Chapter 1

Functions and Graphs

1.2 Basics of Functions and Their Graphs

Chapter 1

Homework

1.2 Pg 159 5, 9, 17, 21, 31, 37, 53, 63. 65, 67, 69, 77, 81, 83, 87, 89. 91

Chapter 1.2

Learning Target

F-IF.2 Understand the concept of a function and use function notation.

Chapter 1.2

Success Criteria:

- I can find the domain and range of a relation.
- I can determine whether a relation is a function.
- I can determine whether an equation represents a function.
- I can graph functions by plotting points.
- I can evaluate a function.
- I can use the vertical line test to identify functions.
- I can obtain information about a function from its graph.
- I can identify the domain and range of a function.
- I can identify intercepts from a function's graph.

Find the domain and range of a relation.

Definition of a Relation

- Find the domain and range of a relation.

- A **relation** is any set of **ordered pairs**.
  - The **set** of all first components **(X)** of the ordered pairs is called the **domain** of the relation
  - The **set** of all second components **(Y)** is called the **range** of the relation.
- Find the domain and range of the relation:

{(0, 9.1), (10, 6.7), (20, 10.7), (30, 13.2), (40, 21.2)}.

domain: {0, 10, 20, 30, 40}

range: {9.1, 6.7, 10.7, 13.2, 21.2}

I can determine whether a relation is a function.

Definition of a Function

I can determine whether a relation is a function.

- A **function** is a correspondence from a first set, called the **domain**, to a second set, called the **range**, …

  such that each element in the domain corresponds to exactly one element in the range.

- Consider a trip to the doughnut shoppe. If you are allowed to choose more than one doughnut that would be a relation, but not a function. One person from the domain can be matched with more than one doughnut from the range of doughnuts. If, howsomever, you are only allowed one doughnut, that would define a function. Even if your friends choose the same doughnut it is still a function. One person, one doughnut.

- A function is a special kind of **relation**.

Definition of Function

A function **f** from a set **A** to a set **B** is a relation that assigns to each element **x** in the set **A** **exactly** one element **y** in the set **B**. The set **A** is the **domain** (or set of inputs) of the function **f**, and the set **B** contains the range (or set of outputs).

- Read this statement carefully and consider everything that is implied.

Determining Whether a Relation is a Function

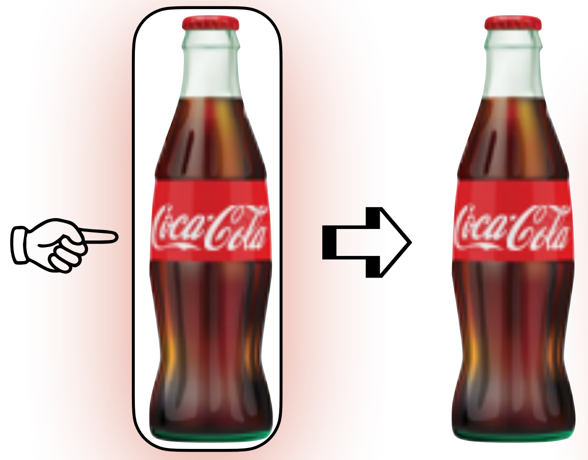
- I can determine whether a relation is a function.

- Determine whether the relation is a function: {(1, 2), (3, 4), (6, 5), (8, 5)}.
  - Yeperdoo - No two ordered pairs in the given relation have the same first component and different second components. Thus, the relation is a function.
  - If an equation is solved for y and **more than one value of y can be obtained for a given x**, then the equation does **not** define y as a function of x.

Functions

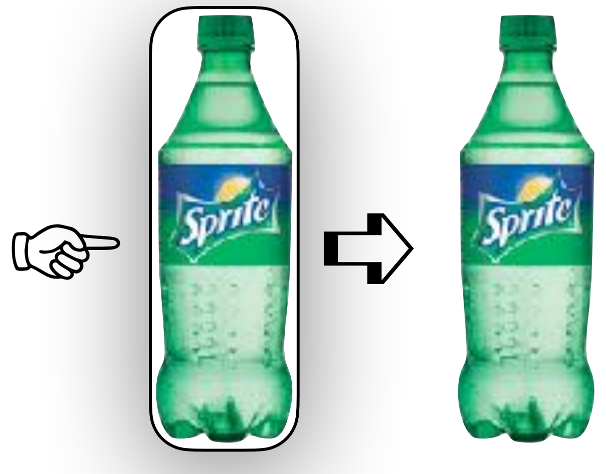
I can determine whether a relation is a function.

- Consider a vending machine that dispenses cold drinks.
  - This can be considered a function machine. If you punch the button showing a Coke, that machine dispenses a Coke.



  - The input value is the button you push, the output is the bottle of Coke.
  - It does not matter if there is more than one button labeled Coke. Each button returns the same ouput, the bottle of Coke.
  - If you push any button labeled Fanta, you get a Fanta. All buttons labeled Fanta return a Fanta.

  - If you push any button labeled Sprite, you get a Sprite. All buttons labeled Sprite return a Sprite.



- When properly filled, the Coke machine is a function.

Functions

I can determine whether a relation is a function.

- Now let us suppose the employee filling the machine did not fill the machine properly and put Sprite in some of the racks that were supposed to be Coke, and Fanta in some of the racks that were supposed to be Sprite.

- This is no longer a function machine. If you punch one button showing a Coke, the machine dispenses a Coke, but if you push another button showing a Coke the machine dispenses a Fanta.

- The input value Coke may return a Coke, but may also return a Fanta. So the input value Coke can return more that one output value. Thus, the machine is no longer a function.

Characteristics of a Function from Set A to Set B

1. Each element in set A must be matched with an element in set B .
2. Some elements in B may not be matched with any element in A.
3. Two or more elements in A may be matched with the same element in B.
4. An element in A (the domain) cannot be matched with two different elements in B.

The set A is the **domain** of the function. The set B is the **range**. of the function.

Determining if a Relation is a Function

- I can determine whether a relation is a function.

- Determine whether the relation is a function: {(1, 2), (3, 4), (6, 5), (8, 5)}.
  - Yep - No two ordered pairs in the given relation have the same first component and different second components (no x is assigned more than one y). Thus, the relation is a function.
- Determine whether the relation is a function: {(5, 2), (6, 4), (6, 5), (8, 8)}.
  - Nope - the element of the domain (6) is matched with two elements of the range {4, 5}. Thus the relation is NOT a function.
- If an equation is solved for y and more than one value of y can be obtained for a given x, then the equation does **not** define y as a function of x.
  - Howsomever multiple values of x can have the same value of y, and the equation may define y as a function of x.

- I can determine whether a relation is a function.

- Determine whether the relation is a function:

A.

x	y
2	6
5	7
6	9
6	10

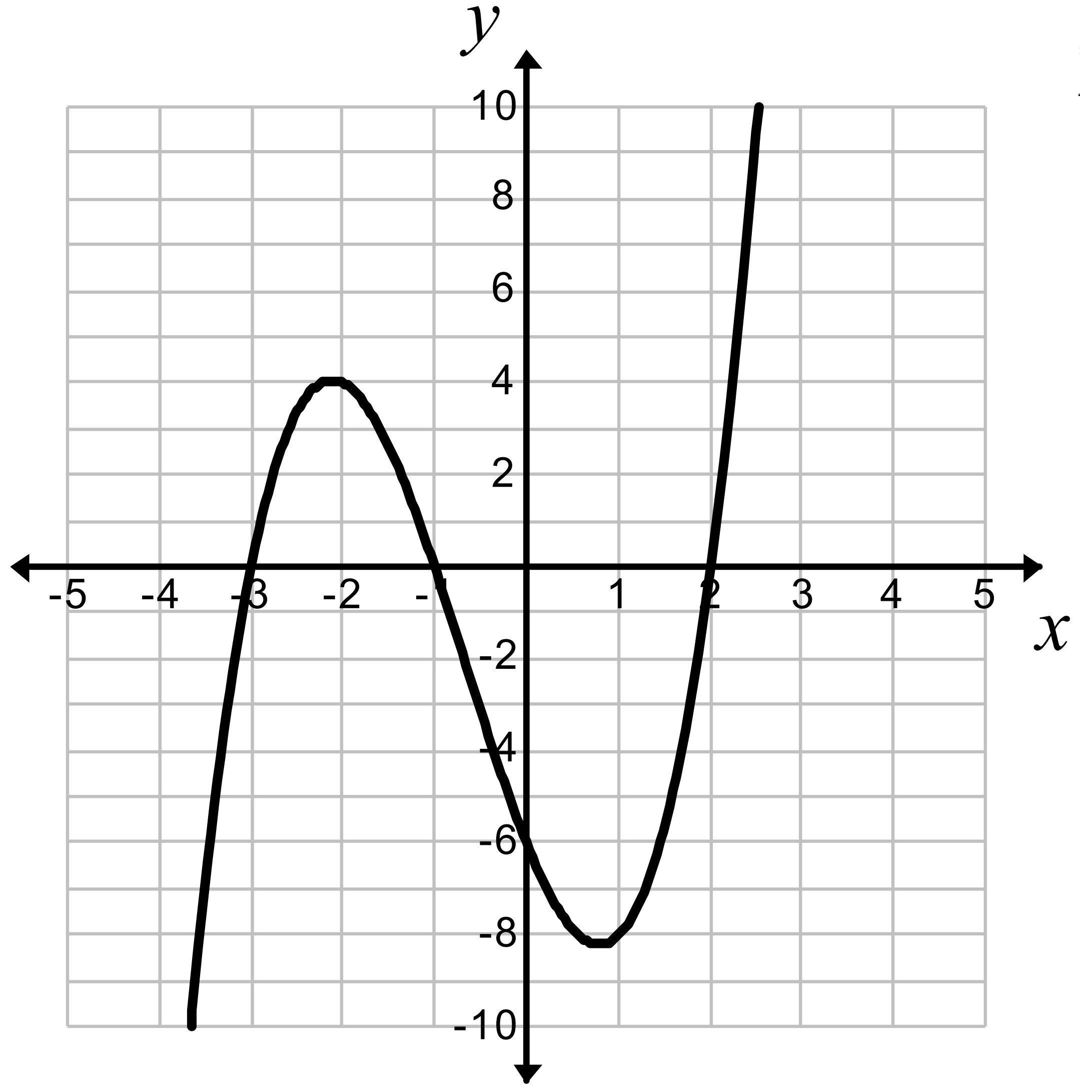
- Nope.
- Input 6 has two output 9, 10.

