

# **AP Statistics**

## **Review #4** **Statistical Inference**

**Name** \_\_\_\_\_

**IV. Statistical Inference: Estimating population parameters and testing hypotheses (30%-40%)  
Statistical inference guides the selection of appropriate models.**

- A. Estimation (point estimators and confidence intervals)
  - 1. Estimating population parameters and margins of error
  - 2. Properties of point estimators, including unbiasedness and variability
  - 3. Logic of confidence intervals, meaning of confidence level and confidence intervals, and properties of confidence intervals.
  - 4. Large sample confidence interval for a proportion
  - 5. Large sample confidence interval for a difference between two Proportions
  - 6. Confidence interval for a mean
  - 7. Confidence interval for a difference between two means (unpaired)
  - 8. Confidence Interval for mean of differences (paired).
  - 8. Confidence interval for the slope of a least-squares regression line

B. Test of significance

1. Logic of significance testing, null and alternative hypotheses; p-values; one- and two-tailed tests
2. Concepts of Type I and Type II errors and concept of power
3. Large sample test for a proportion
4. Large sample test for a difference between two proportions
5. Test for a mean
6. Test for a difference between two means
7. Test for a mean of differences.
8. Chi-square test for goodness of fit, homogeneity of proportions and independence (one- and two-way tables)
9. Test for the slope of a least-squares regression line.

Here is a quick review of the topics we've covered in Chapters 17-26.

## **Chapter 17**

- Sampling Distributions for Proportions: When we take multiple samples and find each sample's p-hat and graph them, we have created a **sampling distribution** for proportions. As long as certain conditions are met (randomness, independence, and at least 10 successes and 10 failures), we can use the Normal model to calculate a z-score and use our normal tables to find a probability of an event occurring. From the formula sheet we know that:

$$\mu_{\hat{p}} = p$$
$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

- Sampling Distributions for Means: When we take multiple samples and find each sample's x-bar and graph them, we have created a **sampling distribution** for means. As long as certain conditions are met (randomness, independence, and either knowledge of Population Normality or the CLT when the sample size is large), we can use the Normal model  $N(\mu, SE)$  to calculate a z-score and use our normal tables to find a probability of an event occurring. From our formula sheet we know that:

$$\mu_{\bar{x}} = \mu$$
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

## **Chapter 18**

- When we have to estimate the standard deviation using sample data, then we have found the "standard error".
- A confidence interval is an estimate of the true population parameter (either proportions or means) that is based on data collected from the population of interest.
- The margin of error (ME) is the extent of the interval on either side of the sample **statistic**. The ME gives us a bit of leeway in our prediction. The general form of a CI:  
Sample Statistic +/- Margin of Error
- When working with proportions data, we use the  $z^*$  (infinity) line on the T-table. These values are called critical values.
- PANIC

- P We are trying to estimate the true proportion of \_\_\_\_\_
- A Randomization – either randomly chosen or randomly assigned  
10% condition – we've sampled less than 10% of the population  
Success/Failure – we have at least 10 successes and 10 failures
- N If the conditions are met, proceed with a 1-proportion z-interval

$$I \quad \hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

- C We are \_\_\_% confident that the true proportion of \_\_\_\_\_ is between \_\_\_ and \_\_\_.

- Using the TI for a 1-proportion z-interval - Stat:Tests:1-PropZInt
- Interpreting the confidence level... The "95% confidence" means that if we were take multiple samples and calculate an interval for each sample, approximately 95% of all of the intervals calculated would contain the true value. Nothing is special about THIS particular interval, the 95% is a long run probability about our method.
- Be able to calculate a minimum sample size for a specific ME. This is an algebra issue! If a previous value for p-hat isn't given, then use 0.5 as a conservative estimate.
- When we change the confidence level or the sample size, ME changes. As the confidence level increases, the ME also has to widen. As the sample size increases, the ME gets smaller because a larger sample allows us to make more precise estimates.

## Chapter 19

- When testing a hypothesis, we first have to determine our null and alternate hypotheses. The null hypothesis is a statement of “no difference, no change, nothing is going on.” The alternate hypothesis is a statement of change – either a decrease, an increase, or just plain ol’ different (not equal to)
- When we make a decision about a hypothesis, we have to decide whether to reject the null hypothesis or not. After we’ve decided what our hypothesis is, we gather data and analyze it. If our data is consistent with the null (not far from what we expected), then we “do not reject  $H_0$ ”. If our data is inconsistent with the null (really different from what we expected), then we “reject  $H_0$ ”. When we reject the null, the probability of getting data like this is really low ( $< \alpha$ ), so we say that the data indicates a “significant difference from what we expected.”

