# 1.1 Rectangular Coordinates

## What you should learn

- Plot points in the Cartesian plane.
- Use the Distance Formula to find the distance between two points.
- Use the Midpoint Formula to find the midpoint of a line segment.
- Use a coordinate plane and geometric formulas to model and solve real-life problems.

## Why you should learn it

The Cartesian plane can be used to represent relationships between two variables. For instance, in Exercise 60 on page 12, a graph represents the minimum wage in the United States from 1950 to 2004.



## The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** named after the French mathematician René Descartes (1596–1650).

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure 1.1. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four parts called **quadrants.**

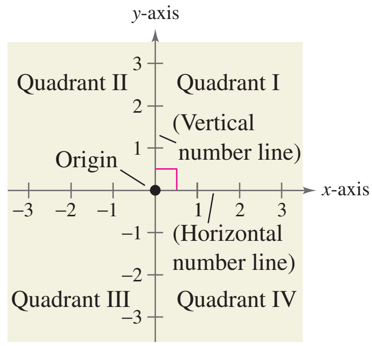



FIGURE 1.1

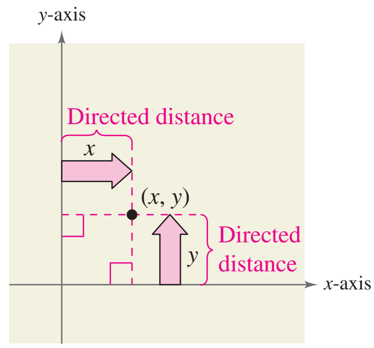
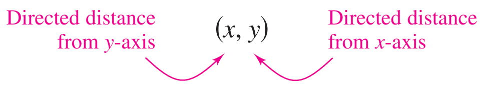



FIGURE 1.2

Each point in the plane corresponds to an **ordered pair** $(x, y)$ of real numbers $x$ and $y$, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the $y$-axis to the point, and the ***y*-coordinate** represents the directed distance from the $x$-axis to the point, as shown in Figure 1.2.




The notation $(x, y)$ denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

### Example 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

### Solution

To plot the point $(-1, 2)$, imagine a vertical line through $-1$ on the $x$-axis and a horizontal line through 2 on the $y$-axis. The intersection of these two lines is the point $(-1, 2)$. The other four points can be plotted in a similar way, as shown in Figure 1.3.

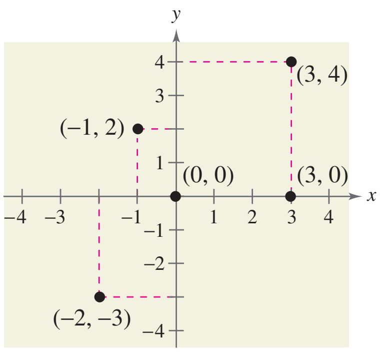



FIGURE 1.3

CHECKPOINT Now try Exercise 3.

The beauty of a rectangular coordinate system is that it allows you to *see* relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

## Example 2 Sketching a Scatter Plot

From 1990 through 2003, the amounts $A$ (in millions of dollars) spent on skiing equipment in the United States are shown in the table, where $t$ represents the year. Sketch a scatter plot of the data. (Source: National Sporting Goods Association)

| Year, $t$ | Amount, $A$ |
|---|---|
| 1990 | 475 |
| 1991 | 577 |
| 1992 | 521 |
| 1993 | 569 |
| 1994 | 609 |
| 1995 | 562 |
| 1996 | 707 |
| 1997 | 723 |
| 1998 | 718 |
| 1999 | 648 |
| 2000 | 495 |
| 2001 | 476 |
| 2002 | 527 |
| 2003 | 464 |

### Solution

To sketch a *scatter plot* of the data shown in the table, you simply represent each pair of values by an ordered pair $(t, A)$ and plot the resulting points, as shown in Figure 1.4. For instance, the first pair of values is represented by the ordered pair (1990, 475). Note that the break in the $t$-axis indicates that the numbers between 0 and 1990 have been omitted.

Amount Spent on Skiing Equipment

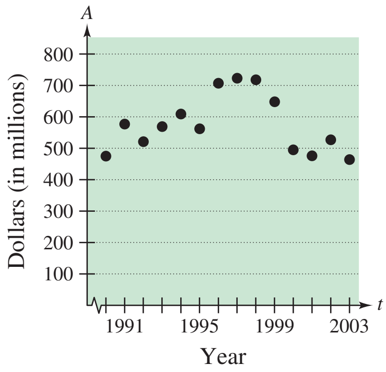



FIGURE 1.4

CHECKPOINT Now try Exercise 21.

In Example 2, you could have let $t = 1$ represent the year 1990. In that case, the horizontal axis would not have been broken, and the tick marks would have been labeled 1 through 14 (instead of 1990 through 2003).

**Technology**

The scatter plot in Example 2 is only one way to represent the data graphically. You could also represent the data using a bar graph and a line graph. If you have access to a graphing utility, try using it to represent graphically the data given in Example 2.

The *HM mathSpace®* CD-ROM and *Eduspace®* for this text contain additional resources related to the concepts discussed in this chapter.

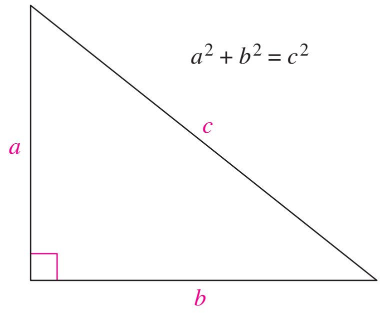



FIGURE 1.5

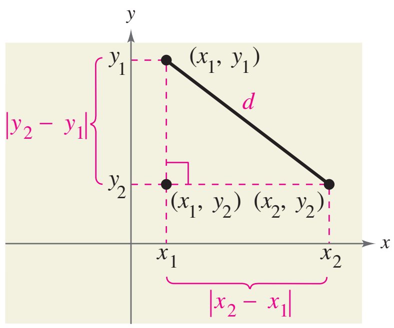



FIGURE 1.6

## The Pythagorean Theorem and the Distance Formula

The following famous theorem is used extensively throughout this course.

### Pythagorean Theorem

For a right triangle with hypotenuse of length $c$ and sides of lengths $a$ and $b$, you have $a^2 + b^2 = c^2$, as shown in Figure 1.5. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

Suppose you want to determine the distance $d$ between two points $(x_1, y_1)$ and $(x_2, y_2)$ in the plane. With these two points, a right triangle can be formed, as shown in Figure 1.6. The length of the vertical side of the triangle is $|y_2 - y_1|$, and the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem, you can write

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is the **Distance Formula.**

### The Distance Formula

The distance $d$ between the points $(x_1, y_1)$ and $(x_2, y_2)$ in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

### Example 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

#### Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= \sqrt{(5)^2 + (3)^2} && \text{Simplify.} \\ &= \sqrt{34} && \text{Simplify.} \\ &\approx 5.83 && \text{Use a calculator.} \end{aligned}$$

So, the distance between the points is about 5.83 units. You can use the Pythagorean Theorem to check that the distance is correct.

$$\begin{aligned} d^2 &\stackrel{?}{=} 3^2 + 5^2 && \text{Pythagorean Theorem} \\ (\sqrt{34})^2 &\stackrel{?}{=} 3^2 + 5^2 && \text{Substitute for } d. \\ 34 &= 34 && \text{Distance checks. } \checkmark \end{aligned}$$

#### Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from $A$ to $B$. Then use a centimeter ruler to measure the length of the segment.

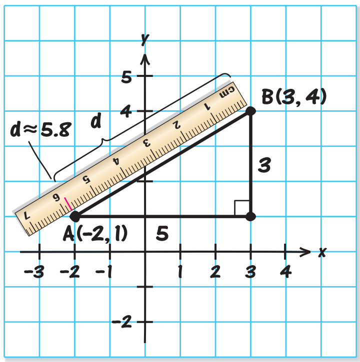



FIGURE 1.7

The line segment measures about 5.8 centimeters, as shown in Figure 1.7. So, the distance between the points is about 5.8 units.

CHECKPOINT Now try Exercises 31(a) and (b).

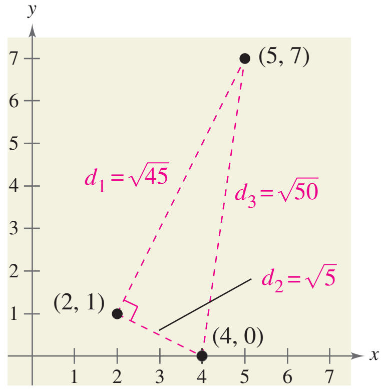



FIGURE 1.8

## Example 4 Verifying a Right Triangle

Show that the points (2, 1), (4, 0), and (5, 7) are vertices of a right triangle.

### Solution

The three points are plotted in Figure 1.8. Using the Distance Formula, you can find the lengths of the three sides as follows.

$$d_1 = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45}$$
$$d_2 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$$
$$d_3 = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1+49} = \sqrt{50}$$

Because

$$(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$$

you can conclude by the Pythagorean Theorem that the triangle must be a right triangle.

CHECKPOINT Now try Exercise 41.

## The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, you can simply find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula.**

### The Midpoint Formula

The midpoint of the line segment joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

For a proof of the Midpoint Formula, see Proofs in Mathematics on page 124.

## Example 5 Finding a Line Segment's Midpoint

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$.

### Solution

Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \text{Midpoint Formula}$$
$$= \left(\frac{-5+9}{2}, \frac{-3+3}{2}\right) \qquad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$
$$= (2, 0) \qquad \text{Simplify.}$$

The midpoint of the line segment is (2, 0), as shown in Figure 1.9.

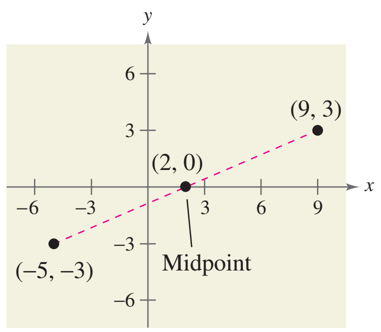



FIGURE 1.9

CHECKPOINT Now try Exercise 31(c).

## Applications

### Example 6 Finding the Length of a Pass

During the third quarter of the 2004 Sugar Bowl, the quarterback for Louisiana State University threw a pass from the 28-yard line, 40 yards from the sideline. The pass was caught by a wide receiver on the 5-yard line, 20 yards from the same sideline, as shown in Figure 1.10. How long was the pass?

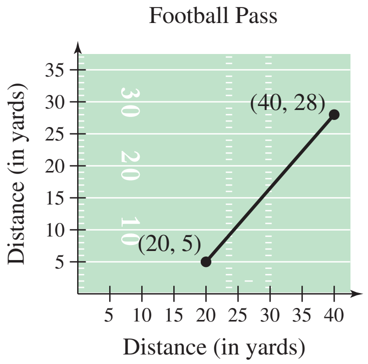



FIGURE 1.10

#### Solution

You can find the length of the pass by finding the distance between the points (40, 28) and (20, 5).

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{(40 - 20)^2 + (28 - 5)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= \sqrt{400 + 529} && \text{Simplify.} \\ &= \sqrt{929} && \text{Simplify.} \\ &\approx 30 && \text{Use a calculator.} \end{aligned}$$

So, the pass was about 30 yards long.

CHECKPOINT Now try Exercise 47.

In Example 6, the scale along the goal line does not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you are free to place the coordinate system in any way that is convenient for the solution of the problem.

### Example 7 Estimating Annual Revenue

FedEx Corporation had annual revenues of \$20.6 billion in 2002 and \$24.7 billion in 2004. Without knowing any additional information, what would you estimate the 2003 revenue to have been? (Source: FedEx Corp.)

#### Solution

One solution to the problem is to assume that revenue followed a linear pattern. With this assumption, you can estimate the 2003 revenue by finding the midpoint of the line segment connecting the points (2002, 20.6) and (2004, 24.7).

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula} \\ &= \left(\frac{2002 + 2004}{2}, \frac{20.6 + 24.7}{2}\right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= (2003, 22.65) && \text{Simplify.} \end{aligned}$$

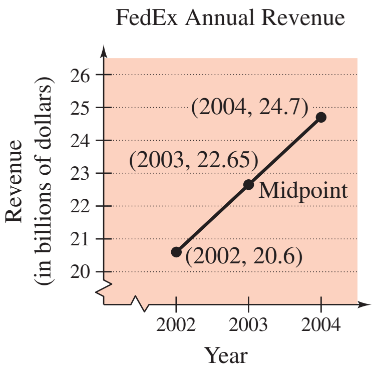



FIGURE 1.11

So, you would estimate the 2003 revenue to have been about \$22.65 billion, as shown in Figure 1.11. (The actual 2003 revenue was \$22.5 billion.)

CHECKPOINT Now try Exercise 49.

Paul Morrell

*Much of computer graphics, including this computer-generated goldfish tessellation, consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types include reflections, rotations, and stretches.*

## Example 8 Translating Points in the Plane

The triangle in Figure 1.12 has vertices at the points $(-1, 2)$, $(1, -4)$, and $(2, 3)$. Shift the triangle three units to the right and two units upward and find the vertices of the shifted triangle, as shown in Figure 1.13.