B.

x	y
2	6
5	7
2	6
-1	3

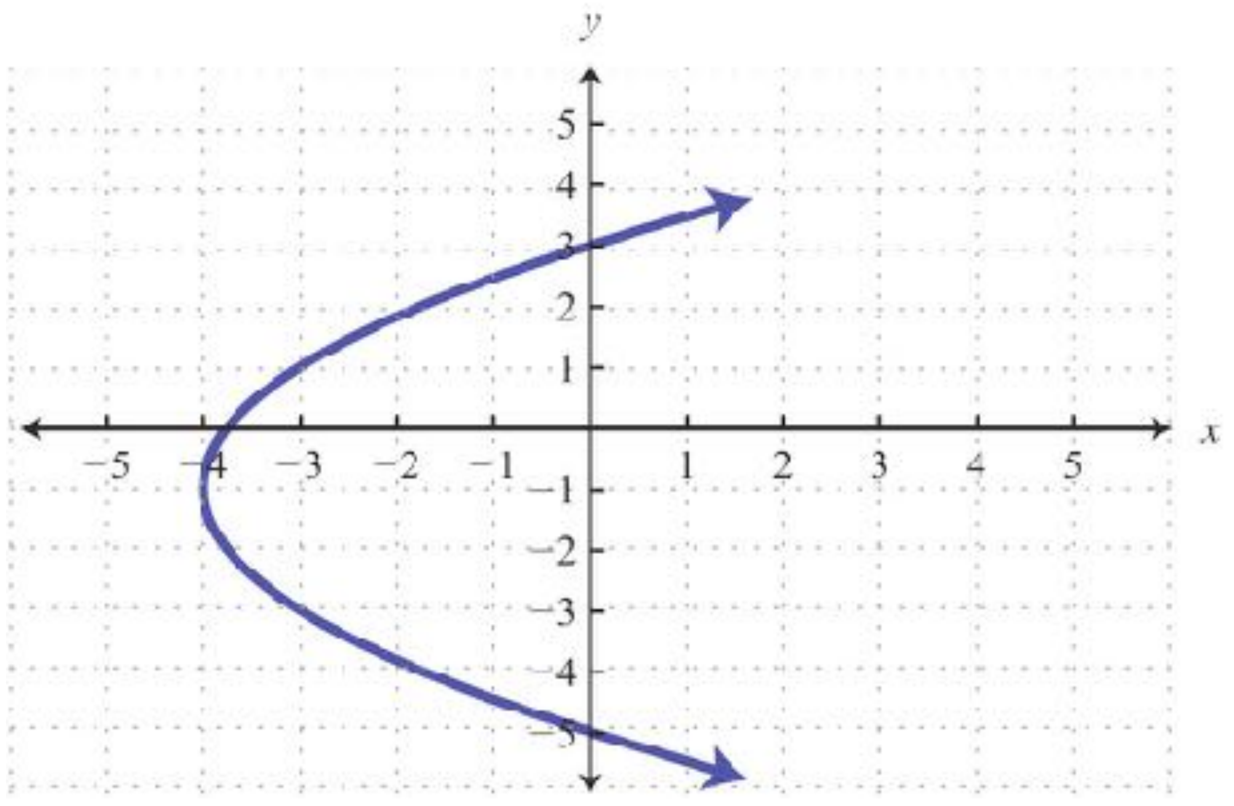
- Yep.
- No x value has more than **one** y value

C.



- Yep.
- No x value on the graph has more than **one** y value

D.



- Nope.
- Some x values have more than **one** y value

- Determine whether the equation defines y as a function of x.

$$x^2 + y^2 = 1 \qquad y^2 = 1 - x^2 \qquad y = \pm\sqrt{1 - x^2}$$

- The ± indicates that for certain values of x, there are two values of y. For this reason, the equation does not define y as a function of x.

- Please note: $\sqrt{1 - x^2} \neq 1 - x$

- Making this mistake will make my head explode.

I can graph functions by plotting points.

Functions

- A function with discrete values of x, that is with gaps between x values, is a **discrete function**. In an interval of potential x-values only certain x-values are allowed as input values in a **discrete function**. The graph of a **discrete function** will only consist of discrete points.

  - {(0, 1), (1, 2), (3, 5), (4, 7)} is a discrete function because there are no values of x between 0 and 1, between 1 and 3, or between 3 and 4.

- A function with continuous values of x, no gaps between consecutive values of x, is a **continuous function**. In an interval of potential x-values every value of x is allowed to be an input value in a **continuous function**. The graph of a **continuous function** will be points connected by a continuous line.

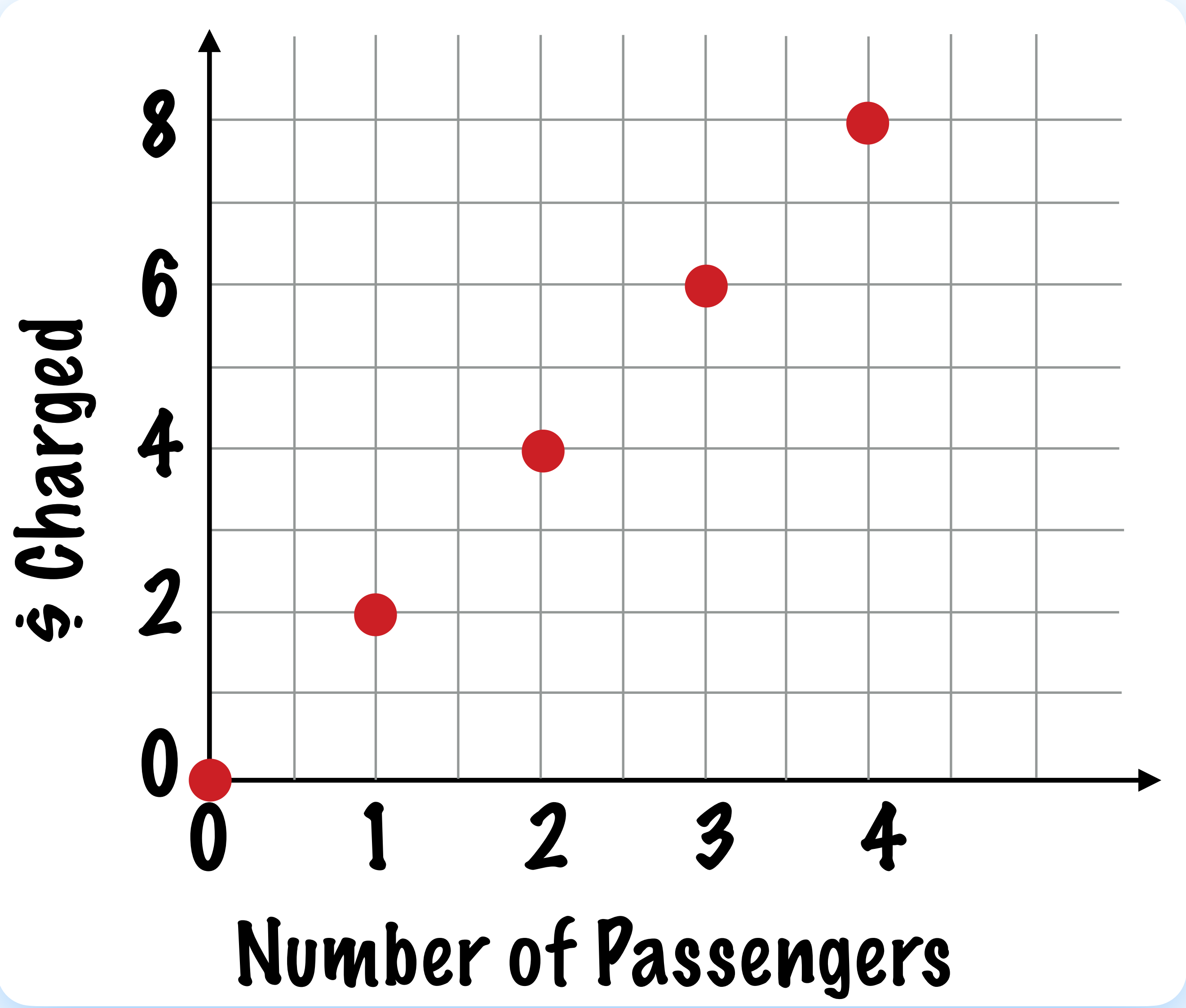
  - $f(x) = 2x - 5$ is the equation of a continuous function. x can be any value between any two other values of x.