### ➤ PHANTOMS

P/H	$H_0: p = \underline{\hspace{2cm}}$	where p represents the true proportion of $\underline{\hspace{2cm}}$
	$H_a: p <, >, \neq \underline{\hspace{2cm}}$	
A	Randomization – either randomly chosen or randomly assigned 10% condition – we’ve sampled less than 10% of the population Success/Failure – we have at least 10 successes and 10 failures	
N	If the conditions are met, proceed with a 1-proportion z-test	
T	$z = \frac{\hat{p} - p_o}{\sqrt{\frac{p_o(1-p_o)}{n}}}$	
O	Obtain a p-value using our normal tables OR from your calculator.	
M	If the p-value is HIGH – do NOT reject $H_0$ If the p-value is LOW – reject $H_0$	
S	We do/do not have evidence of $\underline{\hspace{2cm}}$	

- The p-value is a conditional probability. We start by assuming the null hypothesis is true – under this assumption, the p-value is the probability of obtaining similar data (or more extreme) IF the null is true. If the p-value is low, that indicates that data like this is rare when the  $H_0$  is true. \*The p-value is NOT the probability of the null hypothesis being true!!!!\*
- One-sided vs. Two-sided – If we are testing less than ( $<$ ) or greater than ( $>$ ), we have a one-sided alternate. The p-value is then found using our normal tables. However, if we are testing “not equal to”, we have a two-sided alternate which means we have to take that one-sided p-value and double it to account for both tails. If you are using the TI, the p-value for a two sided test is automatically doubled and you don’t have to worry about it.
- Be able to use your TI to run a 1-proportion z-test... Stat: Tests: 1propZTest
- A confidence interval can be used to test a hypothesis. If the null hypothesis is in the confidence interval, we do NOT reject because the  $H_0$  is a plausible value. If the null hypothesis is NOT in the confidence interval, then we do reject  $H_0$  because it is outside our range of plausible values.
- YOU NEVER **ACCEPT** THE NULL HYPOTHESIS!!!!

## Chapter 20

- An Alpha level is also called the Significance Level of the test. Common alpha levels are 1%, 5%, and 10%. The alpha level is the “cutoff” point for our p-value. A p-value that is less than alpha leads to rejection of the null hypothesis. A p-value that is greater than the alpha level would lead to failing to reject the null hypothesis.
- Back before using calculators and computers, statisticians used the critical values from the T-table to help them make a decision about rejection. The critical value from the table was the “cutoff” point between the rejection region and the non-rejection region. If their test statistic (z-score) was more extreme than the critical value, then we would reject the null hypothesis.
- Whenever we test a hypothesis, it is possible that we will make an error.
  - A Type I error is when we reject the null hypothesis and the null is true – we take action and we shouldn’t – we can control the Type I error because it is our alpha level.

- A Type II error is when we fail to reject the null and the null is false – we don't take action and we should – the probability of a Type II error is Beta
- The POWER of the test is the ability of our test to detect a difference when there actually is a difference  $\beta$  – symbolically, the Power is  $1 - \beta$
- There is a relationship between the Errors and Power. As we increase the alpha level (Type I error), then the Type II error (beta) decreases, meaning that Power increases.
- We would like a test with HIGH POWER.. that means we would like to make good decisions (DUH!). The easiest way to increase power and decrease our errors is to increase our sample size – KNOWLEDGE IS POWER!!
- Power also increases the further apart the true value is from our hypothesized value (greater effect size).

## Chapter 21

### ➤ PANIC

- P We are trying to estimate the true \*difference\* in the proportion of \_\_\_\_ between \_\_\_\_ and \_\_\_\_
- A Randomization – either randomly chosen or randomly assigned – BOTH SAMPLES  
 10% condition – we've sampled less than 10% of the population – BOTH SAMPLES  
 Success/Failure – we have at least 10 successes and 10 failures – BOTH SAMPLES  
 Independent Groups
- N If the conditions are met, proceed with a 2-proportion z-interval
- I 
$$(\hat{p}_1 - \hat{p}_2) \pm z^* \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}}$$
- C We are \_\_\_\_% confident that the true difference in the proportion of \_\_\_\_ between \_\_\_\_ and \_\_\_\_ lies within \_\_\_\_ and \_\_\_\_.