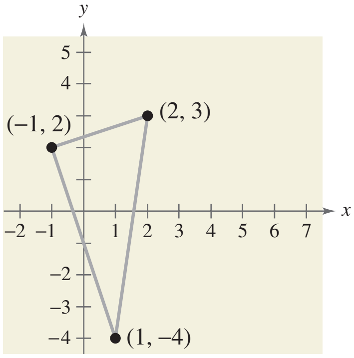



FIGURE 1.12

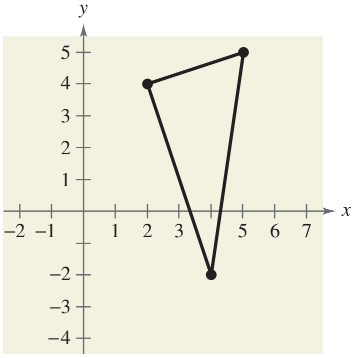



FIGURE 1.13

### Solution

To shift the vertices three units to the right, add 3 to each of the $x$-coordinates. To shift the vertices two units upward, add 2 to each of the $y$-coordinates.

| *Original Point* | *Translated Point* |
|---|---|
| $(-1, 2)$ | $(-1 + 3, 2 + 2) = (2, 4)$ |
| $(1, -4)$ | $(1 + 3, -4 + 2) = (4, -2)$ |
| $(2, 3)$ | $(2 + 3, 3 + 2) = (5, 5)$ |

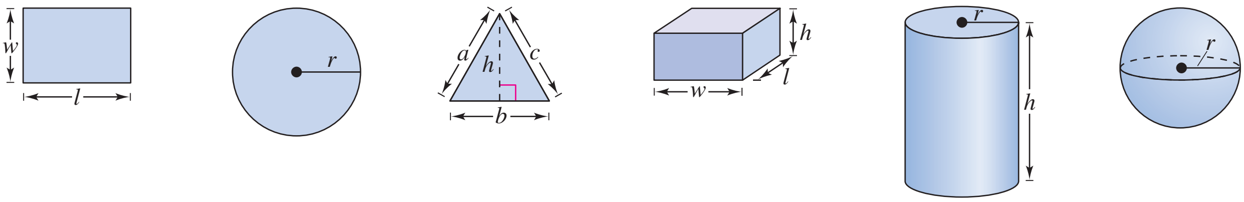
CHECKPOINT Now try Exercise 51.

The figures provided with Example 8 were not really essential to the solution. Nevertheless, it is strongly recommended that you develop the habit of including sketches with your solutions—even if they are not required.

The following geometric formulas are used at various times throughout this course. For your convenience, these formulas along with several others are also provided on the inside back cover of this text.

### Common Formulas for Area *A*, Perimeter *P*, Circumference *C*, and Volume *V*

| *Rectangle* | *Circle* | *Triangle* | *Rectangular Solid* | *Circular Cylinder* | *Sphere* |
|---|---|---|---|---|---|
| $A = lw$ | $A = \pi r^2$ | $A = \frac{1}{2}bh$ | $V = lwh$ | $V = \pi r^2 h$ | $V = \frac{4}{3}\pi r^3$ |
| $P = 2l + 2w$ | $C = 2\pi r$ | $P = a + b + c$ | | | |



## Example 9 Using a Geometric Formula

A cylindrical can has a volume of 200 cubic centimeters ($cm^3$) and a radius of 4 centimeters (cm), as shown in Figure 1.14. Find the height of the can.

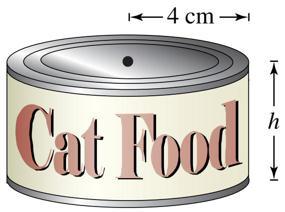



FIGURE 1.14

### Solution

The formula for the *volume of a cylinder* is $V = \pi r^2 h$. To find the height of the can, solve for $h$.

$$h = \frac{V}{\pi r^2}$$

Then, using $V = 200$ and $r = 4$, find the height.

$$h = \frac{200}{\pi(4)^2} \qquad \text{Substitute 200 for } V \text{ and 4 for } r.$$

$$= \frac{200}{16\pi} \qquad \text{Simplify denominator.}$$

$$\approx 3.98 \qquad \text{Use a calculator.}$$

Because the value of $h$ was rounded in the solution, a check of the solution will not result in an equality. If the solution is valid, the expressions on each side of the equal sign will be approximately equal to each other.

$$V = \pi r^2 h \qquad \text{Write original equation.}$$

$$200 \stackrel{?}{\approx} \pi(4)^2(3.98) \qquad \text{Substitute 200 for } V\text{, 4 for } r\text{, and 3.98 for } h.$$

$$200 \approx 200.06 \qquad \text{Solution checks. } \checkmark$$

You can also use unit analysis to check that your answer is reasonable.

$$\frac{200\ \cancel{cm^3}}{16\pi\ \cancel{cm^2}} \approx 3.98\ \text{cm}$$

CHECKPOINT Now try Exercise 63.

### WRITING ABOUT MATHEMATICS

**Extending the Example** Example 8 shows how to translate points in a coordinate plane. Write a short paragraph describing how each of the following transformed points is related to the original point.

| *Original Point* | *Transformed Point* |
|---|---|
| $(x, y)$ | $(-x, y)$ |
| $(x, y)$ | $(x, -y)$ |
| $(x, y)$ | $(-x, -y)$ |

## 1.1 Exercises

The *HM mathSpace*® CD-ROM and *Eduspace*® for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

### VOCABULARY CHECK

**1.** Match each term with its definition.

| | |
|---|---|
| (a) $x$-axis | (i) point of intersection of vertical axis and horizontal axis |
| (b) $y$-axis | (ii) directed distance from the $x$-axis |
| (c) origin | (iii) directed distance from the $y$-axis |
| (d) quadrants | (iv) four regions of the coordinate plane |
| (e) $x$-coordinate | (v) horizontal real number line |
| (f) $y$-coordinate | (vi) vertical real number line |

**In Exercises 2–4, fill in the blanks.**

**2.** An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the ________ plane.

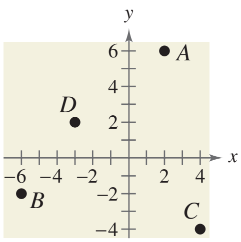
**3.** The ________ ________ is a result derived from the Pythagorean Theorem.

**4.** Finding the average values of the representative coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the ________ ________.

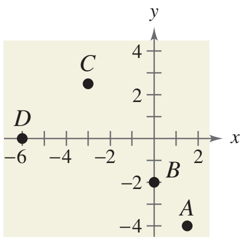
**PREREQUISITE SKILLS REVIEW:** Practice and review algebra skills needed for this section at **www.Eduspace.com.**

**In Exercises 1 and 2, approximate the coordinates of the points.**

**1.**




**2.**




**In Exercises 3–6, plot the points in the Cartesian plane.**

**3.** $(-4, 2), (-3, -6), (0, 5), (1, -4)$

**4.** $(0, 0), (3, 1), (-2, 4), (1, -1)$

**5.** $(3, 8), (0.5, -1), (5, -6), (-2, 2.5)$

**6.** $\left(1, -\frac{1}{3}\right), \left(\frac{3}{4}, 3\right), (-3, 4), \left(-\frac{4}{3}, -\frac{3}{2}\right)$

**In Exercises 7–10, find the coordinates of the point.**

**7.** The point is located three units to the left of the $y$-axis and four units above the $x$-axis.

**8.** The point is located eight units below the $x$-axis and four units to the right of the $y$-axis.

**9.** The point is located five units below the $x$-axis and the coordinates of the point are equal.

**10.** The point is on the $x$-axis and 12 units to the left of the $y$-axis.

**In Exercises 11–20, determine the quadrant(s) in which $(x, y)$ is located so that the condition(s) is (are) satisfied.**

**11.** $x > 0$ and $y < 0$

**12.** $x < 0$ and $y < 0$

**13.** $x = -4$ and $y > 0$

**14.** $x > 2$ and $y = 3$

**15.** $y < -5$

**16.** $x > 4$

**17.** $x < 0$ and $-y > 0$

**18.** $-x > 0$ and $y < 0$

**19.** $xy > 0$

**20.** $xy < 0$

**In Exercises 21 and 22, sketch a scatter plot of the data shown in the table.**

**21.** ***Number of Stores*** The table shows the number $y$ of Wal-Mart stores for each year $x$ from 1996 through 2003. (Source: Wal-Mart Stores, Inc.)

| Year, $x$ | Number of stores, $y$ |
|---|---|
| 1996 | 3054 |
| 1997 | 3406 |
| 1998 | 3599 |
| 1999 | 3985 |
| 2000 | 4189 |
| 2001 | 4414 |
| 2002 | 4688 |
| 2003 | 4906 |

**22.** ***Meteorology*** The table shows the lowest temperature on record $y$ (in degrees Fahrenheit) in Duluth, Minnesota for each month $x$, where $x = 1$ represents January. (Source: NOAA)

| Month, $x$ | Temperature, $y$ |
|---|---|
| 1 | $-39$ |
| 2 | $-39$ |
| 3 | $-29$ |
| 4 | $-5$ |
| 5 | 17 |
| 6 | 27 |
| 7 | 35 |
| 8 | 32 |
| 9 | 22 |
| 10 | 8 |
| 11 | $-23$ |
| 12 | $-34$ |

**In Exercises 23–26, find the distance between the points. (*Note:* In each case, the two points lie on the same horizontal or vertical line.)**

**23.** $(6, -3), (6, 5)$

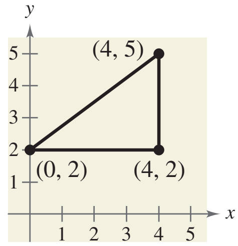
**24.** $(1, 4), (8, 4)$

**25.** $(-3, -1), (2, -1)$

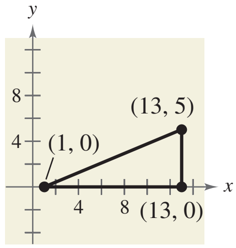
**26.** $(-3, -4), (-3, 6)$

**In Exercises 27–30, (a) find the length of each side of the right triangle, and (b) show that these lengths satisfy the Pythagorean Theorem.**

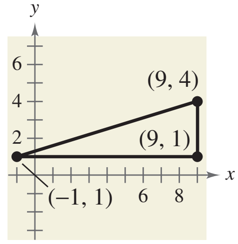
**27.**



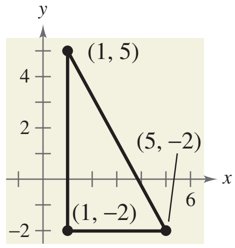

**28.**




**29.**




**30.**




**In Exercises 31–40, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.**

**31.** $(1, 1), (9, 7)$

**32.** $(1, 12), (6, 0)$

**33.** $(-4, 10), (4, -5)$

**34.** $(-7, -4), (2, 8)$

**35.** $(-1, 2), (5, 4)$

**36.** $(2, 10), (10, 2)$

**37.** $\left(\frac{1}{2}, 1\right), \left(-\frac{5}{2}, \frac{4}{3}\right)$

**38.** $\left(-\frac{1}{3}, -\frac{1}{3}\right), \left(-\frac{1}{6}, -\frac{1}{2}\right)$

**39.** $(6.2, 5.4), (-3.7, 1.8)$

**40.** $(-16.8, 12.3), (5.6, 4.9)$

**In Exercises 41 and 42, show that the points form the vertices of the indicated polygon.**

**41.** Right triangle: $(4, 0), (2, 1), (-1, -5)$

**42.** Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$

**43.** A line segment has $(x_1, y_1)$ as one endpoint and $(x_m, y_m)$ as its midpoint. Find the other endpoint $(x_2, y_2)$ of the line segment in terms of $x_1$, $y_1$, $x_m$, and $y_m$.

**44.** Use the result of Exercise 43 to find the coordinates of the endpoint of a line segment if the coordinates of the other endpoint and midpoint are, respectively,

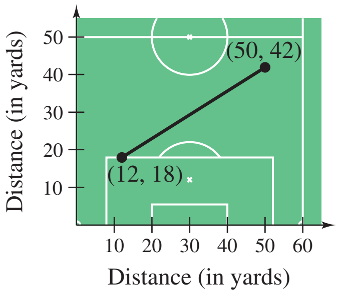
(a) $(1, -2), (4, -1)$ and (b) $(-5, 11), (2, 4)$.

**45.** Use the Midpoint Formula three times to find the three points that divide the line segment joining $(x_1, y_1)$ and $(x_2, y_2)$ into four parts.

**46.** Use the result of Exercise 45 to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$ (b) $(-2, -3), (0, 0)$

**47.** ***Sports*** A soccer player passes the ball from a point that is 18 yards from the endline and 12 yards from the sideline. The pass is received by a teammate who is 42 yards from the same endline and 50 yards from the same sideline, as shown in the figure. How long is the pass?




**48.** ***Flying Distance*** An airplane flies from Naples, Italy in a straight line to Rome, Italy, which is 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

***Sales*** **In Exercises 49 and 50, use the Midpoint Formula to estimate the sales of Big Lots, Inc. and Dollar Tree Stores, Inc. in 2002, given the sales in 2001 and 2003. Assume that the sales followed a linear pattern.** (Source: Big Lots, Inc.; Dollar Tree Stores, Inc.)