Functions

- Chariah is driving her friends to the football game. She can only fit 5 people, including herself, in her car. She charges $2 per person for gas. Does this scenario define a discrete function or a continuous function?
  - This is a discrete function, only counting numbers of people can be input values, 0, 1, 2, 3, or 4
  - The function has ordered pair {(0, 0), (1, 2), (2, 4), (3, 6), (4, 8)}
  - Table of values

x	y
0	0
1	2
2	4
3	6
4	8

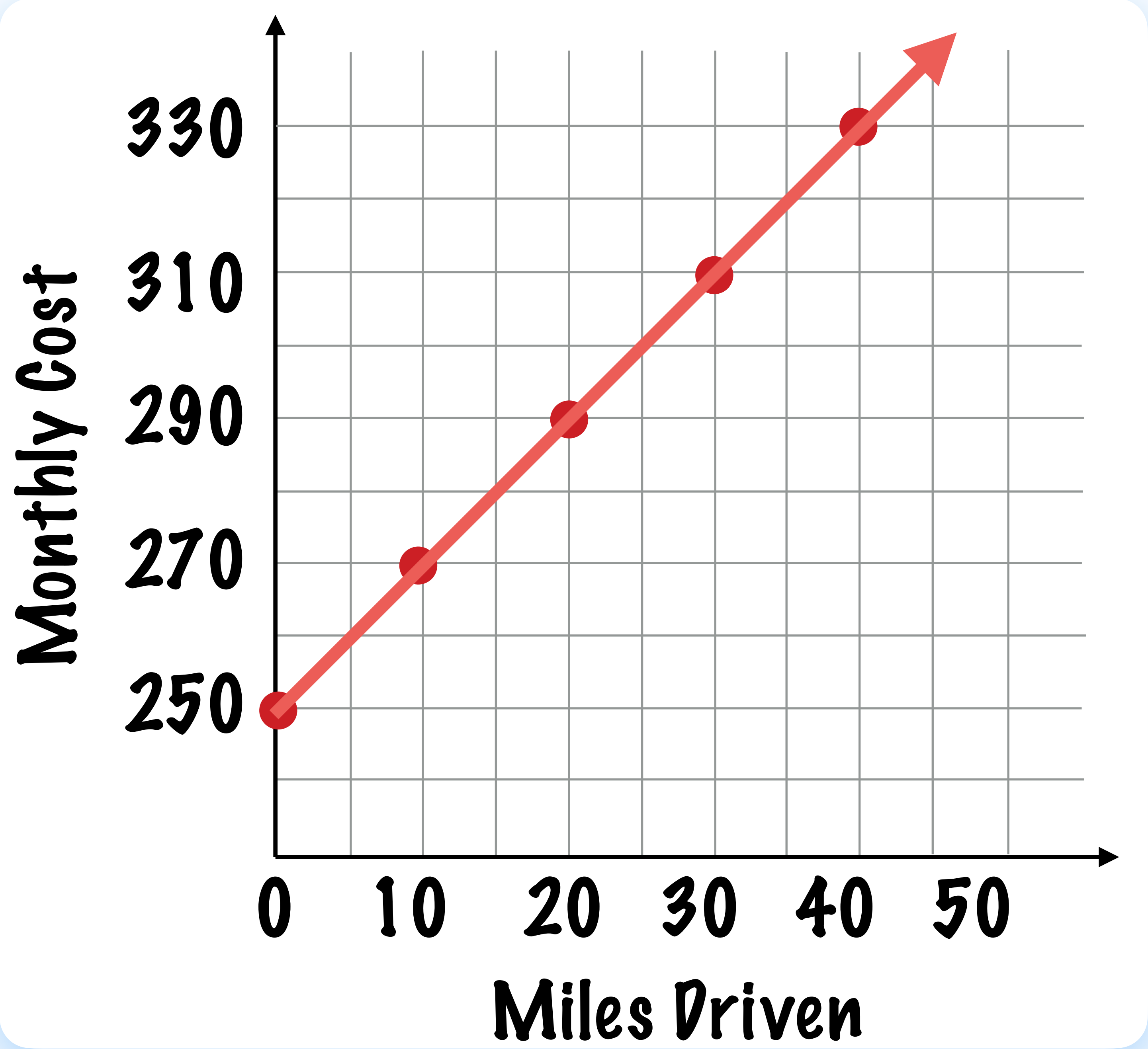
  - Graph is collection of discrete points.



- Chariah leases her car. She pays $250 per month and $2 per mile for each mile driven in the month. Does the function of total paid for a month describe a discrete function or a continuous function?
  - The function would be $f(x) = 2x + 250$, where x is the number of miles driven.
  - This is a continuous function because miles could be any number from 0 to whatever.
  - Partial table of values

x	y
0	250
10	270
20	290
30	310
40	330

  - The graph is a continuous ray.



I can evaluate a function.

Function Notation

I can evaluate a function.

- The special notation f(x), read "f of x" or "f at x", represents the value of the function (commonly known as "y") at the number x (when x is the input value).
  - Students are often confused by function notation. Especially when the input value is an expression.

