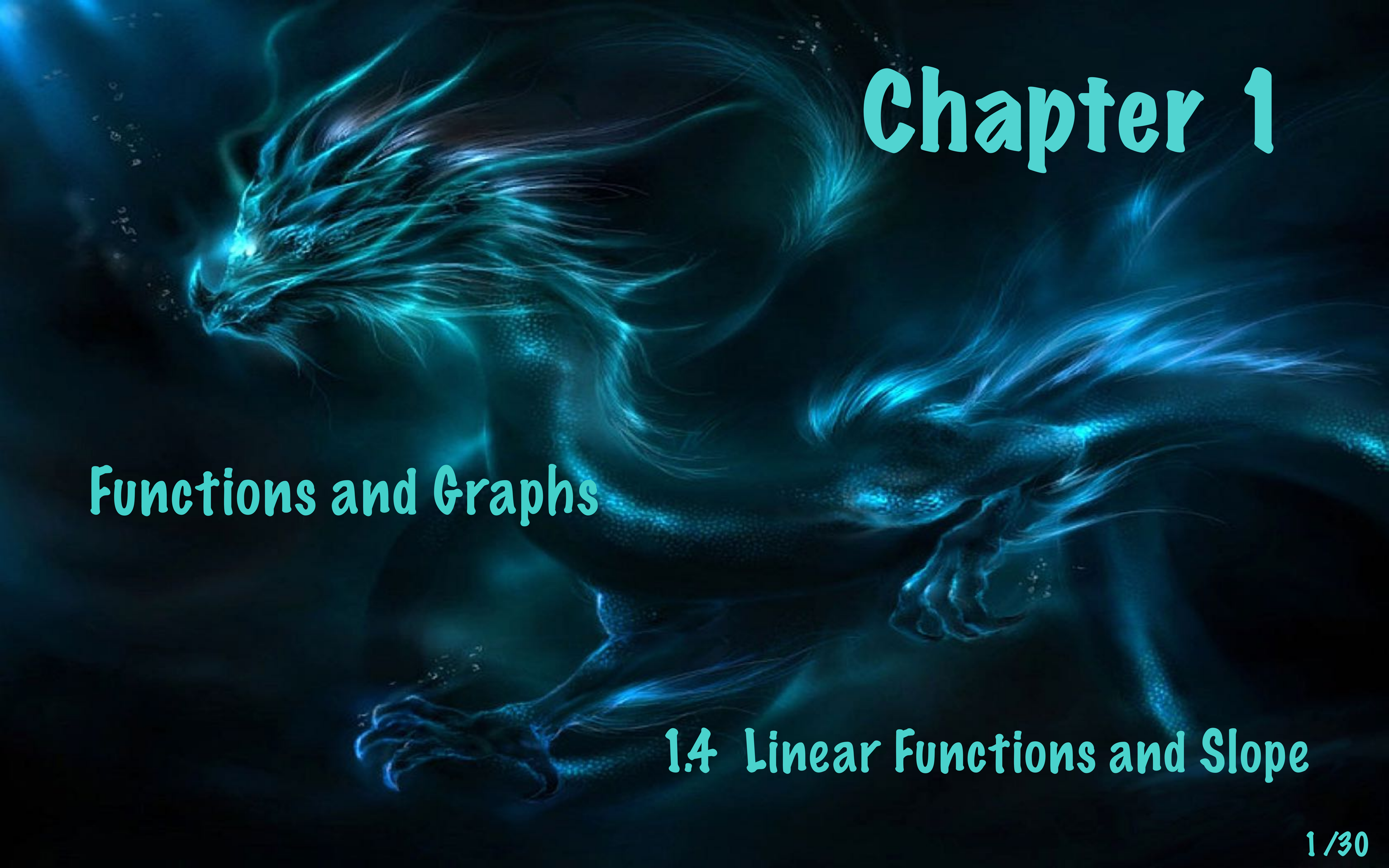
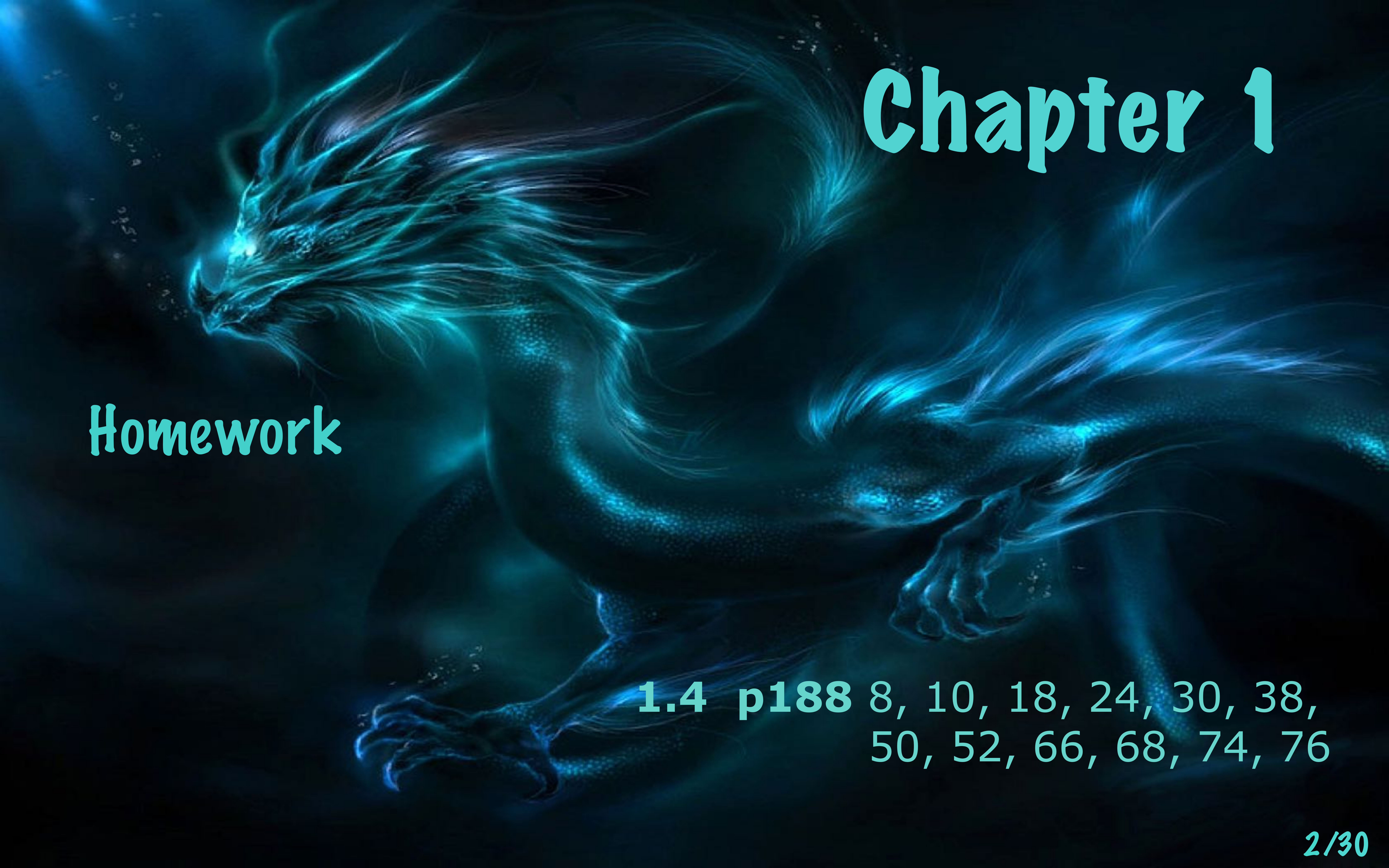