### ➤ PHANTOMS

- P/H  $H_0: p_1 = p_2$  where  $p_1$  and  $p_2$  represent the true proportion of \_\_\_\_  
 $H_a: p_1 <, >, \neq p_2$
- A SAME AS ABOVE
- N If the conditions are met, proceed with a 2-proportion z-test
- T 
$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$
- O Obtain a p-value from your calculator.
- M If the p-value is HIGH – do NOT reject  $H_0$
- S If the p-value is LOW – reject  $H_0$   
 We do/do not have evidence of \_\_\_\_

- Be able to do the above PANIC and PHANTOMS using your TI – Stat: Tests: 2PropZTest or 2PropZInt
- Again, you can use a confidence interval to help you test a hypothesis. If the interval contains 0, then there is no evidence of a difference between the groups. If the interval is all positive or all negative, then there is evidence that a difference exists.

## Chapter 22

- When working with means (quantitative data), you usually won't know sigma (the population standard deviation). In the rare chance that you DO know sigma (like when working with IQs or SAT data), you will use z-tests and intervals for means. However, a VAST majority of the time, you will NOT know sigma, so we have to do some extra estimating using S (the sample's standard deviation). Since having this extra estimation means that we will have a little extra error, we can't use the normal model, we have to use a model that allows for extra error. This model is called the T-distribution.
- The T-distribution is a family of curves that is based on degrees of freedom. The T-distribution is still unimodal, symmetric, and bell-shaped, but it has much thicker "tails" than the Normal model. As the degrees of freedom increases (which is related to an increase in our sample size), the T-distribution looks more and more Normal-like.

- When looking up a critical value on the table, make sure you use the DF line rather than the infinity line! For 1-sample means, the degrees of freedom is n-1. The table only goes through line 30 before it starts jumping around, so you can use the TSTAR program I gave you
- PANIC
  - P We are trying to estimate the true mean of \_\_\_\_
  - A Randomization – either randomly chosen or randomly assigned  
10% condition – we’ve sampled less than 10% of the population  
Normality – Either we know the population was Normally distribution OR we have a large enough sample size (n>30) so that the Central Limit Theorem Applies OR we have the sample data and we \*graph\* it to check for normality
  - N If the conditions are met, proceed with a 1 sample t-interval with df = n-1
  - I  $\bar{x} \pm t * \left( \frac{s}{\sqrt{n}} \right)$
  - C We are \_\_\_\_% confident that the true mean \_\_\_\_ is between \_\_\_\_ and \_\_\_\_
- Be able to do the above PANIC using your TI – Stat: Tests: TInterval
- Be able to calculate a minimum sample size for a specific MoE. This is an algebra issue! Since you don’t know the sample size, use the infinity line to get your critical value for your confidence level.

➤ PHANTOMS

- P/H  $H_0: \mu = \underline{\hspace{2cm}}$  where mu represents the true average of \_\_\_\_  
 $H_a: \mu <, >, \neq \underline{\hspace{2cm}}$
- A Randomization – either randomly chosen or randomly assigned  
10% condition – we’ve sampled less than 10% of the population  
Normality – Either we know the population was Normally distribution OR we have a large enough sample size (n>30) so that the Central Limit Theorem Applies OR we have the sample data and we \*graph\* it to check for symmetry/outliers
- N If the conditions are met, proceed with a 1-sample t-test with df = n-1
- T  $t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
- O Obtain a p-value using your calculator.
- M If the p-value is HIGH – do NOT reject  $H_0$   
If the p-value is LOW – reject  $H_0$
- S We do/do not have evidence of \_\_\_\_

- Be able to do the above PHANTOMS using your TI – Stat: Tests: TTest

**Chapter 23**

➤ PANIC

▪

- P We are trying to estimate the true \*difference\* in the mean \_\_\_\_ between \_\_\_\_ and \_\_\_\_
- A Randomization – either randomly chosen or randomly assigned – BOTH SAMPLES  
10% condition – we’ve sampled less than 10% of the population – BOTH SAMPLES  
Normality – Either we know the population was Normally distribution OR we have a large enough sample size (n>30) so that the Central Limit Theorem Applies OR we have the sample data and we \*graph\* it to check for symmetry/outliers – BOTH SAMPLES  
Independent Groups
- N If the conditions are met, proceed with a 2-proportion t-interval with df coming from your TI
- I  $(\bar{x}_1 - \bar{x}_2) \pm t * \left( \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \right)$
- C We are \_\_\_\_% confident that the true difference in the mean \_\_\_\_ between \_\_\_\_ and \_\_\_\_ lies between \_\_\_\_ and \_\_\_\_

