Chapter 1

Functions and Graphs

1.3 More on Functions and Their Graphs

Chapter 1

Homework

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39, 42, 44, 50, 56, 66

Chapter 1.3

Objectives

- Identify intervals on which a function increases, decreases, or is constant.
- Use graphs to locate relative maxima or minima.
- Identify even or odd functions and recognize their symmetries.
- Understand and use piecewise functions.
- Find and simplify a function's difference quotient.

Increasing, Decreasing, and Constant Functions

1. A function is increasing on an open interval, I, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.

2. A function is decreasing on an open interval, I, if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.

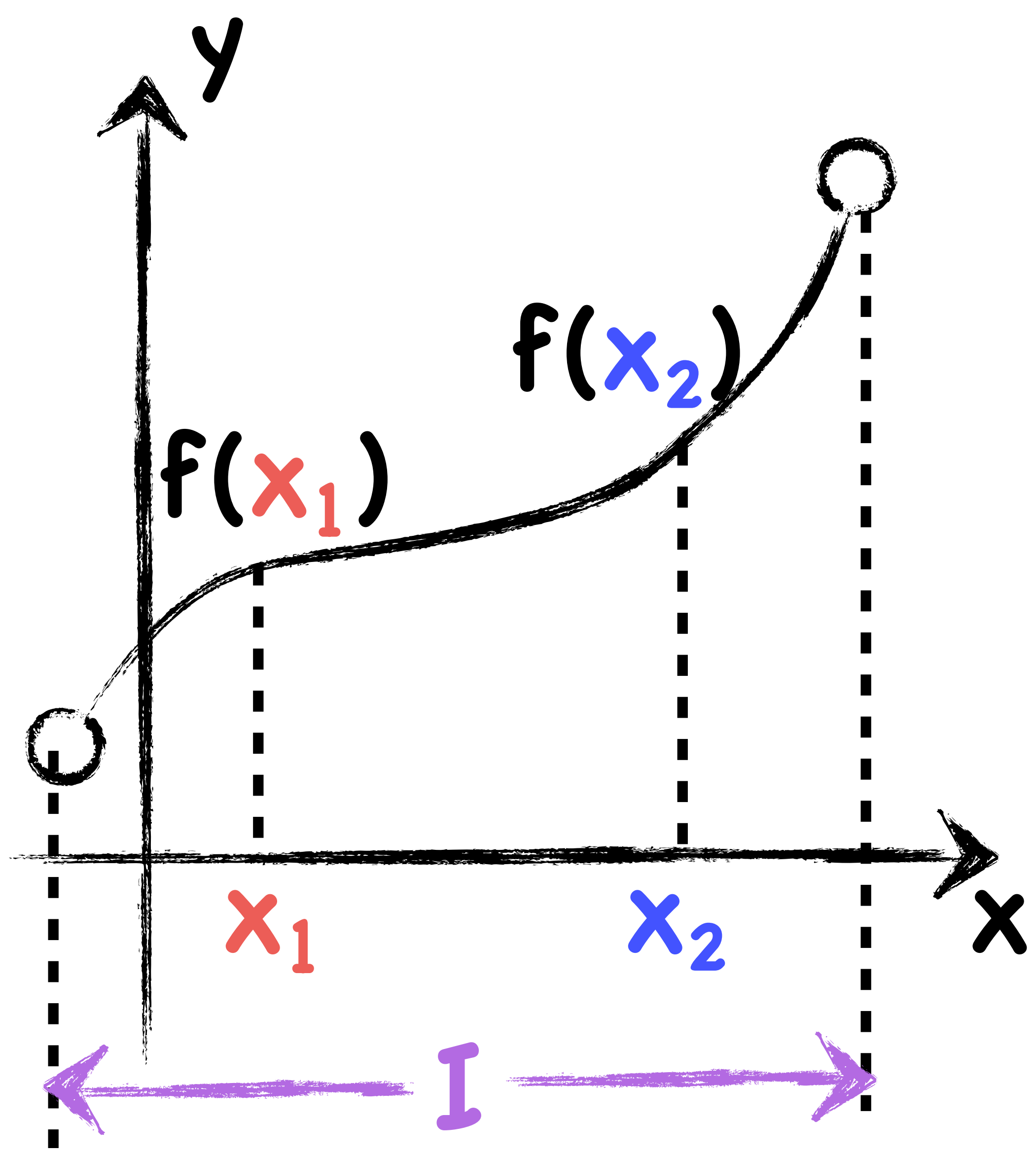
3. A function is constant on an open interval, I, if $f(x_1) = f(x_2)$ for any x_1 and x_2 in the interval.

Increasing, Decreasing, and Constant Functions

The open intervals, I, describing where functions increase, decrease, or are constant, use x-coordinates and not the y-coordinates.