Chapter 1

Functions and Graphs

1.4 Linear Functions and Slope



Chapter 1

Homework

1.4 p188 8, 10, 18, 24, 30, 38, 50, 52, 66, 68, 74, 76

Chapter 1

Objectives

- Calculate a line's slope.
- Write the point-slope form of the equation of a line.
- Write and graph the slope-intercept form of the equation of a line.
- Graph horizontal or vertical lines.
- Recognize and use the general form of a line's equation.
- Use intercepts to graph the general form of a line's equation.
- Model data with linear functions and make predictions.

Definition of Slope

Slope

The constant rate of change for a linear function is its slope. The slope of a linear function is the ratio $\frac{\text{change in } f(x)}{\text{change in } x}$, or $\frac{\Delta f(x)}{\Delta x} = \frac{\Delta y}{\Delta x}$, or, for the non-educated, $\frac{\text{rise}}{\text{run}}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

The slope of a line is the same between any two points on the line. You can graph lines by using two points, or the slope and a point.

Slope

- For a linear function, slope may be interpreted as the rate or ratio of change in the dependent variable (y) per **unit change** in the independent variable (x).

If x and y are measured in the same units, slope is a ratio with no units.

The ratio of Baptists to total population is about 1/10

If x and y are measured in the different units, slope is a rate of change.

The rate of growth of Baptists churches is about .006 per year

Example: Using the Definition of Slope

- Find the slope of the line passing through the points (4, -2) and (-1, 5)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - -2}{-1 - 4} = \frac{7}{-5} = -\frac{7}{5}$$

- The slope of the line passing through the points (4, -2) and (-1, 5) is $-\frac{7}{5}$

- This tells us that for every 5 units x increases, f(x), or y, decreases 7 units.

Example: Slope

- In 1990, 9 million adult men in the United States lived alone. In 2008, 14.7 million adult men in the United States lived alone. Use this information to find the slope of the linear function representing adult men living alone in the United States. Express the slope correct to two decimal places and describe what it represents.

We form the ordered pairs (year, number living alone). (1990, 9) and (2008, 14.7). Using these points, find the slope.

$$m = \frac{\Delta y}{\Delta x} = \frac{14.7 - 9}{2008 - 1990} = \frac{5.7}{18} = .31\overline{6}$$

The number of men living alone increased at a rate of about 0.32 million per year. (316667/year)

Possibilities for the Slope of a Line

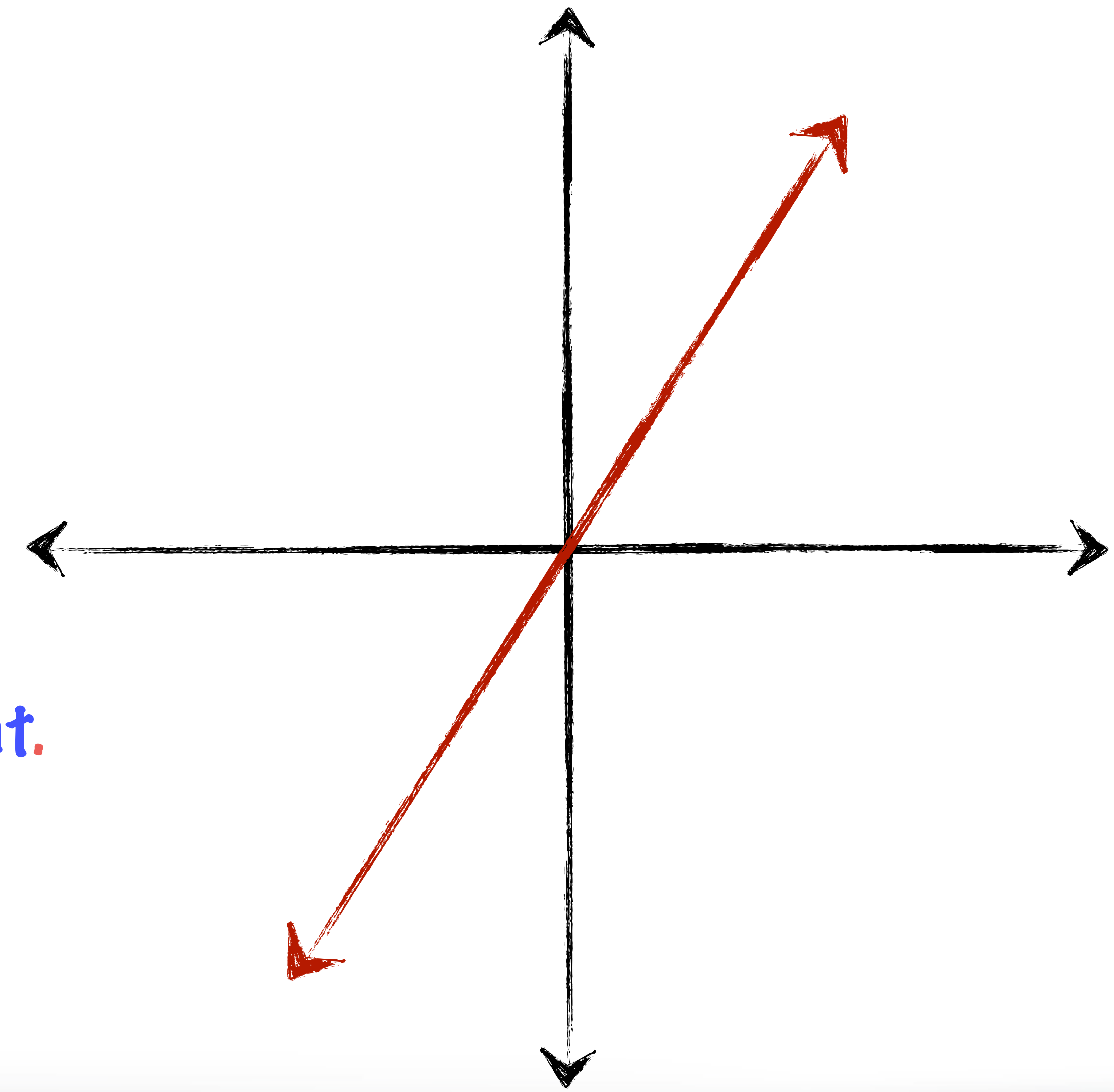
- The slope of a linear relation can be positive, negative, zero, or undefined.
  - Positive Slope m > 0