## ➤ PHANTOMS

- P/H  $H_0: \mu_1 = \mu_2$  where  $\mu_1$  and  $\mu_2$  represent the true average of \_\_\_\_\_  
 $H_a: \mu_1 <, >, \neq \mu_2$
- A SAME AS ABOVE
- N If the conditions are met, proceed with a 2-proportion t-test with df coming from your TI
- T 
$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$
- O Obtain a p-value using your calculator.
- M If the p-value is HIGH – do NOT reject  $H_0$
- S If the p-value is LOW – reject  $H_0$   
We do/do not have evidence of \_\_\_\_\_

- Be able to do the above PANIC and PHANTOMS using your TI – Stat:Tests:2sampTInt or 2sampTTest
- On your calculator, remember to always say NO to the Pooled? question.
- Again, you can use a confidence interval to help you test a hypothesis. If the interval contains 0, then there is no evidence of a difference between the groups. If the interval is all positive or all negative, then there is evidence that a difference exists.

## **Chapter 24**

- In Chapter 24, one of the requirements of working with the 2 samples was that the data must be independent groups. The reason for that was due to the standard deviations... Remember back in Chapter 16 that in order to combine two variables by adding or subtracting, we had to ADD the variances, but that ONLY worked for independent data...
- However, not all data is independent. Sometimes we have dependent data, often called PAIRED data. Pairing actually allows us to do a much better job of seeing the differences between the data values because that difference becomes our focal point. Paired data occurs when we have data that is intentionally matched with a partner. Some examples of this would be a pre/post test, a taste test, etc. In a paired situation, we don't really care about the two sets of data (the pre and the post test scores), we really want to know about the \*differences\* (or growth) between those two data values PER PERSON.
- A Paired test or interval focuses on those differences only, so we really have a 1-sample t test or t-interval... we just have to write the start and finish a bit differently 😊 Do note that in a 2-sample test we want to know the "difference of the averages" but in a paired t-test we want to know the "average difference" – these are NOT the same thing!!!
- PANIC
  - P We are trying to estimate the true average \*difference\* of \_\_\_\_\_
  - A Paired Data – somehow matched/related
    - Randomization – either randomly chosen or randomly assigned
    - 10% condition – we've sampled less than 10% of the population
    - Normality – Either we know the population was Normally distribution OR we have a large enough sample size ( $n > 30$ ) so that the Central Limit Theorem Applies OR we have the sample data and we \*graph\* it to check for symmetry/outliers (usually we have data– GRAPH the DIFFERENCES!!!)
  - N If the conditions are met, proceed with a paired t-interval with  $df = n-1$
  - I 
$$\bar{x}_{diff} \pm t * \left( \frac{s_{diff}}{\sqrt{n}} \right)$$
  - C We are \_\_\_% confident that the true mean difference of \_\_\_\_\_ is between \_\_\_ and \_\_\_\_\_

## ➤ PHANTOMS

- P/H  $H_0: \mu_{diff} = 0$  where  $\mu$  represents the true average of \_\_\_\_\_  
 $H_a: \mu_{diff} <, >, \neq 0$
- A SAME AS ABOVE
- N If the conditions are met, proceed with a paired t-test with  $df = n-1$
- T 
$$t = \frac{\bar{x}_{diff} - \mu_{diff}}{\frac{s_{diff}}{\sqrt{n}}}$$
- O Obtain a p-value using your calculator.
- M If the p-value is HIGH – do NOT reject  $H_0$   
 If the p-value is LOW – reject  $H_0$
- S We do/do not have evidence of \_\_\_\_\_

- Be able to do the above PANIC and PHANTOMS using your TI – Stat: Tests: TInterval or TTest
- HUGE NOTE: When you subtract the two datasets to get your paired data, make sure you take note of how you subtracted!!! For example, if I am analyzing your pre/post data for this class, I would probably want to do PostTest – PreTest so I would end up with positive values (a > test)... If I reversed that, I would have a < test.