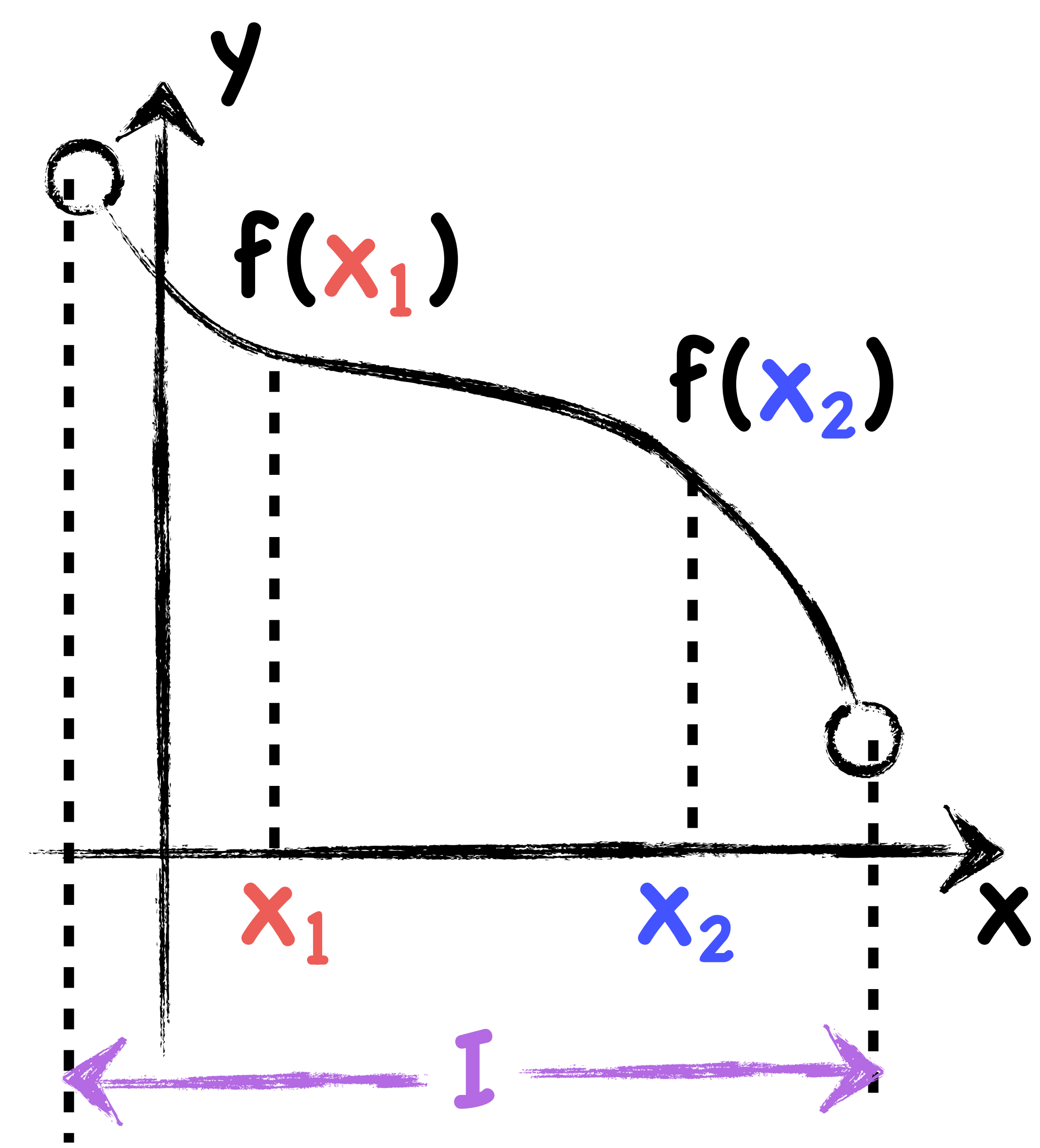
Increasing, Decreasing, and Constant Functions

Increasing



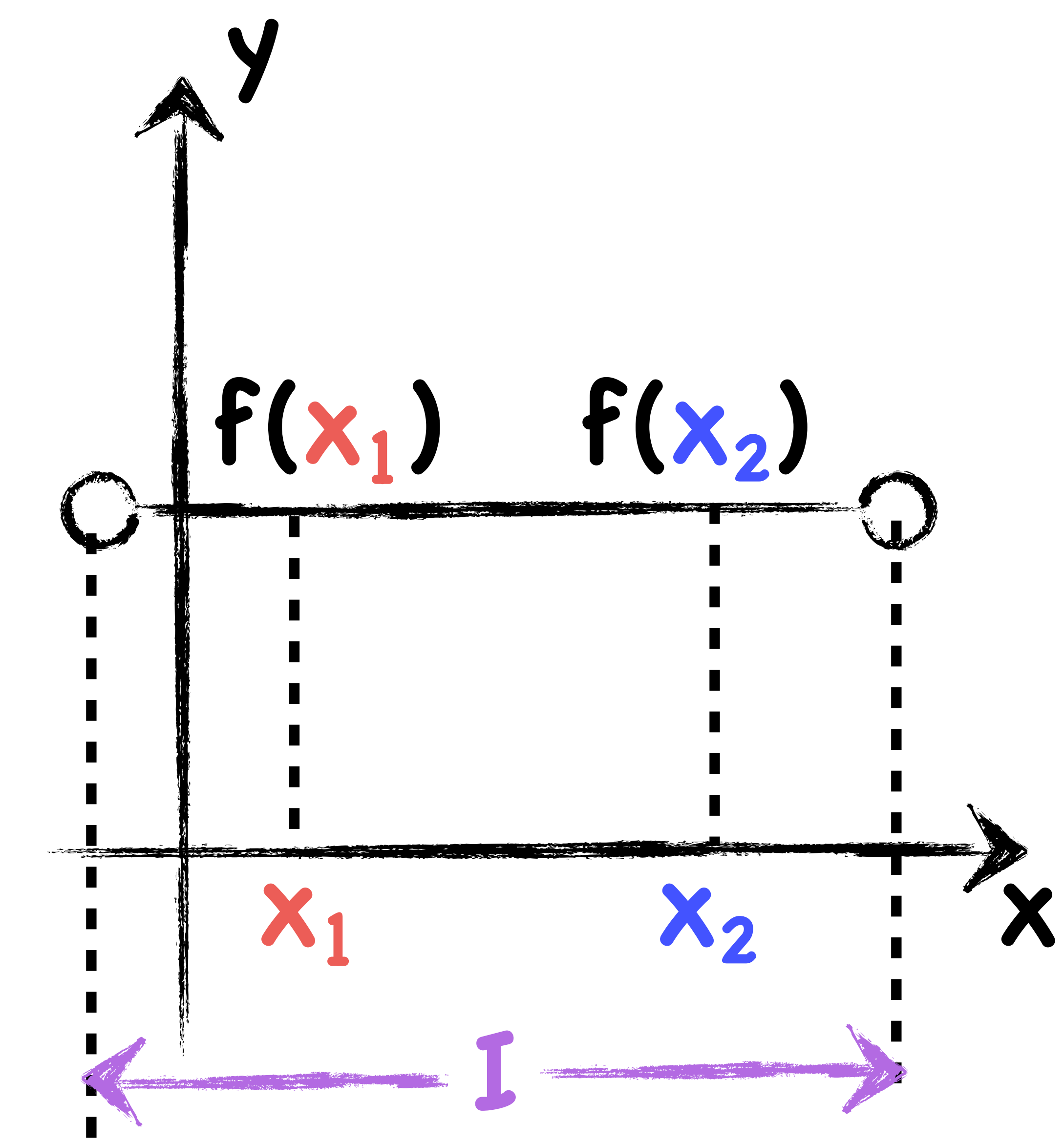
In I, $f(x_1) < f(x_2)$ whenever $x_1 < x_2$

