications.

Construct viable arguments and critique the reasoning of others

Calculus students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data in the appropriate context from which the data arose. Students listen respectfully to the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. Students are continually asked to reason, explain, and justify their solutions on a regular and daily basis. Bald answers are not accepted.

Model with mathematics

Calculus students use functions to describe how quantities are increasing, decreasing, concave up, and concave down. They will analyze maxima, minima, and points of inflection, as well as intervals where a function is increasing and decreasing. Calculus students make accurate assumptions and approximations to simplify complicated situations. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, tables, graphs, charts and formulas, two-dimensional models of area, and 3-D models of volume. Students are graphing position, velocity, and other functions on a regular basis. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Use appropriate tools strategically

Calculus students use appropriate tools strategically when solving mathematical problems. These tools include pencil and paper, concrete models, diagrams and charts, a graphing calculator, and "Calculus in Motion" software. Calculus students make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. They detect possible errors by strategically using estimation and other mathematical analysis. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Calculus students use technological tools to explore and deepen their understanding of concepts.

Attend to precision

In all units and assignments, calculus students must attend to precision. Calculus students must communicate precisely to others. They use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They calculate accurately and efficiently and specify units of measure, accurate to at least three decimal places, either rounded or truncated in accordance with AP standards. They know how to use the STO feature on their graphing calculator and they learn not to round off intermediate results. Calculus students clearly label axes to clarify the correspondence with quantities in a problem. All answers must be accurate, preferably in fraction or radical form. Use of e and π notation is encouraged.

Look for and make use of structure

Calculus students look closely to discern a pattern or structure. They also can step back for an overview and shift perspective. They can see complicated things, such as piecewise functions, as single objects or as being composed of the sum of its pieces. Rules, properties, and theorems are developed sequentially and proved.

Look for and express regularity in repeated reasoning

Calculus students look for and express regularity in repeated reasoning throughout all units. They notice if calculations are repeated, and look both for general methods and for shortcuts. Calculus students find patterns and repetition in implicit differentiation, solving separable differential equations and Newton's method of approximating zeros of functions, just to name a few specific instances. As they work to solve a problem, calculus students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

The eight common core state standards are embedded in assessments.

1. Students make sense of problems and persevere in solving them throughout all units of calculus. Novel multi-step problems occur regularly on assessments.
2. Students reason abstractly and quantitatively throughout all units of calculus. Assessments include free-response questions which allow for both abstract and quantitative reasoning to be demonstrated.
3. Calculus students construct viable arguments and critique the reasoning of others. Students are continually asked to justify and explain their reasoning. Bare-bone answers, without justification, are not accepted on assignments or assessments.
4. Calculus students model with mathematics through the use of graphs, diagrams, illustrations, sign charts, tables, formulas, slope fields, two-dimensional models of area, and 3-D models of volume. Students are graphing position, velocity, and other functions on a regular basis. Students analyze functions from formulae, graphs, and data tables on assessments involving derivatives and anti-derivatives.

5. Calculus students use appropriate tools strategically. They use their graphing calculator regularly. Calculus students use technological tools to explore and deepen their understanding of concepts. Smart Board technology and responders are utilized for assessments. The AP Calculus website provides a wealth of outstanding test bank items, which are used for both direct and free response assessment.
6. In all units and assignments, calculus students must attend to precision. All answers must be accurate, preferably in fraction or radical form. Use of e, pi, and radical notation in solutions is encouraged. Decimal approximations must be accurate to three decimal places, either rounded or truncated in accordance with AP standards. Students learn NOT to round off intermediate results.
7. Calculus students look closely to discern a pattern or structure throughout all units. Rules, properties, and theorems are developed sequentially, proved, and assessed.
8. Calculus students look for and express regularity in repeated reasoning on assessments throughout all units.

