Chapter 2

Polynomial and Rational Functions

2.1 Complex Numbers

Chapter 2

Homework

2.1 p284 3,, 11, 17, 19, 27, 31, 41, 47, 51, 57

Objectives:

- Add and subtract complex numbers.
- Multiply complex numbers.
- Divide complex numbers.
- Perform operations with square roots of negative numbers.
- Solve quadratic equations with complex imaginary solutions.

Complex and Imaginary Numbers

- The imaginary unit, *i*, is defined as

$$i = \sqrt{-1} \qquad \text{where} \qquad i^2 = -1$$

The set of all numbers in the form **a + bi** with real numbers a, b, and **i**, the imaginary unit, is called the set of **complex numbers**.

The **standard form** of a complex number is **a + bi**

Real and Imaginary Numbers

- The set of all complex numbers is in the form $a + bi$

  The set of all **real** numbers have the form $a + 0i$

  Thus a is called the real part of $a + bi$.

  The set of **pure imaginary** numbers have the form $0 + bi$

  Thus b is called the imaginary part of $a + bi$.

- Note that the additive identity of the complex numbers is zero, $0 + 0i = 0$.
- Additionally, the additive inverse of $a + bi$ is $-(a + bi) = -a - bi$.

Operations on Complex Numbers

- The form of a complex number $a + bi$ is like the binomial $a + bx$. To add, subtract, and multiply complex numbers, we use the same methods that we use for binomials.
- To add two complex numbers, we add the two real parts and then add the two imaginary parts. That is, $(a + bi) + (c + di) = (a + c) + (b + d)i$.

$$a.\ (5 - 2i) + (3 + 3i)$$

$= 5 + (-2i + 3) + 3i$	Associative Property of Addition
$= 5 + (3 + -2i) + 3i$	Commutative Property of Addition
$= (5 + 3) + (-2i + 3i)$	Associative Property of Addition
$= 8 + (-2 + 3)i$	Distributive Property
$= 8 + i$	

Example: Subtracting Complex Numbers

- Perform the indicated operations, writing the result in standard form:

$$a.\ (2-6i)-(12-i)$$
$$=(2-6i)+(-12+i)$$
$$=(2+-12)+(-6i+i)$$
$$=-10-5i$$

Example: Multiplying Complex Numbers

- Perform the indicated operations, writing the result in standard form:

a. $7i(2-9i)$

$$= 14i - 63i^2$$

$$= 14i - 63(-1)$$

$$= 63 + 14i$$

b. $(5+4i)(6-7i)$

$$= 5(6-7i) + 4i(6-7i)$$

$$= 30 - 35i + 24i - 28i^2$$

$$= 30 - 11i - 28(-1)$$

$$= 30 - 11i + 28$$

$$= 58 - 11i$$

The F**L Word

- Do NOT let me hear anyone use the "F" Word in my classroom.
- When we multiply two binomials, or any two polynomials, …

WE USE THE DISTRIBUTIVE PROPERTY

- We, most assuredly, most emphatically, **DO NOT** use the F **L Method, which is not a "method", but simply a mnemonic device for the mathematically challenged.

Powers of Complex Numbers

- Perform the indicated operation and write the result in standard form.

$$(2-3i)^2$$

$$=(2-3i)(2-3i)$$

$$=4-6i-6i+(3i)^2$$

$$=4-12i+9i^2$$

$$=4-12i+9(-1)$$

$$=-5-12i$$

$$(a+b)^2=a^2+2ab+b^2$$

$$(2-3i)^2 \quad =4-12i+(3i)^2$$

Equity of Complex Numbers

- Two complex numbers a + bi and c + di are equal if and only if (iff) the real and imaginary parts are equal.

$$a + bi = c + di \text{ iff } a = c \text{ and } b = d$$

Special Products

- Perform the indicated operation and write the result in standard form.

$$(4+5i)(4-5i) \qquad (a+b)(a-b)=a^2-b^2$$

$$=4^2-(5i)^2$$

$$=16-25(-1)$$

$$=41$$

- A real number

Conjugate of a Complex Number

- For the complex number **a + bi**, its **complex conjugate** is defined to be **a - bi**.

