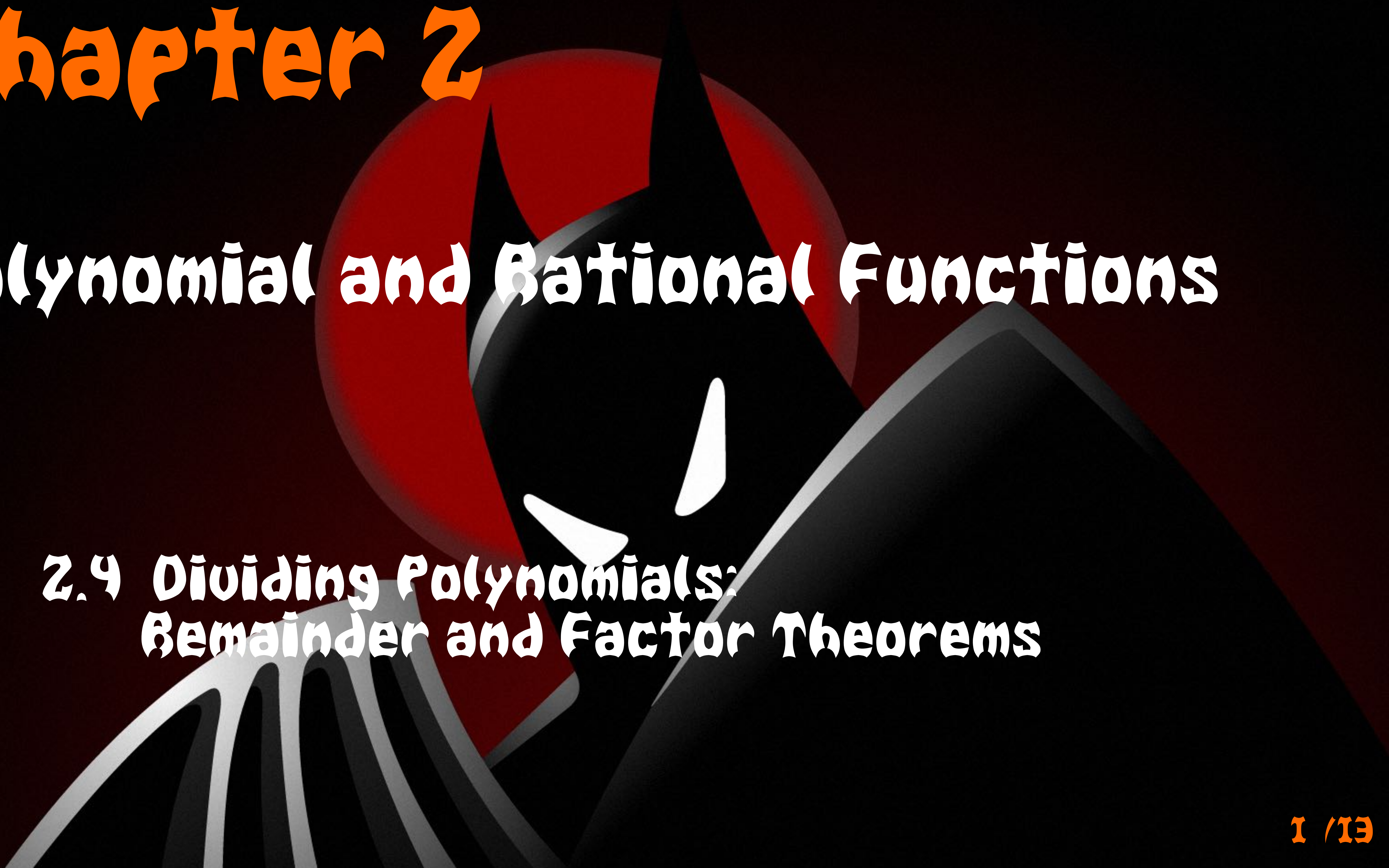
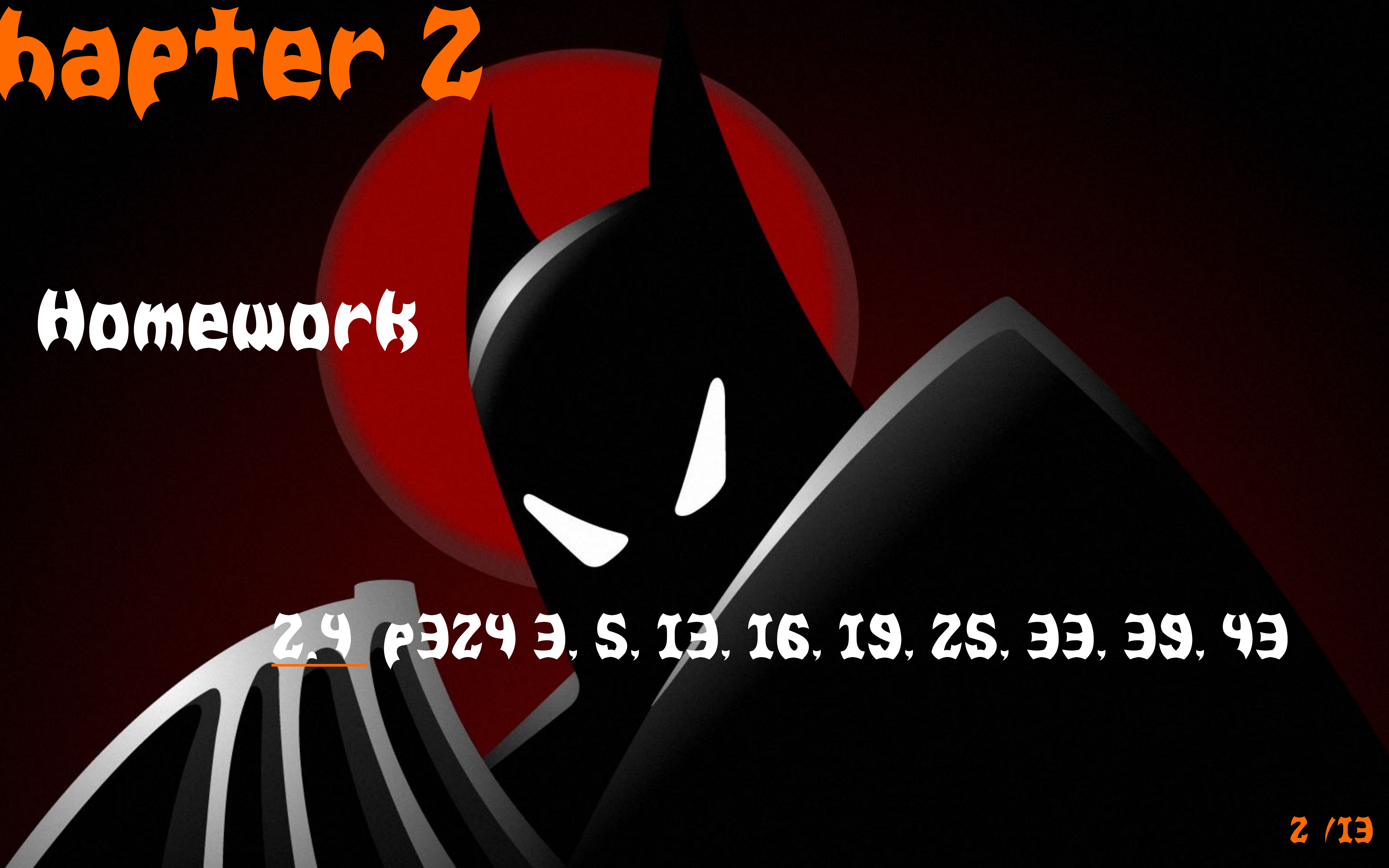
Chapter 2

