Chapter 1

Functions and Graphs

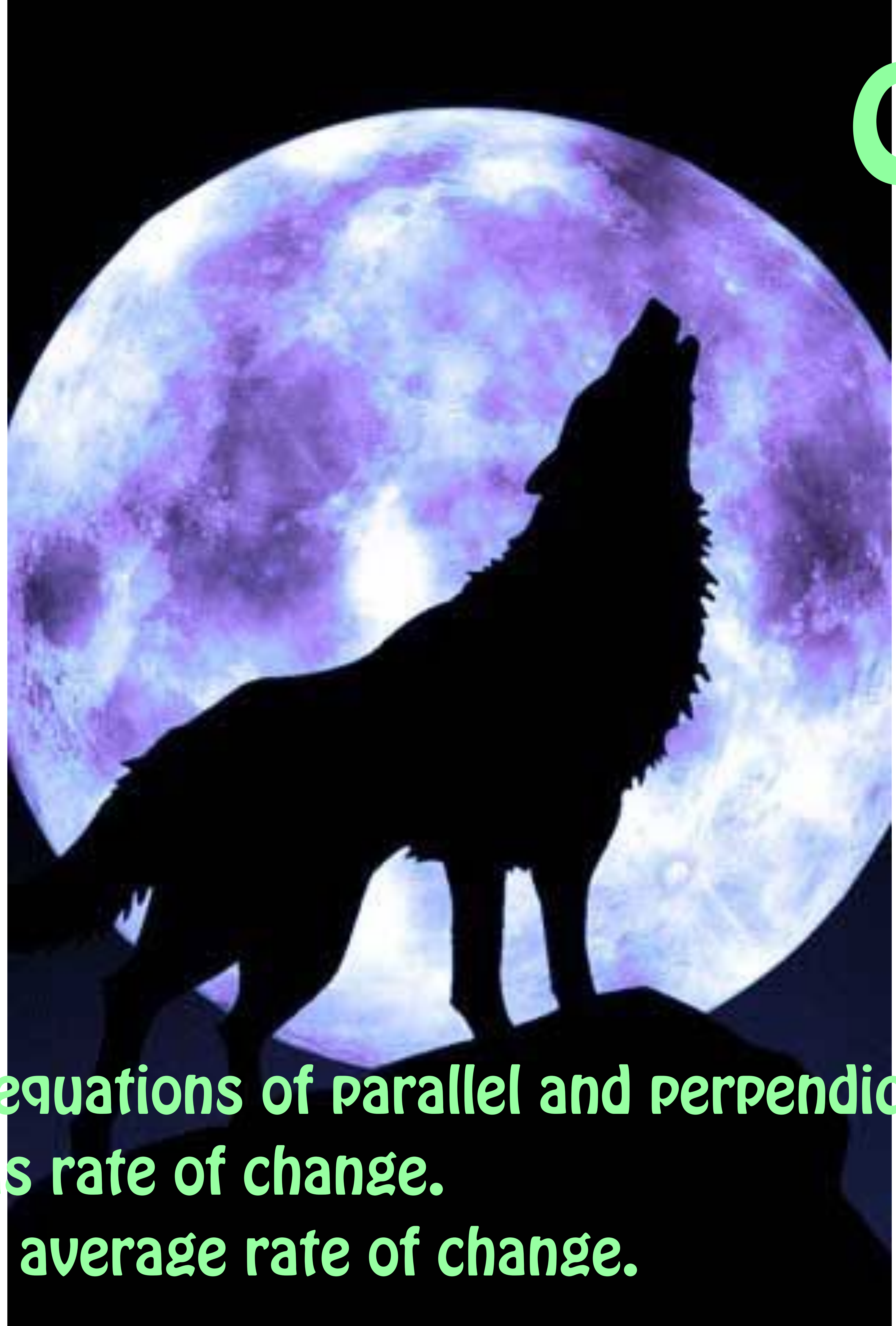
1.5 More on Slope

Chapter 1

Homework

1.5 p200 2, 4, 6, 8, 12, 14, 16, 18, 22, 24, 26, 29, 30, 32, 46, 48

Chapter 1



Objectives

- Find slopes and equations of parallel and perpendicular lines.
- Interpret slope as rate of change.
- Find a function's average rate of change.

Parallel and Perpendicular Lines

- By examining slopes, you can determine if lines are parallel or perpendicular.

- Using that information you can also write equations of lines that meet certain criteria.

Slope and Parallel Lines

- 1. If two non-vertical lines are parallel, then they have the same slope.
- 2. If two distinct non-vertical lines have the same slope, then they are parallel.
- 3. Two distinct vertical lines, both with undefined slopes, are parallel.