    As x increases, f(x) increases.
  - Negative Slope m < 0

    As x increases, f(x) decreases.
  - Zero Slope m = 0

    As x increases, f(x) remains constant.
  - Undefined Slope

    x remains constant



Point-Slope Form of the Equation of a Line

point-slope form

- The point-slope form of the equation of a nonvertical line with slope m that passes through the point (x_1, y_1) is

$$y - y_1 = m(x - x_1)$$

Point-Slope Form

If you are given two points and you need to write an equation in point-slope form, then you can use either point for (x_1, y_1).

Writing an Equation for a Line in Point-Slope Form

- Write an equation in point-slope form for the line with slope 6 that passes through the point (2, -5). Then solve the equation for y.

$$y - y_1 = m(x - x_1)$$

$$y - (-5) = 6(x - 2)$$

$$y + 5 = 6(x - 2)$$

The equation in point-slope form.

Solving for y...

$$y + 5 = 6x - 12$$

$$y = 6x - 17$$

The equation in slope-intercept form.

Slope-Intercept Form of the Equation of a Line

Slope-Intercept Form

- Linear functions can also be expressed as linear equations of the form $y = mx + b$. When a linear function is written in the form $y = mx + b$, the function is said to be in slope-intercept form because m is the slope of the graph and b is the y-intercept.

$$y = mx + b$$

Notice that slope-intercept form is the equation solved for y in terms of x.

Graphing y = mx + b Using the Slope (m) and the y-Intercept (b)

Graphing y = mx + b

1. Plot the point containing the y-intercept on the y-axis. This is the point (0, b).

2. Obtain a second point using the slope, m. Write m as a fraction, and use $\Delta y/\Delta x$, starting at the point containing the y-intercept, to plot this point.

3. Use a straightedge to draw a line through the two points. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

Example: Graphing Using the Slope and y-Intercept

- Graph the linear function: $f(x) = \frac{3}{5}x + 1$

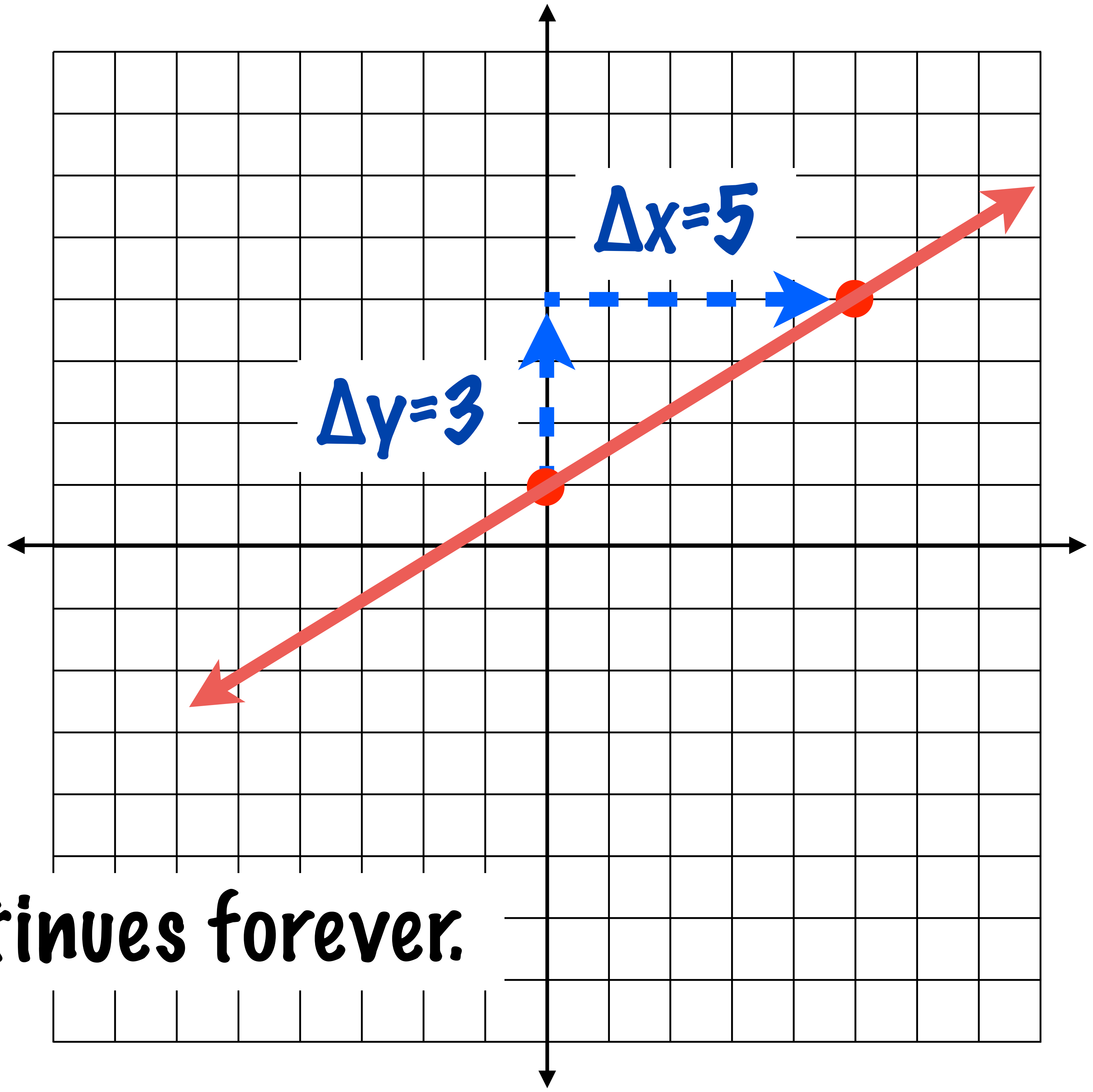
Step 1 Plot the point containing the y-intercept on the y-axis.

The y-intercept is 1. We plot the point (0, 1).

Step 2 Using the slope, plot another point.

$$m = \frac{3}{5} = \frac{\Delta y}{\Delta x}$$

Step 3 Draw the line, indicating that the line continues forever.