**49.** Big Lots

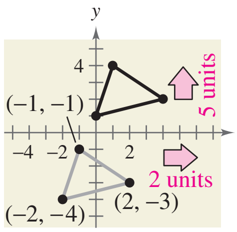
| Year | Sales (in millions) |
|---|---|
| 2001 | \$3433 |
| 2003 | \$4174 |

**50.** Dollar Tree

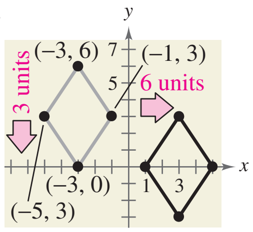
| Year | Sales (in millions) |
|---|---|
| 2001 | \$1987 |
| 2003 | \$2800 |

**In Exercises 51–54, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in its new position.**

**51.**




**52.**



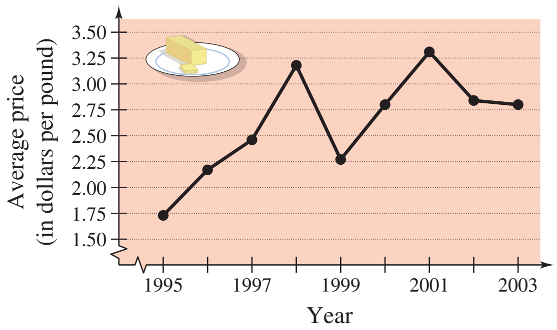

**53.** Original coordinates of vertices: $(-7, -2), (-2, 2)$, $(-2, -4)$, $(-7, -4)$

Shift: eight units upward, four units to the right

**54.** Original coordinates of vertices: $(5, 8)$, $(3, 6)$, $(7, 6)$, $(5, 2)$

Shift: 6 units downward, 10 units to the left

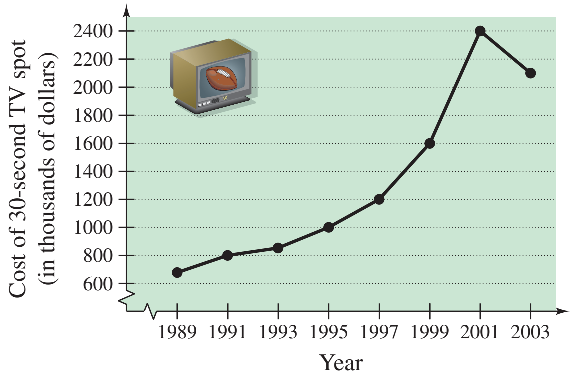
***Retail Price*** **In Exercises 55 and 56, use the graph below, which shows the average retail price of 1 pound of butter from 1995 to 2003.** (Source: U.S. Bureau of Labor Statistics)




**55.** Approximate the highest price of a pound of butter shown in the graph. When did this occur?

**56.** Approximate the percent change in the price of butter from the price in 1995 to the highest price shown in the graph.

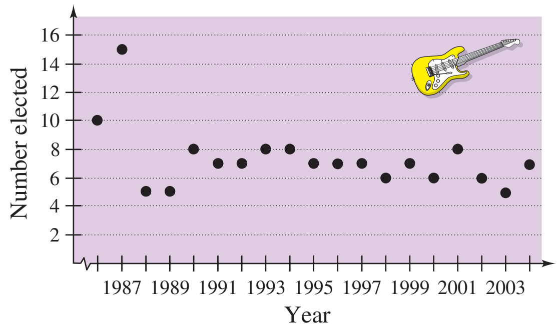
***Advertising*** **In Exercises 57 and 58, use the graph below, which shows the cost of a 30-second television spot (in thousands of dollars) during the Super Bowl from 1989 to 2003.** (Source: USA Today Research and CNN)




**57.** Approximate the percent increase in the cost of a 30-second spot from Super Bowl XXIII in 1989 to Super Bowl XXXV in 2001.

**58.** Estimate the percent increase in the cost of a 30-second spot (a) from Super Bowl XXIII in 1989 to Super Bowl XXVII in 1993 and (b) from Super Bowl XXVII in 1993 to Super Bowl XXXVII in 2003.

**59.** ***Music*** The graph shows the numbers of recording artists who were elected to the Rock and Roll Hall of Fame from 1986 to 2004.

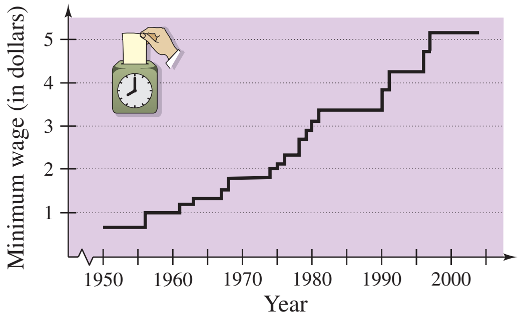



(a) Describe any trends in the data. From these trends, predict the number of artists elected in 2008.

(b) Why do you think the numbers elected in 1986 and 1987 were greater in other years?

## Model It

**60.** ***Labor Force*** Use the graph below, which shows the minimum wage in the United States (in dollars) from 1950 to 2004. (Source: U.S. Department of Labor)




(a) Which decade shows the greatest increase in minimum wage?

(b) Approximate the percent increases in the minimum wage from 1990 to 1995 and from 1995 to 2004.

(c) Use the percent increase from 1995 to 2004 to predict the minimum wage in 2008.

(d) Do you believe that your prediction in part (c) is reasonable? Explain.

**61.** ***Sales*** The Coca-Cola Company had sales of \$18,546 million in 1996 and \$21,900 million in 2004. Use the Midpoint Formula to estimate the sales in 1998, 2000, and 2002. Assume that the sales followed a linear pattern. (Source: The Coca-Cola Company)

**62.** ***Data Analysis: Exam Scores*** The table shows the mathematics entrance test scores $x$ and the final examination scores $y$ in an algebra course for a sample of 10 students.

| $x$ | 22 | 29 | 35 | 40 | 44 |
|---|---|---|---|---|---|
| $y$ | 53 | 74 | 57 | 66 | 79 |

| $x$ | 48 | 53 | 58 | 65 | 76 |
|---|---|---|---|---|---|
| $y$ | 90 | 76 | 93 | 83 | 99 |

(a) Sketch a scatter plot of the data.

(b) Find the entrance exam score of any student with a final exam score in the 80s.

(c) Does a higher entrance exam score imply a higher final exam score? Explain.

**63.** ***Volume of a Billiard Ball*** A billiard ball has a volume of 5.96 cubic inches. Find the radius of a billiard ball.

**64.** ***Length of a Tank*** The diameter of a cylindrical propane gas tank is 4 feet. The total volume of the tank is 603.2 cubic feet. Find the length of the tank.

**65.** ***Geometry*** A "Slow Moving Vehicle" sign has the shape of an equilateral triangle. The sign has a perimeter of 129 centimeters. Find the length of each side of the sign. Find the area of the sign.

**66.** ***Geometry*** The radius of a traffic cone is 14 centimeters and the lateral surface of the cone is 1617 square centimeters. Find the height of the cone.

**67.** ***Dimensions of a Room*** A room is 1.5 times as long as it is wide, and its perimeter is 25 meters.

(a) Draw a diagram that represents the problem. Identify the length as $l$ and the width as $w$.

(b) Write $l$ in terms of $w$ and write an equation for the perimeter in terms of $w$.

(c) Find the dimensions of the room.

**68.** ***Dimensions of a Container*** The width of a rectangular storage container is 1.25 times its height. The length of the container is 16 inches and the volume of the container is 2000 cubic inches.

(a) Draw a diagram that represents the problem. Label the height, width, and length accordingly.

(b) Write $w$ in terms of $h$ and write an equation for the volume in terms of $h$.

(c) Find the dimensions of the container.

**69.** ***Data Analysis: Mail*** The table shows the number $y$ of pieces of mail handled (in billions) by the U.S. Postal Service for each year $x$ from 1996 through 2003. (Source: U.S. Postal Service)

| Year, $x$ | Pieces of mail, $y$ |
|---|---|
| 1996 | 183 |
| 1997 | 191 |
| 1998 | 197 |
| 1999 | 202 |
| 2000 | 208 |
| 2001 | 207 |
| 2002 | 203 |
| 2003 | 202 |

(a) Sketch a scatter plot of the data.

(b) Approximate the year in which there was the greatest decrease in the number of pieces of mail handled.

(c) Why do you think the number of pieces of mail handled decreased?

**70.** ***Data Analysis: Athletics*** The table shows the numbers of men's $M$ and women's $W$ college basketball teams for each year $x$ from 1994 through 2003. (Source: National Collegiate Athletic Association)

| Year, $x$ | Men's teams, $M$ | Women's teams, $W$ |
|---|---|---|
| 1994 | 858 | 859 |
| 1995 | 868 | 864 |
| 1996 | 866 | 874 |
| 1997 | 865 | 879 |
| 1998 | 895 | 911 |
| 1999 | 926 | 940 |
| 2000 | 932 | 956 |
| 2001 | 937 | 958 |
| 2002 | 936 | 975 |
| 2003 | 967 | 1009 |

(a) Sketch scatter plots of these two sets of data on the same set of coordinate axes.

(b) Find the year in which the numbers of men's and women's teams were nearly equal.

(c) Find the year in which the difference between the numbers of men's and women's teams was the greatest. What was this difference?

**71.** ***Make a Conjecture*** Plot the points $(2, 1)$, $(-3, 5)$, and $(7, -3)$ on a rectangular coordinate system. Then change the sign of the $x$-coordinate of each point and plot the three new points on the same rectangular coordinate system. Make a conjecture about the location of a point when each of the following occurs.

(a) The sign of the $x$-coordinate is changed.

(b) The sign of the $y$-coordinate is changed.

(c) The signs of both the $x$- and $y$-coordinates are changed.

**72.** ***Collinear Points*** Three or more points are *collinear* if they all lie on the same line. Use the steps below to determine if the set of points $\{A(2, 3), B(2, 6), C(6, 3)\}$ and the set of points $\{A(8, 3), B(5, 2), C(2, 1)\}$ are collinear.

(a) For each set of points, use the Distance Formula to find the distances from $A$ to $B$, from $B$ to $C$, and from $A$ to $C$. What relationship exists among these distances for each set of points?

(b) Plot each set of points in the Cartesian plane. Do all the points of either set appear to lie on the same line?

(c) Compare your conclusions from part (a) with the conclusions you made from the graphs in part (b). Make a general statement about how to use the Distance Formula to determine collinearity.

## Synthesis

***True or False?*** **In Exercises 73 and 74, determine whether the statement is true or false. Justify your answer.**

**73.** In order to divide a line segment into 16 equal parts, you would have to use the Midpoint Formula 16 times.

**74.** The points $(-8, 4)$, $(2, 11)$, and $(-5, 1)$ represent the vertices of an isosceles triangle.

**75.** ***Think About It*** When plotting points on the rectangular coordinate system, is it true that the scales on the $x$- and $y$-axes must be the same? Explain.

**76.** ***Proof*** Prove that the diagonals of the parallelogram in the figure intersect at their midpoints.

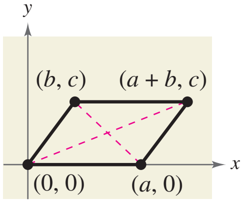



FIGURE FOR 76

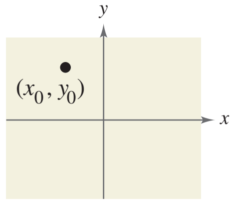
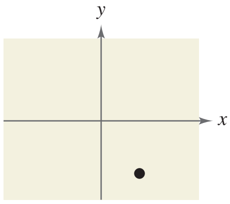


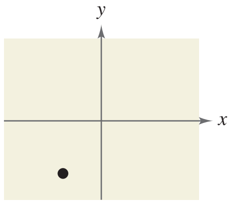

FIGURE FOR 77–80

**In Exercises 77–80, use the plot of the point $(x_0, y_0)$ in the figure. Match the transformation of the point with the correct plot. [The plots are labeled (a), (b), (c), and (d).]**

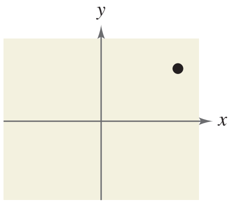
(a)



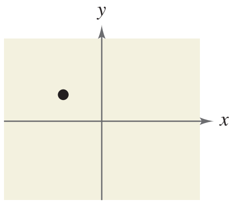

(b)




(c)




(d)




**77.** $(x_0, -y_0)$

**78.** $(-2x_0, y_0)$

**79.** $\left(x_0, \frac{1}{2}y_0\right)$

**80.** $(-x_0, -y_0)$

## Skills Review

**In Exercises 81–88, solve the equation or inequality.**

**81.** $2x + 1 = 7x - 4$

**82.** $\frac{1}{3}x + 2 = 5 - \frac{1}{6}x$

**83.** $x^2 - 4x - 7 = 0$

**84.** $2x^2 + 3x - 8 = 0$

**85.** $3x + 1 < 2(2 - x)$

**86.** $3x - 8 \geq \frac{1}{2}(10x + 7)$

**87.** $|x - 18| < 4$

**88.** $|2x + 15| \geq 11$

# 1.2 Graphs of Equations

### What you should learn

- Sketch graphs of equations.
- Find $x$- and $y$-intercepts of graphs of equations.
- Use symmetry to sketch graphs of equations.
- Find equations of and sketch graphs of circles.
- Use graphs of equations in solving real-life problems.

