Lesson 9: Momentum and Impulse

Name Period REMEMBER, PARALLEL PARKING IS JUST A MATTER OF GEOMETRY. PRIVING STUDENT DRIVER

9.1. Experiment: What is Momentum?

(a) Use two lab carts with about the same mass for this experiment. With the Velcro sides facing inward, give the two carts a push into each other so they'll move slowly then stick together and just stop. Make sure they are both at rest immediately after the collision. How fast did they have to go relative to each other?

(b) Now add a bit more mass on one cart (Cart B). Push the other cart the same as before, but also push Cart B toward the other cart so that they come to rest immediately after the collision. How fast did Cart A (the cart with less mass) have to go compared to before (more, less, or the same)?

(c) Now remove the extra masses from the cart and push Cart A so that it moves a little faster than before. How fast did Cart B have to go this time – compared to part (a) – to bring them both to rest?

(d) Momentum is how hard it is to stop something, like the lab cart. What are the two quantities it depends on?

*To calculate momentum (p), all you have to do is multiply mass and velocity: p = mv

9.2. Experiment: What is Impulse?

(a) Set up the cart, meter stick, pulley, hanging mass, and sensors as you did in lesson 6.1 and 8.1. Use a 100g hanging mass to pull on the cart and <u>calculate the final velocity</u> of the cart as it speeds up across the meter stick. You will examine the time and applied force. Let the string pull the cart and measure the force. Fill in the table. (release it from rest so that its initial velocity is zero)

| Mass of Cart (kg): | | | | | | | | | | |
|----------------------|--------------------------|---|--|---|---------------|---------------|---------------|---------------------|--|---|
| Mass on Cart (kg) | Hangin g Mass (kg) | Mass of System (kg) m _{cart} +m _{on cart} +m _h | Average Tension Force Applied (N) [from graph] | Accel. =Force/ m _{system} (m/s ²) | Time 1 (s) | Time 2 (s) | Time 3 (s) | Avg. time (s) | Final velo- city calc. (m/s) | Final velo- city from graph |

- (b) Calculate the final velocities and fill in the table. Use mass, force, and time. (show a calculation below)
- (c) Calculate the initial momentum and final momentum of the cart by multiplying the total mass of the system by the final velocity.
- (d) Now multiply the force of tension times the average time it was moving to get a value in Newton-seconds.
- (e) How do the number of Newton-seconds (the impulse you just calculated) and kilogram-meters-per-second (the momentum you calculated) compare? (Which is bigger?)
- (f) What does this say about the way the Force-times-time compares to the difference in final momentum compared to initial momentum (the change in momentum)?

(g) Solve the same problem you did in (b), but for the general case (using only variables) where the initial velocity is v_i , the final velocity is v_f , the time is t, the force is F, and the mass is m. Solve the problem for F times t.

*The average force times the time it is applied is defined as the *impulse*. The impulse causes a change in momentum of the same size. This handy equation you derived is called the *impulse-momentum theorem*.

9.3. Force versus Time Graphs

- (a) Recall Force versus distance graphs. The area under a graph of constant force is shaped like a rectangle. The area is the base times height of the rectangle, which is distance times force. What does the *area* between the data line on the force-vs-distance graph and the distance axis represent?
- (b) Now consider a graph of force on the y-axis and time on the x-axis. Draw a force-versus-time graph of a constant force of 5 Newtons acting from 1s to 5s.

| Force | |
|-------|------|
| | - |
| | time |

(c) Calculate the area under the graph. Remember to include units.

Area under this Force vs time graph = _____

(d) What do these units represent? (Are they force units? energy units? what are they?)

*The <u>area</u> under a <u>force versus time graph is</u> the <u>impulse</u> acting on an object, which is equivalent to the <u>change in momentum</u> that the impulse causes. This works out in general, even if the force is not constant.

(e) In car crashes, we want to minimize the damage to occupants without forcing everyone to drive slower. How? (consider a graph) Draw graphs and make comments to describe.

(f) Athletes want to maximize collisions without increasing the force. How? (consider a graph) Draw graphs and make comments to describe.