Decreasing



In I, $f(x_1) > f(x_2)$ whenever $x_1 < x_2$

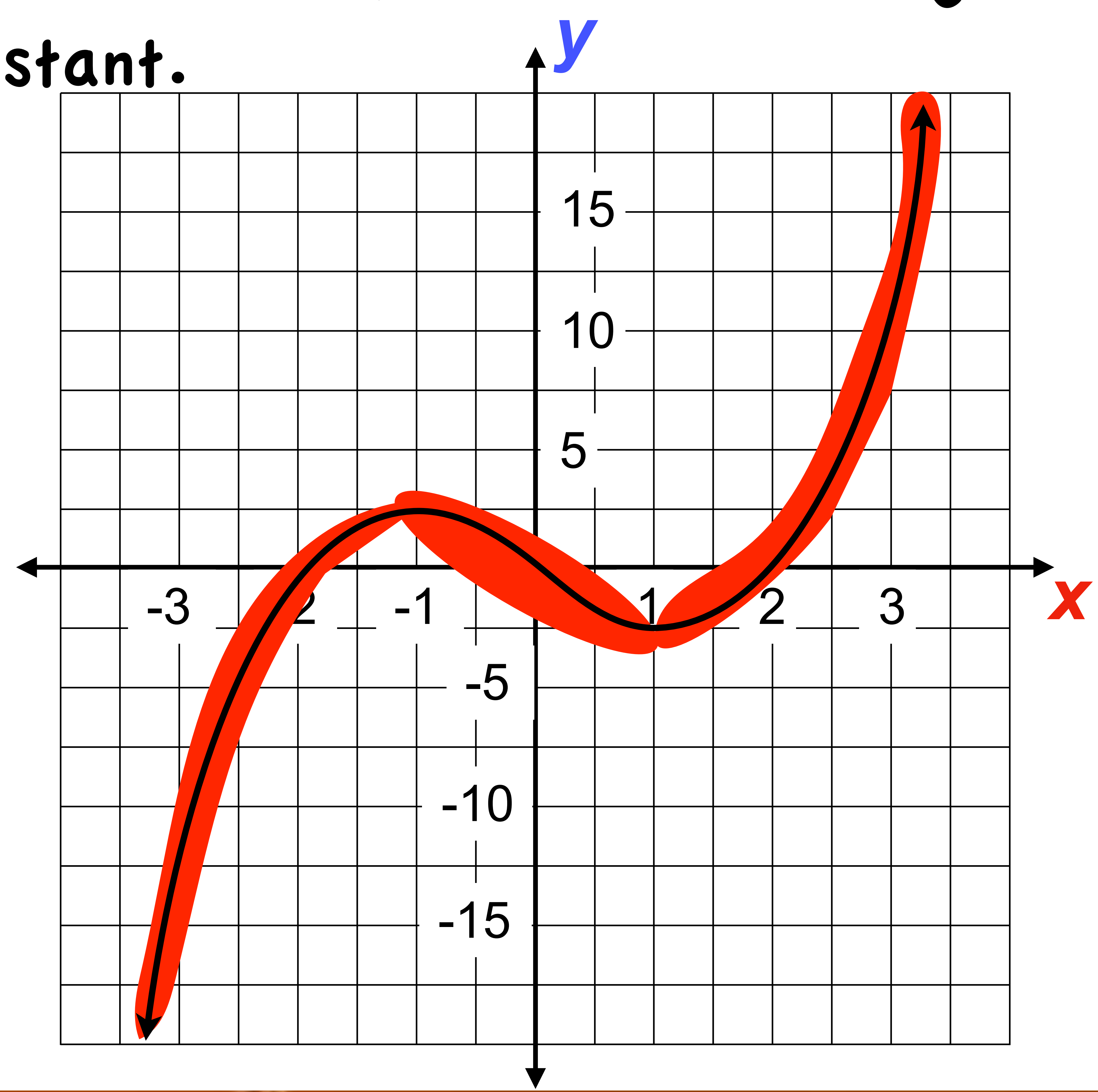
Constant



In I, $f(x_1) = f(x_2)$ whenever $x_1 < x_2$

Example: Intervals on Which a Function Increases, Decreases, or is Constant

State the intervals on which the given function is increasing, decreasing, or constant.



Increasing on $(-\infty, -1)$

Decreasing on $(-1, 1)$

Increasing on $(1, \infty)$

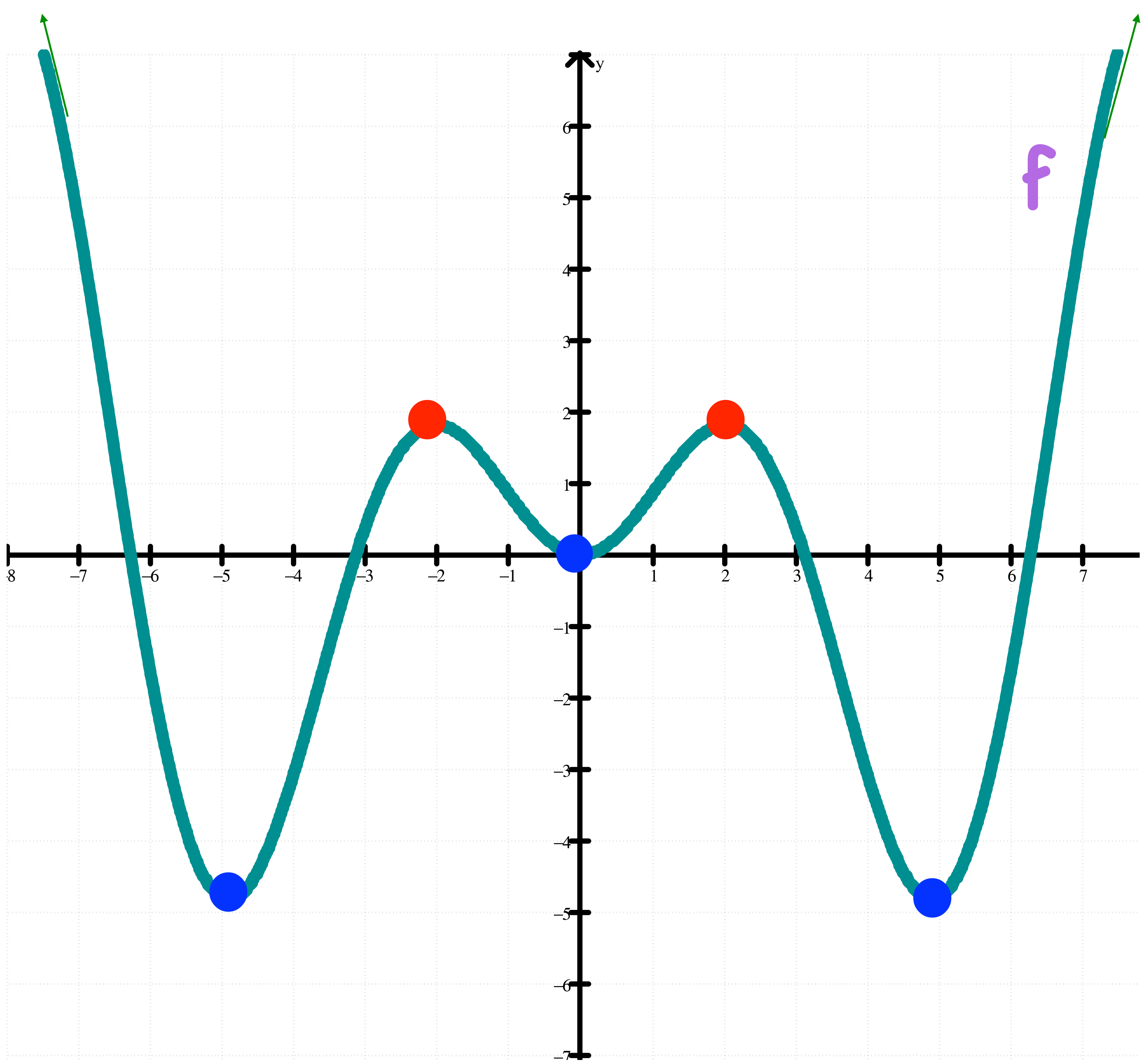
Definitions of Relative Extrema (Releative Maximum and Relative Minimum)