Write an equation of the line passing through $(-2, 5)$ and parallel to the line whose equation is $y = 3x + 1$.

The slope of the line $y = 3x + 1$ is 3. A parallel line will also have slope of 3.

In point-slope form, the equation of the new line is

$$y - 5 = 3(x - -2) \qquad y - 5 = 3(x + 2)$$

Slope and Perpendicular Lines

1. If two non-vertical lines are perpendicular, then the product of their slopes is -1 (opposite reciprocal).

2. If the product of slopes of two lines is -1, then the lines are perpendicular.

3. A horizontal line having zero slope is perpendicular to a vertical line having undefined slope.

Example: Writing Equations of a Line

- Find the slope of any line that is perpendicular to the line whose equation is $x + 3y - 12 = 0$.

First we must determine the slope of the line. There are a few ways to find the slope from an equation, but perhaps the most common is to put the equation in slope-intercept form.

$$x + 3y - 12 = 0$$

$$3y = -x + 12$$

$$y = -\frac{1}{3}x + 4$$

The slope of our original line is $-\frac{1}{3}$

The slope of any line perpendicular to the our original line is 3.

$$-\frac{1}{3} \bullet 3 = -1$$

Parallel and Perpendicular Lines

Parallel lines have equal slopes.

$\parallel$ $m_1 = m_2$

Perpendicular lines have opposite reciprocal slopes.

$\perp$ $m_1 \cdot m_2 = -1$

Slope as Rate of Change

- Slope is defined as the ratio of the change in y to a corresponding change in x.

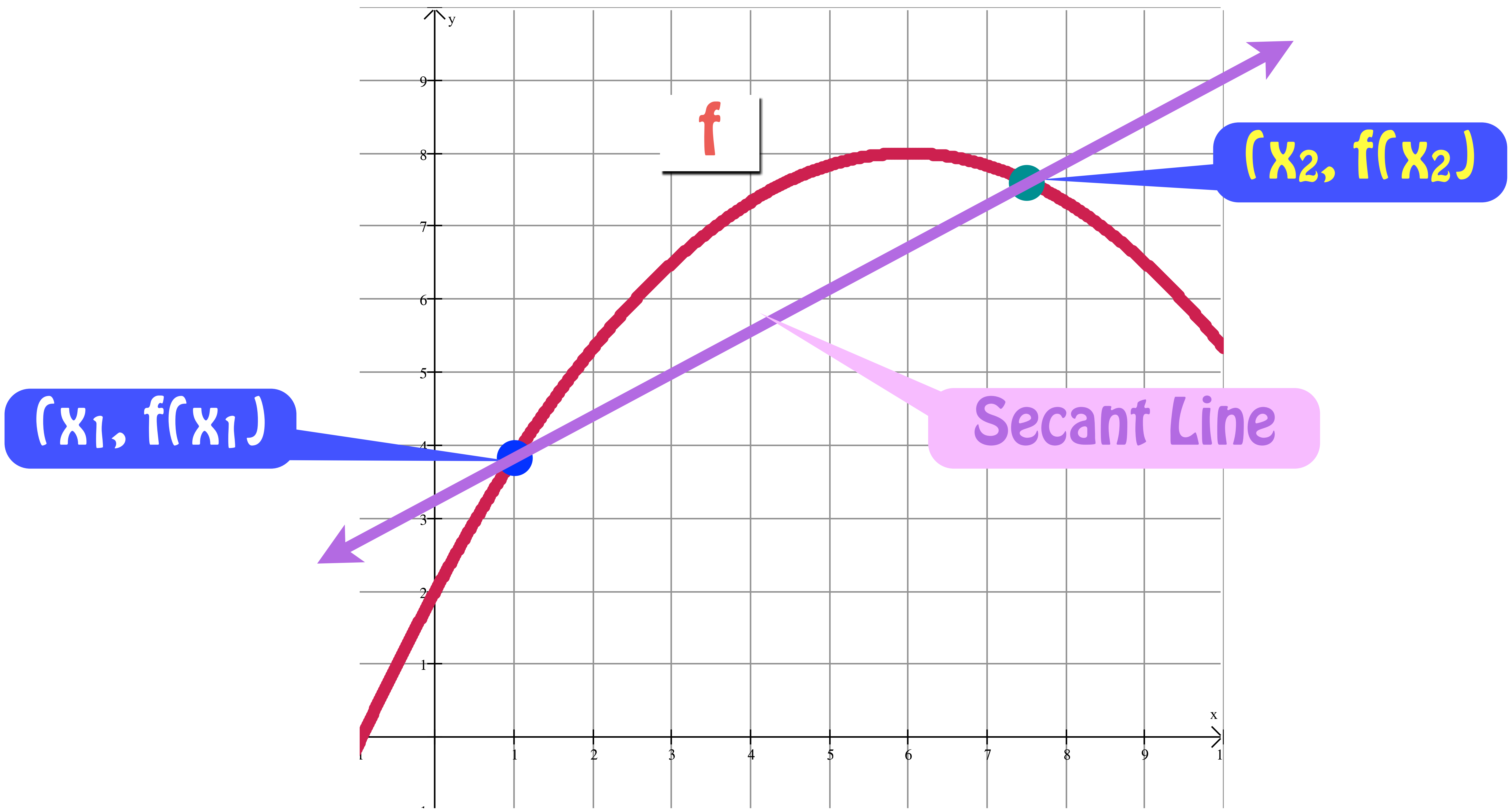
$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta f(x)}{\Delta x}$$