### Why you should learn it

The graph of an equation can help you see relationships between real-life quantities. For example, in Exercise 75 on page 24, a graph can be used to estimate the life expectancies of children who are born in the years 2005 and 2010.



## The Graph of an Equation

In Section 1.1, you used a coordinate system to represent graphically the relationship between two quantities. There, the graphical picture consisted of a collection of points in a coordinate plane.

Frequently, a relationship between two quantities is expressed as an **equation in two variables.** For instance, $y = 7 - 3x$ is an equation in $x$ and $y$. An ordered pair $(a, b)$ is a **solution** or **solution point** of an equation in $x$ and $y$ if the equation is true when $a$ is substituted for $x$ and $b$ is substituted for $y$. For instance, $(1, 4)$ is a solution of $y = 7 - 3x$ because $4 = 7 - 3(1)$ is a true statement.

In this section you will review some basic procedures for sketching the graph of an equation in two variables. The **graph of an equation** is the set of all points that are solutions of the equation.

### Example 1 Determining Solutions

Determine whether (a) $(2, 13)$ and (b) $(-1, -3)$ are solutions of the equation $y = 10x - 7$.

#### Solution

**a.**

$y = 10x - 7$ Write original equation.

$13 \stackrel{?}{=} 10(2) - 7$ Substitute 2 for $x$ and 13 for $y$.

$13 = 13$ $(2, 13)$ is a solution. ✓

Because the substitution does satisfy the original equation, you can conclude that the ordered pair $(2, 13)$ *is* a solution of the original equation.

**b.**

$y = 10x - 7$ Write original equation.

$-3 \stackrel{?}{=} 10(-1) - 7$ Substitute $-1$ for $x$ and $-3$ for $y$.

$-3 \neq -17$ $(-1, -3)$ is not a solution.

Because the substitution does not satisfy the original equation, you can conclude that the ordered pair $(-1, -3)$ *is not* a solution of the original equation.

✓CHECKPOINT Now try Exercise 1.

The basic technique used for sketching the graph of an equation is the **point-plotting method.**

#### Sketching the Graph of an Equation by Point Plotting

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

## Example 2 Sketching the Graph of an Equation

Sketch the graph of

$$y = 7 - 3x.$$

### Solution

Because the equation is already solved for $y$, construct a table of values that consists of several solution points of the equation. For instance, when $x = -1$,

$$\begin{aligned} y &= 7 - 3(-1) \\ &= 10 \end{aligned}$$

which implies that $(-1, 10)$ is a solution point of the graph.

| $x$ | $y = 7 - 3x$ | $(x, y)$ |
|---|---|---|
| $-1$ | 10 | $(-1, 10)$ |
| 0 | 7 | $(0, 7)$ |
| 1 | 4 | $(1, 4)$ |
| 2 | 1 | $(2, 1)$ |
| 3 | $-2$ | $(3, -2)$ |
| 4 | $-5$ | $(4, -5)$ |

From the table, it follows that

$$(-1, 10), (0, 7), (1, 4), (2, 1), (3, -2), \text{ and } (4, -5)$$

are solution points of the equation. After plotting these points, you can see that they appear to lie on a line, as shown in Figure 1.15. The graph of the equation is the line that passes through the six plotted points.

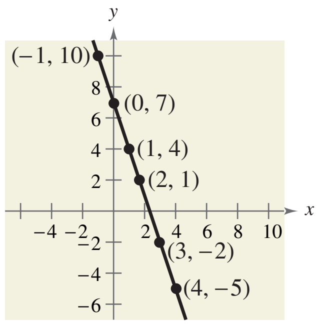



FIGURE 1.15

CHECKPOINT Now try Exercise 5.

## Example 3 Sketching the Graph of an Equation

Sketch the graph of

$$y = x^2 - 2.$$

### Solution

Because the equation is already solved for $y$, begin by constructing a table of values.

| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $y = x^2 - 2$ | $2$ | $-1$ | $-2$ | $-1$ | $2$ | $7$ |
| $(x, y)$ | $(-2, 2)$ | $(-1, -1)$ | $(0, -2)$ | $(1, -1)$ | $(2, 2)$ | $(3, 7)$ |

**STUDY TIP**

One of your goals in this course is to learn to classify the basic shape of a graph from its equation. For instance, you will learn that the *linear equation* in Example 2 has the form

$$y = mx + b$$

and its graph is a line. Similarly, the *quadratic equation* in Example 3 has the form

$$y = ax^2 + bx + c$$

and its graph is a parabola.

Next, plot the points given in the table, as shown in Figure 1.16. Finally, connect the points with a smooth curve, as shown in Figure 1.17.

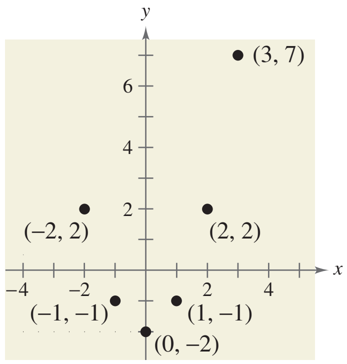



FIGURE 1.16

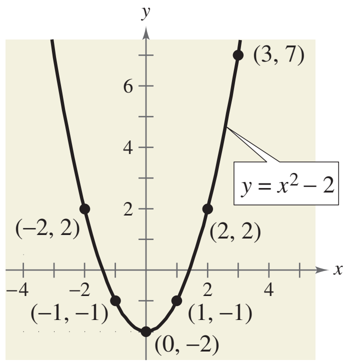



FIGURE 1.17

CHECKPOINT Now try Exercise 7.

The point-plotting method demonstrated in Examples 2 and 3 is easy to use, but it has some shortcomings. With too few solution points, you can misrepresent the graph of an equation. For instance, if only the four points

$$(-2, 2), (-1, -1), (1, -1), \text{ and } (2, 2)$$

in Figure 1.16 were plotted, any one of the three graphs in Figure 1.18 would be reasonable.

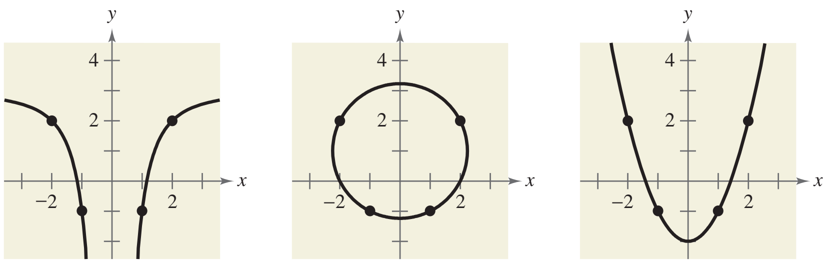


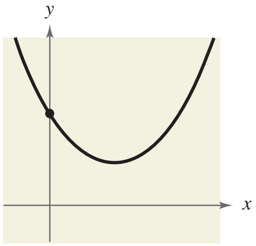

FIGURE 1.18

## Technology

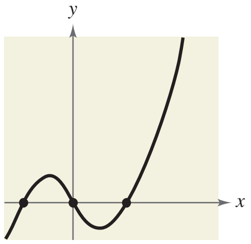
To graph an equation involving *x* and *y* on a graphing utility, use the following procedure.

1. Rewrite the equation so that *y* is isolated on the left side.
2. Enter the equation into the graphing utility.
3. Determine a *viewing window* that shows all important features of the graph.
4. Graph the equation.

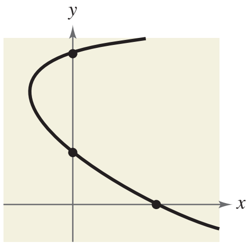
For more extensive instructions on how to use a graphing utility to graph an equation, see the *Graphing Technology Guide* on the text website at *college.hmco.com*.



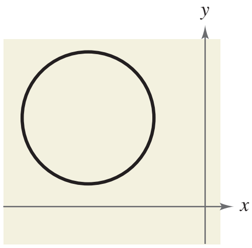

*No x-intercepts; one y-intercept*




*Three x-intercepts; one y-intercept*




*One x-intercept; two y-intercepts*




*No intercepts*

FIGURE 1.19

## Intercepts of a Graph

It is often easy to determine the solution points that have zero as either the $x$-coordinate or the $y$-coordinate. These points are called **intercepts** because they are the points at which the graph intersects or touches the $x$- or $y$-axis. It is possible for a graph to have no intercepts, one intercept, or several intercepts, as shown in Figure 1.19.

Note that an $x$-intercept can be written as the ordered pair $(x, 0)$ and a $y$-intercept can be written as the ordered pair $(0, y)$. Some texts denote the $x$-intercept as the $x$-coordinate of the point $(a, 0)$ [and the $y$-intercept as the $y$-coordinate of the point $(0, b)$] rather than the point itself. Unless it is necessary to make a distinction, we will use the term *intercept* to mean either the point or the coordinate.

### Finding Intercepts

1. To find $x$-intercepts, let $y$ be zero and solve the equation for $x$.
2. To find $y$-intercepts, let $x$ be zero and solve the equation for $y$.

### Example 4 Finding *x*- and *y*-Intercepts

Find the $x$- and $y$-intercepts of the graph of $y = x^3 - 4x$.

**Solution**

Let $y = 0$. Then

$$0 = x^3 - 4x = x(x^2 - 4)$$

has solutions $x = 0$ and $x = \pm 2$.

$$x\text{-intercepts: } (0, 0), (2, 0), (-2, 0)$$

Let $x = 0$. Then

$$y = (0)^3 - 4(0)$$

has one solution, $y = 0$.

$$y\text{-intercept: } (0, 0)$$ See Figure 1.20.

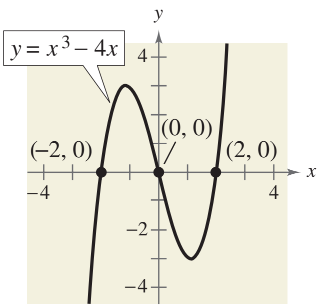
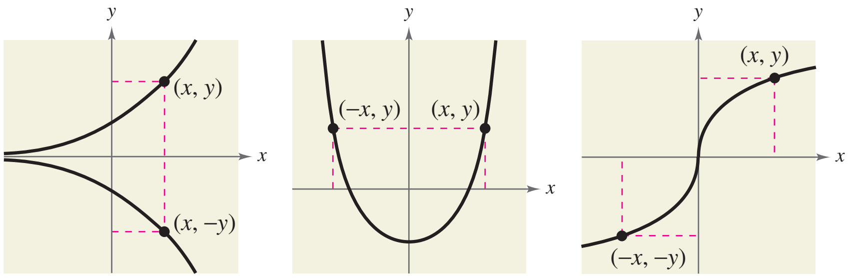



FIGURE 1.20

CHECKPOINT Now try Exercise 11.

## Symmetry

Graphs of equations can have **symmetry** with respect to one of the coordinate axes or with respect to the origin. Symmetry with respect to the $x$-axis means that if the Cartesian plane were folded along the $x$-axis, the portion of the graph above the $x$-axis would coincide with the portion below the $x$-axis. Symmetry with respect to the $y$-axis or the origin can be described in a similar manner, as shown in Figure 1.21.




*x-axis symmetry* *y-axis symmetry* *Origin symmetry*

FIGURE 1.21

Knowing the symmetry of a graph *before* attempting to sketch it is helpful, because then you need only half as many solution points to sketch the graph. There are three basic types of symmetry, described as follows.

### Graphical Tests for Symmetry

1. A graph is **symmetric with respect to the *x*-axis** if, whenever $(x, y)$ is on the graph, $(x, -y)$ is also on the graph.
2. A graph is **symmetric with respect to the *y*-axis** if, whenever $(x, y)$ is on the graph, $(-x, y)$ is also on the graph.
3. A graph is **symmetric with respect to the origin** if, whenever $(x, y)$ is on the graph, $(-x, -y)$ is also on the graph.

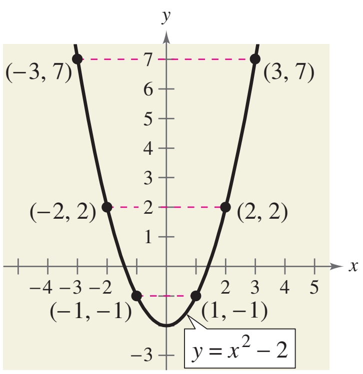



FIGURE 1.22 *y-axis symmetry*

### Example 5 Testing for Symmetry

The graph of $y = x^2 - 2$ is symmetric with respect to the $y$-axis because the point $(-x, y)$ is also on the graph of $y = x^2 - 2$. (See Figure 1.22.) The table below confirms that the graph is symmetric with respect to the $y$-axis.

| $x$ | $-3$ | $-2$ | $-1$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|---|
| $y$ | $7$ | $2$ | $-1$ | $-1$ | $2$ | $7$ |
| $(x, y)$ | $(-3, 7)$ | $(-2, 2)$ | $(-1, -1)$ | $(1, -1)$ | $(2, 2)$ | $(3, 7)$ |

CHECKPOINT Now try Exercise 23.