## Chapter 25

- When working with categorical data that is provided in \*counts\*, then we have to use some new inference methods to deal with this data. This data must be in counts (like # of blue, orange, red, etc M&Ms), it cannot be in weights, percentages, etc. This type of data is measured using a Chi-Squared Distribution.
- The Chi-Squared Distribution is also a family of curves that is dependent on degrees of freedom. The Chi Squared Distribution is always skewed to the right, although that skew does get less noticeable as the  $df$  increases.
- There are three Chi-Squared Tests that we work with – Goodness of Fit, Test of Homogeneity, and Test of Independence. The latter two use a contingency table (from Ch 3) to organize the data. The Chi-Squared tests are non-parametric tests, which means that with all three tests, we are comparing distributions of categorical counts rather than to a single population parameter.
- Goodness of Fit Test – Whenever we wish to see if our sample data \*fits\* a theoretical distribution of data, we will use a GOF test. Your actual data is your “observed” data and your theoretical distribution helps you find your “expected” data. Be careful with the expected data – Sometimes we want to know if all of the categories are equally likely and sometimes we want to know if they follow a specific distribution (like the M&M data) and we have to calculate the expected using a percentage of the total.

- P/H  $H_0$ : The observed data FITS the expected distribution  
 $H_a$ : The observed data DOES NOT FIT the expected distribution
- A Randomness – again, either a random sample or random assignment  
 10% condition – we again need our sample to be less than 10% of the population  
 Expected Values > 5 – You must SHOW the expected values!!!
- N If the conditions are met, proceed with a Chi Squared GOF Test with  $df=k-1$  ( $k$  is the # of categories)
- T 
$$\chi^2 = \sum \frac{(O - E)^2}{E}$$
- O Obtain a p-value using  $X^2cdf(\text{____}, \text{infinity}, df)$
- M If the p-value is HIGH – do NOT reject  $H_0$   
 If the p-value is LOW – reject  $H_0$
- S We do/do not have evidence of \_\_\_\_\_

- As always we cannot prove our null hypothesis is true!! You can only say the data doesn't disprove it
- Tests of Homogeneity and Independence – These two tests are virtually identical, other than the data collection itself and the question it is trying to answer. A Test of Homogeneity takes 2 (or more) samples and wants to know if the distributions of the results are the same or not. A Test of Independence takes one sample and gathers data on 2 variables to see if the variables are related.

P/H  $H_0$ : The distributions are the same OR  $H_0$ : The variables are independent  
 $H_a$ : The distributions are different OR  $H_a$ : The variables are dependent  
 A Randomness – again, either a random sample or random assignment  
 10% condition – we again need our sample to be less than 10% of the population  
 Expected Values > 5 – You must SHOW the expected values!!!

$$\frac{(\text{RowTotal})(\text{ColumnTotal})}{\text{GrandTotal}}$$

To find the expected values in a chi-squared table:

N If the conditions are met, proceed with a Chi Squared GOF Test with  $df=(\#rows-1)(\#columns-1)$   
 T

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

O Obtain a p-value using  $X^2cdf(\_, \text{infinity}, df)$  OR using your calculator  
 M

If the p-value is HIGH – do NOT reject  $H_0$

If the p-value is LOW – reject  $H_0$

S We do/do not have evidence of \_\_\_\_\_

- Be able to do the above PHANTOMS using your TI – Enter your data into the matrix first!
- Note: When choosing between the tests of Homogeneity or Independence, pay attention to the wording of the question!!! A test of Homogeneity would have words like “are they the same”, “is there evidence of a difference”, etc. A test of Independence would say things like “are the variables related”, “is there evidence of an association”, etc.

## **Chapter 26**

- When working with bivariate data (ordered pairs  $x, y$ ), we can also test to see if there is a relationship between the two variables. We’ve already explored scatterplots, regression lines, residual plots, etc in Chapters 7 and 8, so this chapter takes that information one step further and looks at the question of “Is there a statistically significant relationship between X and Y?”
- Most of the time, in regression analysis, you will be given computer output, so make sure you look at your well-labeled regression output in your notes. When working with the computer output, you really want to focus on the slope line. That is the p-value you are interested in!
- The S on the computer output refers to the standard deviation of the residuals. If this number is small, that indicates that the data isn’t very spread out. If s is large, the data is very scattered around the line.
- Test for Slope – In a hypothesis test for the slope of a line, we are testing to see if a relationship exists.