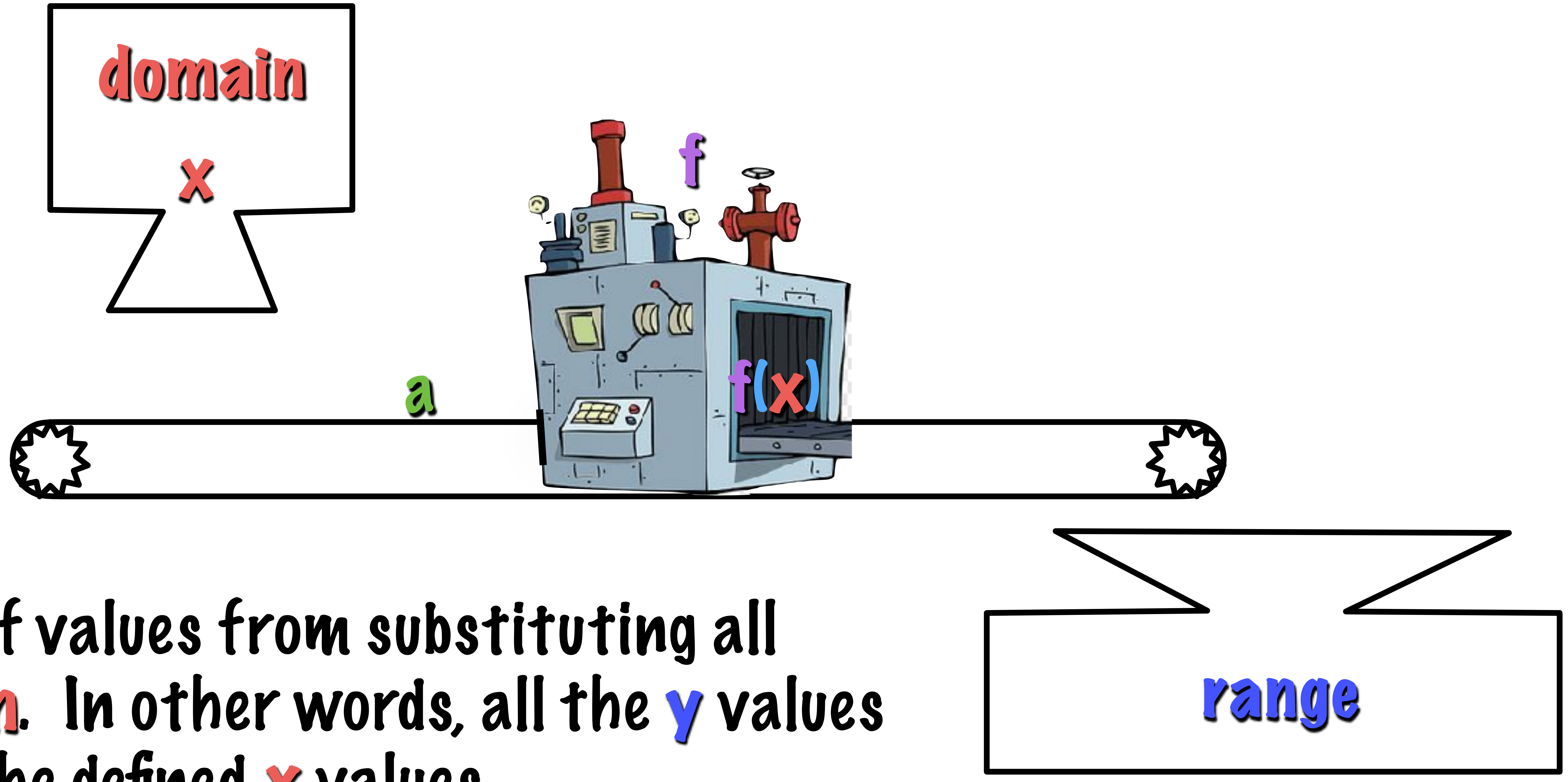
f (x) does not mean f times x or f • x

- If $f(x) = x^2$, then f is the name of the function. The input value is x and the output value is f(x).
  - f is the name of the function. The input value is x and the output value is f(x).
  - The input value is x.
  - The output value is f(x). This is the same as y in equation notation.
  - The function rule is to square the input value.

- Remember; f(x), (commonly known as "y"), represents the value of the function f at the number x (in other words, when x is the input value).
  - If $g(x) = x - 3$, then g is the name of the function.
    - The input value is x and the output value is g(x).
  - The function rule defined by the function g is to take the input value (in this case x) and subtract 3 to get the output value g(x).
    - g is the name of the function.
    - The input value is x
    - The output value is g(x). This is the same as y in equation notation.
    - The function rule is to subtract 3 from the input value to obtain the output value.

Domain and Range

- The domain of a function is the set of all possible input values (all possible x values).
- The range of a function is the set of all possible output values (all possible f(x) or y values).



- The range is the resulting set of values from substituting all defined values (x) of the domain. In other words, all the y values that result from inputting all the defined x values.

Functions

I can evaluate a function.

- Given $f(x) = 2x - 8$
  - What is the input value? x
  - What is the name of the function? f
  - What is the output value? $f(x)$
  - What is the function rule?

    Multiply the input value by 2 then subtract 8.

- Given $g(a) = 3a + 1$
  - What is the input value? a
  - What is the name of the function? g
  - What is the output value? $g(a)$
  - What is the function rule?

    Multiply the input value by 3, then add 1.

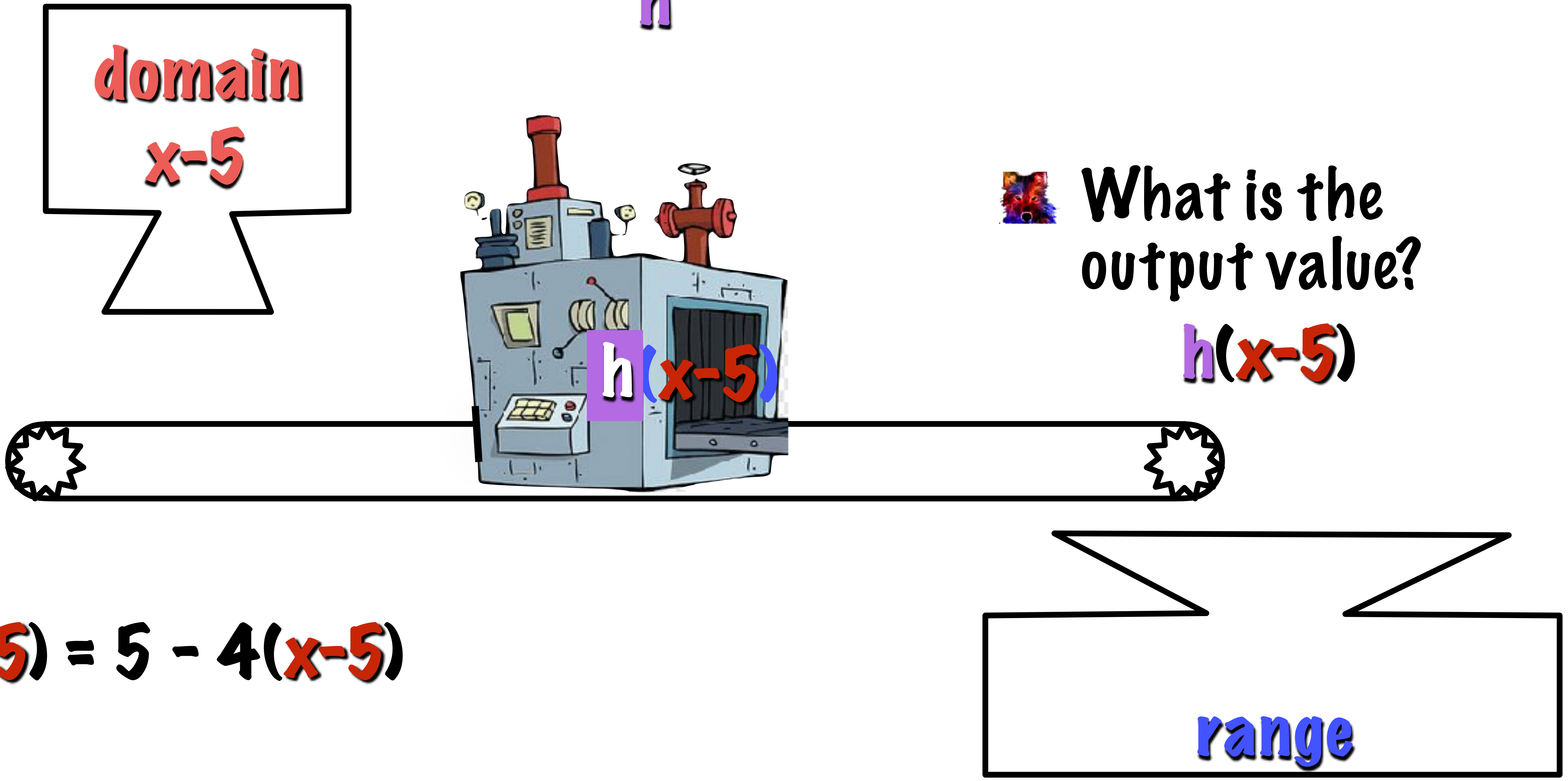
- Given h(x) = 5 - 4x, find h(x-5)

  - What is the input value?

    x-5

- What is the name of the function?

  h



- What is the output value?

  h(x-5)

- What is the function rule?

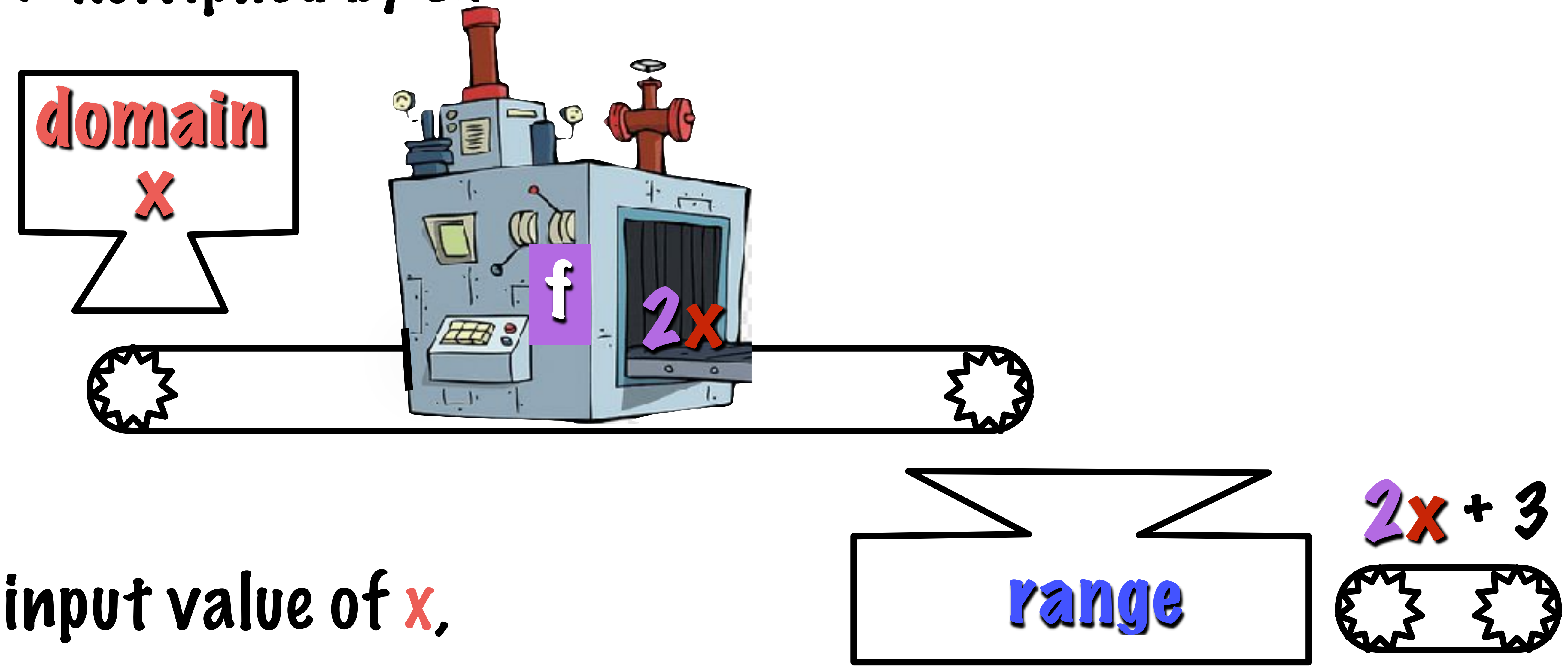
  Subtract 4 input values from 5. h(x-5) = 5 - 4(x-5)