Polynomial and Rational Functions

2.4 Dividing Polynomials: Remainder and Factor Theorems



Chapter 2



Homework

2.4 p324 3, 5, 13, 16, 19, 25, 33, 39, 43

Chapter 2.4

Objectives

- Use long division to divide polynomials.
- Use synthetic division to divide polynomials.
- Evaluate a polynomial using the Remainder Theorem.
- Use the Factor Theorem to solve a polynomial equation.

Long Division of Polynomials

1. **Arrange** the terms of both the dividend and the divisor in descending powers of any variable.
2. **Divide** the first term in the dividend by the first term in the divisor. The result is the first term of the quotient.
3. **Multiply** every term in the divisor by the first term in the quotient. Write the resulting product beneath the dividend with like terms lined up.
4. **Subtract** the product from the dividend.
5. **Bring down** the next term in the original dividend and write it next to the remainder to form a new dividend.
6. **Repeat** until the remainder can no longer be divided

The Division Algorithm

- If $f(x)$ and $d(x)$ are polynomials, with $d(x) \neq 0$, the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x) \cdot q(x) + r(x) \qquad \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The remainder, $r(x)$, equals 0 or it is of degree less than the degree of $d(x)$.

If $r(x) = 0$, we say that $d(x)$ divides evenly into $f(x)$ and that $d(x)$ and $q(x)$ are factors of $f(x)$.

Example: Long Division of Polynomials

Divide $7 - 11x - 3x^2 + 2x^3$ by $x - 3$.

Begin by writing the dividend in general form (descending powers of x).

$$7 - 11x - 3x^2 + 2x^3 = 2x^3 + -3x^2 + -11x + 7$$

Divide

$$\begin{array}{r|l} & 2x^2 + 3x - 2 \\ \hline x - 3 & 2x^3 + -3x^2 + -11x + 7 \\ & 2x^3 + -6x^2 \\ \hline & 3x^2 \\ & 3x^2 + -9x \\ \hline & -2x \\ & -2x + 6 \\ \hline & 1 \end{array}$$

The quotient is

$$2x^2 + 3x - 2 + \frac{1}{x-3}$$

$$q(x) = 2x^2 + 3x - 2$$

$$r(x) = 1$$

Example: Long Division of Polynomials

- Divide $3x - 5 + 6x^4 + 5x^3$ by $3x^2 - 2x$.