Equation of a Horizontal Line

- An equation with only one variable can be represented by either a vertical line or a horizontal line.

$$y = b$$

A horizontal line is given by an equation of the form $y = b$, where b is the y-intercept of the line.
The slope of a horizontal line is zero.

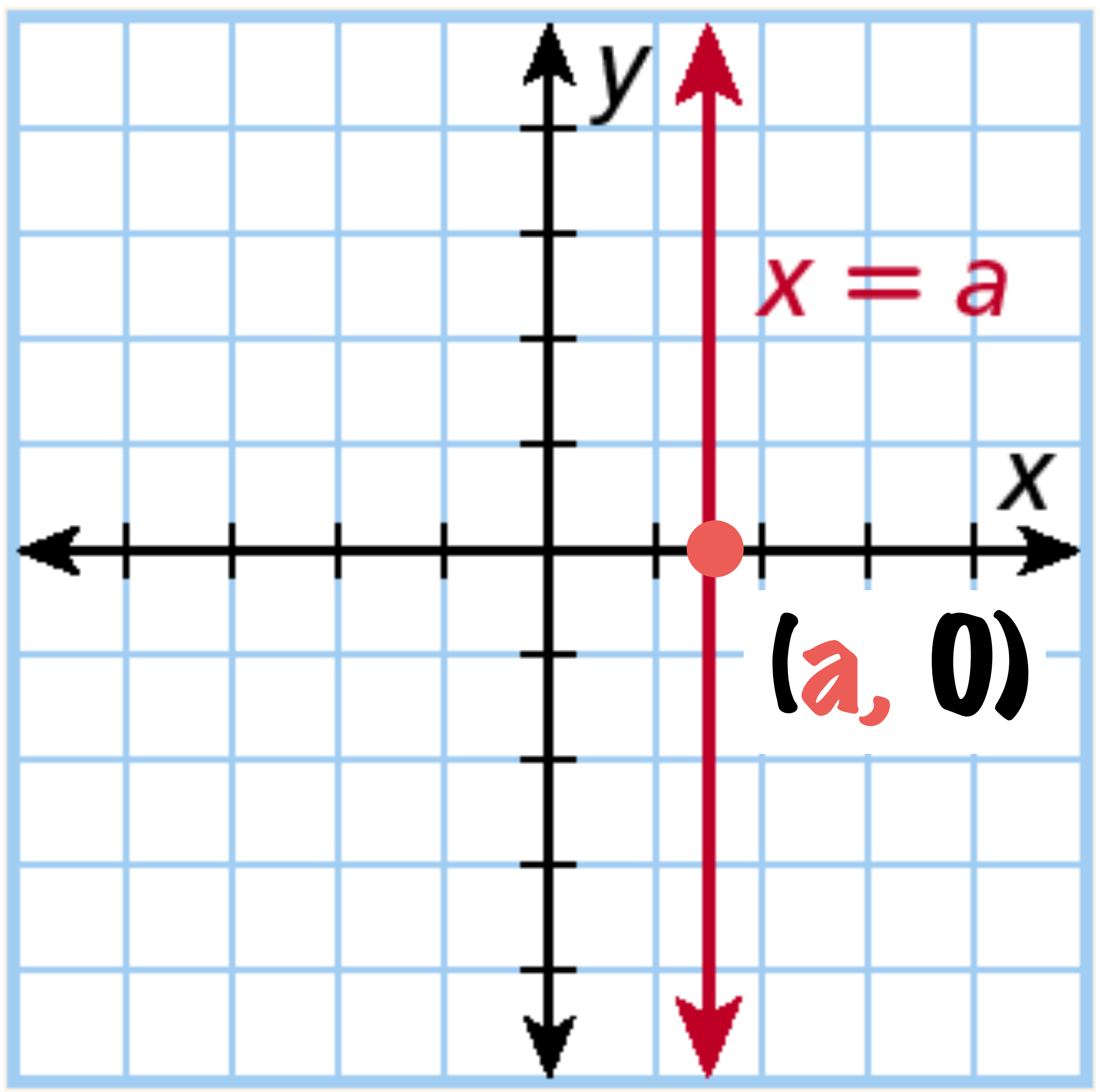
$$x = a$$

A vertical line is given by an equation of the form $x = a$, where a is the x-intercept of the line.
The slope of a vertical line is undefined.