The product of a complex number and its **conjugate** is a **real number**.

$$
\begin{aligned}
&(a+bi)(a-bi) \\
&= a^2 - (bi)^2 \\
&= a^2 - b^2(-1) \\
&= a^2 + b^2
\end{aligned}
$$

Complex Number Division

- The goal of complex number division is to obtain a real number in the denominator (rationalize the denominator).

- We multiply the numerator and denominator of a complex number quotient by a value (usually the conjugate of the denominator) to obtain a real number in the denominator.

Example: Dividing Complex Numbers

- Divide and express the result in standard form:

$$\frac{5+4i}{4-i} = \frac{5+4i}{4-i} \bullet \frac{4+i}{4+i}$$

$$= \frac{20+5i+16i+4i^2}{4^2-i^2}$$

$$= \frac{16+21i}{17} \quad \text{In standard form} \quad \frac{16}{17}+\frac{21}{17}i$$

STUDY TIP

Note that when you multiply the numerator and denominator of a quotient of complex numbers by

$$\frac{c-di}{c-di}$$

you are actually multiplying the quotient by a form of 1. You are not changing the original expression, you are only creating an expression that is equivalent to the original expression.

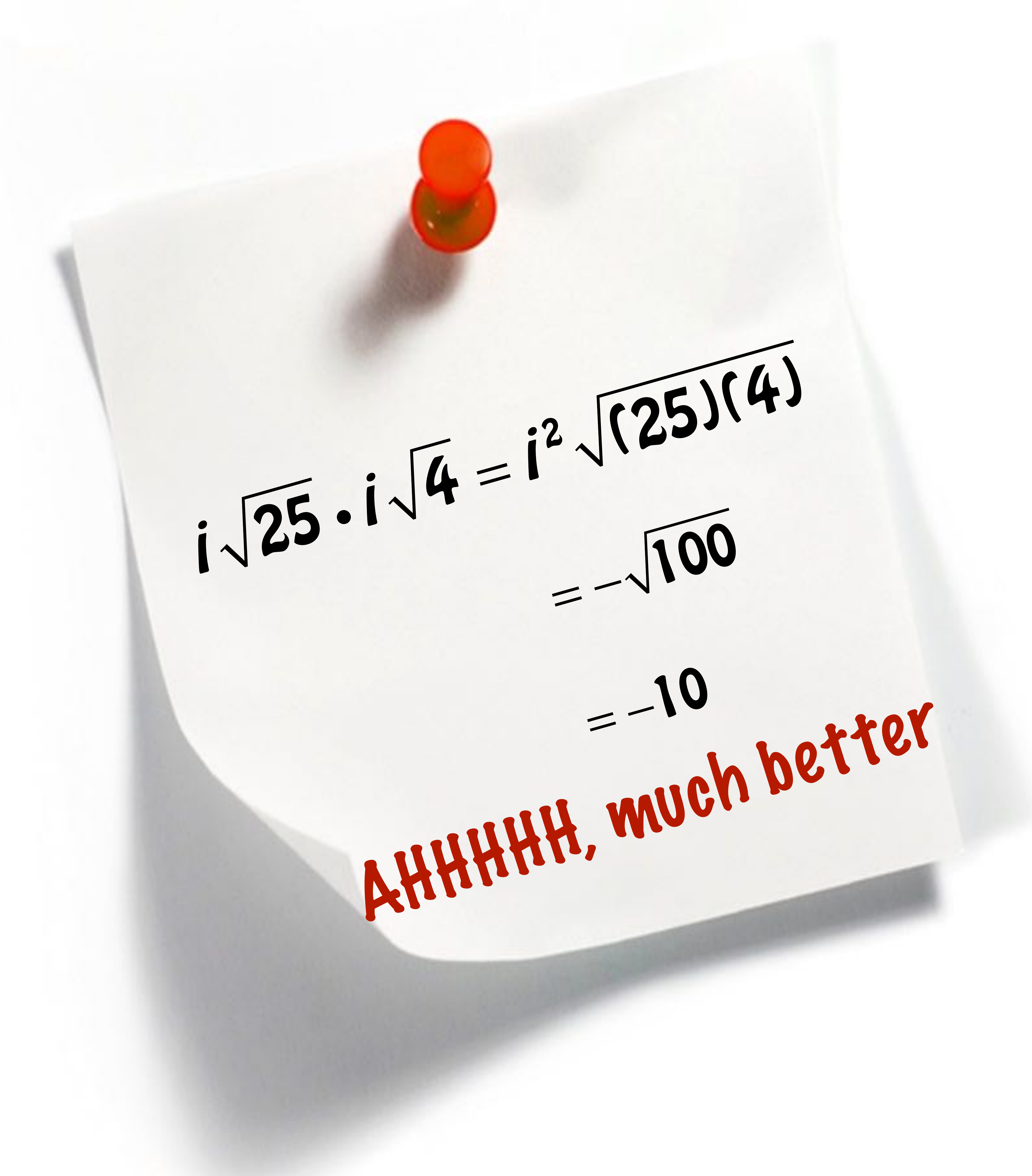
Principal Square Root of a Negative

- For any positive real number b, the principal square root of the negative number **-b** is defined by

$$\sqrt{-b} = i\sqrt{b}$$

- Remember the order of operations, square root is an exponent and must be done first, thus

take care of the negative square root before any other operation.



STUDY TIP

The definition of principal square root uses the rule

$$\sqrt{ab} = \sqrt{a}\sqrt{b}$$

for $a > 0$ and $b < 0$. This rule is not valid if *both* a and b are negative. For example,

$$\begin{aligned}\sqrt{-5}\sqrt{-5} &= \sqrt{5(-1)}\sqrt{5(-1)}\\ &= \sqrt{5}i\sqrt{5}i\\ &= \sqrt{25i^2}\\ &= 5i^2 = -5\end{aligned}$$

whereas