- The slope of a linear function describes how fast y (or f(x)) is changing with respect to x.

- For a linear function, slope may be interpreted as the rate of change of the dependent variable per unit change in the independent variable.

Secant Line

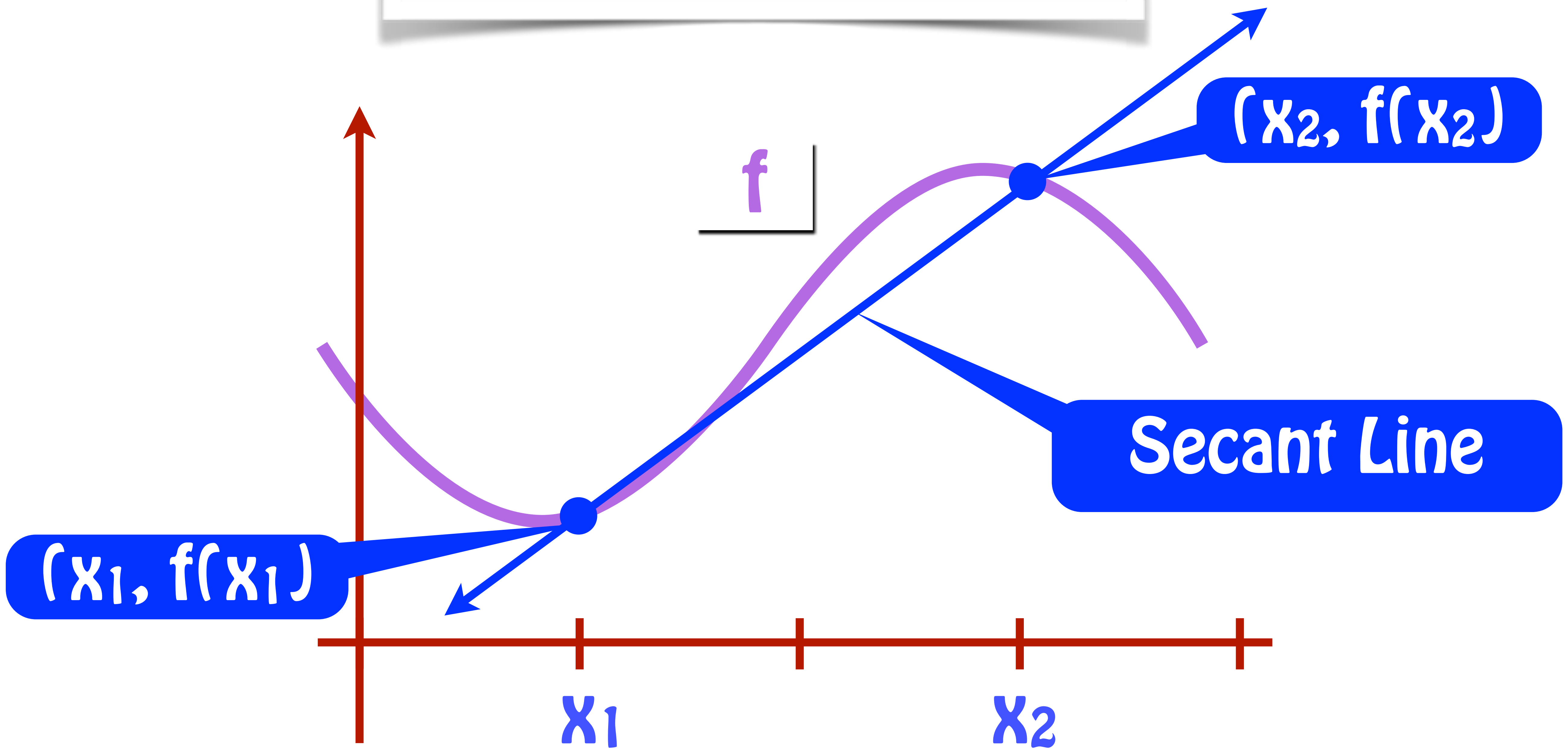
- If the graph of a function is not a straight line, the average rate of change between any two points is the slope of the line containing the two points. This line is called a secant line.