17. Textbooks:	Calculus Graphical, Numerical, Algebraic Pearson Prentice Hall

B. COURSE CONTENT

Course Purpose:

The purpose of this course is to provide a pathway for students who have taken the necessary prerequisites to take AP Calculus AB but do not want to rush through the course. The class is similar in rigor to the AP Calculus course, but is taught at a slower pace. This course does not have the same time restriction as the AP test, allowing student to have more time with the material. The course will also provide students who are not being successful with 1st semester AP Calculus AB another path to take calculus.

Course Outline:

Unit 1: Pre-Requisites for Calculus

Slopes, lines, and linear equations will be reviewed in this unit. Average rate of change and slope of the secant line will lead to instantaneous rate of change and the slope of the tangent line. Difference quotient will be used and then we will progress to the limit of the difference quotient as approaches zero. Point-slope form will be emphasized. The relationship of the slopes of a function and its inverse will be revisited. We will conduct a quick and concise review of trigonometric ratios and basic trigonometric identities. Functions and their graphs will be reviewed including domain and range, odd and even functions, the graph of a semi-circle, the absolute value function, greatest integer function, piecewise functions, exponential functions, logarithmic functions, inverse functions, and composite functions. The difference quotient will be used to find average rate of change of a function. In the next unit, we will explore how the limit of the difference quotient leads to the derivative.

Unit 2: Limits and Continuity

We begin this unit with rates of change: average rate of change and instantaneous rate of change. Rates of change will be analyzed by table, by graph, or by equation. Limits will be introduced graphically, algebraically, and numerically. Limits will be defined informally and formally. One-sided limits, when a limit fails to exist, limit theorems, limits involving infinity, and limits involving the difference quotient will all be explored. This is where we make connections as we transition from slope to difference quotient then derivative.

Continuity will be defined informally and formally. Continuity will be explored for piecewise functions. Continuity will be analyzed at a hole, at a jump discontinuity, and at vertical asymptotes. Students will also apply the Intermediate Value Theorem and the Extreme Value Theorem to continuous, differentiable functions on a closed interval.

Unit 3: Derivatives: graphically, algebraically, and numerically

This unit begins with the concept of local linearity. Then we learn that differentiability implies continuity. We will transition from the limit of the difference quotient (or the limit of the slope) definition of derivative to some of the more efficient techniques for finding derivatives. We will use point-slope form to find tangent and normal lines. We will use the power rule to find first derivatives, second derivatives, and higher order derivatives. Application problems in this unit include particle motion involving position, displacement, total distance travelled, velocity, speed and acceleration. Students will be expected to determine if an object is speeding up or slowing down, where a function is increasing or decreasing, and the concavity of a function. Students will analyze functions graphically, algebraically, and numerically.

Techniques of differentiation will be explored and practiced to proficiency. These include the power rule, product rule, quotient rule, and the chain rule. Students will discover the six trig derivatives and the six inverse trig derivatives. These rules will be derived, proved, and used to solve variety of problems, both theoretical and applied. Students will learn implicit differentiation and use implicit techniques to solve related rates problems. They will also find derivatives of exponential and logarithmic functions.

Students will use curve sketching to analyze functions visually and graphically. They will find exact maxima, minima, and points of inflection. They will use the first derivative test and the second derivative test to find extrema. They will determine when a function is increasing, decreasing, concave up, and concave down. Students will also analyze the behavior of a function at a cusp, corner, vertical asymptote or other discontinuity, and the end behavior of the function. Students will use L'Hopital's Rule to find the limit of a function at a point resulting in indeterminate form.

Calculator Use**:** Students are introduced to the 2nd-calc-6 feature on their graphing calculator to find dy/dx at a particular value and the nDeriv function to graph derivatives at one or all x-values. The calculator helps facilitate class discussion comparing a function with its first derivative and second derivative, comparing extrema of *f* (x) with zeros of *f* '(x), comparing points of inflection of *f* (x) with extrema of the *f* '(x), comparing the degree of *f* (x) with the degree of *f* '(x), and the overall discussion of increasing and decreasing slope.