Vertical and Horizontal Lines

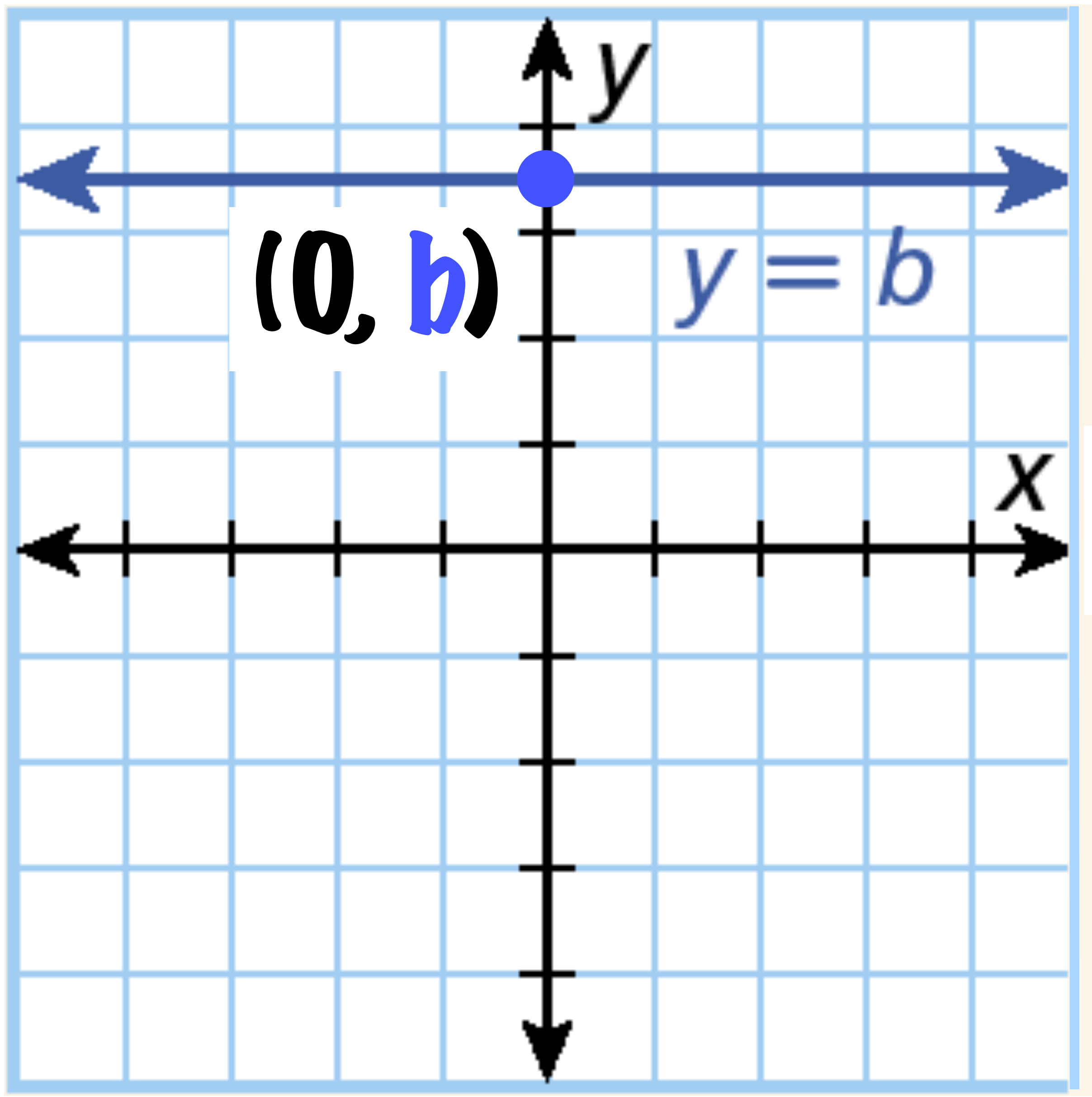
Vertical Lines

The line x = a is a vertical line at a.



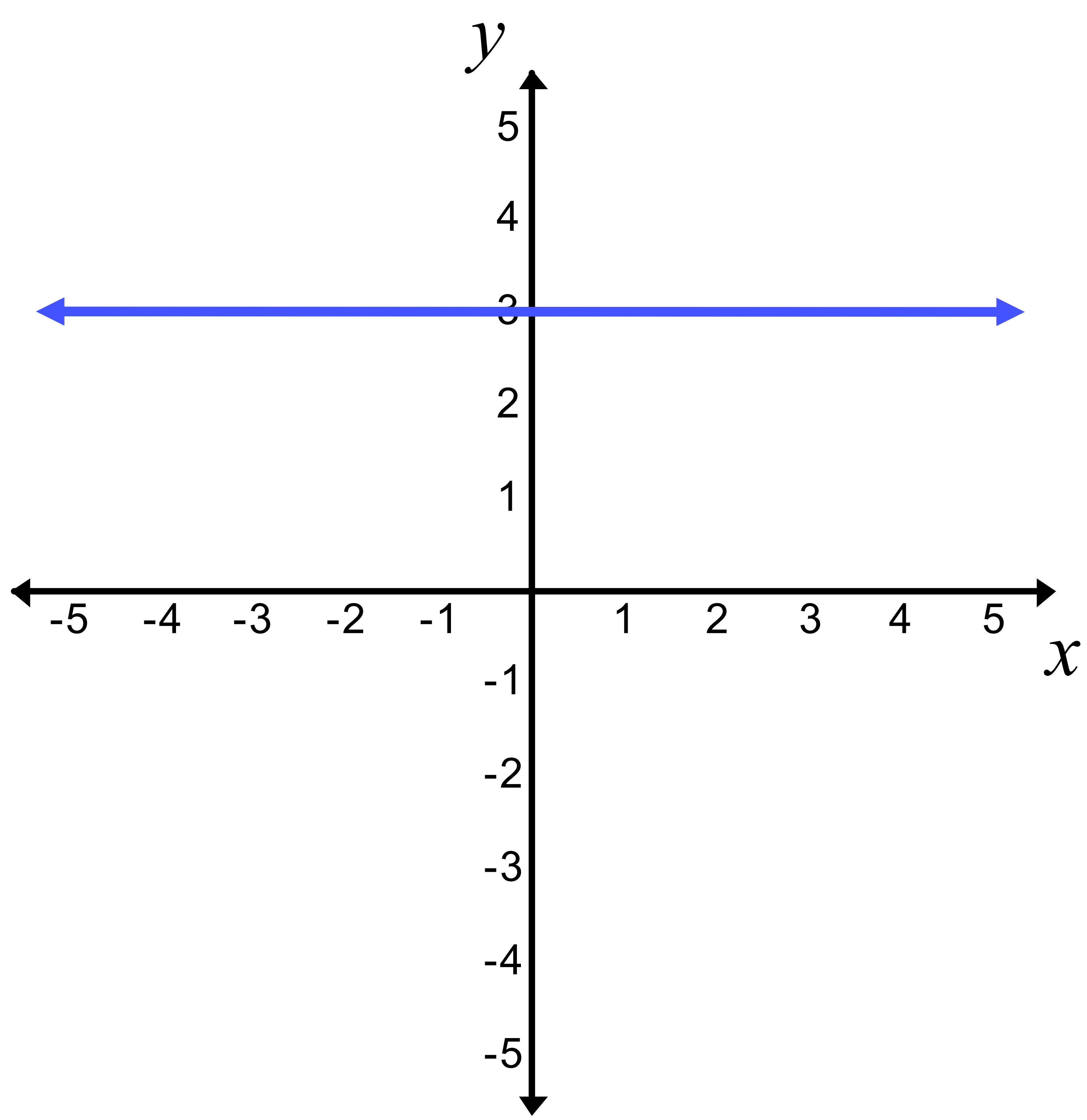
Horizontal Lines

The line y = b is a horizontal line at b.