The Average Rate of Change of a Function

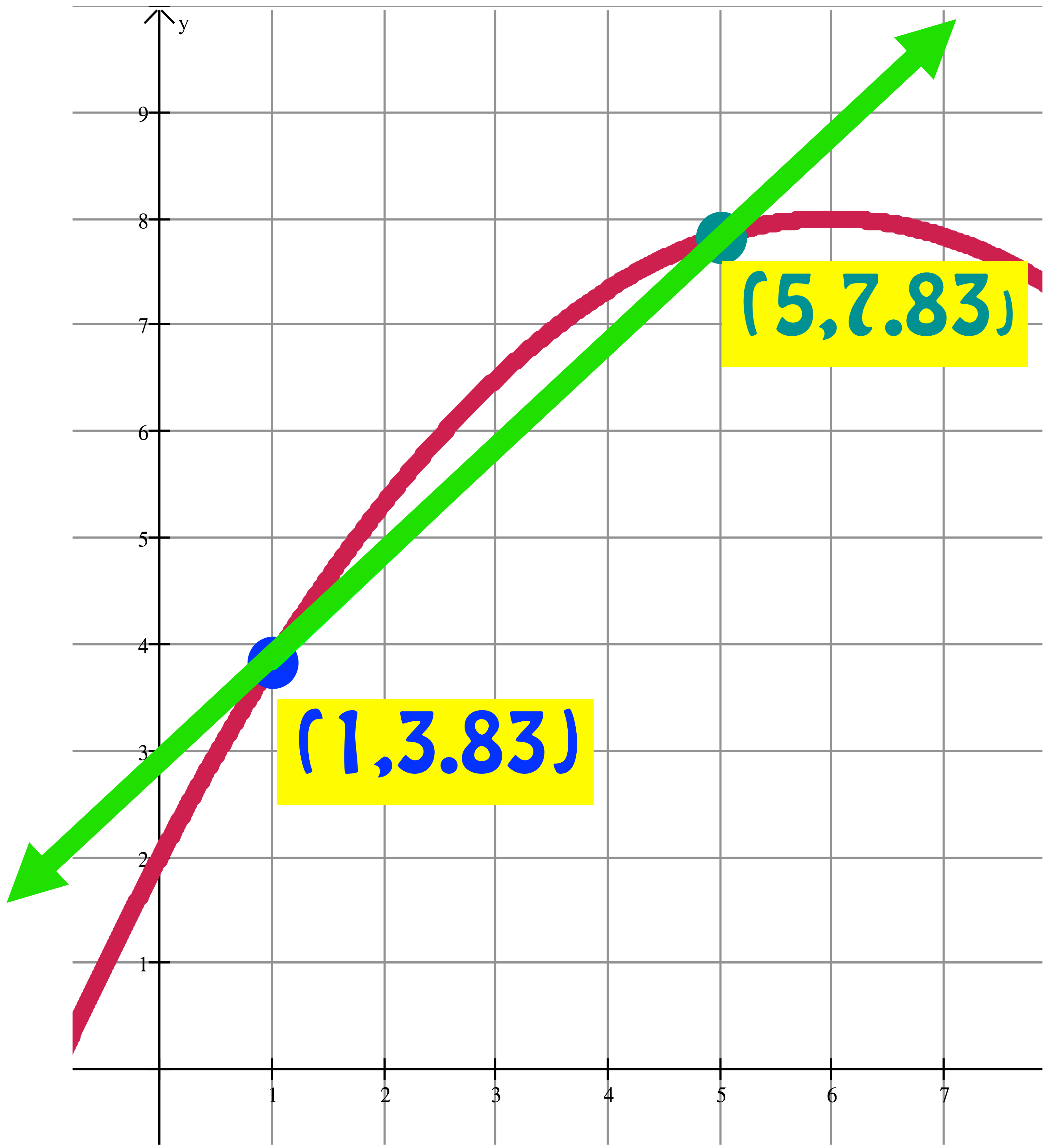
- Let $(x_1, f(x_1))$ and $(x_2, f(x_2))$ be distinct points on the graph of a function f. The average rate of change of f from x_1 to x_2 is:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