## Algebraic Tests for Symmetry

1. The graph of an equation is symmetric with respect to the $x$-axis if replacing $y$ with $-y$ yields an equivalent equation.
2. The graph of an equation is symmetric with respect to the $y$-axis if replacing $x$ with $-x$ yields an equivalent equation.
3. The graph of an equation is symmetric with respect to the origin if replacing $x$ with $-x$ and $y$ with $-y$ yields an equivalent equation.

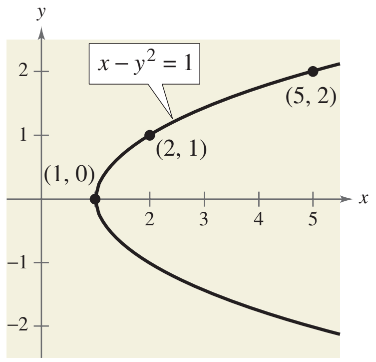



FIGURE 1.23

### Example 6 Using Symmetry as a Sketching Aid

Use symmetry to sketch the graph of

$$x - y^2 = 1.$$

#### Solution

Of the three tests for symmetry, the only one that is satisfied is the test for $x$-axis symmetry because $x - (-y)^2 = 1$ is equivalent to $x - y^2 = 1$. So, the graph is symmetric with respect to the $x$-axis. Using symmetry, you only need to find the solution points above the $x$-axis and then reflect them to obtain the graph, as shown in Figure 1.23.

| $y$ | $x = y^2 + 1$ | $(x, y)$ |
|---|---|---|
| 0 | 1 | (1, 0) |
| 1 | 2 | (2, 1) |
| 2 | 5 | (5, 2) |

CHECKPOINT Now try Exercise 37.

**STUDY TIP**

Notice that when creating the table in Example 6, it is easier to choose $y$-values and then find the corresponding $x$-values of the ordered pairs.

### Example 7 Sketching the Graph of an Equation

Sketch the graph of

$$y = |x - 1|.$$

#### Solution

This equation fails all three tests for symmetry and consequently its graph is not symmetric with respect to either axis or to the origin. The absolute value sign indicates that $y$ is always nonnegative. Create a table of values and plot the points as shown in Figure 1.24. From the table, you can see that $x = 0$ when $y = 1$. So, the $y$-intercept is $(0, 1)$. Similarly, $y = 0$ when $x = 1$. So, the $x$-intercept is $(1, 0)$.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|
| $y = \|x - 1\|$ | 3 | 2 | 1 | 0 | 1 | 2 | 3 |
| $(x, y)$ | $(-2, 3)$ | $(-1, 2)$ | (0, 1) | (1, 0) | (2, 1) | (3, 2) | (4, 3) |

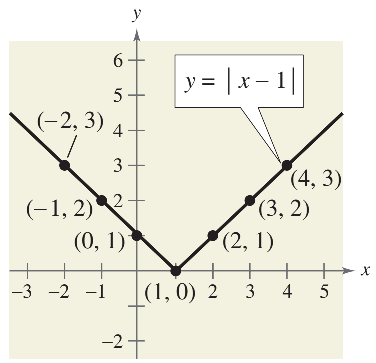



FIGURE 1.24

CHECKPOINT Now try Exercise 41.

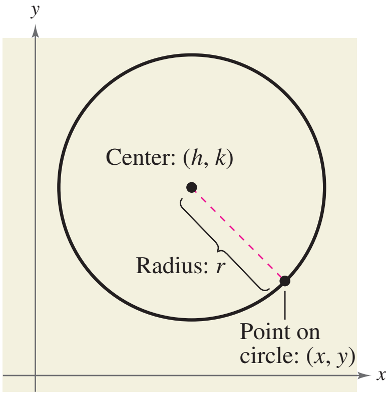



FIGURE 1.25

Throughout this course, you will learn to recognize several types of graphs from their equations. For instance, you will learn to recognize that the graph of a second-degree equation of the form

$$y = ax^2 + bx + c$$

is a parabola (see Example 3). The graph of a **circle** is also easy to recognize.

## Circles

Consider the circle shown in Figure 1.25. A point $(x, y)$ is on the circle if and only if its distance from the center $(h, k)$ is $r$. By the Distance Formula,

$$\sqrt{(x-h)^2 + (y-k)^2} = r.$$

By squaring each side of this equation, you obtain the **standard form of the equation of a circle.**

### Standard Form of the Equation of a Circle

The point $(x, y)$ lies on the circle of **radius** $r$ and **center** $(h, k)$ if and only if

$$(x-h)^2 + (y-k)^2 = r^2.$$

From this result, you can see that the standard form of the equation of a circle *with its center at the origin,* $(h, k) = (0, 0)$, is simply

$$x^2 + y^2 = r^2. \qquad \text{Circle with center at origin}$$

### STUDY TIP

To find the correct $h$ and $k$, from the equation of the circle in Example 8, it may be helpful to rewrite the quantities $(x + 1)^2$ and $(y - 2)^2$, using subtraction.

$$(x+1)^2 = [x - (-1)]^2,$$

$$(y-2)^2 = [y - (2)]^2$$

So, $h = -1$ and $k = 2$.

### Example 8 Finding the Equation of a Circle

The point $(3, 4)$ lies on a circle whose center is at $(-1, 2)$, as shown in Figure 1.26. Write the standard form of the equation of this circle.

**Solution**

The radius of the circle is the distance between $(-1, 2)$ and $(3, 4)$.

$$\begin{aligned} r &= \sqrt{(x-h)^2 + (y-k)^2} && \text{Distance Formula} \\ &= \sqrt{[3-(-1)]^2 + (4-2)^2} && \text{Substitute for } x, y, h, \text{ and } k. \\ &= \sqrt{4^2 + 2^2} && \text{Simplify.} \\ &= \sqrt{16 + 4} && \text{Simplify.} \\ &= \sqrt{20} && \text{Radius} \end{aligned}$$

Using $(h, k) = (-1, 2)$ and $r = \sqrt{20}$, the equation of the circle is

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 && \text{Equation of circle} \\ [x-(-1)]^2 + (y-2)^2 &= \left(\sqrt{20}\right)^2 && \text{Substitute for } h, k, \text{ and } r. \\ (x+1)^2 + (y-2)^2 &= 20. && \text{Standard form} \end{aligned}$$

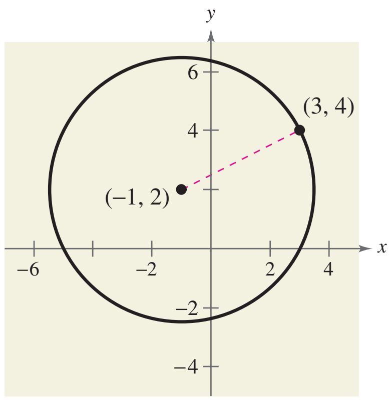



FIGURE 1.26

CHECKPOINT Now try Exercise 61.

**STUDY TIP**

You should develop the habit of using at least two approaches to solve every problem. This helps build your intuition and helps you check that your answer is reasonable.

## Application

In this course, you will learn that there are many ways to approach a problem. Three common approaches are illustrated in Example 9.

A *Numerical Approach:* Construct and use a table.
A *Graphical Approach:* Draw and use a graph.
An *Algebraic Approach:* Use the rules of algebra.

### Example 9 Recommended Weight

The median recommended weight $y$ (in pounds) for men of medium frame who are 25 to 59 years old can be approximated by the mathematical model

$$y = 0.073x^2 - 6.99x + 289.0, \quad 62 \le x \le 76$$

where $x$ is the man's height (in inches). (Source: Metropolitan Life Insurance Company)

**a.** Construct a table of values that shows the median recommended weights for men with heights of 62, 64, 66, 68, 70, 72, 74, and 76 inches.

**b.** Use the table of values to sketch a graph of the model. Then use the graph to estimate *graphically* the median recommended weight for a man whose height is 71 inches.

**c.** Use the model to confirm *algebraically* the estimate you found in part (b).

### Solution

**a.** You can use a calculator to complete the table, as shown at the left.

| Height, $x$ | Weight, $y$ |
|---|---|
| 62 | 136.2 |
| 64 | 140.6 |
| 66 | 145.6 |
| 68 | 151.2 |
| 70 | 157.4 |
| 72 | 164.2 |
| 74 | 171.5 |
| 76 | 179.4 |

**b.** The table of values can be used to sketch the graph of the equation, as shown in Figure 1.27. From the graph, you can estimate that a height of 71 inches corresponds to a weight of about 161 pounds.

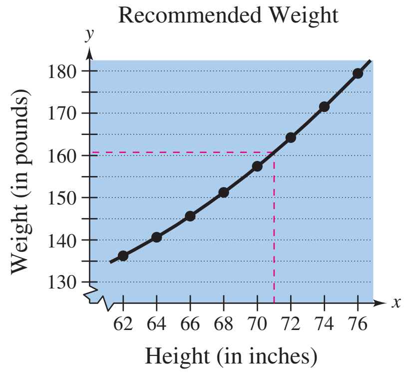



FIGURE 1.27

**c.** To confirm algebraically the estimate found in part (b), you can substitute 71 for $x$ in the model.

$$y = 0.073(71)^2 - 6.99(71) + 289.0 \approx 160.70$$

So, the graphical estimate of 161 pounds is fairly good.

CHECKPOINT Now try Exercise 75.

# 1.2 Exercises

**VOCABULARY CHECK:** Fill in the blanks.

1. An ordered pair $(a, b)$ is a ________ of an equation in $x$ and $y$ if the equation is true when $a$ is substituted for $x$ and $b$ is substituted for $y$.
2. The set of all solution points of an equation is the ________ of the equation.
3. The points at which a graph intersects or touches an axis are called the ________ of the graph.
4. A graph is symmetric with respect to the ________ if, whenever $(x, y)$ is on the graph, $(-x, y)$ is also on the graph.
5. The equation $(x - h)^2 + (y - k)^2 = r^2$ is the standard form of the equation of a ________ with center ________ and radius ________.
6. When you construct and use a table to solve a problem, you are using a ________ approach.

**PREREQUISITE SKILLS REVIEW:** Practice and review algebra skills needed for this section at **www.Eduspace.com.**

**In Exercises 1–4, determine whether each point lies on the graph of the equation.**

| | *Equation* | *Points* | |
|---|---|---|---|
| **1.** | $y = \sqrt{x + 4}$ | (a) $(0, 2)$ | (b) $(5, 3)$ |
| **2.** | $y = x^2 - 3x + 2$ | (a) $(2, 0)$ | (b) $(-2, 8)$ |
| **3.** | $y = 4 - \lvert x - 2 \rvert$ | (a) $(1, 5)$ | (b) $(6, 0)$ |
| **4.** | $y = \frac{1}{3}x^3 - 2x^2$ | (a) $\left(2, -\frac{16}{3}\right)$ | (b) $(-3, 9)$ |

**In Exercises 5–8, complete the table. Use the resulting solution points to sketch the graph of the equation.**

**5.** $y = -2x + 5$

| $x$ | $-1$ | $0$ | $1$ | $2$ | $\frac{5}{2}$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $(x, y)$ | | | | | |

**6.** $y = \frac{3}{4}x - 1$

| $x$ | $-2$ | $0$ | $1$ | $\frac{4}{3}$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $(x, y)$ | | | | | |

**7.** $y = x^2 - 3x$

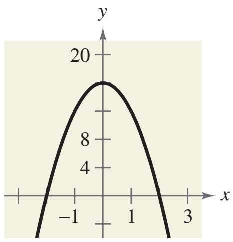
| $x$ | $-1$ | $0$ | $1$ | $2$ | $3$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $(x, y)$ | | | | | |

**8.** $y = 5 - x^2$

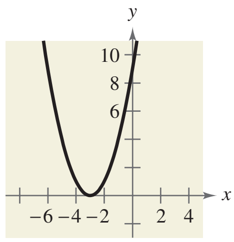
| $x$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|
| $y$ | | | | | |
| $(x, y)$ | | | | | |

**In Exercises 9–20, find the *x*- and *y*-intercepts of the graph of the equation.**

**9.** $y = 16 - 4x^2$




**10.** $y = (x + 3)^2$




**11.** $y = 5x - 6$

**12.** $y = 8 - 3x$

**13.** $y = \sqrt{x + 4}$

**14.** $y = \sqrt{2x - 1}$

**15.** $y = \lvert 3x - 7 \rvert$

**16.** $y = -\lvert x + 10 \rvert$

**17.** $y = 2x^3 - 4x^2$

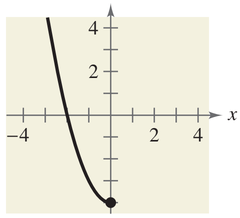
**18.** $y = x^4 - 25$

**19.** $y^2 = 6 - x$

**20.** $y^2 = x + 1$

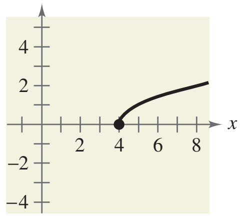
**In Exercises 21–24, assume that the graph has the indicated type of symmetry. Sketch the complete graph of the equation. To print an enlarged copy of the graph, go to the website *www.mathgraphs.com*.**