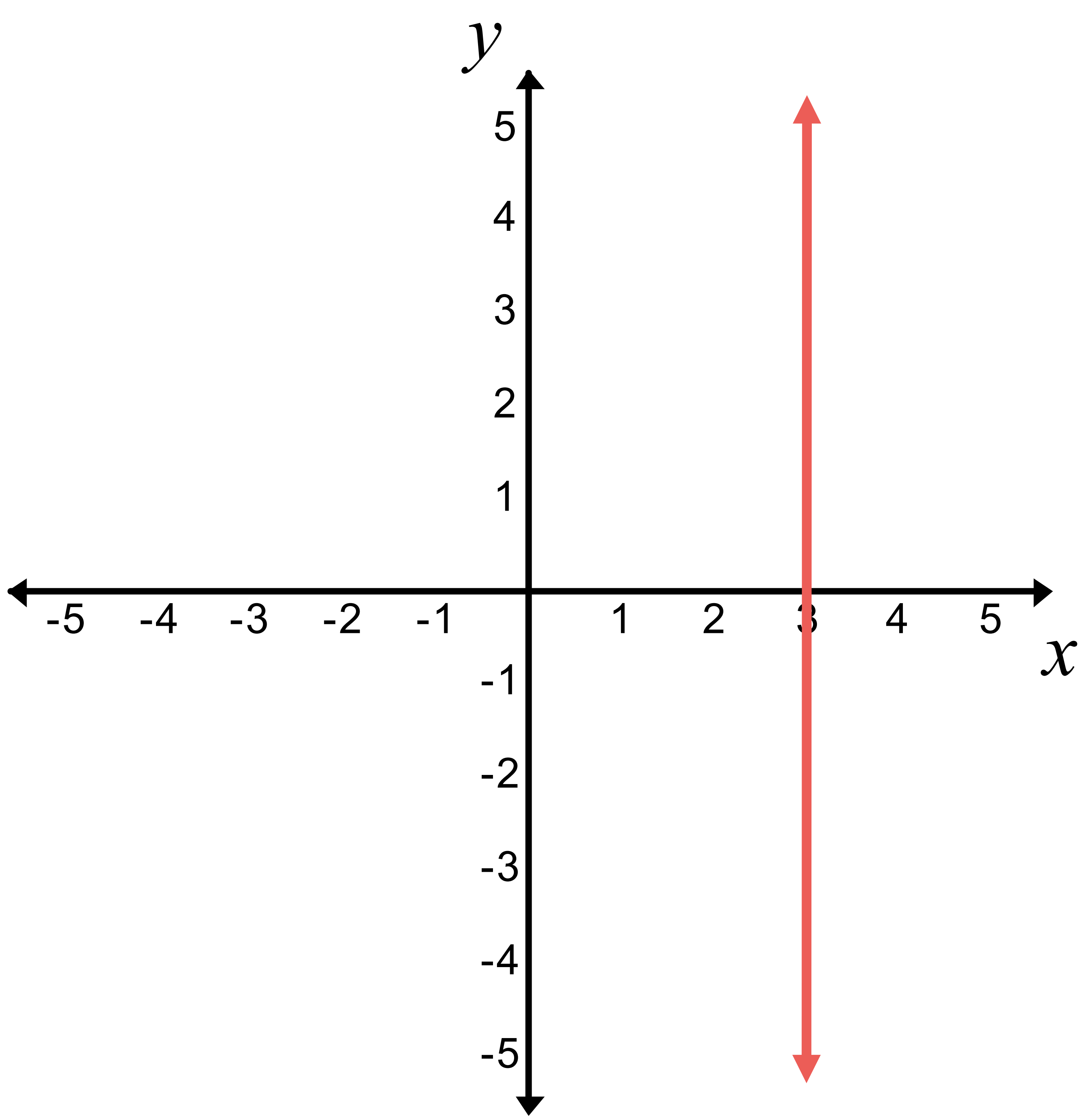


Example: Graphing a Horizontal Line

- Graph y = 3 in the rectangular coordinate system.



Graph x = 3 in the rectangular coordinate system.



General Form of the Equation of a Line

- Every line has an equation that can be written in the general form $ax + by + c = 0$ where a, b, and c are real numbers, a and b are not both zero.

$$ax + by + c = 0$$

Example: Finding the Slope and the y-Intercept

- Find the slope and the y-intercept of the line whose equation is: $3x + 6y - 12 = 0$

Rewrite the equation in slope intercept form by solving for y in terms of x.

$$3x + 6y - 12 = 0$$

$$6y = -3x + 12$$

$$y = -\frac{1}{2}x + 2$$

The slope is $-\frac{1}{2}$

NOT $-\frac{1}{2}x$

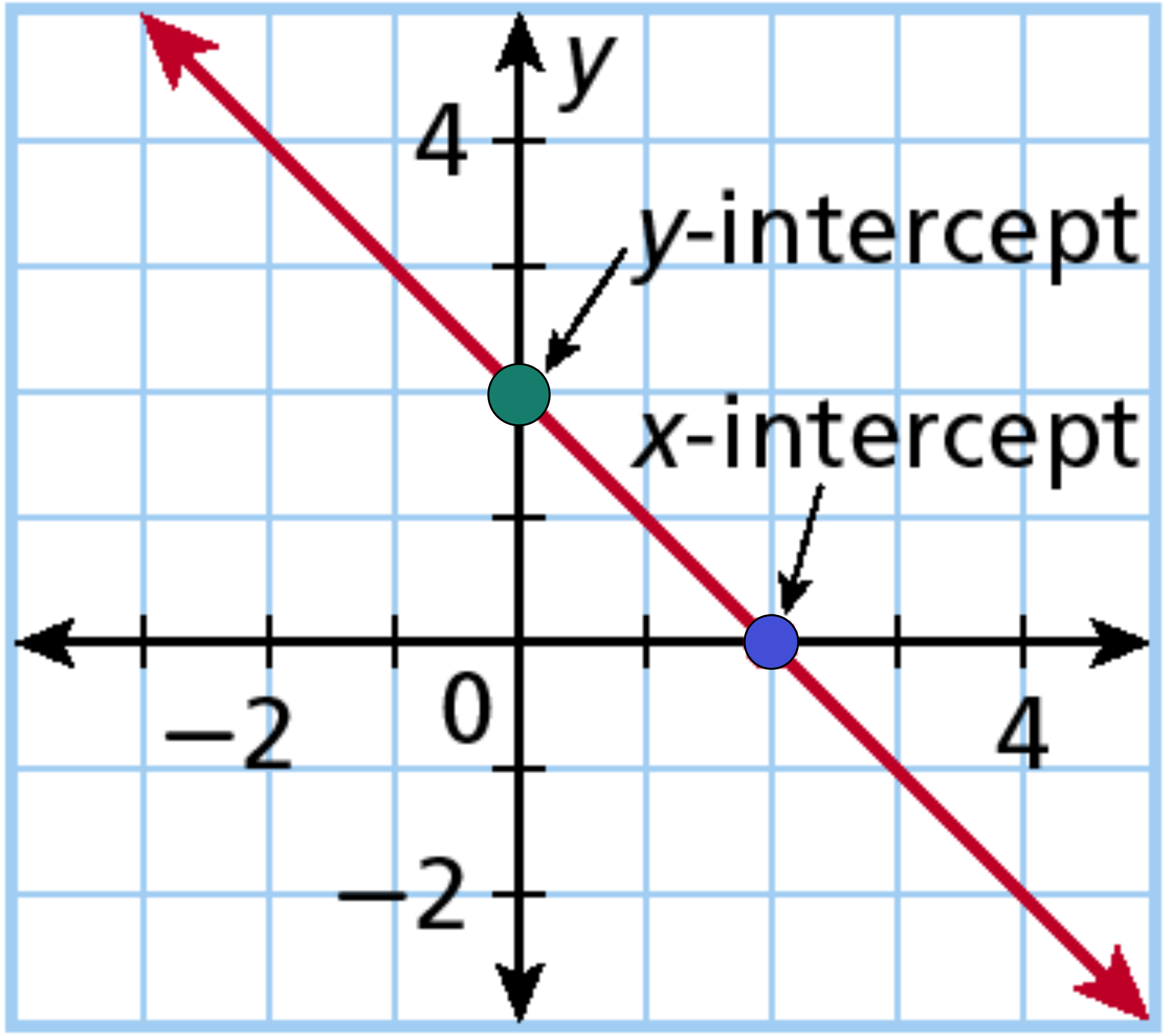
The y-intercept is 2.