Functions

I can evaluate a function.

- Be careful when using f(x). Remember f(x) is the output value when x is the input value.
  - f(x) = 2x means the function f has the rule that the input value of x is multiplied by 2.
  - The function rule for the function f is to multiply any input value by 2.
  - f(x) + 3 means the input value of x is multiplied by 2, then 3 is added to the output value.
  - That is NOT the same as f(x + 3), which is (x + 3) multiplied by 2.

  - $f(x) = 2x$
  - $f(x) + 3 = 2x + 3$
  - $f(x + 3) = 2x + 6$



- f(x) + 3 means apply the function rule to the input value of x, then add 3 to the output value.

- If $f(x) = x^2 - 2x + 7$ evaluate $f(-5)$

$$f(x) = x^2 - 2x + 7$$

$$f(-5) = (-5)^2 - 2(-5) + 7$$

$$f(-5) = 25 + 10 + 7 = 42$$

- Thus $f(-5) = 42$

- If $f(x) = x^2 - 2x + 7$ evaluate $f(x+2)$

$$f(x+2) = (x+2)^2 - 2(x+2) + 7$$

$$f(x+2) = (x^2 + 4x + 4) - 2x - 4 + 7 = x^2 + 2x + 7$$

- Thus $f(x+2) = x^2+2x+7$

Evaluating Functions

I can evaluate a function.

- Let us make things a little more interesting.
  - If $g(x) = 3x - 2$ evaluate $g(-5)$

    $g(x) = 3x - 2$ $g(-5) = 3(-5) - 2$

    - Thus $g(-5) = -17$
  - If $g(x) = 3x - 2$ evaluate $g(a)$

    $g(x) = 3x - 2$ $g(a) = 3(a) - 2$

    - Thus $g(a) = 3a - 2$
  - If $g(x) = 3x - 2$ evaluate $g(x - 4)$

    $g(x) = 3x - 2$ $g(x-4) = 3(x-4) - 2$

    - Thus $g(x-4) = 3x - 14$

I can identify the domain and range of a function from its graph.

Finding Domain and Range From a Function's Graph

- I can identify the domain and range of a function from its graph.

- To find the domain of a function **from it's graph**, look for all the inputs on the x-axis that correspond to points on the graph.

- To find the range of a function **from it's graph**, look for all the outputs on the y-axis that correspond to points on the graph.

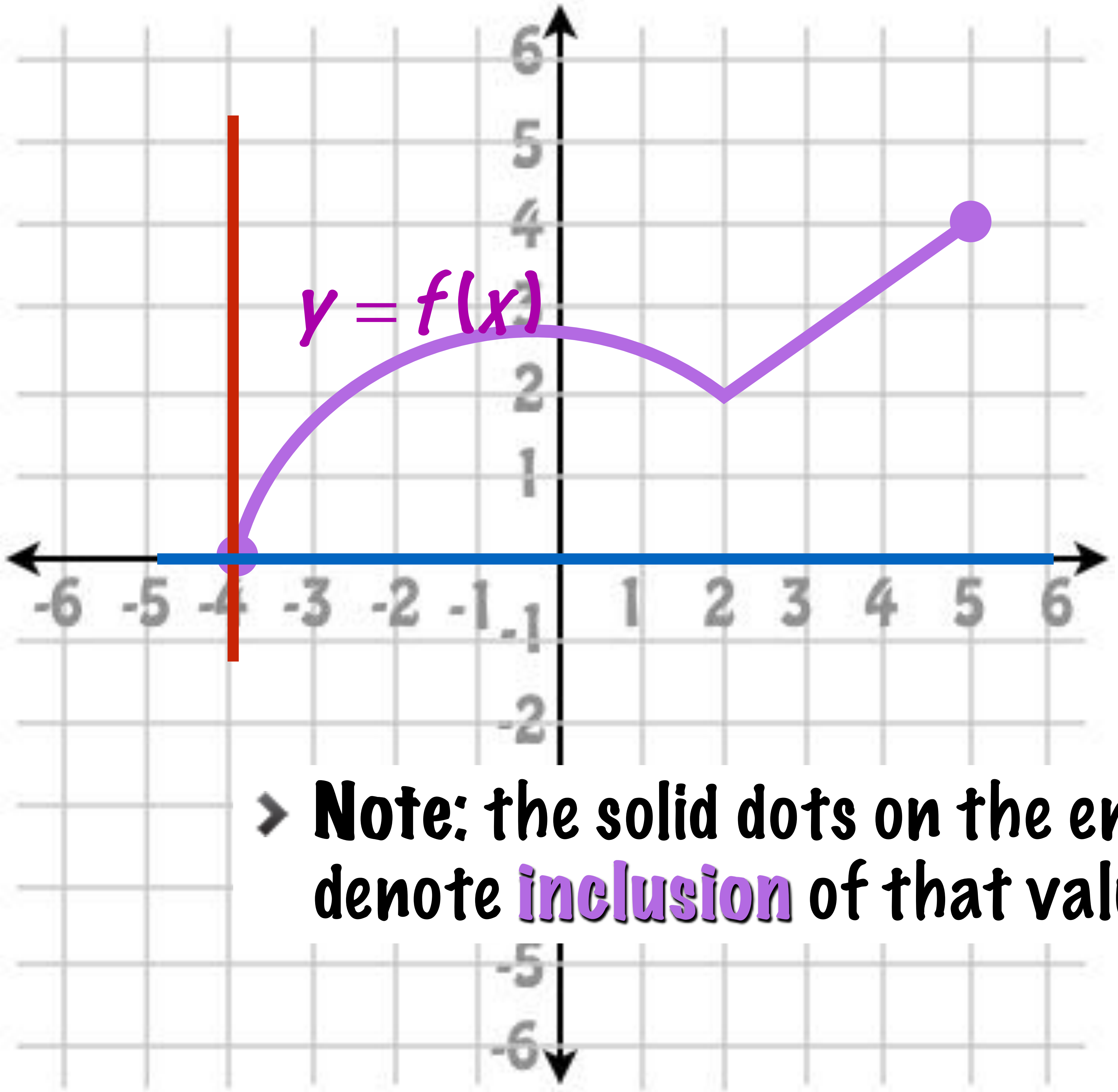
- A function may have more than one x-intercept, but a function can have only one y-intercept.

Finding Domain and Range From a Function's Graph

- Use the graph of the function to identify its domain and its range.
  - Domain $\{x \mid -4 \leq x \leq 5\}$

    $[-4, 5]$
  - Range $\{y \mid 0 \leq y \leq 4\}$

    $[0, 4]$



- **Note:** the solid dots on the ends denote **inclusion** of that value.