**21.**



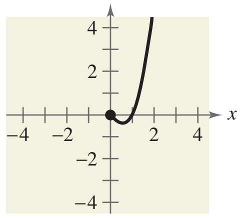

$y$-axis symmetry

**22.**



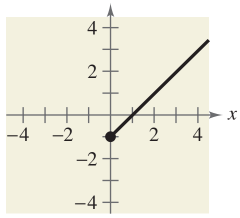

$x$-axis symmetry

**23.**




Origin symmetry

**24.**




$y$-axis symmetry

**In Exercises 25–32, use the algebraic tests to check for symmetry with respect to both axes and the origin.**

**25.** $x^2 - y = 0$ **26.** $x - y^2 = 0$

**27.** $y = x^3$ **28.** $y = x^4 - x^2 + 3$

**29.** $y = \dfrac{x}{x^2 + 1}$ **30.** $y = \dfrac{1}{x^2 + 1}$

**31.** $xy^2 + 10 = 0$ **32.** $xy = 4$

**In Exercises 33–44, use symmetry to sketch the graph of the equation.**

**33.** $y = -3x + 1$ **34.** $y = 2x - 3$

**35.** $y = x^2 - 2x$ **36.** $y = -x^2 - 2x$

**37.** $y = x^3 + 3$ **38.** $y = x^3 - 1$

**39.** $y = \sqrt{x - 3}$ **40.** $y = \sqrt{1 - x}$

**41.** $y = |x - 6|$ **42.** $y = 1 - |x|$

**43.** $x = y^2 - 1$ **44.** $x = y^2 - 5$

**In Exercises 45–56, use a graphing utility to graph the equation. Use a standard setting. Approximate any intercepts.**

**45.** $y = 3 - \frac{1}{2}x$ **46.** $y = \frac{2}{3}x - 1$

**47.** $y = x^2 - 4x + 3$ **48.** $y = x^2 + x - 2$

**49.** $y = \dfrac{2x}{x - 1}$ **50.** $y = \dfrac{4}{x^2 + 1}$

**51.** $y = \sqrt[3]{x}$ **52.** $y = \sqrt[3]{x + 1}$

**53.** $y = x\sqrt{x + 6}$ **54.** $y = (6 - x)\sqrt{x}$

**55.** $y = |x + 3|$ **56.** $y = 2 - |x|$

**In Exercises 57–64, write the standard form of the equation of the circle with the given characteristics.**

**57.** Center: $(0, 0)$; radius: 4 **58.** Center: $(0, 0)$; radius: 5

**59.** Center: $(2, -1)$; radius: 4

**60.** Center: $(-7, -4)$; radius: 7

**61.** Center: $(-1, 2)$; solution point: $(0, 0)$

**62.** Center: $(3, -2)$; solution point: $(-1, 1)$

**63.** Endpoints of a diameter: $(0, 0)$, $(6, 8)$

**64.** Endpoints of a diameter: $(-4, -1)$, $(4, 1)$

**In Exercises 65–70, find the center and radius of the circle, and sketch its graph.**

**65.** $x^2 + y^2 = 25$ **66.** $x^2 + y^2 = 16$

**67.** $(x - 1)^2 + (y + 3)^2 = 9$

**68.** $x^2 + (y - 1)^2 = 1$

**69.** $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

**70.** $(x - 2)^2 + (y + 3)^2 = \frac{16}{9}$

**71.** ***Depreciation*** A manufacturing plant purchases a new molding machine for \$225,000. The depreciated value $y$ (reduced value) after $t$ years is given by $y = 225{,}000 - 20{,}000t$, $0 \le t \le 8$. Sketch the graph of the equation.

**72.** ***Consumerism*** You purchase a jet ski for \$8100. The depreciated value $y$ after $t$ years is given by $y = 8100 - 929t$, $0 \le t \le 6$. Sketch the graph of the equation.

**73.** ***Geometry*** A regulation NFL playing field (including the end zones) of length $x$ and width $y$ has a perimeter of $346\frac{2}{3}$ or $\dfrac{1040}{3}$ yards.

(a) Draw a rectangle that gives a visual representation of the problem. Use the specified variables to label the sides of the rectangle.

(b) Show that the width of the rectangle is $y = \dfrac{520}{3} - x$ and its area is $A = x\left(\dfrac{520}{3} - x\right)$.

(c) Use a graphing utility to graph the area equation. Be sure to adjust your window settings.

(d) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

(e) Use your school's library, the Internet, or some other reference source to find the actual dimensions and area of a regulation NFL playing field and compare your findings with the results of part (d).

The symbol indicates an exercise or a part of an exercise in which you are instructed to use a graphing utility.

**74.** ***Geometry*** A soccer playing field of length $x$ and width $y$ has a perimeter of 360 meters.

(a) Draw a rectangle that gives a visual representation of the problem. Use the specified variables to label the sides of the rectangle.

(b) Show that the width of the rectangle is $w = 180 - x$ and its area is $A = x(180 - x)$.

(c) Use a graphing utility to graph the area equation. Be sure to adjust your window settings.

(d) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

(e) Use your school's library, the Internet, or some other reference source to find the actual dimensions and area of a regulation Major League Soccer field and compare your findings with the results of part(d).

## Model It

**75.** ***Population Statistics*** The table shows the life expectancies of a child (at birth) in the United States for selected years from 1920 to 2000. (Source: U.S. National Center for Health Statistics)

| Year | Life expectancy, $y$ |
|---|---|
| 1920 | 54.1 |
| 1930 | 59.7 |
| 1940 | 62.9 |
| 1950 | 68.2 |
| 1960 | 69.7 |
| 1970 | 70.8 |
| 1980 | 73.7 |
| 1990 | 75.4 |
| 2000 | 77.0 |

A model for the life expectancy during this period is

$$y = -0.0025t^2 + 0.574t + 44.25, \quad 20 \le t \le 100$$

where $y$ represents the life expectancy and $t$ is the time in years, with $t = 20$ corresponding to 1920.

(a) Sketch a scatter plot of the data.

(b) Graph the model for the data and compare the scatter plot and the graph.

(c) Determine the life expectancy in 1948 both graphically and algebraically.

(d) Use the graph of the model to estimate the life expectancies of a child for the years 2005 and 2010.

(e) Do you think this model can be used to predict the life expectancy of a child 50 years from now? Explain.

**76.** ***Electronics*** The resistance $y$ (in ohms) of 1000 feet of solid copper wire at 68 degrees Fahrenheit can be approximated by the model $y = \dfrac{10,770}{x^2} - 0.37, \; 5 \le x \le 100$ where $x$ is the diameter of the wire in mils (0.001 inch). (Source: American Wire Gage)

(a) Complete the table.

| $x$ | 5 | 10 | 20 | 30 | 40 | 50 |
|---|---|---|---|---|---|---|
| $y$ | | | | | | |

| $x$ | 60 | 70 | 80 | 90 | 100 |
|---|---|---|---|---|---|
| $y$ | | | | | |

(b) Use the table of values in part (a) to sketch a graph of the model. Then use your graph to estimate the resistance when $x = 85.5$.

(c) Use the model to confirm algebraically the estimate you found in part (b).

(d) What can you conclude in general about the relationship between the diameter of the copper wire and the resistance?

## Synthesis

***True or False?*** **In Exercises 77 and 78, determine whether the statement is true or false. Justify your answer.**

**77.** A graph is symmetric with respect to the $x$-axis if, whenever $(x, y)$ is on the graph, $(-x, y)$ is also on the graph.

**78.** A graph of an equation can have more than one $y$-intercept.

**79.** ***Think About It*** Suppose you correctly enter an expression for the variable $y$ on a graphing utility. However, no graph appears on the display when you graph the equation. Give a possible explanation and the steps you could take to remedy the problem. Illustrate your explanation with an example.

**80.** ***Think About It*** Find $a$ and $b$ if the graph of $y = ax^2 + bx^3$ is symmetric with respect to (a) the $y$-axis and (b) the origin. (There are many correct answers.)

## Skills Review

**81.** Identify the terms: $9x^5 + 4x^3 - 7$.

**82.** Rewrite the expression using exponential notation.

$$-(7 \times 7 \times 7 \times 7)$$

**In Exercises 83–88, simplify the expression.**

**83.** $\sqrt{18x} - \sqrt{2x}$

**84.** $\sqrt[4]{x^5}$

**85.** $\dfrac{70}{\sqrt{7x}}$

**86.** $\dfrac{55}{\sqrt{20} - 3}$

**87.** $\sqrt[6]{t^2}$

**88.** $\sqrt[3]{\sqrt{y}}$

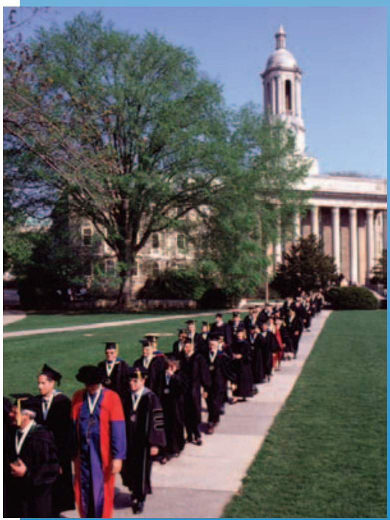
# 1.3 Linear Equations in Two Variables

### What you should learn

- Use slope to graph linear equations in two variables.
- Find slopes of lines.
- Write linear equations in two variables.
- Use slope to identify parallel and perpendicular lines.
- Use slope and linear equations in two variables to model and solve real-life problems.

### Why you should learn it

Linear equations in two variables can be used to model and solve real-life problems. For instance, in Exercise 109 on page 37, you will use a linear equation to model student enrollment at the Pennsylvania State University.



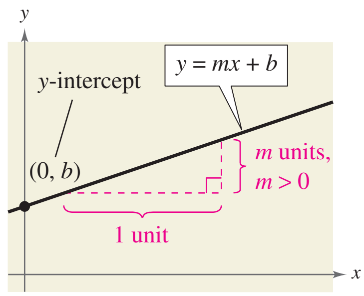
Courtesy of Pennsylvania State University

## Using Slope

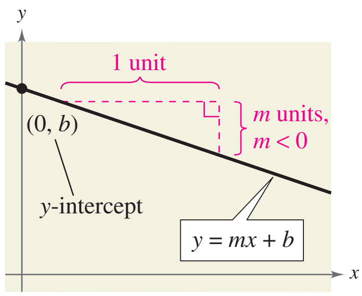
The simplest mathematical model for relating two variables is the **linear equation in two variables** $y = mx + b$. The equation is called *linear* because its graph is a line. (In mathematics, the term *line* means *straight line*.) By letting $x = 0$, you can see that the line crosses the $y$-axis at $y = b$, as shown in Figure 1.28. In other words, the $y$-intercept is $(0, b)$. The steepness or slope of the line is $m$.

$$y = \underset{\text{Slope}}{m}x + \underset{y\text{-Intercept}}{b}$$

The **slope** of a nonvertical line is the number of units the line rises (or falls) vertically for each unit of horizontal change from left to right, as shown in Figure 1.28 and Figure 1.29.




*Positive slope, line rises.*
FIGURE 1.28




*Negative slope, line falls.*
FIGURE 1.29

A linear equation that is written in the form $y = mx + b$ is said to be written in **slope-intercept form.**

### The Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is $m$ and whose $y$-intercept is $(0, b)$.

### *Exploration*

Use a graphing utility to compare the slopes of the lines $y = mx$, where $m = 0.5, 1, 2,$ and 4. Which line rises most quickly? Now, let $m = -0.5, -1, -2,$ and $-4$. Which line falls most quickly? Use a square setting to obtain a true geometric perspective. What can you conclude about the slope and the "rate" at which the line rises or falls?

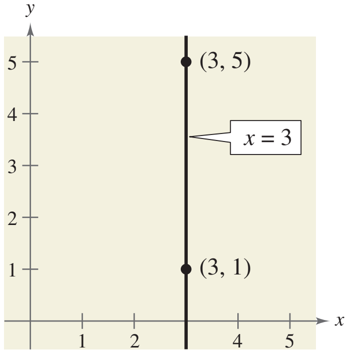



FIGURE 1.30 *Slope is undefined.*

Once you have determined the slope and the $y$-intercept of a line, it is a relatively simple matter to sketch its graph. In the next example, note that none of the lines is vertical. A vertical line has an equation of the form

$$x = a. \qquad \text{Vertical line}$$

The equation of a vertical line cannot be written in the form $y = mx + b$ because the slope of a vertical line is undefined, as indicated in Figure 1.30.

## Example 1 Graphing a Linear Equation

Sketch the graph of each linear equation.

**a.** $y = 2x + 1$

**b.** $y = 2$

**c.** $x + y = 2$

### Solution

**a.** Because $b = 1$, the $y$-intercept is $(0, 1)$. Moreover, because the slope is $m = 2$, the line *rises* two units for each unit the line moves to the right, as shown in Figure 1.31.