Intercepts

- Recall from geometry that two points determine a line. Often the easiest points to find are the points where a line crosses the axes. The x- and y-intercepts.

The x-intercept is the x-coordinate of the point with y-coordinate 0, also where the line crosses the y-axis. (Note that y = 0.)

The y-intercept is the y-coordinate of the point with x-coordinate 0, also where the line crosses the x-axis. (Note that x = 0.)



Using Intercepts to Graph $ax + by + c = 0$

Graphing With Intercepts

- 1. To find the x-intercept: Let $y = 0$ and solve for x. Plot the point containing the x-intercept on the x-axis.
- 2. Find the y-intercept. Let $x = 0$ and solve for y. Plot the point containing the y-intercept on the y-axis.
- 3. Use a straightedge to draw a line through the points containing the intercepts. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

Example: Using Intercepts to Graph a Linear Equation

- Graph using intercepts: $3x - 2y - 6 = 0$

Step 1: Find the y-intercept.

Let $x = 0$ and solve for y.

$$3x - 2y - 6 = 0$$
$$3(0) - 2y - 6 = 0$$
$$-2y = 6$$
$$y = -3$$

The y-intercept is -3, the line passes through (0, -3).

Step 2: Find the x-intercept.

Let $y = 0$ and solve for x.

$$3x - 2y - 6 = 0$$
$$3x - 2(0) - 6 = 0$$
$$3x = 6$$
$$x = 2$$

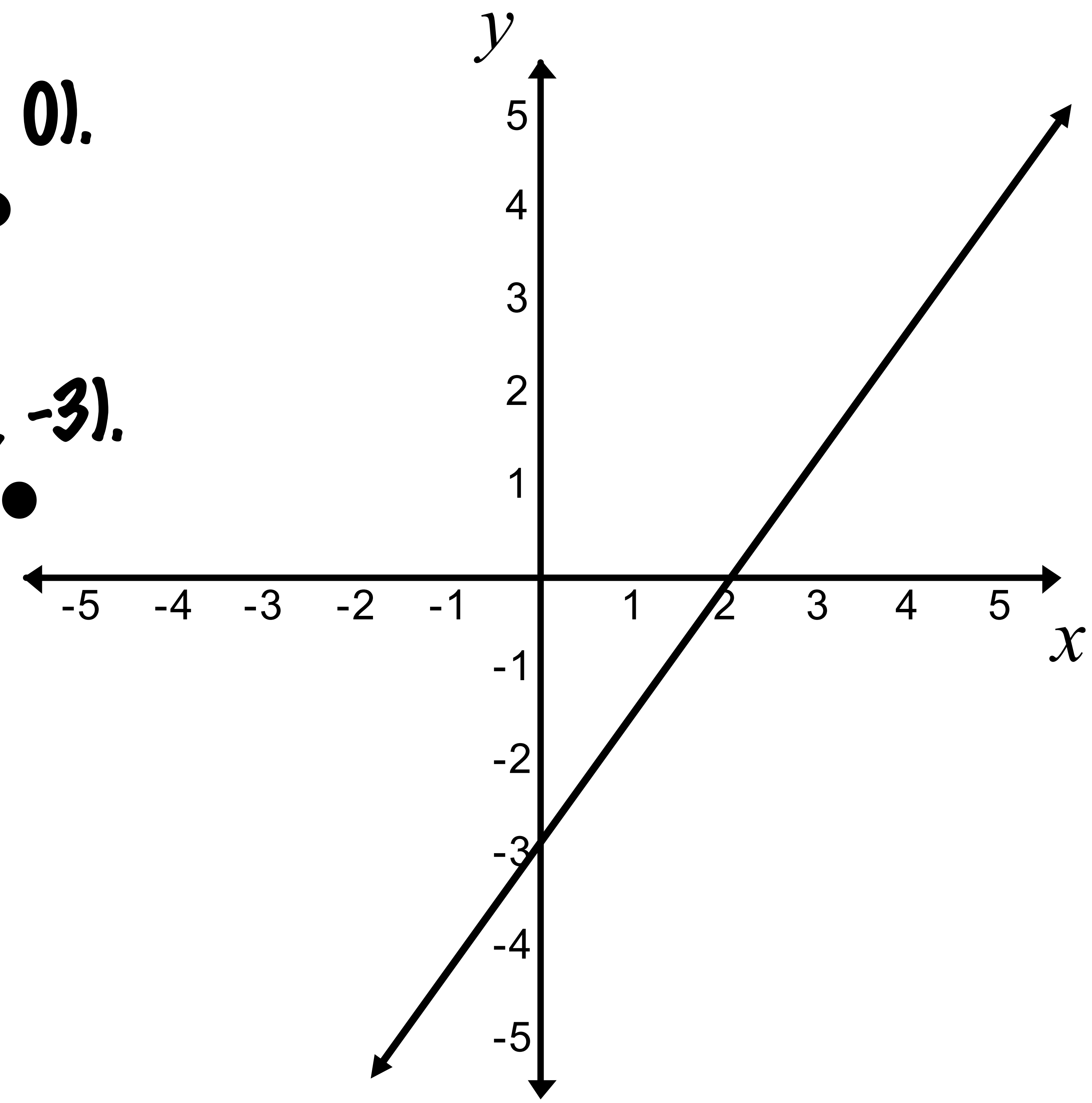
The x-intercept is 2, the line passes through (2, 0).

Example: Using Intercepts to Graph a Linear Equation

- **Step 3:** Graph the equation by drawing a line through the two points containing the intercepts.

The x-intercept is 2, the line passes through (2, 0).

The y-intercept is -3, the line passes through (0, -3).