$$\sqrt{(-5)(-5)} = \sqrt{25} = 5.$$

To avoid problems with square roots of negative numbers, be sure to convert complex numbers to standard form *before* multiplying.

Example

- Rewrite in standard form $\left(1-\sqrt{-14}\right)^2$

$$=\left(1-i\sqrt{14}\right)^2$$

$$=1^2-2i\sqrt{14}+\left(i\sqrt{14}\right)^2$$

$$=1-2i\sqrt{14}+14i^2$$

$$=1-2i\sqrt{14}+14(-1)$$

$$=-13-2i\sqrt{14}$$

- Remember, take care of the negative first!

Example: Square Roots of Negatives

- Perform the indicated operations and write the result in standard form.

a. $\sqrt{-27}+\sqrt{-48}$

$=i\sqrt{27}+i\sqrt{48}$

$=3i\sqrt{3}+4i\sqrt{3}$

$=7i\sqrt{3}$

b. $\left(-2+\sqrt{-3}\right)^2$

$=\left(-2+i\sqrt{3}\right)^2$

$=(-2)^2+2(-2)(i\sqrt{3})+(i\sqrt{3})^2$

$=4+(-4i\sqrt{3})+3i^2$

$=1-4i\sqrt{3}$

Quadratic Equations with complex roots

- Solve $x^2 + 6x + 15 = 0$

$$x^2 + 6x + \_\_ = -15 + \_\_$$

- Complete the square

$$x^2 + 6x + 9 = -15 + 9$$

$$(x+3)^2 = -6$$

$$x + 3 = \pm\sqrt{-6}$$

$$x + 3 = \pm i\sqrt{6}$$

$$x = -3 \pm i\sqrt{6}$$

- Complex Conjugates

Quadratic Equations with Complex Imaginary Solutions

- A quadratic equation may be expressed in the general form

$$ax^2 + bx + c = 0$$

- The quadratic can be solved using the quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- $b^2 - 4ac$ is called the **discriminant**. If the discriminant is negative, a quadratic equation has no real solutions. Quadratic equations with negative discriminants have two solutions that are complex conjugates.

Example: A Quadratic Equation with Imaginary Solutions

- Solve using the quadratic formula: $x^2 - 2x + 2 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{--2 \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2} = 1 \pm i$$

The solutions are complex conjugates.

The solution set is {1 + i, 1 - i}.