9.4. Experiment: Conservation of Momentum

(a) For this experiment, face the spring-plunger end of one cart toward the end of the other cart without the plunger. The carts need to start <u>at rest</u> touching each other so that when the spring plunger is released the carts will <u>move away</u> from each other.

| Masses (kg) | | Distance moved by cart (cm) | | Time (s) | Final Velocity (m/s) [distance/time] | | Final Momentum (kg*m/s) [mass*velocity _{final}] | | Total Final Momentum (kg*m/s) [p ₁ +p ₂] |
|-------------|--------|--------------------------------|--------|-------------|--|--|--|--|--|
| Cart 1 | Cart 2 | Cart 1 | Cart 2 | | Cart 1 Cart 2 | | Cart 1 Cart 2 | | |
| | | | | | | | | | |
| | | | | | | | | | |
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- (b) Carry out three "explosions": (a) 2 empty carts, (b) 1 empty cart and 1 carrying one mass, and (c) 1 empty cart and 1 carrying two masses. For each explosion determine by trial and error the starting point for the two carts so that when the explosion is triggered each cart reaches the end of the table <u>at the same time</u>.
- (c) Perform the calculations to fill in the table.
- (d) Calculate the total initial and total final momentum values for each case and fill in the 3 tables below:

(e)

| (k | <i>ass x velocity</i> g*m/s) velocity _{initial}] | Initial Total <i>mv</i> (kg*m/s) [m1v1 initial +m2v2 initial] | | | | | |
|--------|--|--|--|--|--|--|--|
| Cart 1 | Cart 2 | | | | | | |
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| Fina | al <i>mass x velocity</i> (kg*m/s) | Final Total <i>mv</i> (kg*m/s) |
|--------|---------------------------------------|---|
| [m: | ass*velocity _{final}] | (kg*m/s) [m ₁ v _{1 final} +m ₂ v _{2 final}] |
| Cart 1 | Cart 2 | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

| Initial Total mv | Final Total mv |
|-------------------------------------|---|
| (kg*m/s) | (kg*m/s) |
| $[m_1v_1 initial + m_2v_2 initial]$ | $[m_1v_{1 \text{ final}}+m_2v_{2 \text{ final}}]$ |
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(f) How much total momentum did the carts have at the beginning in each case?

Total Initial Momentum = _____ (kg)*(m/s)

(g) If you add the momentum of the leftward moving cart (negative momentum) to the momentum of the rightward moving cart, how much total momentum did the carts have at the end in each case?

Total Final Momentum = _____ (kg)*(m/s)

(h) Compare the total momentum before and after each launch. Was there a significant difference?

*When there is no net external force acting on a system of objects, all the momentum before the collision is "saved" for after the collision takes place: the total momentum of the system is constant (the same before and after a collision) and we say that "momentum is conserved." Mathematically: $m_{1i}v_{1i} + m_{2i}v_{2i} = m_{1f}v_{1f} + m_{2f}v_{2f}$ because $p_{1i} + p_{2i} = p_{1f} + p_{2f}$

9.5. Exercise: Determining Conservation of Momentum

- (a) Two lab carts bump into each other as they did in 9.1. Consider the forces between lab cart 1 and lab cart 2.
 Which is bigger, the force of cart 1 on 2 or the force of cart 2 on 1? (Remember Newton's 3rd Law of motion)
- (b) Mathematically, you should have $F_{1on2} = -F_{2on1}$. Multiply both sides of the equation by the time they are in contact "t" and write that step here.
- (c) What is Force-times-time called? Rewrite the equation, substituting impulse in for "Ft" on each side. You can shorten the subscripts from "1on2" to just "2" and "2on1" to just "1".
- (d) Recall your result from 9.1. By the Impulse Momentum Theorem the impulse produces a change in momentum of the same size. Substitute the change in momentum (Δp)for the impulse (J) on each side of the equation. Keep the subscripts from the last step.

- (e) Rewrite the delta in the form of final initial values.
- (f) Now rearrange the equation so that all the initial values are on one side and all the final values are on the other side.
- (g) What does this show about the total momentum when you add up all the parts over time?

*You should have derived that $p_{1,i} + p_{2,i} = p_{1,f} + p_{2,f} = constant$ in time. This verifies what we found in 9.4.

- 9.6. Exercise: Energy and Momentum
 - (a) The total momentum of a system stays constant when nothing pushes from the outside, but what about the total energy of the system? PREDICT: Do you think the total energy of the system is "conserved" (stays constant) also?

Explain.

(b) Use your data from the previous exercise to fill in the data table below that keeps track of energy during the various "explosions" between the spring carts.

| Masses (kg) | | Final Velocity | | Final Kine | etic Energy (J) | Total Final Kinetic | |
|-------------|--------|----------------|--------|------------|--|-------------------------------------|--|
| | | (m/s) | | [1/2mass | [*] V _{final} ²] | Energy (kg*m/s) | |
| Cart 1 | Cart 2 | Cart 1 | Cart 2 | Cart 1 | Cart 2 | [KE ₁ +KE ₂] | |

| Masses (kg) | | Initial Velocity (m/s) One's at rest while the one that's measured comes in and hits and sticks to it | | Final Velocity (m/s) they move off together after the collision | | Initial Kinetic Energy (J) [1/2mass*v _{final} ²] | | Final Kinetic Energy (J) [1/2mass*v _{final} ²] | |
|-------------|--------|--|--------|---|--|---|--------|--|--------|
| Cart 1 | Cart 2 | Cart 1 | Cart 1 | Cart 1 Cart 2 | | Cart 1 | Cart 2 | Cart 1 | Cart 2 |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