1. A function value $f(a)$ is a relative maximum of f if there exists an open interval containing a such that $f(a) > f(x)$ for all $x \neq a$ in the open interval.

2. A function value $f(b)$ is a relative minimum of f if there exists an open interval containing b such that $f(b) < f(x)$ for all $x \neq b$ in the open interval.

Use Graphs to Locate Relative Maxima or Minima

Identify the relative maxima and minima for the graph of f.



f has a relative maximum at $x = -2$ and $x = 2$.

f has a relative minimum at $x = -5$, $x = 0$, and $x = 5$

Relative extrema

Definitions of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \le f(x).$$

A function value $f(a)$ is called a **relative maximum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \ge f(x).$$

- Using your calculator, graph $f(x) = -3x^2 - 2x + 1$ to estimate the relative extrema.

Y= (-) 3 X,T,θ,n x^2 − 2 X,T,θ,n + 1 ZOOM 6

2nd TRACE CALC ⋎ 4:Maximum Set your left boundary by moving the cursor. ENTER

Set your right boundary by moving the cursor. ENTER Guess? ENTER

$$x = -.3333314 \qquad y = 1.3333333$$

- The relative maximum is 1 1/3 at the point (-1/3, 1 1/3)

Definitions of Even and Odd Functions

The function f is an even function if $f(-x) = f(x)$ for all x in the domain of f. The right side of the equation of an even function does not change if x is replaced with −x.

Even: $f(-x) = f(x)$

The function f is an odd function if $f(-x) = -f(x)$ for all x in the domain of f. Every term on the right side of the equation of an odd function changes its sign (becomes the opposite) if x is replaced with −x.

Odd: $f(-x) = -f(x)$

Example: Identifying Even or Odd Functions

Determine whether the function is even, odd, or neither.

$$h(x) = x^5 + 1$$

$$h(-x) = (-x)^5 + 1 = -x^5 + 1$$

$$h(-x) \neq h(x)$$

The function is not even.

$$-h(x) = -(x^5 + 1) = -x^5 - 1$$

$$h(-x) \neq -h(x)$$

The function is not odd.

The function is neither odd nor even.

- Determine whether the function is even, odd, or neither.

$$f(x) = x^4 - |x|$$

$$f(-x) = (-x)^4 - |-x| = x^4 - |x|$$

$$f(-x) = f(x)$$

The function is even.