Identifying the Domain and Range of a Function from Its Graph

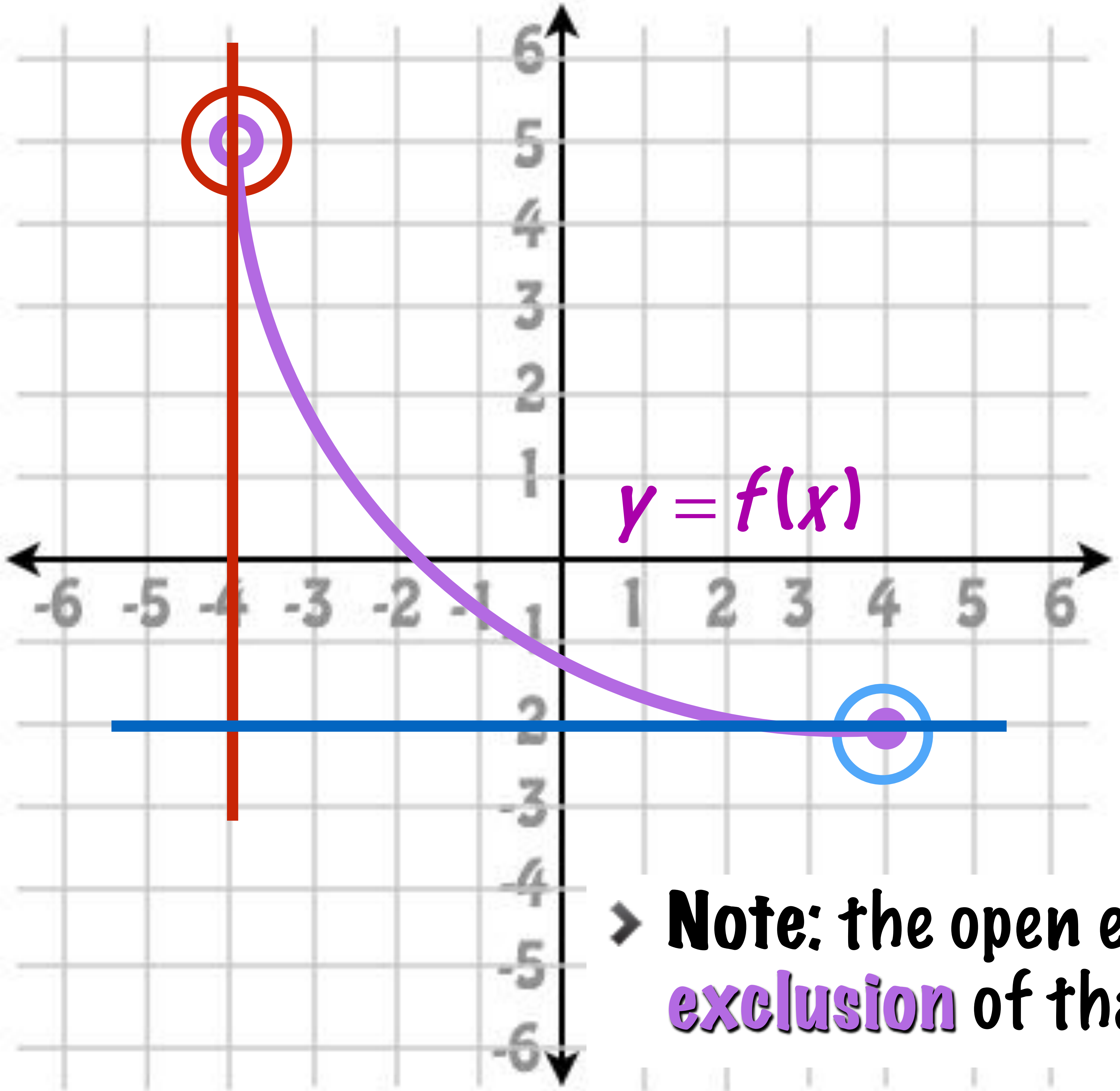
- Use the graph of the function to identify its domain and its range.

  - Domain $\{x \mid -4 < x \leq 4\}$

    $(-4, 4]$

  - Range $\{y \mid -2 \leq y < 5\}$

    $[-2, 5)$



> **Note:** the open ends denote **exclusion** of that value.

I can obtain information about a function from its graph.

Graphs of Functions

I can obtain information about a function from its graph.

- The **graph of a function** is the graph of its ordered pairs.

Graph the functions $f(x) = 2x$ and $g(x) = 2x - 3$ in the same rectangular coordinate system. Select integers for x, starting with -2 and ending with 2.

- Why do you think we choose -2 to 2 for our domain values?

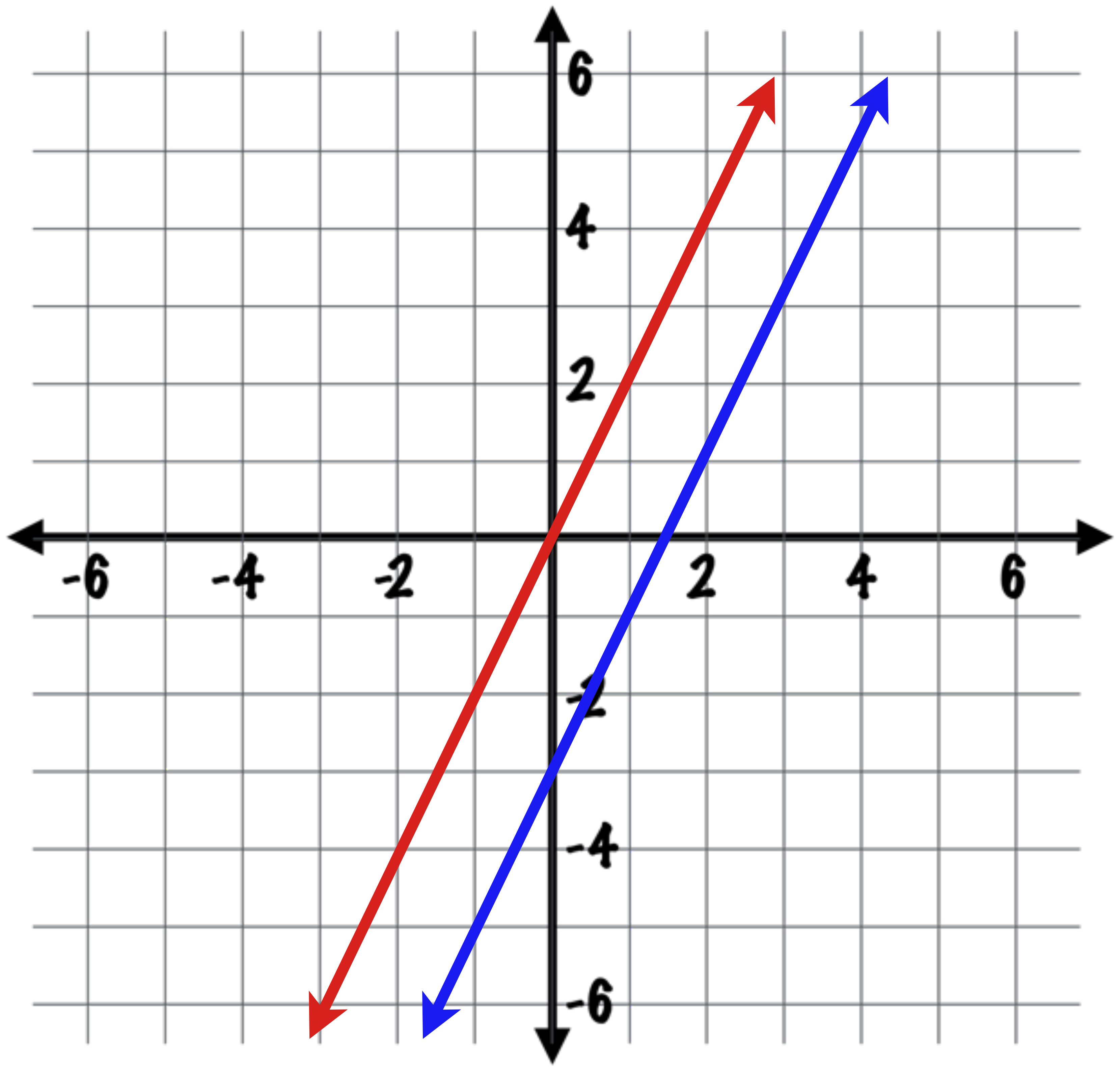
Graphing Functions

I can obtain information about a function from its graph.

- We set up a partial table of coordinates for each function, plot the points, and connect the points.