The Average Rate of Change of a Function

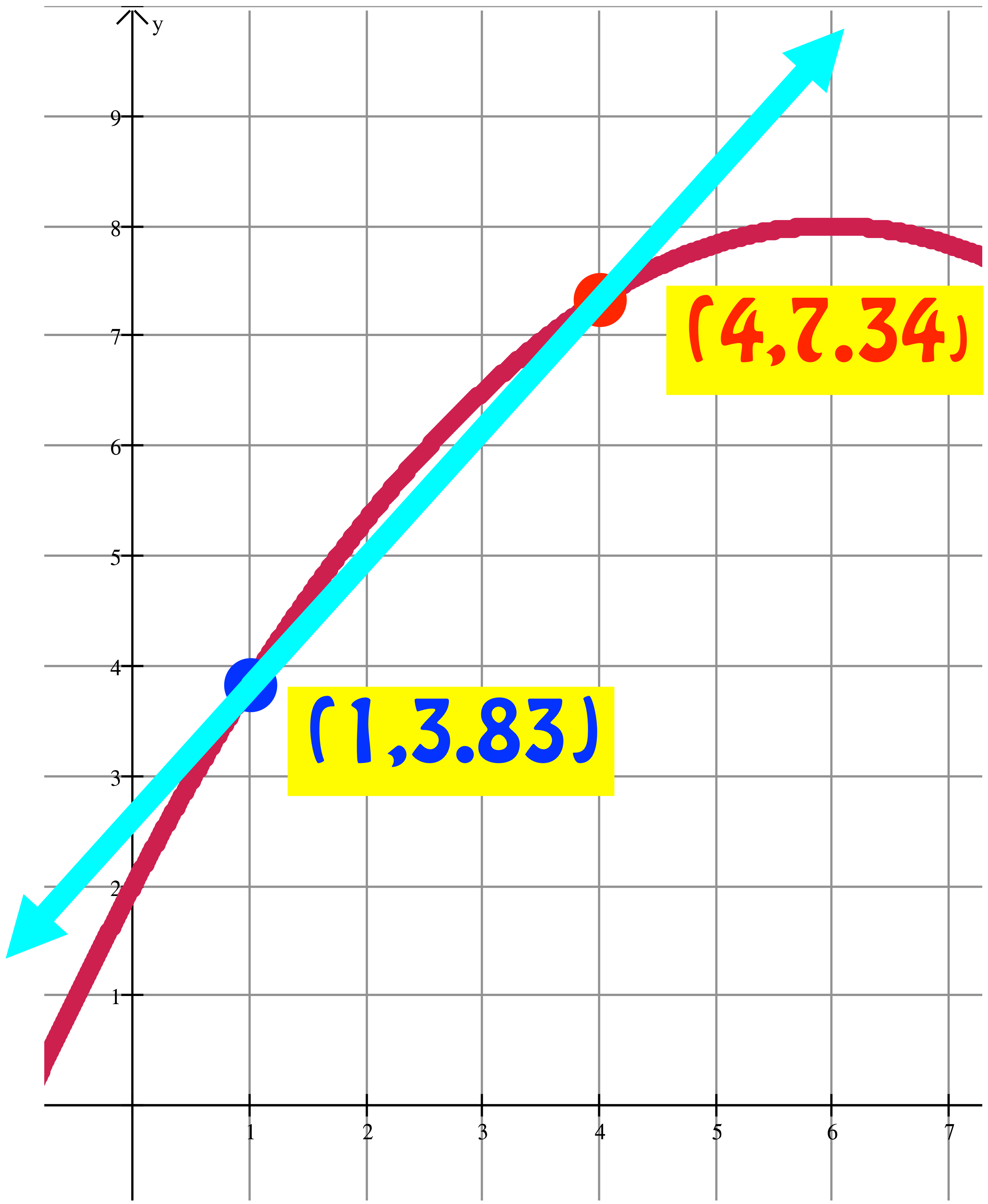
- The slope of the secant line between the points (1,3.83) and (5,7.83) is:



$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{7.83 - 3.83}{5 - 1} = \frac{4}{4} = 1$$

The slope of the secant line between the points (1,3.83) and (4,7.34) is



$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{7.34 - 3.83}{4 - 1} = \frac{3.51}{3} = 1.17$$