$$g(x) = \frac{x}{x^2 + 1}$$

$$g(-x) = \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1} = -g(x)$$

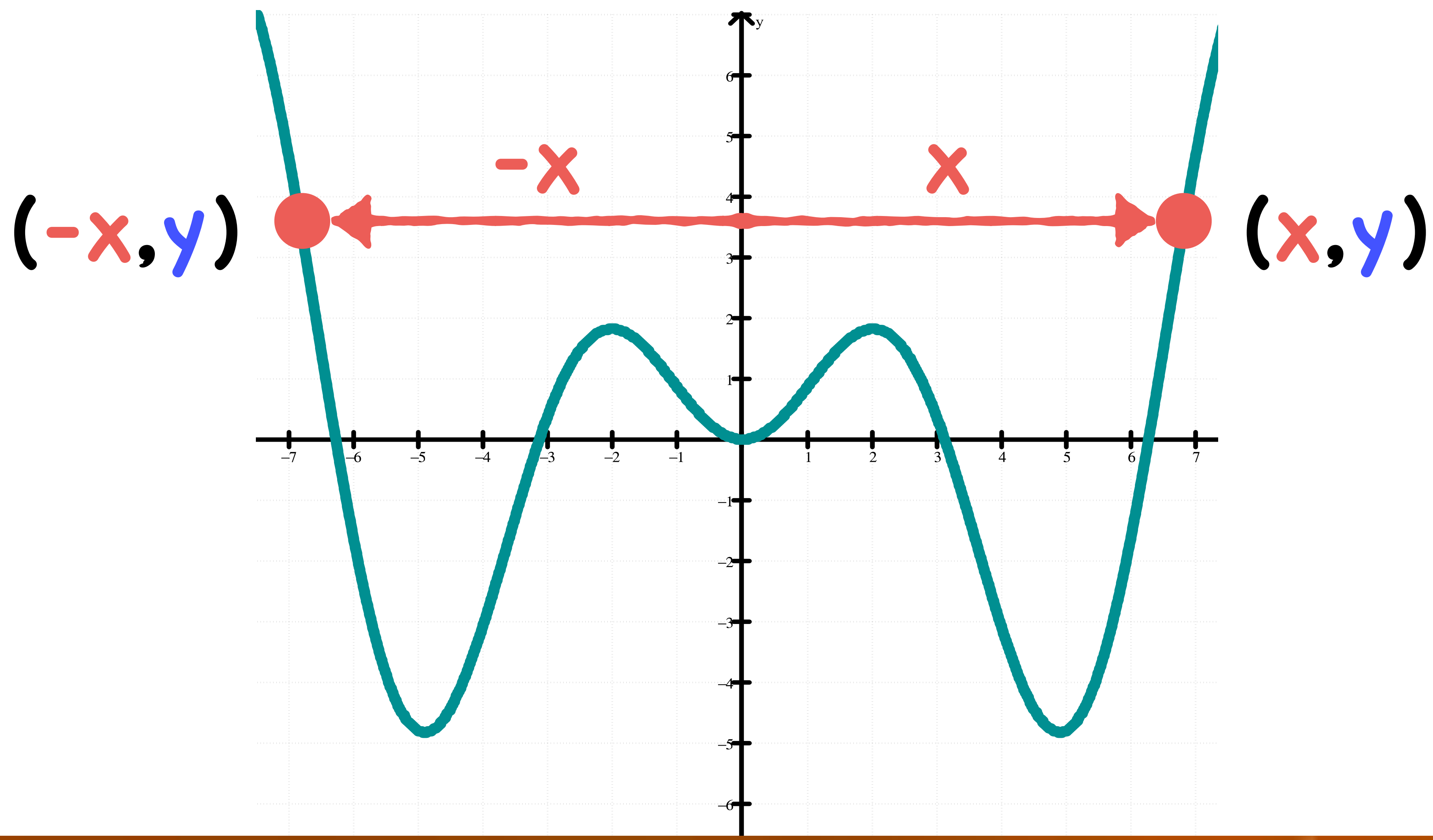
$$g(-x) = -g(x)$$

The function is odd.

Even Functions and *y*-Axis Symmetry

The graph of an even function in which $f(-x) = f(x)$ is symmetric with respect to the y-axis.

A graph is symmetric with respect to the y-axis if, for every point (x,y) on the graph, the point (-x,y) is also on the graph. All even functions have graphs with this kind of symmetry.

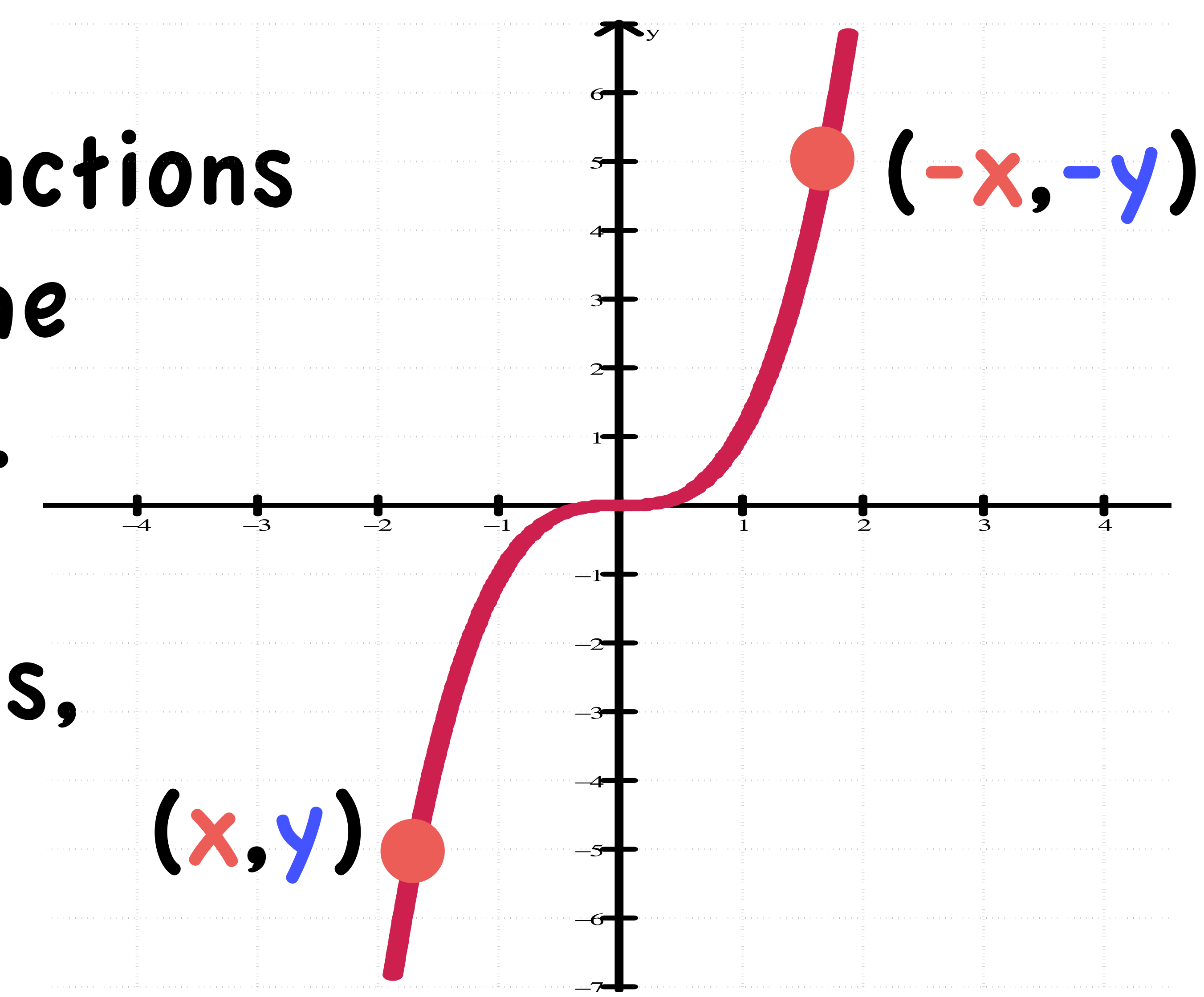


Odd Functions and Origin Symmetry

The graph of an odd function in which $f(-x) = -f(x)$ is symmetric with respect to the origin.

A graph is symmetric with respect to the origin if, for every point (x,y) on the graph, the point $(-x,-y)$ is also on the graph. All odd functions have graphs with origin symmetry.

Note that the 1st and 3rd quadrants of odd functions are reflections of each other with respect to the origin. The same is true for 2nd and 4th quads.