$f(x) = 2x$

x	y = f(x)
-2	-4
-1	-2
0	0
1	2
2	4



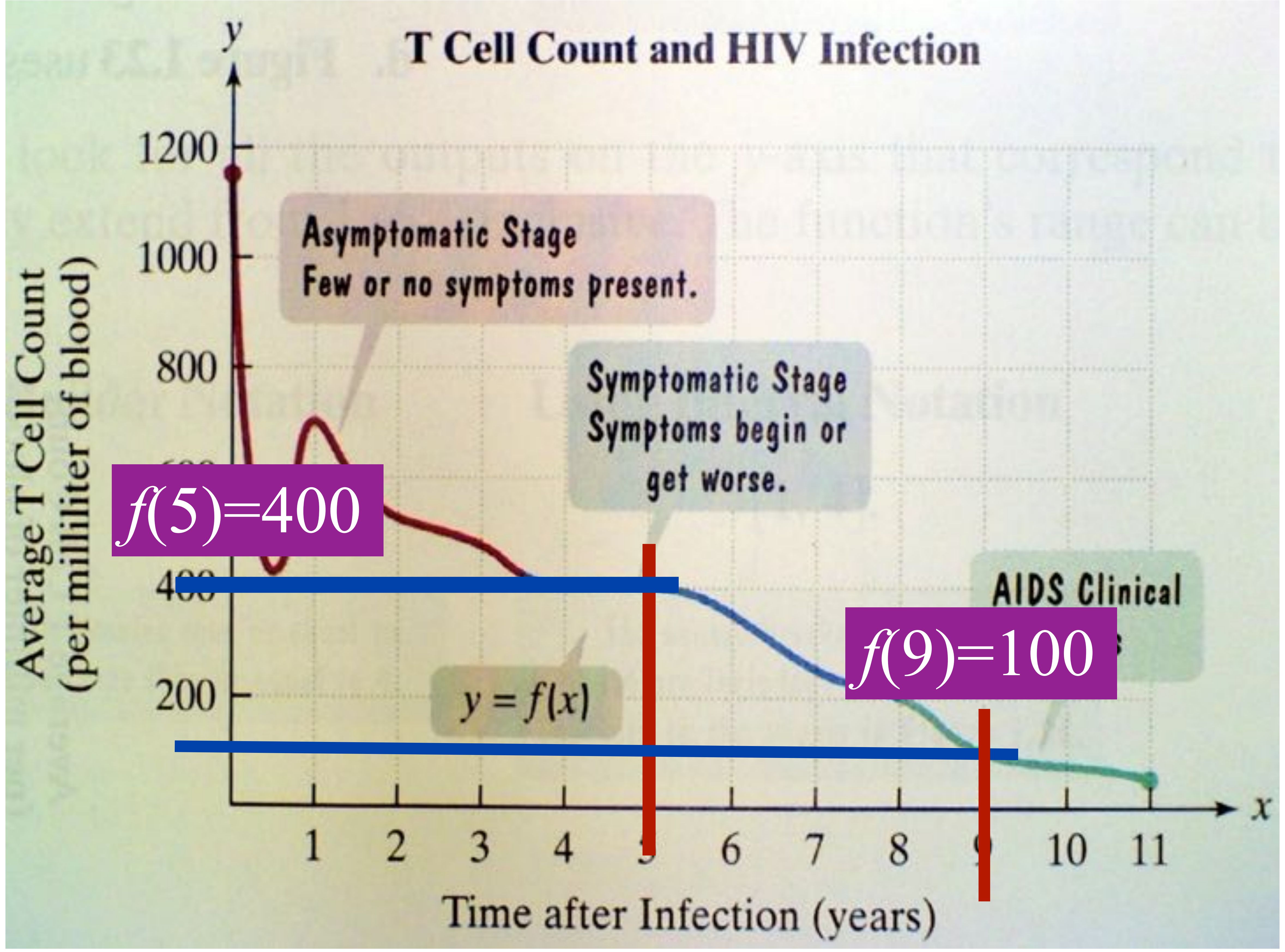
$g(x) = 2x - 3$

x	y = g(x)
-2	-7
-1	-5
0	-3
1	-1
2	1

Graphs of Functions

I can obtain information about a function from its graph.

- Use the graph to find f(5)
  - f(5) = 400
- For what value of x is f(x) = 100?
  - f(9) ≈ 100, so x ≈ 9
- What is the domain of the function?
  - $0 \le x \le 11$
- What is the range of the function?
  - $50 \le f(x) \le 1150$

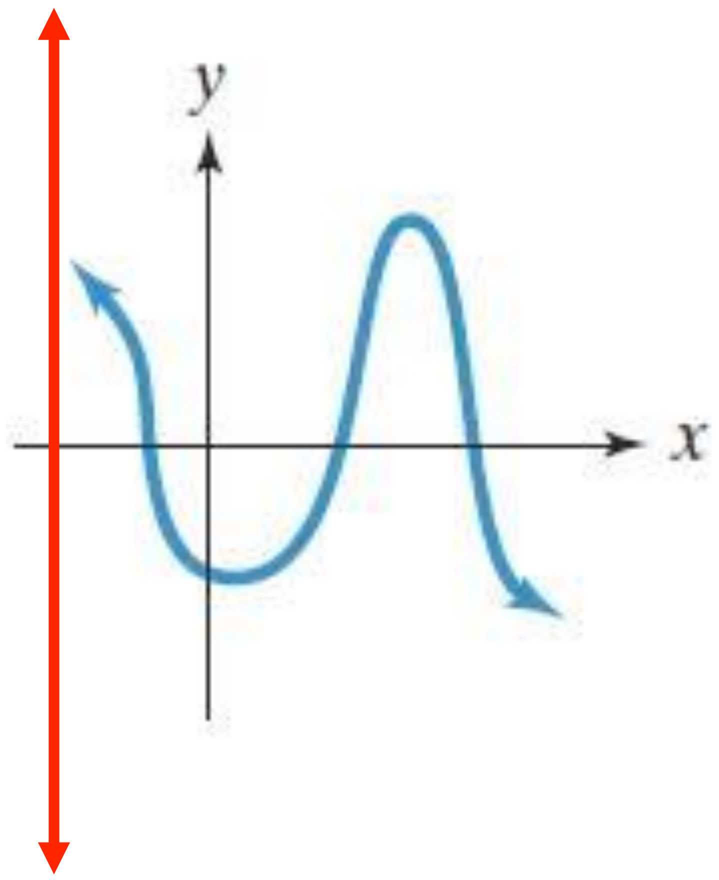


I can use the vertical line test to identify functions.

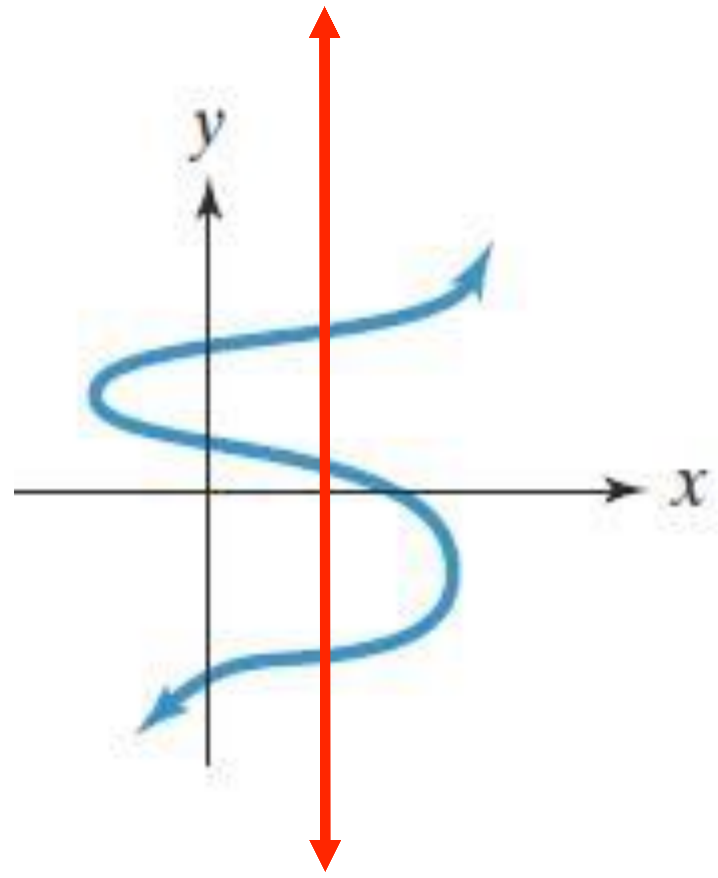
The Vertical Line Test for Functions

I can use the vertical line test to identify functions.

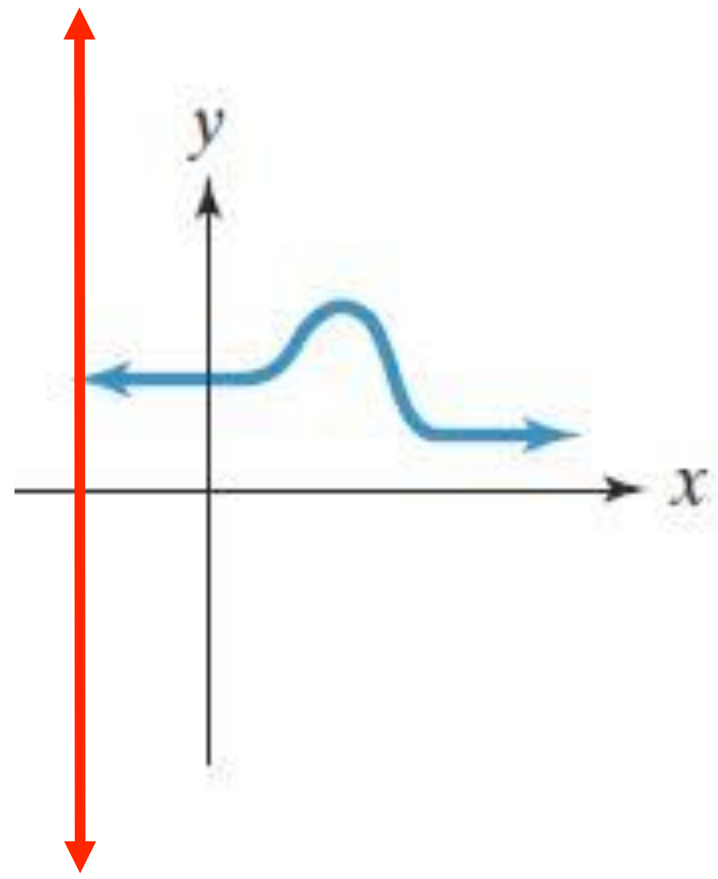
- If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x.
  - Use the vertical line test to identify graphs in which y is a function of x.