P/H  $H_0$ : There is no linear relationship between X and Y ( $\beta=0$ )

$H_a$ : There is a linear relationship between X and Y (could also be positive or negative) ( $\beta <, >, \neq 0$ )

A Randomness – again, either a random sample or random assignment

Linearity of the scatterplot – do the points fit a fairly straight line?

No pattern in the residuals – look at the residual plot

\*Note\* Usually on the AP exam, it says “Assume the conditions necessary for inference are met”

N If the conditions are met, proceed with a t-test for slope with  $df=n-2$   
 T

$$t = \frac{\text{slope} - \beta}{SE_{\text{slope}}}$$

O Obtain a p-value using the computer output on the slope line – Remember that the computer output always uses a  
 M 2-sided pvalue, so you may need to divide by 2 if you are only doing > or <.

If the p-value is HIGH – do NOT reject  $H_0$

If the p-value is LOW – reject  $H_0$

S We do/do not have evidence of \_\_\_\_\_

➤ We can also calculate an interval to estimate the true slope of the regression line.

- P We wish to estimate the true slope of the line between X and Y (put in your context!!)  
 A Randomness – again, either a random sample or random assignment  
 Linearity of the scatterplot – do the points fit a fairly straight line?  
 No pattern in the residuals – look at the residual plot  
 \*Note\* Usually on the AP exam, it says “Assume the conditions necessary for inference are met”  
 N If the conditions are met, proceed with a t-interval for slope with  $df=n-2$   
 I  $slope \pm t^*(SE_{slope})$  ← slope and SE can be found on the computer output on the slope line  
 C We are \_\_\_% confident that the true slope of the line between X and Y is between \_\_\_ and \_\_\_.

➤ In the rare case that you wish to do a t-test or t-interval for the slope by calculator rather than by using computer output, please see the instructions and screen shots in Chapter 27 of your textbook

### 2011B #5

5. During a flu vaccine shortage in the United States, it was believed that 45 percent of vaccine-eligible people received flu vaccine. The results of a survey given to a random sample of 2,350 vaccine-eligible people indicated that 978 of the 2,350 people had received flu vaccine.
- (a) Construct a 99 percent confidence interval for the proportion of vaccine-eligible people who had received flu vaccine. Use your confidence interval to comment on the belief that 45 percent of the vaccine-eligible people had received flu vaccine.
- (b) Suppose a similar survey will be given to vaccine-eligible people in Canada by Canadian health officials. A 99 percent confidence interval for the proportion of people who will have received flu vaccine is to be constructed. What is the smallest sample size that can be used to guarantee that the margin of error will be less than or equal to 0.02 ?
5. A large pet store buys the identical species of adult tropical fish from two different suppliers—Buy-Rite Pets and Fish Friends. Several of the managers at the pet store suspect that the lengths of the fish from Fish Friends are consistently greater than the lengths of the fish from Buy-Rite Pets. Random samples of 8 adult fish of the species from Buy-Rite Pets and 10 adult fish of the same species from Fish Friends were selected and the lengths of the fish, in inches, were recorded, as shown in the table below.

	Length of Fish	Mean	Standard Deviation
Buy-Rite Pets ( $n_B = 8$ )	3.4 2.7 3.3 4.1 3.5 3.4 3.0 3.8	3.40	0.434
Fish Friends ( $n_F = 10$ )	3.3 2.9 4.2 3.1 4.2 4.0 3.4 3.2 3.7 2.6	3.46	0.550

Do the data provide convincing evidence that the mean length of the adult fish of the species from Fish Friends is greater than the mean length of the adult fish of the same species from Buy-Rite Pets?

**2010 #3**

3. A humane society wanted to estimate with 95 percent confidence the proportion of households in its county that own at least one dog.
- (a) Interpret the 95 percent confidence level in this context.

The humane society selected a random sample of households in its county and used the sample to estimate the proportion of all households that own at least one dog. The conditions for calculating a 95 percent confidence interval for the proportion of households in this county that own at least one dog were checked and verified, and the resulting confidence interval was  $0.417 \pm 0.119$ .

- (b) A national pet products association claimed that 39 percent of all American households owned at least one dog. Does the humane society's interval estimate provide evidence that the proportion of dog owners in its county is different from the claimed national proportion? Explain.
- (c) How many households were selected in the humane society's sample? Show how you obtained your answer.