Another Example

Graph the linear function: $2y + 4x = 7$

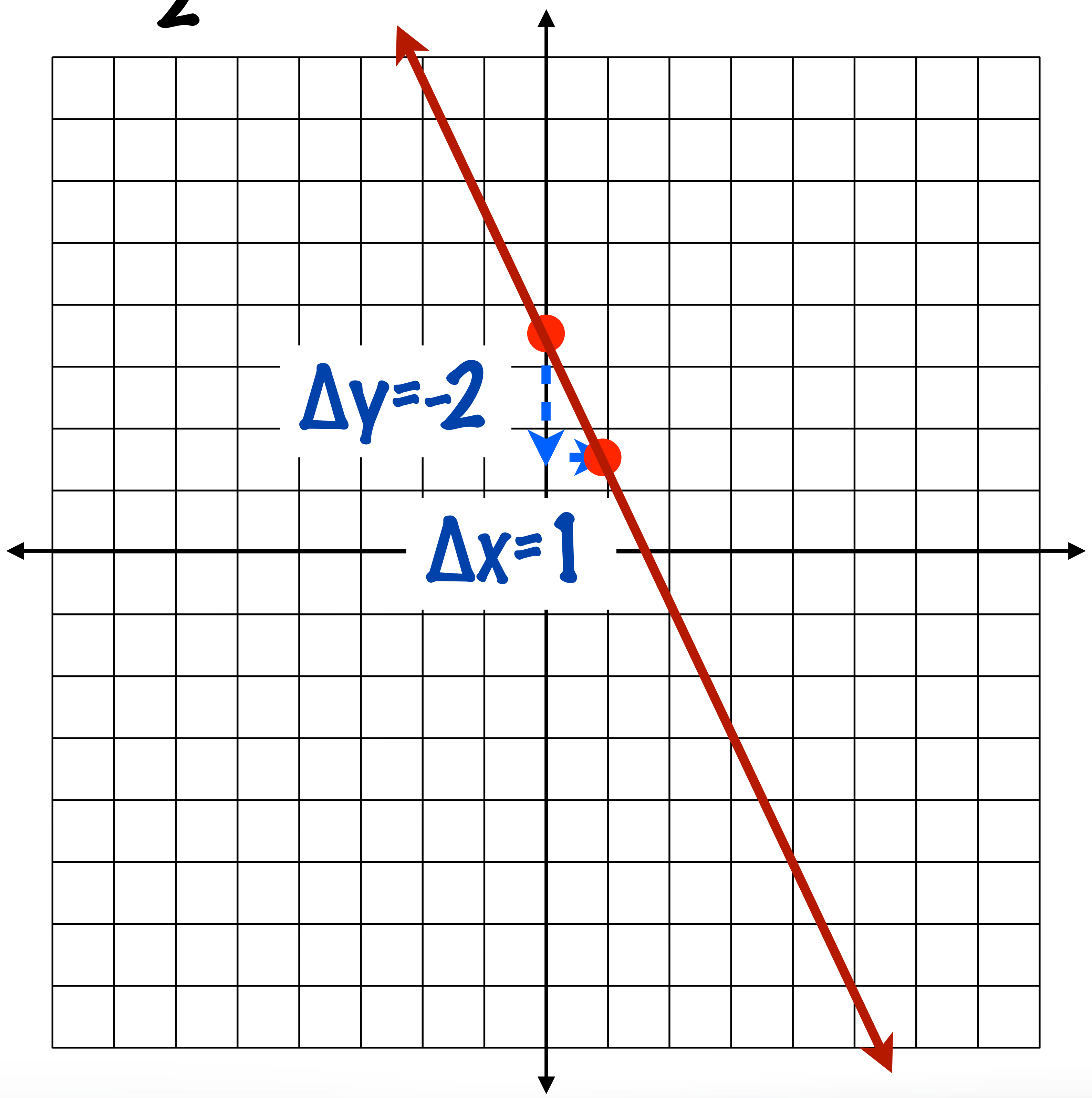
Step 0 Write the equation in slope intercept form. $y = -2x + \frac{7}{2}$

Step 1 Plot the y-intercept on the y-axis.

We plot the point $\left(0, \frac{7}{2}\right)$.

Step 2 Using the slope, plot a second point. $m = \frac{-2}{1} = \frac{\Delta y}{\Delta x}$

Step 3 Draw the line.



TI-84

- Be careful when graphing linear equations (or any equations) on the TI. The window is not square. To ensure a square graph use ZOOM 5:Zsquare, or set the window with the y max and min to be 2/3 the x values.

Doing so will ensure the slope of your line looks as you would expect.

Technology

On a graphing utility, lines will not appear to have the correct slope unless you use a viewing window that has a square setting. For instance, try graphing the lines in Example 4 using the standard setting $-10 \le x \le 10$ and $-10 \le y \le 10$. Then reset the viewing window with the square setting $-9 \le x \le 9$ and $-6 \le y \le 6$. On which setting do the lines $y = \frac{2}{3}x - \frac{5}{3}$ and $y = -\frac{3}{2}x + 2$ appear to be perpendicular?

A Summary of the Various Forms of Linear Equations

1. Point-Slope Form	$y - y_1 = m(x - x_1)$
2. Slope-Intercept Form	$y = mx + b$
3. Horizontal Line	$y = b$
4. Vertical Line	$x = a$
5. General Form	$ax + by + c = 0$

Example: Application

- Use the data points (CO_2, °F) = (317, 57.04) and (354, 57.64) to obtain a linear function that models average global temperature, f(x), in °F for an atmospheric carbon dioxide concentration of x parts per million. Round m to three decimal places and b to one decimal place.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{57.64 - 57.04}{354 - 317} = \frac{0.60}{37} \approx 0.016$$

$m \approx 0.016$ What does this value indicate?

It is the change in average global temperature (.016°F) for each increase of one part per million in CO_2 concentration.

Example: Application

- Now find the equation of the line: $m \approx 0.016$ (317, 57.04) (354, 57.64)

We have the slope and a choice of points so we can use point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 57.64 = 0.016(x - 354)$$

$$y - 57.64 = 0.016x - 5.664$$

$$y = 0.016x + 51.976$$

$$f(x) = 0.016x + 52.0$$

What does the value 52.0 represent?

It is the average global temperature expected for a CO_2 concentration of 0.

The function f models average global temperature, f(x), for an atmospheric carbon dioxide concentration of x parts per million.

Example: Application

- Use the function to project average global temperature at a CO_2 concentration of 600 parts per million.

$$f(x) = 0.016x + 52.0$$

$$f(600) = 0.016(600) + 52.0$$

$$f(600) \approx 61.6°F$$

The average global temperature at a concentration of 600 parts ppm CO_2 would be 61.6°F.

In June 2017 the concentration of CO_2 was 408.84 ppm. Intended Nationally Determined Contributions (INDCs) predict a possible 670 ppm by 2100.

The 2015 estimate of average global temperature was about 59°F. The hottest year on record until 2016. And 2017?

CO_2

- Shown is the graph indicating the increase (positive slope) of atmospheric CO_2 since 1950s.
- Anyone think it will begin to decrease (negative slope) any time soon?