Unit 4: Applications of the Derivative

Students will learn to find extreme values on a closed interval, examining candidates at critical points and endpoints. They will apply the Extreme Value Theorem. Students will also apply the Mean Value Theorem and Rolle's Theorem to differentiable functions on a given interval. Students will solve optimization problems, related rates problems, linearization problems and Newton's Method problems

Unit 4 Calculator Use: Calculator use will be ongoing and regular as students solve application problems in this unit.

Unit 5: Definite and Indefinite Integrals

Students will learn to solve definite and indefinite integrals. First they will be introduced to the Trapezoid Rule and Riemann Sums – left, right, and midpoint. They will learn the Fundamental Theorem of Calculus, both Part 1 and Part 2. They will understand the Mean Value Theorem for definite integrals, properties of definite and indefinite integrals. They will learn to solve definite integrals with a negative integrand and definite integrals where the lower bound is greater than the upper bound.

Students will find anti-derivatives of powers, trig functions, exponential and logarithmic functions. Students will find anti-derivatives that result in inverse trig functions. Students will become proficient at the techniques of u-substitution, integration by parts, and integration by using partial fractions. U-substitution will be used for both definite and indefinite integrals.
Students will complete a Riemann Sum investigation which will include in-depth analysis of a function and the area between the curve and the x-axis. Students will compare the left Riemann sum, the right Riemann sum, the midpoint Riemann sum, the Trapezoid sum, and the definite integral result. Students will find when an approximation overestimates or underestimates the actual area and they will make and justify conclusions about the accuracy of each sum as an estimate of the area.

Students will use anti-differentiation to solve a variety of application problems including accumulation (input-output) problems and particle motion problems.

Unit 5 Calculator Use: Using graphing technology and available programs, we explore Riemann sums and the Trapezoid Rule. Calculators are used to confirm definite integrals that we have first found numerically. We also discuss regions below the x-axis and regions that are found both above and below the x-axis. Students use the 2nd-calc-7 function to find the anti-derivative of a function on a closed interval. Class discussion emphasizes the concept of area under the curve.

The calculator is also used in the process of adding Riemann sums, using STAT, the Lists, formulae in the title bar of the lists, SUM (L4)--under 2nd List, MATH.

<u>Unit 6: The Calculus of Growth and Decay</u>
Slope Fields will be drawn and analyzed in this unit. Separable differential equations will be solved using a four step process (separate the variables, integrate both sided, find the general solution including the constant of integration, solve for the particular solution). Applications will include exponential growth and decay, predator-prey population problems and direct proportion differential equations that lead to the exponential function (where C is the constant of integration and k is the constant of proportion), and other differential equations for real-world applications. Students will learn Euler's Method for solving differential equations in this unit. Unit 6 also includes the technique of u-substitution for solving indefinite and definite integrals.

Unit 6 Calculator Use: Calculator use will be embedded in unit 6. The graphing calculator will be allowed for five primary purposes.

1. to plot the graph of a function, as needed
2. to find x-intercepts or zeros of a function
3. to numerically calculate a derivative
4. to numerically calculate the value of a definite integral

<u>Unit 7: The Calculus of Plane and Solid Figures</u>
In this unit, students will learn to find the area between two curves. Cross sections may be perpendicular to the x-axis or the y-axis. Some problems will be calculator enabled and some will not. Students will learn to find the volume of a solid of revolution with both horizontal and vertical axes. Students will also find the volume of a solid with known cross-sections. Cross sections may be perpendicular to the x-axis or the y-axis. Students will learn to find volumes of revolution by using the disk method, the washer method, and by cylindrical shells. This unit will also include finding the length of a plane curve, aka finding arc length.

Unit 8: Additional Techniques for Integration

Students will use integration by parts to find the integral of a product of two functions. Students will learn the ILATE acronym for choosing the parts in integration by parts. Students will also learn tabular integration, aka rapid repeated integration by parts. In this unit, students will find the integral of the natural logarithm function and the common logarithmic function. Students will also use trigonometric substitution, specifically the power reduction formulas to find integrals of powers of trigonometric functions. Students will also use advanced trigonometric substitution to find integrals of trigonometric functions. Integration of Rational Functions will be solved using the technique of partial fraction decomposition. Partial fractions will include proper and improper fractions in the integrand. Students will also integrate piecewise continuous functions.