**2010 #5**

5. A large pet store buys the identical species of adult tropical fish from two different suppliers—Buy-Rite Pets and Fish Friends. Several of the managers at the pet store suspect that the lengths of the fish from Fish Friends are consistently greater than the lengths of the fish from Buy-Rite Pets. Random samples of 8 adult fish of the species from Buy-Rite Pets and 10 adult fish of the same species from Fish Friends were selected and the lengths of the fish, in inches, were recorded, as shown in the table below.

	Length of Fish	Mean	Standard Deviation
Buy-Rite Pets ( $n_B = 8$ )	3.4 2.7 3.3 4.1 3.5 3.4 3.0 3.8	3.40	0.434
Fish Friends ( $n_F = 10$ )	3.3 2.9 4.2 3.1 4.2 4.0 3.4 3.2 3.7 2.6	3.46	0.550

Do the data provide convincing evidence that the mean length of the adult fish of the species from Fish Friends is greater than the mean length of the adult fish of the same species from Buy-Rite Pets?

**2009B #3**

3. A French study was conducted in the 1990s to compare the effectiveness of using an instrument called a cardiopump with the effectiveness of using traditional cardiopulmonary resuscitation (CPR) in saving lives of heart attack victims. Heart attack patients in participating cities were treated with either a cardiopump or CPR, depending on whether the individual's heart attack occurred on an even-numbered or an odd-numbered day of the month. Before the start of the study, a coin was tossed to determine which treatment, a cardiopump or CPR, was given on the even-numbered days. The other treatment was given on the odd-numbered days. In total, 754 patients were treated with a cardiopump, and 37 survived at least one year; while 746 patients were treated with CPR, and 15 survived at least one year.
- (a) The conditions for inference are satisfied in the study. State the conditions and indicate how they are satisfied.
- (b) Perform a statistical test to determine whether the survival rate for patients treated with a cardiopump is significantly higher than the survival rate for patients treated with CPR.

## 2009 #5

5. For many years, the medically accepted practice of giving aid to a person experiencing a heart attack was to have the person who placed the emergency call administer chest compression (CC) plus standard mouth-to-mouth resuscitation (MMR) to the heart attack patient until the emergency response team arrived. However, some researchers believed that CC alone would be a more effective approach.

In the 1990s a study was conducted in Seattle in which 518 cases were randomly assigned to treatments: 278 to CC plus standard MMR and 240 to CC alone. A total of 64 patients survived the heart attack: 29 in the group receiving CC plus standard MMR, and 35 in the group receiving CC alone. A test of significance was conducted on the following hypotheses.

$H_0$ : The survival rates for the two treatments are equal.

$H_a$ : The treatment that uses CC alone produces a higher survival rate.

This test resulted in a  $p$ -value of 0.0761.

- Interpret what this  $p$ -value measures in the context of this study.
- Based on this  $p$ -value and study design, what conclusion should be drawn in the context of this study? Use a significance level of  $\alpha = 0.05$ .
- Based on your conclusion in part (b), which type of error, Type I or Type II, could have been made? What is one potential consequence of this error?

## 2007 #4

4. Investigators at the U.S. Department of Agriculture wished to compare methods of determining the level of *E. coli* bacteria contamination in beef. Two different methods (A and B) of determining the level of contamination were used on each of ten randomly selected specimens of a certain type of beef. The data obtained, in millimicrobes/liter of ground beef, for each of the methods are shown in the table below.

		Specimen									
		1	2	3	4	5	6	7	8	9	10
Method	A	22.7	23.6	24.0	27.1	27.4	27.8	34.4	35.2	40.4	46.8
	B	23.0	23.1	23.7	26.5	26.6	27.1	33.2	35.0	40.5	47.8

Is there a significant difference in the mean amount of *E. coli* bacteria detected by the two methods for this type of beef? Provide a statistical justification to support your answer.

**2003 #5**

5. A random sample of 200 students was selected from a large college in the United States. Each selected student was asked to give his or her opinion about the following statement.

“The most important quality of a person who aspires to be the President of the United States is a knowledge of foreign affairs.”

Each response was recorded in one of five categories. The gender of each selected student was noted. The data are summarized in the table below.

	Response Category				
	Strongly Disagree	Somewhat Disagree	Neither Agree nor Disagree	Somewhat Agree	Strongly Agree
Male	10	15	15	25	25
Female	20	25	25	25	15

Is there sufficient evidence to indicate that the response is dependent on gender? Provide statistical evidence to support your conclusion.