Also note that f(x) and f(-x) have opposite signs, so that $f(-x) = -f(x)$.

Piecewise Functions

A function that is defined by two (or more) equations over a specified domain is called a piecewise function.

Example: Evaluating a Piecewise Function

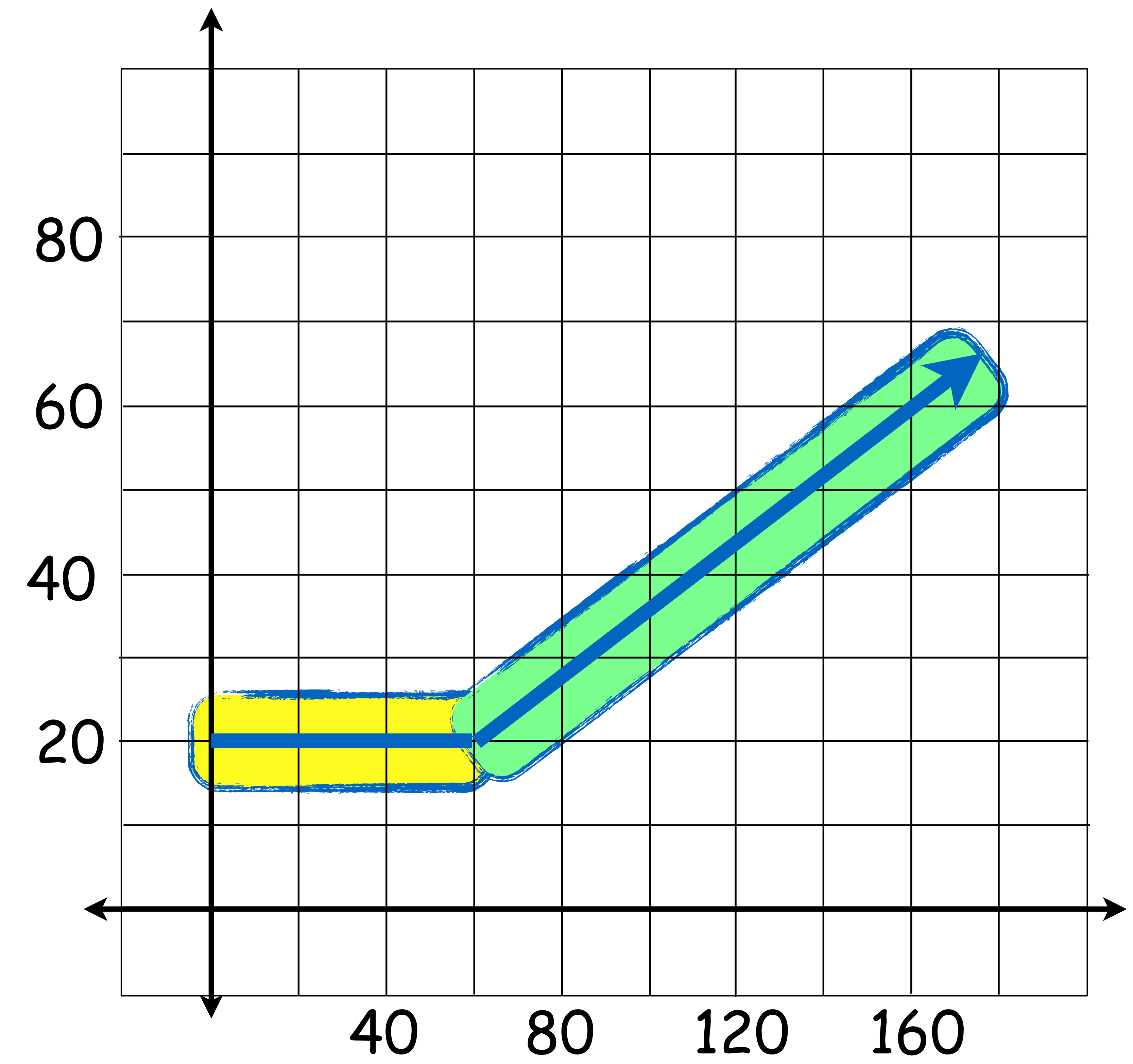
Given the function $C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$

Find C(40)

$0 \le 40 \le 60 \text{ so } C(40) = 20$

Find C(80)

$$80 > 60 \text{ so } C(80) = 20 + 0.40(80 - 60)$$
$$= 20 + 0.4(20)$$
$$= 20 + 8 = 28$$



Example: Graphing a Piecewise Function

Graph the piecewise function defined by

$$C(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x - 2 & \text{if } x > -1 \end{cases}$$

We will graph C in two parts, using a partial table of coordinates for each piece of the graph.

x	C(x)=3	(x,C(x))
-1	3	(-1,3)
-2	3	(-2,3)
-3	3	(-3,3)

x	C(x)=x-2	(x,C(x))
-1	-3	(-1,-3)
0	-2	(0,-2)
1	-1	(1,-1)