Average Slope

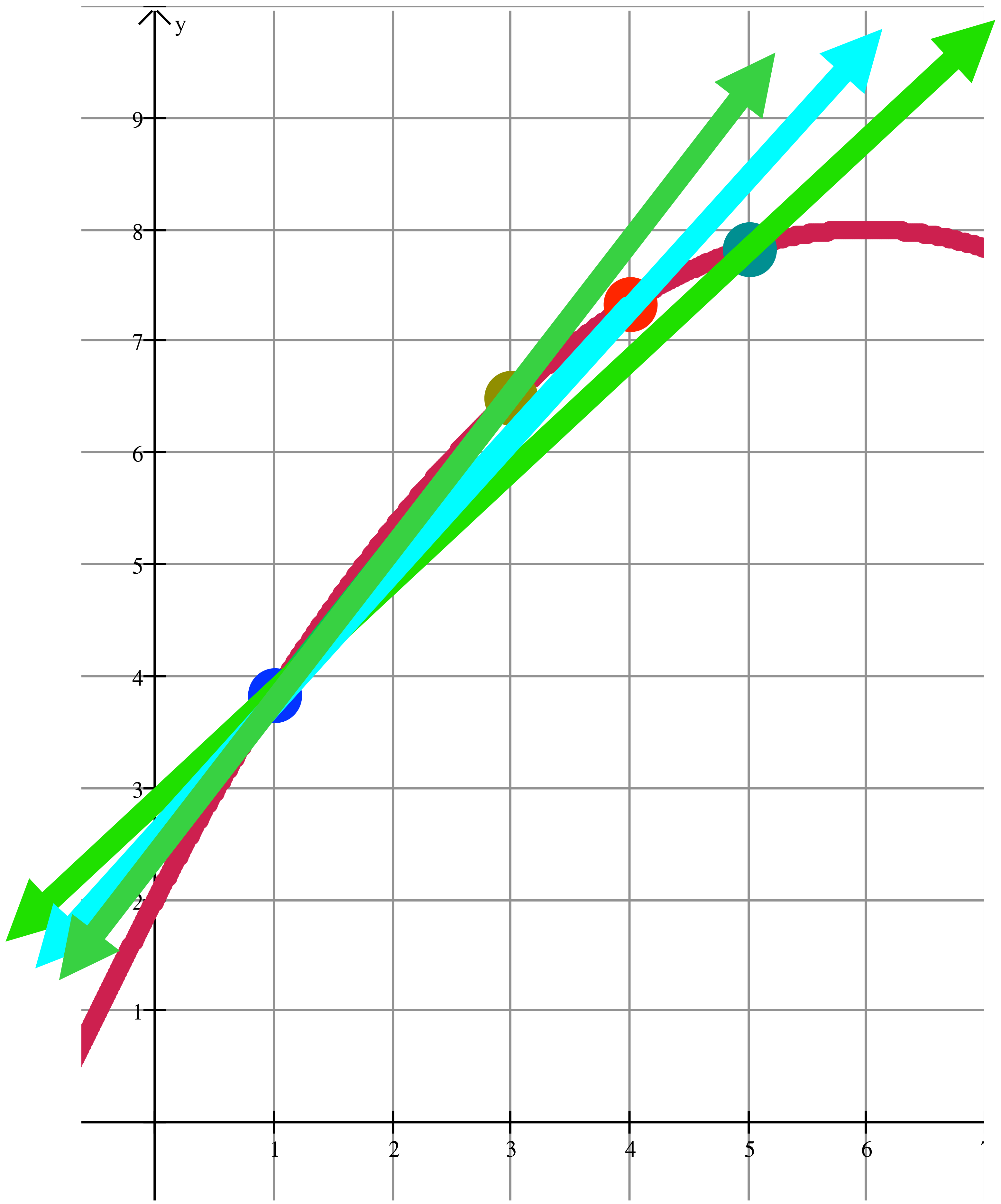
- The slope of the secant line between the points (1,3.83) and (4,7.34) is

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{6.5 - 3.83}{3 - 1} = \frac{2.67}{2} = 1.34$$

Average Slope

- Let us look at the 3 cases together



x	f(x)	Slope of the secant line
3	6.5	1.34
4	7.34	1.17
5	7.83	1

- Notice how the secant slope changes depending upon the points that you choose because this function is a curve, not a line. So the average rate of change varies depending upon which points you may choose.