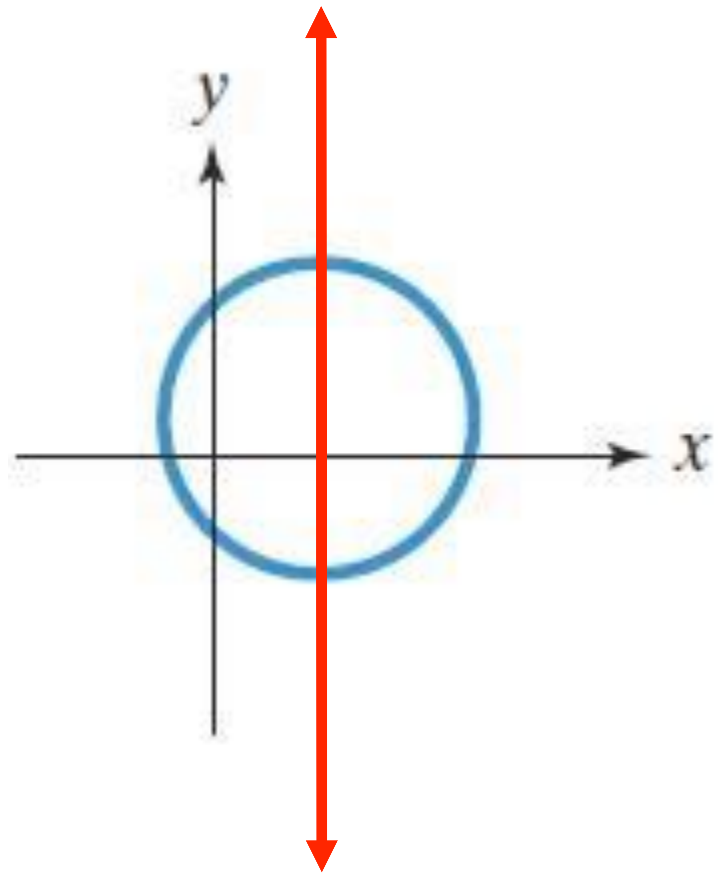
function



not a function



function



not a function


I can identify the domain and range of a function.

Domain

- The domain of a function is the set of all possible input values (all possible x values).
  - The domain can be explicit, meaning that it is decided apriori or defined for the function. Such as deciding ahead of time (apriori) that we will restrict the domain to positive integers.
  - The domain can be implicit, meaning that the function is not defined for some values. Taking the square root of negative numbers result in imaginary values, so if we are only interested in real numbers the domain of the square root function is implicitly defined as positive real numbers.

Domain and Range

- I can identify the domain and range of a function.

- What is the domain of the function? $f(x) = \sqrt{4 - x^2}$ [-2, 2]
- What is the range of the function? [0, 2]

- What is the domain of the function? $f(x) = \sqrt{x^2 - 4}$ $(-\infty, -2] \cup [2, \infty)$
- What is the range of the function? $[0, \infty)$

Domain

- I can identify the domain and range of a function.

- When determining the domain of a function ask yourself a couple of questions.
  - 1. What values make sense in the problem.
    - Do negative values make sense? Do fractional values make sense?
  - 2. What values are prohibited?
    - Even roots of negative values? Denominators of 0?

Domain

Find the appropriate domains.

$$f(x)=\frac{1}{x^2-2x-8} \qquad =\frac{1}{(x+2)(x-4)}$$

D: All reals except -2 and 4,
(-∞, -2)∪(-2, 4)∪(4, ∞)

$$f(x)=\sqrt{x^2-8} \qquad x^2 \geq 8$$

$$D: x \leq -\sqrt{8}, x \geq \sqrt{8}$$

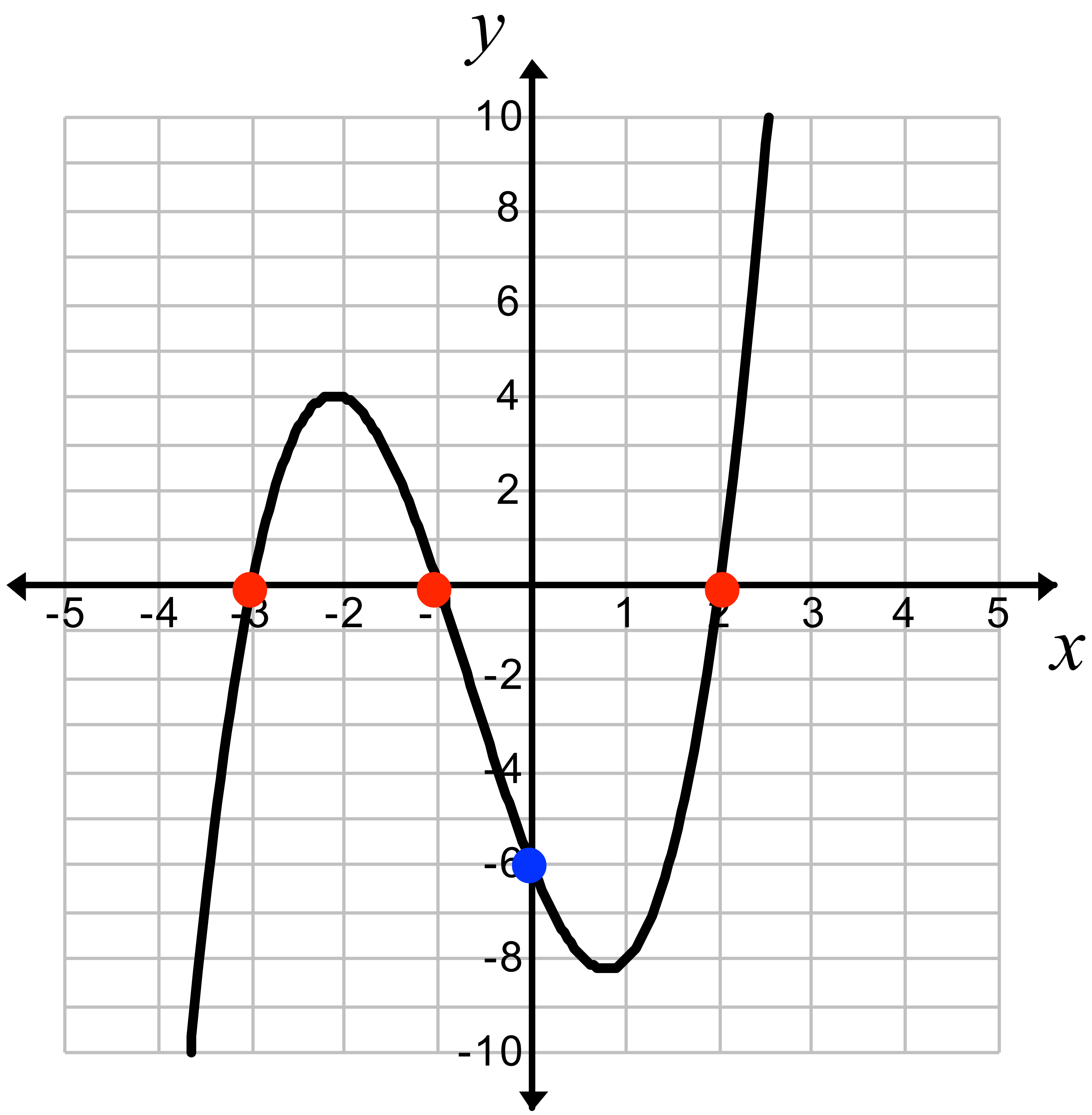
$$\left(-\infty, -\sqrt{8}\right] \cup \left[\sqrt{8}, \infty\right)$$


I can identify intercepts from a function's graph.

Identifying Intercepts from a Function's Graph

- I can identify intercepts from a function's graph.

- Identify the x- and y-intercepts for the graph of f(x).



- x-intercepts

$$(-3,0)\quad(-1,0)\quad(2,0)$$

- y-intercept

$$(0,-6)$$