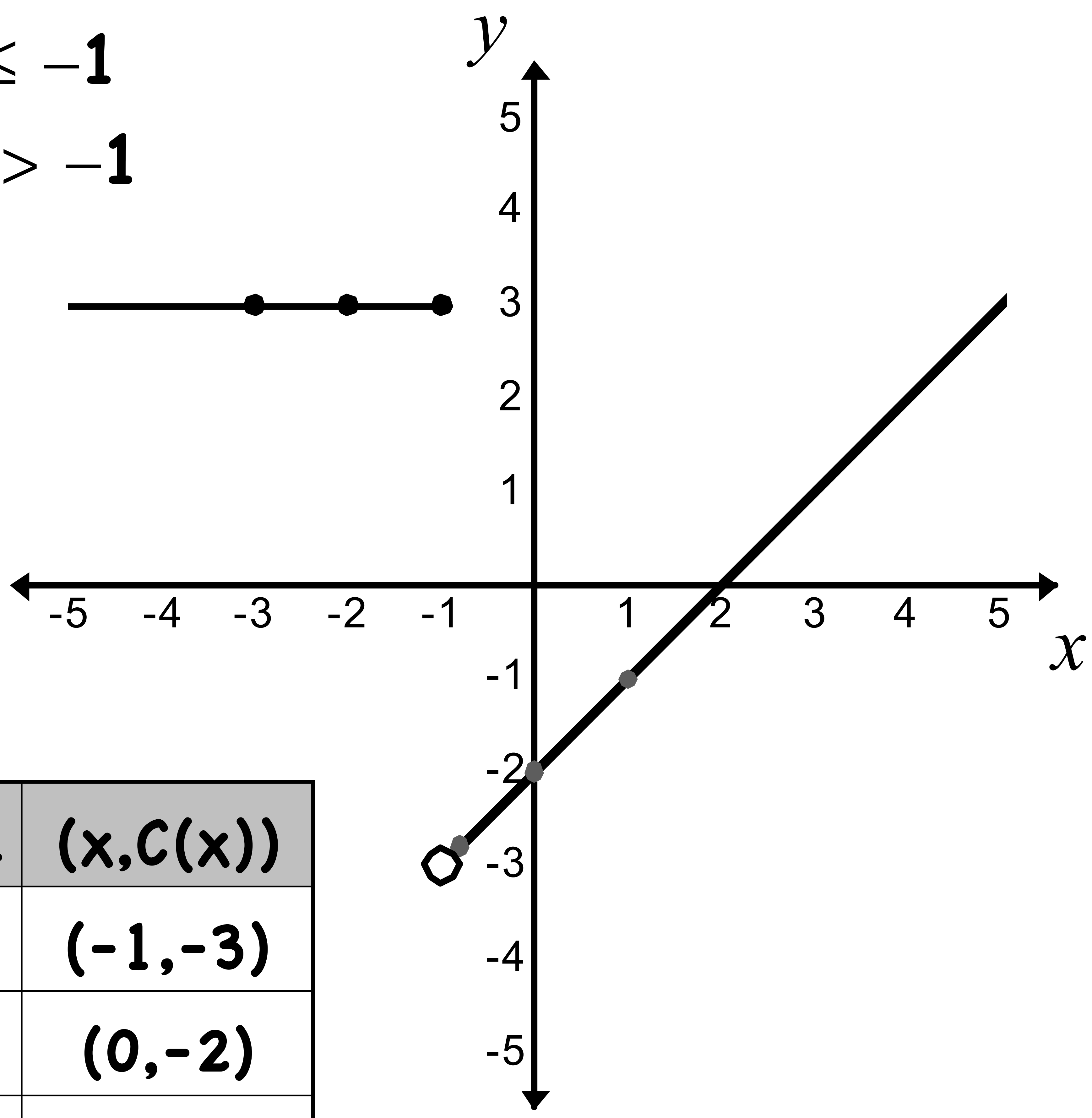
Example: Graphing a Piecewise Function

The graph of

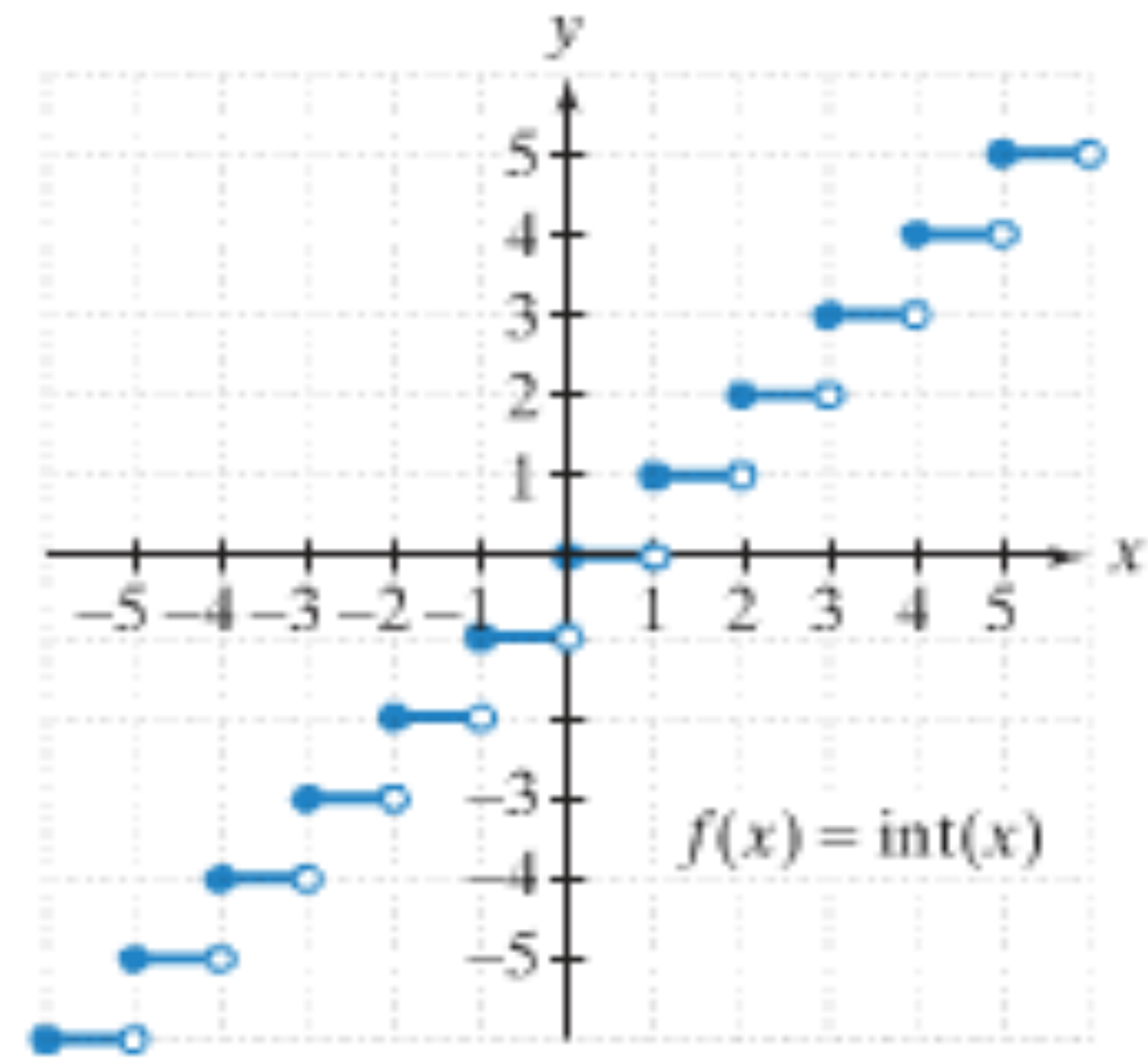
$$C(t) = \begin{cases} 3 & \text{if } x \le -1 \\ x - 2 & \text{if } x > -1 \end{cases}$$

x	C(x)=3	(x,C(x))
-1	3	(-1,3)
-2	3	(-2,3)
-3	3	(-3,3)

x	C(x)=x-2	(x,C(x))
-1	-3	(-1,-3)
0	-2	(0,-2)
1	-1	(1,-1)