Key Assignments: Key Assignments and Student Activities

Unit 1: Pre-Requisites for Calculus

Any student who has completed the pre-requisites may take this class, reinforcing equity and access for all students. Because some students are coming from honors classes and other students are coming from regular class, students will fill out an inventory of concepts, labeling their level of learning: Introduction, Reinforcement and extension, Mastery, and Technology Interface (A Pre-AP Mathematics Curriculum Plan by Benita Albert). There are columns covering their mathematics education from 6th grade to present. This shows the students their strengths and weaknesses, as well as providing a terrific tool for the teacher to supplement the curriculum to address areas of weakness.

Graph match will be another key assignment, where 25 graphs will be matched with their equations. This is an activity done in pairs as an in-class activity. It is done without calculators and requires the students' critical thinking about asymptotes, intercepts, symmetry, absolute value, domain and range and extrema in order to create the match. The goal is for the student to produce a match for all the equations. The student learns there are many tools at his/her disposal and not just making an x-y table. All the previous knowledge listed above is synthesized to be able to complete the worksheet.

Unit 2: Limits of Trigonometric Functions

In this activity, students explore the graph of $y = (\sin x)/x$ as $x \to 0$. First students use their graphing calculators to investigate the function graphically. Second, they confirm their observations by analyzing the function numerically. Next, they consider the data numerically from a data table in the neighborhood of. Finally, students extend their finding to other functions such as $y = (\cos(x) - 1)/x$ as $x \to 0$ and $y = (\sin(ax))/bx$ as $x \to 0$. The student learns that different tools are necessary to be able to find answers. Also the answer can be confirmed in various modalities: graphing and numerically using the data. They will experience one of the examples of limits, where there is no value on the graph where the x is approaching.

Unit 3: Graphing the Derivative of a Function

Students explore derivatives by analyzing and discovering the derivative of a mystery function. Mystery functions include $y = x^2$, $y = \sin(x)$, $y = \cos(x)$, $y = \tan(x)$, and $y = e^x$. Students take turns coming to the Smart Board to plot points on the graph of the derivative of each mystery function; in time, the derivative function comes into view. Students will often begin by identifying where the derivative is 0. Then they pick out slopes of 1 and –1, and soon the derivative function materializes. The students learn how to use the dy/dx on the graph of a graphing calculator. They are also have an opportunity to hypothesize and make conjectures about derivatives.

Derivative Poster Project

This activity enables students to summarize what they have learned about derivatives from the limit of the difference quotient to derivatives of trigonometric, exponential, and logarithmic functions. Posters are presented to the class and subsequently displayed for all to see.

Unit 4: Optimization
Unit four has plenty of thinking problems between both the optimization and the related rate problems. One of the classic problems that will be discussed and assigned is the fabricating a box problem. A flat piece of tin is given. Squares are to be cut out of the corners to maximize the volume. What is the length of the squares and what is the maximum volume? Students will be given a worksheet to guide them in what is expected: diagram, equation, graph, analytic work, and answer. Then they will have to come up with their own optimization problem and solve it. The students will again use graphical and analytic means to find the answer. Coming up with their own problem helps solidify the process of these types of problems, as well as allowing for creativity.

Unit 5: Riemann Sum and Trapezoid Sum Project
In this activity, students are each given a unique function on a closed interval. Students compare and contrast the methods for calculating the area under a curve (e.g., left Riemann sum, right Riemann sum, midpoint Riemann sum, Trapezoid sum and the definite integral). Students analyze the results of using these various methods in a book format. They chart which methods are overestimates and which are underestimates of the actual definite integral area. They analyze their particular function for increasing intervals, decreasing, intervals, concavity and points of inflection. The book format allows students to illustrate and explain their findings in a finished product.