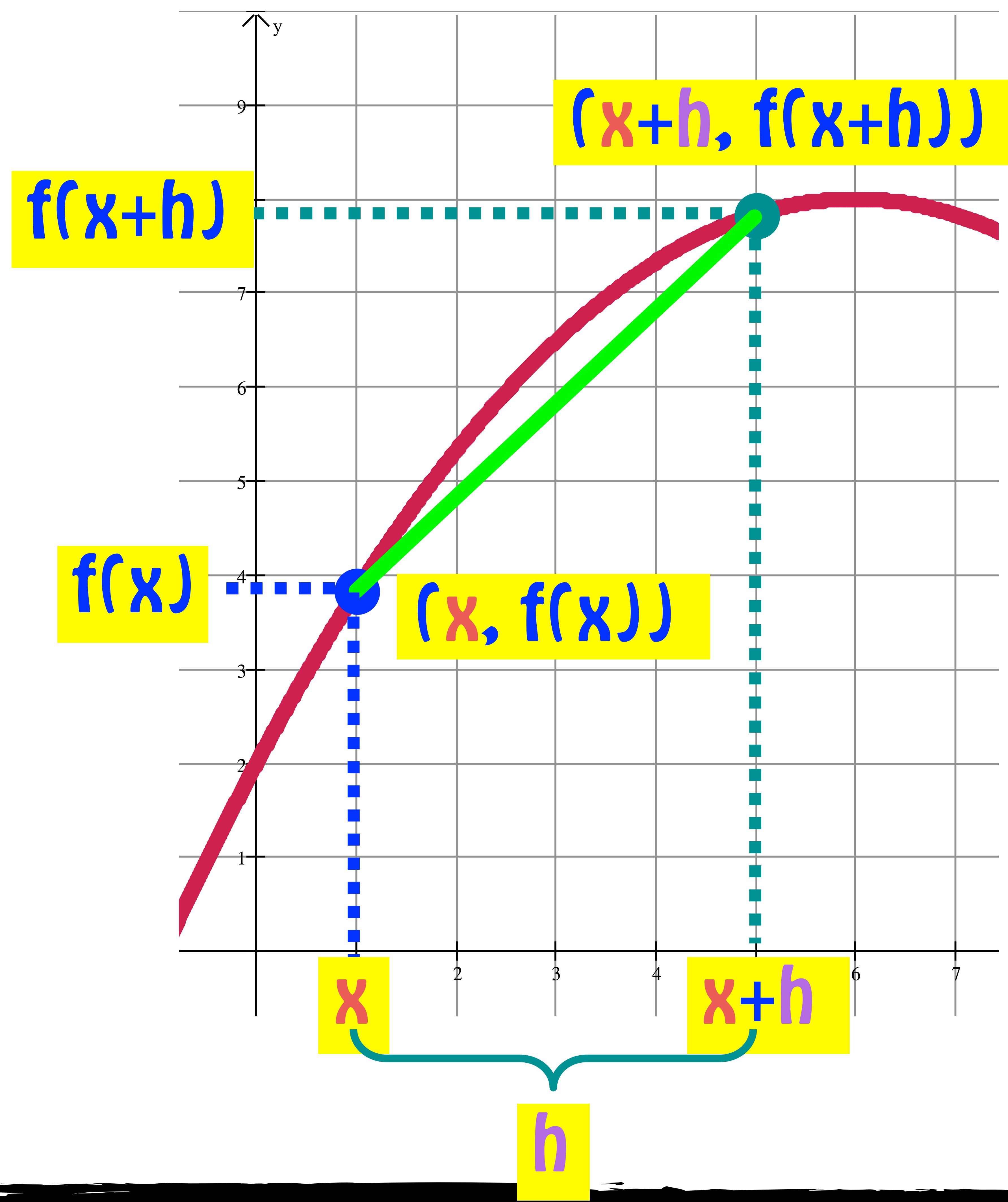
Average Rate of Change

- Suppose we move a little bit along a function from one x value to some new value that is the original x plus an amount h (to $x + h$).
- The average rate of change from $x_1 = x$ to $x_2 = x + h$ is:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

- This is the difference quotient we briefly met in an earlier section.
- The difference quotient gives the average rate of change of a function from x to $x + h$. In the difference quotient, the value h is usually small (a number very close to 0). In this way the average rate of change can be found for a very short interval.

Difference Quotient



$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{f(x+h) - f(x)}{h}$$

The difference quotient is an average rate of change and becomes very important in higher levels of math.