**b.** By writing this equation in the form $y = (0)x + 2$, you can see that the $y$-intercept is $(0, 2)$ and the slope is zero. A zero slope implies that the line is horizontal—that is, it doesn't rise *or* fall, as shown in Figure 1.32.

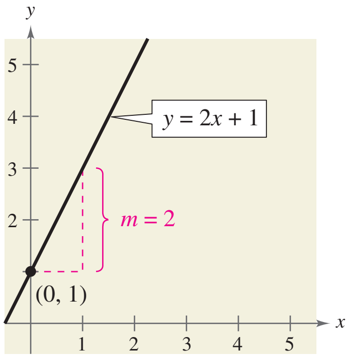
**c.** By writing this equation in slope-intercept form

$$x + y = 2 \qquad \text{Write original equation.}$$

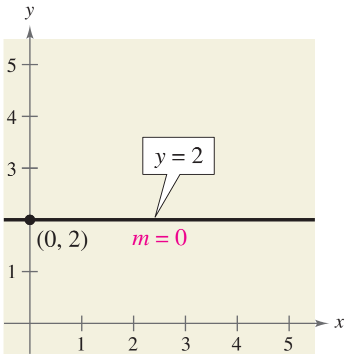
$$y = -x + 2 \qquad \text{Subtract } x \text{ from each side.}$$

$$y = (-1)x + 2 \qquad \text{Write in slope-intercept form.}$$

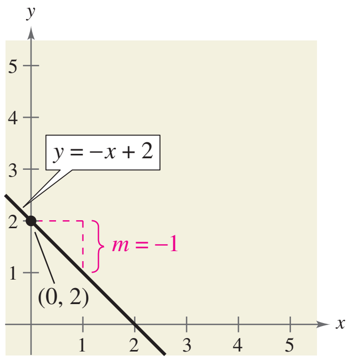
you can see that the $y$-intercept is $(0, 2)$. Moreover, because the slope is $m = -1$, the line *falls* one unit for each unit the line moves to the right, as shown in Figure 1.33.




*When m is positive, the line rises.*
FIGURE 1.31




*When m is 0, the line is horizontal.*
FIGURE 1.32




*When m is negative, the line falls.*
FIGURE 1.33

CHECKPOINT Now try Exercise 9.

## Finding the Slope of a Line

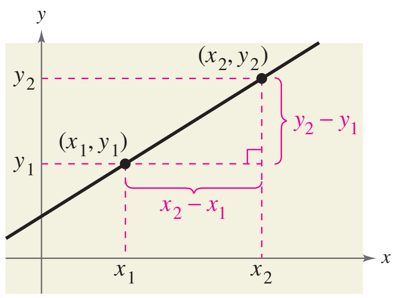



FIGURE 1.34

Given an equation of a line, you can find its slope by writing the equation in slope-intercept form. If you are not given an equation, you can still find the slope of a line. For instance, suppose you want to find the slope of the line passing through the points $(x_1, y_1)$ and $(x_2, y_2)$, as shown in Figure 1.34. As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction.

$$y_2 - y_1 = \text{the change in } y = \text{rise}$$

and

$$x_2 - x_1 = \text{the change in } x = \text{run}$$

The ratio of $(y_2 - y_1)$ to $(x_2 - x_1)$ represents the slope of the line that passes through the points $(x_1, y_1)$ and $(x_2, y_2)$.

$$\begin{aligned}\text{Slope} &= \frac{\text{change in } y}{\text{change in } x}\\ &= \frac{\text{rise}}{\text{run}}\\ &= \frac{y_2 - y_1}{x_2 - x_1}\end{aligned}$$

### The Slope of a Line Passing Through Two Points

The **slope** $m$ of the nonvertical line through $(x_1, y_1)$ and $(x_2, y_2)$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where $x_1 \neq x_2$.

When this formula is used for slope, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as $(x_1, y_1)$ and the other as $(x_2, y_2)$. However, once you have done this, you must form the numerator and denominator using the same order of subtraction.

$$\underbrace{m = \frac{y_2 - y_1}{x_2 - x_1}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_1 - y_2}{x_1 - x_2}}_{\text{Correct}} \qquad \underbrace{\xcancel{m = \frac{y_2 - y_1}{x_1 - x_2}}}_{\text{Incorrect}}$$

For instance, the slope of the line passing through the points $(3, 4)$ and $(5, 7)$ can be calculated as

$$m = \frac{7 - 4}{5 - 3} = \frac{3}{2}$$

or, reversing the subtraction order in both the numerator and denominator, as

$$m = \frac{4 - 7}{3 - 5} = \frac{-3}{-2} = \frac{3}{2}.$$

### Example 2 Finding the Slope of a Line Through Two Points

Find the slope of the line passing through each pair of points.

**a.** $(-2, 0)$ and $(3, 1)$ **b.** $(-1, 2)$ and $(2, 2)$

**c.** $(0, 4)$ and $(1, -1)$ **d.** $(3, 4)$ and $(3, 1)$

#### Solution

**a.** Letting $(x_1, y_1) = (-2, 0)$ and $(x_2, y_2) = (3, 1)$, you obtain a slope of

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{3 - (-2)} = \frac{1}{5}.$$

See Figure 1.35.

**b.** The slope of the line passing through $(-1, 2)$ and $(2, 2)$ is

$$m = \frac{2 - 2}{2 - (-1)} = \frac{0}{3} = 0.$$

See Figure 1.36.

**c.** The slope of the line passing through $(0, 4)$ and $(1, -1)$ is

$$m = \frac{-1 - 4}{1 - 0} = \frac{-5}{1} = -5.$$

See Figure 1.37.

**d.** The slope of the line passing through $(3, 4)$ and $(3, 1)$ is

$$m = \frac{1 - 4}{3 - 3} = \cancel{\frac{-3}{0}}.$$

See Figure 1.38.

Because division by 0 is undefined, the slope is undefined and the line is vertical.

**STUDY TIP**

In Figures 1.35 to 1.38, note the relationships between slope and the orientation of the line.

**a.** Positive slope: line rises from left to right

**b.** Zero slope: line is horizontal

**c.** Negative slope: line falls from left to right

**d.** Undefined slope: line is vertical

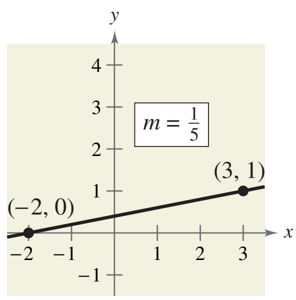



FIGURE 1.35

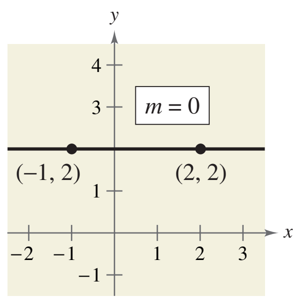



FIGURE 1.36

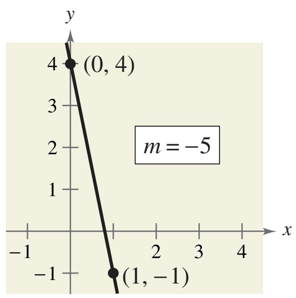



FIGURE 1.37

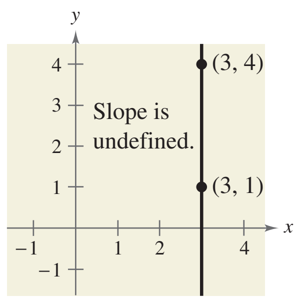



FIGURE 1.38

CHECKPOINT Now try Exercise 21.

## Writing Linear Equations in Two Variables

If $(x_1, y_1)$ is a point on a line of slope $m$ and $(x, y)$ is *any other* point on the line, then

$$\frac{y - y_1}{x - x_1} = m.$$

This equation, involving the variables $x$ and $y$, can be rewritten in the form

$$y - y_1 = m(x - x_1)$$

which is the **point-slope form** of the equation of a line.

### Point-Slope Form of the Equation of a Line

The equation of the line with slope $m$ passing through the point $(x_1, y_1)$ is

$$y - y_1 = m(x - x_1).$$

The point-slope form is most useful for *finding* the equation of a line. You should remember this form.

### Example 3 Using the Point-Slope Form

Find the slope-intercept form of the equation of the line that has a slope of 3 and passes through the point $(1, -2)$.

#### Solution

Use the point-slope form with $m = 3$ and $(x_1, y_1) = (1, -2)$.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - (-2) = 3(x - 1) \qquad \text{Substitute for } m, x_1, \text{ and } y_1.$$

$$y + 2 = 3x - 3 \qquad \text{Simplify.}$$

$$y = 3x - 5 \qquad \text{Write in slope-intercept form.}$$

The slope-intercept form of the equation of the line is $y = 3x - 5$. The graph of this line is shown in Figure 1.39.

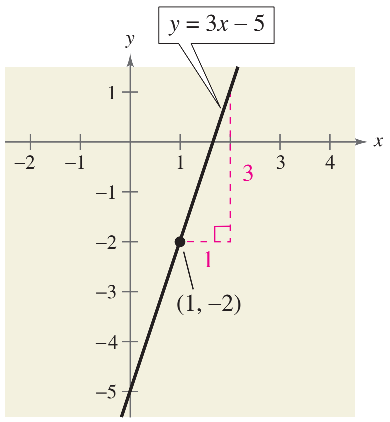



FIGURE 1.39

CHECKPOINT Now try Exercise 39.

The point-slope form can be used to find an equation of the line passing through two points $(x_1, y_1)$ and $(x_2, y_2)$. To do this, first find the slope of the line

$$m = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2$$

and then use the point-slope form to obtain the equation

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1). \qquad \text{Two-point form}$$

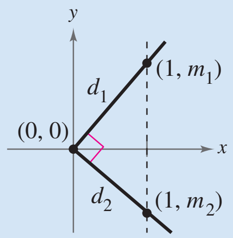
This is sometimes called the **two-point form** of the equation of a line.

**STUDY TIP**

When you find an equation of the line that passes through two given points, you only need to substitute the coordinates of one of the points into the point-slope form. It does not matter which point you choose because both points will yield the same result.

**Exploration**

Find $d_1$ and $d_2$ in terms of $m_1$ and $m_2$, respectively (see figure). Then use the Pythagorean Theorem to find a relationship between $m_1$ and $m_2$.



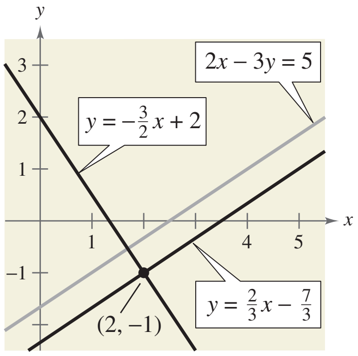




FIGURE 1.40

**Technology**

On a graphing utility, lines will not appear to have the correct slope unless you use a viewing window that has a square setting. For instance, try graphing the lines in Example 4 using the standard setting $-10 \le x \le 10$ and $-10 \le y \le 10$. Then reset the viewing window with the square setting $-9 \le x \le 9$ and $-6 \le y \le 6$. On which setting do the lines $y = \frac{2}{3}x - \frac{5}{3}$ and $y = -\frac{3}{2}x + 2$ appear to be perpendicular?

## Parallel and Perpendicular Lines

Slope can be used to decide whether two nonvertical lines in a plane are parallel, perpendicular, or neither.

### Parallel and Perpendicular Lines

1. Two distinct nonvertical lines are **parallel** if and only if their slopes are equal. That is, $m_1 = m_2$.
2. Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is, $m_1 = -1/m_2$.

### Example 4 Finding Parallel and Perpendicular Lines

Find the slope-intercept forms of the equations of the lines that pass through the point $(2, -1)$ and are (a) parallel to and (b) perpendicular to the line $2x - 3y = 5$.

**Solution**

By writing the equation of the given line in slope-intercept form

$$2x - 3y = 5 \qquad \text{Write original equation.}$$

$$-3y = -2x + 5 \qquad \text{Subtract } 2x \text{ from each side.}$$

$$y = \tfrac{2}{3}x - \tfrac{5}{3} \qquad \text{Write in slope-intercept form.}$$

you can see that it has a slope of $m = \frac{2}{3}$, as shown in Figure 1.40.

**a.** Any line parallel to the given line must also have a slope of $\frac{2}{3}$. So, the line through $(2, -1)$ that is parallel to the given line has the following equation.

$$y - (-1) = \tfrac{2}{3}(x - 2) \qquad \text{Write in point-slope form.}$$

$$3(y + 1) = 2(x - 2) \qquad \text{Multiply each side by 3.}$$

$$3y + 3 = 2x - 4 \qquad \text{Distributive Property}$$

$$y = \tfrac{2}{3}x - \tfrac{7}{3} \qquad \text{Write in slope-intercept form.}$$

**b.** Any line perpendicular to the given line must have a slope of $-\frac{3}{2}$ $\left(\text{because } -\frac{3}{2}\right.$ is the negative reciprocal of $\left.\frac{2}{3}\right)$. So, the line through $(2, -1)$ that is perpendicular to the given line has the following equation.

$$y - (-1) = -\tfrac{3}{2}(x - 2) \qquad \text{Write in point-slope form.}$$

$$2(y + 1) = -3(x - 2) \qquad \text{Multiply each side by 2.}$$

$$2y + 2 = -3x + 6 \qquad \text{Distributive Property}$$

$$y = -\tfrac{3}{2}x + 2 \qquad \text{Write in slope-intercept form.}$$

CHECKPOINT Now try Exercise 69.