Another key assignment is using Larry Peterson's brilliant worksheet on the Fundamental Theorem of Calculus, Using the Rule of Three. This worksheet packet walks the students through the relationship between an area function and an original function. It connects for them in a tangible way the idea of an integral and a definite integral, using graphical, analytical and numerical methods. In the Fundamental Theorem of Calculus Part 2, the worksheet highlights the important fact the the chain rule never goes away.

Unit 6: Exploration Lab "Seeing the Slopes"
Students will explore the differential equation dy/dx = cos x and compare it to a given graph that is the general solution, the family functions y1= sin x + L1, where L1 = (-3, -2, -1, 0, 1, 2, 3,). These are familiar graphs, so students can know where cos x = 0 and compare it in the graph. The y coordinate is explored. Does it affect the value of dy/dx = cos x? Why or why not? The assignment also explores slopes when x = 0 or x=pi. Students complete the exploration, verbally explaining what they discover. They work in pairs. The students starts to "see" the slopes and how to produce a family of curves without solving the differential equation, simply by looking carefully at the slopes. This is the idea behind slope fields (Exploration from Finney, Demana, Waits, Kennedy text, Calculus).

Unit 7: Fruits and Vegetable Lab
Students work in pairs to determine the volume of a banana, squash, potato, etc., using graph paper, their calculators, and the disk or washer method for determining volumes of solids of revolution. They hand in a written report that describes how they determined the volume and their findings. [Borrowed from Kent Tarr, Monte Vista Christian School, Watsonville, CA]

Unit 8: Partial Fractions
Students will use integration by parts to find the integral of a product of two functions. Students will learn the ILATE acronym for choosing the parts in integration by parts. Students will also learn tabular integration, aka rapid repeated integration by parts. In this unit, students will find the integral of the natural logarithm function and the common logarithmic function. Students will also use trigonometric substitution, specifically the power reduction formulas to find integrals of powers of trigonometric functions. Students will also use advanced trigonometric substitution to find integrals of trigonometric functions. Integration of Rational Functions will be solved using the technique of partial fraction decomposition. Partial fractions will include proper and improper fractions in the integrand. Students will also integrate piecewise continuous functions.

Instructional Methods and/or Strategies:

1. Lecture: This instructional pattern features one day of lecture followed by one or two days of follow-up/in-class practice. PowerPoint slides and "Calculus In Motion" software programs often supplement the instruction, and students are encouraged to interact by asking questions, conjecturing, and otherwise articulating their ideas.
2. Group Work: Group work is also an essential aspect of the class, fostering teamwork and problem-solving skills.
3. Hands-on Activities/Assignments: When appropriate, students make use of their graphing calculators to explore functions, graphs, etc., and detailed examples are provided to demonstrate the correct keystrokes.

Assessment Including Methods and/or Tools:

Student grades are determined on a weighted scale. Class work, projects, study team tests and assignments are weighed 30%, and quizzes and individual tests, including the final, are weighed 70%. Class work and assignments: Cornell notes, examples, assignments and homework are the foundation of the assessment process. They provide immediate feedback on content mastery to the instructor, as well as the student. Homework: Students can expect nightly homework to finish any assignments that have not been completed in class. Study Team Tests: A practice test in small groups of 3-4 students is held prior to every unit test. Problems are challenging, require collaboration, and sometimes extend beyond the classroom. Peer tutoring is encouraged and required during team tests. Student Projects and Activities: Throughout the course, students undertake various projects that tie into the course content. These activities are designed to make the math hands-on, relevant, and practical. Activities are frequently in pairs or in small groups. Projects and activities are graded according to a pre-determined rubric.

Quizzes: Students take quizzes weekly, which lead up to a larger exam at the end of every chapter of the text. Tests/Exams: Tests occur at the end of each chapter. The tests are comprised of direct problems, multiple choice questions, and free response problems which include writing explanations and justification. A comprehensive, accumulative final exam is given at the end of each semester and counted as two test grades.