Finding the Average Rate of Change

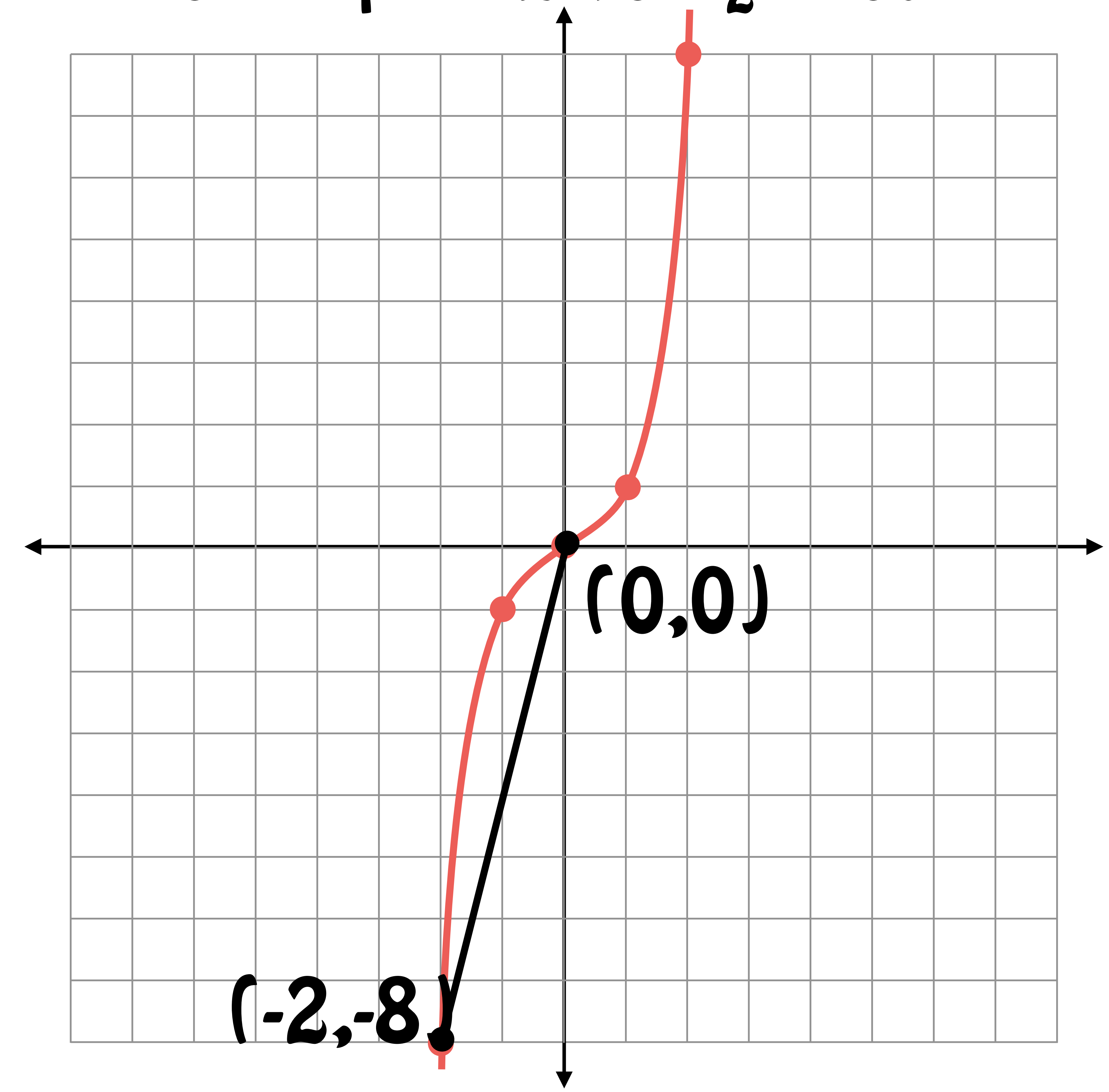
- Find the average rate of change for $f(x) = x^3$ from $x_1 = -2$ to $x_2 = 0$.

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h) - f(x)}{h}$$

$$= \frac{f(0) - f(-2)}{0 - (-2)}$$

$$= \frac{0^3 - (-2)^3}{0 - (-2)}$$

$$= \frac{8}{2} = 4$$



The average rate of change for $f(x) = x^3$ from $x_1 = -2$ to $x_2 = 0$ is 4 units of change in y for every unit of change in x.

Average Velocity of an Object

- Suppose that a function expresses an objects's position, $s(t)$, in terms of time, t. The average velocity of the object from t_1 to t_2 is:

position 2 position 1

$$\Delta v = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

time 2 time 1

Example: Finding Average Velocity

- The distance, $s(t)$, in feet, traveled by a ball rolling down a ramp is given by the function $s(t) = 4t^2$, where t is the time, in seconds, after the ball is released.

The ball starts slowly and accelerates as it moves down the ramp.

Find the ball's average velocity from $t_1 = 1$ second to $t_2 = 2$ seconds.

$$\Delta v = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(2) - s(1)}{2 - 1} = \frac{4(2^2) - 4(1^2)}{1} = \frac{16ft - 4ft}{1sec}$$

The ball's average velocity between 1 and 2 seconds is 12 ft/sec.

Example: Finding Average Velocity

- The distance, s(t), in feet, traveled by a ball rolling down a ramp is given by the function $s(t) = 4t^2$, where t is the time, in seconds, after the ball is released.

Find the ball's average velocity from $t_1 = 1$ sec to $t_2 = 1.5$ sec.

$$\Delta v = \frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \frac{s(1.5) - s(1)}{1.5 - 1} = \frac{4(1.5^2) - 4(1^2)}{.5}$$

$$= \frac{9ft - 4ft}{.5\,sec} = \frac{5ft}{.5\,sec} = \frac{10\,ft}{1\,sec}$$

The ball's average velocity between 1 and 1.5 seconds is 10 ft/sec.