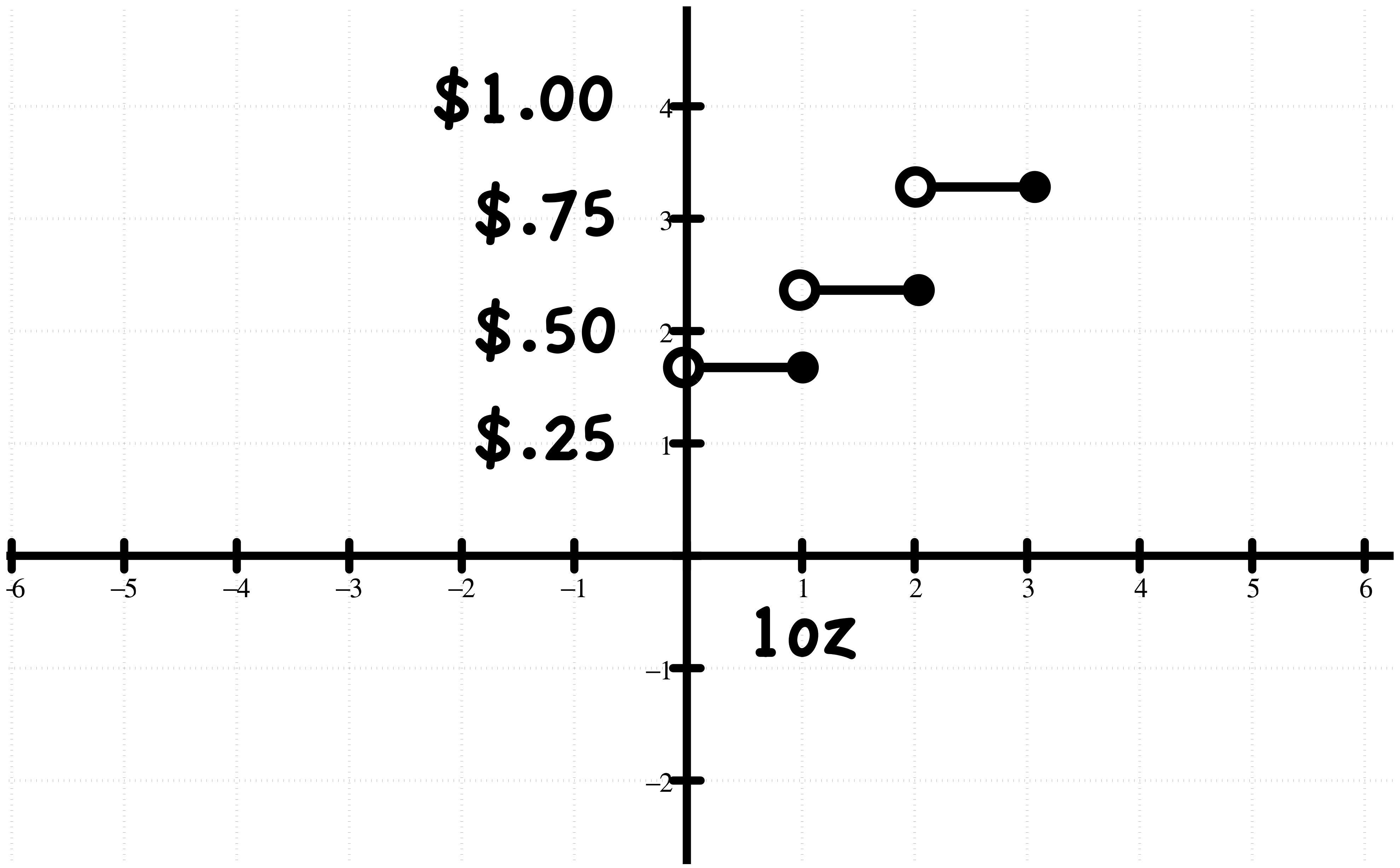
- Some piecewise functions are called step functions because their graphs form discontinuous steps. One such function is called the greatest integer function, symbolized by int(x) or [x], where int(x) equals the greatest integer that is less than or equal to x.
  - For example:
    - int(1) = 1, int(1.3) = 1, int(1.9) = 1
    - int(2) = 2, int(2.15) = 2, int(2.78) = 2



Example: Step Functions

- The USPS charges $0.42 for letters 1 oz or less. For letters 2 oz or less they charge $0.59, and 3 oz or less they charge $0.76.
- Graph this function and then find the following charges.

  - a. The charge for a letter weighing 1.5 oz
    - $0.59
  - b. The charge for a letter weighing 2.3 oz
    - $0.76



Difference Quotient

The expression $\frac{f(x+h)-f(x)}{h}$ for h ≠ 0, is called the difference quotient of the function f.

If $f(x) = -2x^2 + x + 5$ find and simplify the expression. $\frac{f(x+h)-f(x)}{h}$

$$f(x+h) = -2(x+h)^2 + (x+h) + 5$$

$$= -2(x^2 + 2hx + h^2) + (x+h) + 5$$

$$f(x) = -2x^2 + x + 5$$

$$f(x+h) = -2x^2 - 4hx - 2h^2 + x + h + 5$$

Example: Evaluating and Simplifying a Difference Quotient

If $f(x) = -2x^2 + x + 5$ find and simplify the expression. $\dfrac{f(x+h)-f(x)}{h}$

$$f(x) = -2x^2 + x + 5$$

$$f(x+h) = -2x^2 - 4hx - 2h^2 + x + h + 5$$

$$\frac{f(x+h)-f(x)}{h} = \frac{\left(\cancel{-2x^2} - 4hx - 2h^2 + \cancel{x} + h + \cancel{5}\right) - \left(\cancel{-2x^2} + \cancel{x} + \cancel{5}\right)}{h}$$

$$= \frac{-4hx - 2h^2 + h}{h} = \frac{h\left(-4x - 2h + 1\right)}{h}$$

$$= -4x - 2h + 1, h \neq 0$$