Notice in Example 4 how the slope-intercept form is used to obtain information about the graph of a line, whereas the point-slope form is used to write the equation of a line.

## Applications

In real-life problems, the slope of a line can be interpreted as either a *ratio* or a *rate*. If the $x$-axis and $y$-axis have the same unit of measure, then the slope has no units and is a **ratio.** If the $x$-axis and $y$-axis have different units of measure, then the slope is a **rate** or **rate of change.**

### Example 5 Using Slope as a Ratio

The maximum recommended slope of a wheelchair ramp is $\frac{1}{12}$. A business is installing a wheelchair ramp that rises 22 inches over a horizontal length of 24 feet. Is the ramp steeper than recommended? (Source: *Americans with Disabilities Act Handbook*)

### Solution

The horizontal length of the ramp is 24 feet or $12(24) = 288$ inches, as shown in Figure 1.41. So, the slope of the ramp is

$$\text{Slope} = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{22 \text{ in.}}{288 \text{ in.}} \approx 0.076.$$

Because $\frac{1}{12} \approx 0.083$, the slope of the ramp is not steeper than recommended.

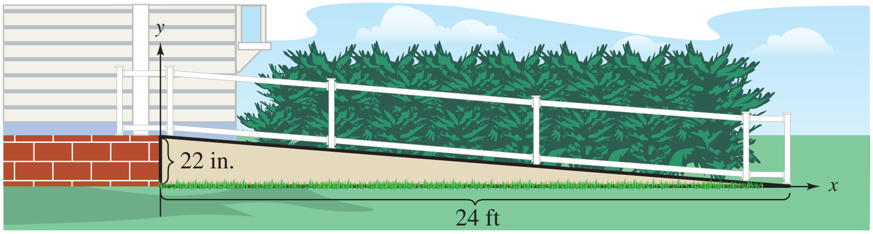



FIGURE 1.41

CHECKPOINT Now try Exercise 97.

### Example 6 Using Slope as a Rate of Change

A kitchen appliance manufacturing company determines that the total cost in dollars of producing $x$ units of a blender is

$$C = 25x + 3500. \qquad \text{Cost equation}$$

Describe the practical significance of the $y$-intercept and slope of this line.

### Solution

The $y$-intercept $(0, 3500)$ tells you that the cost of producing zero units is \$3500. This is the *fixed cost* of production—it includes costs that must be paid regardless of the number of units produced. The slope of $m = 25$ tells you that the cost of producing each unit is \$25, as shown in Figure 1.42. Economists call the cost per unit the *marginal cost*. If the production increases by one unit, then the "margin," or extra amount of cost, is \$25. So, the cost increases at a rate of \$25 per unit.

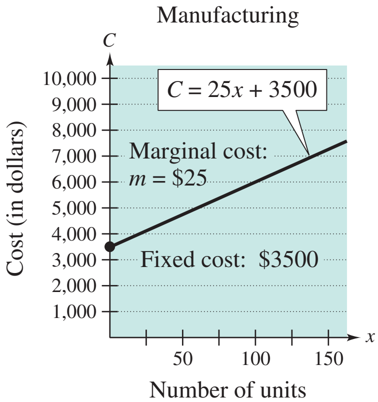



FIGURE 1.42 *Production cost*

CHECKPOINT Now try Exercise 101.

Most business expenses can be deducted in the same year they occur. One exception is the cost of property that has a useful life of more than 1 year. Such costs must be *depreciated* (decreased in value) over the useful life of the property. If the *same amount* is depreciated each year, the procedure is called *linear* or *straight-line depreciation*. The *book value* is the difference between the original value and the total amount of depreciation accumulated to date.

## Example 7 Straight-Line Depreciation

A college purchased exercise equipment worth \$12,000 for the new campus fitness center. The equipment has a useful life of 8 years. The salvage value at the end of 8 years is \$2000. Write a linear equation that describes the book value of the equipment each year.

### Solution

Let $V$ represent the value of the equipment at the end of year $t$. You can represent the initial value of the equipment by the data point (0, 12,000) and the salvage value of the equipment by the data point (8, 2000). The slope of the line is

$$m = \frac{2000 - 12{,}000}{8 - 0} = -\$1250$$

which represents the annual depreciation in *dollars per year*. Using the point-slope form, you can write the equation of the line as follows.

$$V - 12{,}000 = -1250(t - 0) \qquad \text{Write in point-slope form.}$$

$$V = -1250t + 12{,}000 \qquad \text{Write in slope-intercept form.}$$

The table shows the book value at the end of each year, and the graph of the equation is shown in Figure 1.43.

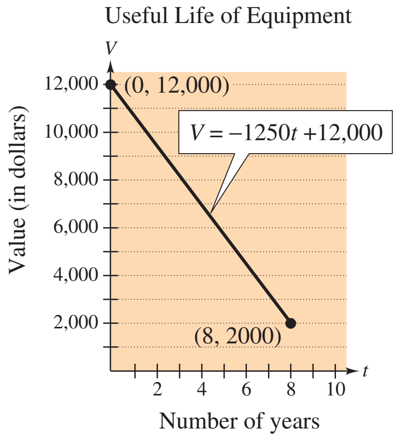



FIGURE 1.43 *Straight-line depreciation*

| Year, $t$ | Value, $V$ |
|---|---|
| 0 | 12,000 |
| 1 | 10,750 |
| 2 | 9,500 |
| 3 | 8,250 |
| 4 | 7,000 |
| 5 | 5,750 |
| 6 | 4,500 |
| 7 | 3,250 |
| 8 | 2,000 |

CHECKPOINT Now try Exercise 107.

In many real-life applications, the two data points that determine the line are often given in a disguised form. Note how the data points are described in Example 7.

## Example 8 Predicting Sales per Share

The sales per share for Starbucks Corporation were \$6.97 in 2001 and \$8.47 in 2002. Using only this information, write a linear equation that gives the sales per share in terms of the year. Then predict the sales per share for 2003. (Source: Starbucks Corporation)

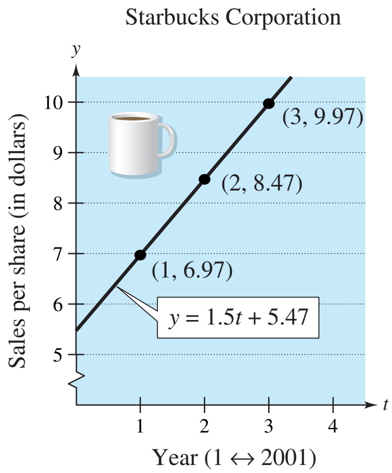



FIGURE 1.44

### Solution

Let $t = 1$ represent 2001. Then the two given values are represented by the data points (1, 6.97) and (2, 8.47). The slope of the line through these points is

$$m = \frac{8.47 - 6.97}{2 - 1}$$

$$= 1.5.$$

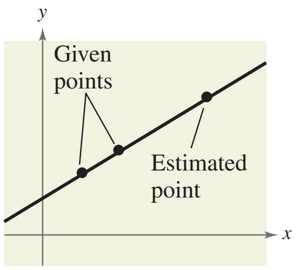
Using the point-slope form, you can find the equation that relates the sales per share $y$ and the year $t$ to be

$$y - 6.97 = 1.5(t - 1) \qquad \text{Write in point-slope form.}$$

$$y = 1.5t + 5.47. \qquad \text{Write in slope-intercept form.}$$

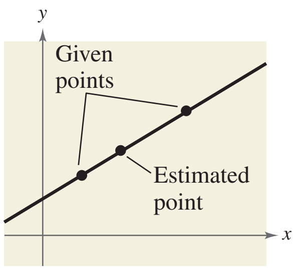
According to this equation, the sales per share in 2003 was $y = 1.5(3) + 5.47 = \$9.97$, as shown in Figure 1.44. (In this case, the prediction is quite good—the actual sales per share in 2003 was \$10.35.)

CHECKPOINT Now try Exercise 109.




*Linear extrapolation*

FIGURE 1.45




*Linear interpolation*

FIGURE 1.46

The prediction method illustrated in Example 8 is called **linear extrapolation.** Note in Figure 1.45 that an extrapolated point does not lie between the given points. When the estimated point lies between two given points, as shown in Figure 1.46, the procedure is called **linear interpolation.**

Because the slope of a vertical line is not defined, its equation cannot be written in slope-intercept form. However, every line has an equation that can be written in the **general form**

$$Ax + By + C = 0 \qquad \text{General form}$$

where $A$ and $B$ are not both zero. For instance, the vertical line given by $x = a$ can be represented by the general form $x - a = 0$.

### Summary of Equations of Lines

1. General form: $Ax + By + C = 0$
2. Vertical line: $x = a$
3. Horizontal line: $y = b$
4. Slope-intercept form: $y = mx + b$
5. Point-slope form: $y - y_1 = m(x - x_1)$
6. Two-point form: $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

# 1.3 Exercises

## VOCABULARY CHECK:

**In Exercises 1–6, fill in the blanks.**

1. The simplest mathematical model for relating two variables is the ________ equation in two variables $y = mx + b$.
2. For a line, the ratio of the change in $y$ to the change in $x$ is called the ________ of the line.
3. Two lines are ________ if and only if their slopes are equal.
4. Two lines are ________ if and only if their slopes are negative reciprocals of each other.
5. When the $x$-axis and $y$-axis have different units of measure, the slope can be interpreted as a ________.
6. The prediction method ________ ________ is the method used to estimate a point on a line that does not lie between the given points.
7. Match each equation of a line with its form.

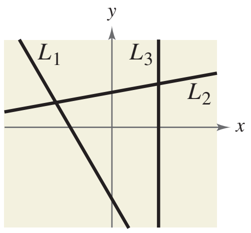
| | |
|---|---|
| (a) $Ax + By + C = 0$ | (i) Vertical line |
| (b) $x = a$ | (ii) Slope-intercept form |
| (c) $y = b$ | (iii) General form |
| (d) $y = mx + b$ | (iv) Point-slope form |
| (e) $y - y_1 = m(x - x_1)$ | (v) Horizontal line |

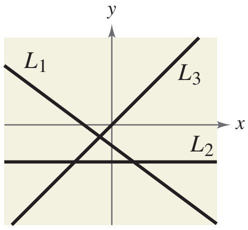
**PREREQUISITE SKILLS REVIEW:** Practice and review algebra skills needed for this section at **www.Eduspace.com.**

**In Exercises 1 and 2, identify the line that has each slope.**

1. (a) $m = \frac{2}{3}$
   (b) $m$ is undefined.
   (c) $m = -2$

2. (a) $m = 0$
   (b) $m = -\frac{3}{4}$
   (c) $m = 1$



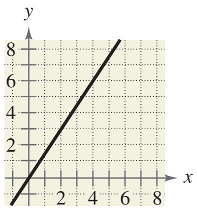




**In Exercises 3 and 4, sketch the lines through the point with the indicated slopes on the same set of coordinate axes.**

| | *Point* | *Slopes* | | | |
|---|---|---|---|---|---|
| 3. | (2, 3) | (a) 0 | (b) 1 | (c) 2 | (d) $-3$ |
| 4. | $(-4, 1)$ | (a) 3 | (b) $-3$ | (c) $\frac{1}{2}$ | (d) Undefined |

**In Exercises 5–8, estimate the slope of the line.**

5.

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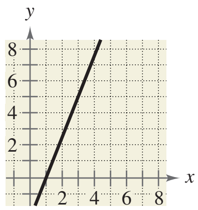


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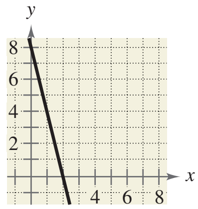
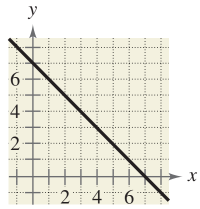



8.




**In Exercises 9–20, find the slope and $y$-intercept (if possible) of the equation of the line. Sketch the line.**

9. $y = 5x + 3$
10. $y = x - 10$
11. $y = -\frac{1}{2}x + 4$
12. $y = -\frac{3}{2}x + 6$
13. $5x - 2 = 0$
14. $3y + 5 = 0$
15. $7x + 6y = 30$
16. $2x + 3y = 9$
17. $y - 3 = 0$
18. $y + 4 = 0$
19. $x + 5 = 0$
20. $x - 2 = 0$

**In Exercises 21–28, plot the points and find the slope of the line passing through the pair of points.**

21. $(-3, -2), (1, 6)$
22. $(2, 4), (4, -4)$
23. $(-6, -1), (-6, 4)$
24. $(0, -10), (-4, 0)$
25. $\left(\frac{11}{2}, -\frac{4}{3}\right), \left(-\frac{3}{2}, -\frac{1}{3}\right)$
26. $\left(\frac{7}{8}, \frac{3}{4}\right), \left(\frac{5}{4}, -\frac{1}{4}\right)$
27. $(4.8, 3.1), (-5.2, 1.6)$
28. $(-1.75, -8.3), (2.25, -2.6)$