Begin by writing the dividend in general form, filling in any missing terms.

$$6x^4 + 5x^3 + 0x^2 + 3x - 5$$

Divide

$$\begin{array}{r} 2x^2 + 3x + 2 \\ 3x^2 - 2x + 0 \overline{\big)\, 6x^4 + 5x^3 + 0x^2 + 3x - 5} \\ \underline{6x^4 - 4x^3 + 0x^2 } \\ 9x^3 + 0x^2 \\ \underline{9x^3 - 6x^2 + 0x } \\ 6x^2 + 3x \\ \underline{6x^2 - 4x + 0} \\ 7x - 5 \end{array}$$

The quotient is

$$2x^2 + 3x + 2 + \frac{7x - 5}{3x^2 - 2x}$$

Synthetic Division

1. Arrange the polynomial in descending powers, with a 0 coefficient for any missing term.
2. Write c for the divisor, x – c. To the right, write the coefficients of the dividend.
3. Write the leading coefficient of the dividend on the bottom row.
4. Multiply c times the value just written on the bottom row. Write the product in the next column in the second row.
5. Add the values in this new column, writing the sum in the bottom row.
6. Repeat this series of multiplications and additions until all columns are filled in.

Synthetic Division

7. Use the numbers in the last row to write the quotient, plus the remainder divided by the divisor. The degree of the first term of the quotient is one less than the degree of the first term of the dividend. The final value in this row is the remainder.

Use synthetic division to divide $x^3 - 7x - 6$ by $x + 2$.

1. $x^3 - 7x - 6 = x^3 + 0x^2 - 7x - 6$
2. $x - c$. $x + 2 = x - -2$
3. leading coefficient
4. Multiply
5. Add
6. Repeat

-2	1	0	-7	-6
		-2	4	6
	1	-2	-3	0

The quotient is $x^2 - 2x - 3$

Long Division vs Synthetic Division

Look at the two techniques side-by-side.

Divide $7 - 11x - 3x^2 + 2x^3$ by $x - 3$.

$$
\begin{array}{r}
2x^2 + 3x - 2 \\
x - 3 \,\big)\, 2x^3 + -3x^2 + -11x + 7 \\
2x^3 + -6x^2 \qquad\qquad\qquad \\
\hline
3x^2 + -11x \qquad \\
3x^2 + -9x \qquad \\
\hline
-2x + 7 \\
-2x + 6 \\
\hline
1
\end{array}
$$

$$
\begin{array}{r|rrrr}
3 & 2 & -3 & -11 & 7 \\
 & & 6 & 9 & -6 \\
\hline
 & 2 & 3 & -2 & 1
\end{array}
$$

The quotient is

$$2x^2 + 3x - 2 + \frac{1}{x - 3}$$

The Remainder Theorem

If the polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

Given $f(x) = 3x^3 + 4x^2 - 5x + 3$ use the Remainder Theorem to find $f(-4)$.

We use synthetic division (now called synthetic substitution) to divide.

$$\begin{array}{r|rrrr} -4 & 3 & 4 & -5 & 3 \\ & & -12 & 32 & -108 \\ \hline & 3 & -8 & 27 & -105 \end{array}$$

The remainder, -105, is the value of $f(-4)$. Thus, $f(-4) = -105$.

$f(-4) = 3(-4)^3 + 4(-4)^2 - 5(-4) + 3 = -192 + 64 + 20 + 3 = -105$

The Factor Theorem

Let $f(x)$ be a polynomial.

a. If $f(c) = 0$, then $x - c$ is a factor of $f(x)$.

b. If $x - c$ is a factor of $f(x)$, then $f(c) = 0$.

Example: Using the Factor Theorem

Solve the equation $15x^3 + 14x^2 - 3x - 2 = 0$ given that -1 is a zero of $f(x) = 15x^3 + 14x^2 - 3x - 2$.

Since -1 is a zero of $f(x) = 15x^3 + 14x^2 - 3x - 2$, then $x + 1$ is a factor of $f(x)$.

Use synthetic division to divide $f(x)$ by $x + 1$

$$\begin{array}{r|rrrr} -1 & 15 & 14 & -3 & -2 \\ & & -15 & 1 & 2 \\ \hline & 15 & -1 & -2 & \boxed{0} \end{array}$$

Another factor of $f(x)$ is $15x^2 - x - 2$

$15x^2 - x - 2 = (5x - 2)(3x + 1)$

$f(x) = (x + 1)(5x - 2)(3x + 1)$

$x = -1$ or $x = 2/5$ or $x = -1/3$)

The solutions set is

$$\left\{-1, -\frac{1}{3}, \frac{2}{5}\right\}$$