- (c) What was the initial velocity of each cart?
- (d) What was the initial kinetic energy of each cart?
- (e) Compare the total initial kinetic energy and total final kinetic energy of the system. Were they the same?
- (f) If you found that the energies were different, what would account for the difference? (In other words, where did the energy go or where did it come from?) If not, why were they the same?

- (g) Was momentum conserved in the "explosion"? ("Conserved" in this context means "the same before and after.")
- (h) Was energy conserved in the explosion?
- (i) If momentum is conserved, does that mean energy is conserved too?

Explain.

* Collision types are based on what happens to the energy during the collision. Collisions where the kinetic energy is the same before and after (<u>no energy lost or gained</u>) are called <u>elastic collisions</u>. If <u>there is a net loss or gain of energy</u> during the collision (which is most of the time), it's called an <u>inelastic collision</u>. If the <u>most amount of kinetic energy possible is lost</u> (which is when they have no motion relative to each other – they <u>hit and stick together</u>), it's called a <u>perfectly inelastic collision</u>.

9.7. Exercise: Determining Conservation of Momentum

- (a) For this experiment, face the Velcro ends of each cart toward each other. One cart will start <u>at rest</u> while the other cart will roll in, hit, and stick both carts together. Position the motion detector behind the cart that moves initially so that the speed just before and just after the collision can be recorded.
- (i) Carry out three collisions: (a) 2 empty carts, (b) 1 empty cart and 1 carrying 500g of mass, and (c) 1 empty cart and 1 carrying 1kg of mass. For each collision determine from the graph what the initial speed of the incoming cart is just before it collides and what the final speed of both carts are just as they begin to move together.
- (b) Perform the calculations to fill in the table.

| Masses (kg) | | Initial Velocity (m/s) One's at rest while the one that's measured comes in and hits and sticks to it | | Final Velocity (m/s) they move off together after the collision | | Initial Kinetic Energy (J) [1/2mass*v _{final} ²] | | Final Kinetic Energy (J) [1/2mass*v _{final} ²] | |
|-------------|--------|--|--------|---|--|---|--------|--|--------|
| Cart 1 | Cart 2 | Cart 1 | Cart 2 | Cart 1 Cart 2 | | Cart 1 | Cart 2 | Cart 1 | Cart 2 |
| | | | | | | | | | |
| | | | | | | | | | |

- (c) Did the carts move after the two of them became stuck together?
- (d) Were the total initial and total final kinetic energies (½mv²) the same before and after the force was applied? (In other words, was kinetic energy "conserved"?)

* It was helpful for us to analyze energy in a table, knowing that it was "conserved" so the total stayed constant as long as no work was done by external nonconservative forces. We could tally the individual parts and keep track of the totals. Here, kinetic energy of the lab carts is dissipated into the environment during the collision as via heat and sound, transferring some of its kinetic energy to the energy of air molecules and molecules within the surfaces of both carts (internal thermal energy).

9.8. Exercise: Some Momentum Thought Experiments

(a) Egg Drop: pillow versus ground PREDICT

What actually happens?

(b) <u>Toyota versus Tank</u> PREDICT What actually happens?

(c) <u>Save Billy (hit and stick or hit and bounce?)</u> PREDICT

What actually happens?

(d) <u>Recoilless gun</u> PREDICT

What actually happens?

9.9. Demonstrations:

To review, momentum represents how hard it is to stop something and it depends on the object's mass and velocity. If nothing pushes on an object, its momentum will stay constant because of its inertia. This works out for one object or for a bunch of objects in a "system." If no force acts on the *system*, the total momentum when you add together all the momenta at any given time is always the same: momentum is "conserved." If something *does* push on it ("if an impulse is exerted on the object"), its momentum changes since its velocity does. To find out exactly how much it changes, it depends on how long the force was pushing, so you multiply force and time. For a given change of momentum, the more time the force acts, the gentler the collision (less force). **For each of the demonstrations briefly explain how it applies to impulse and momentum.**

- 1. Egg thrown against a loose sheet
- 2. Mass is dropped on a plastic ruler

- 3. Air track carts collide: equal masses come together and stick
- 4. Air track carts collide: equal masses where one hits and bounces off of another one that's at rest
- 5. Softball on top of basketball bounced together