Chapter 3

Exponential and Logarithmic Functions

3.2 Logarithmic Functions

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3.2 4, 8, 14, 16, 18, 20, 22, 30, 31, 32, 33, 34, 39, 42, 54, 56, 62, 68, 72, 76, 78, 80, 82, 84, 86, 92, 94, 98, 100, 102, 104, 106, 108, 110, 112

Chapter 3-2

Objectives

Change from logarithmic to exponential form.

Change from exponential to logarithmic form.

Evaluate logarithms.

Use basic logarithmic properties.

Graph logarithmic functions.

Find the domain of a logarithmic function.

Use common logarithms.

Use natural logarithms.

Definition of the Logarithmic Function

🐱 We can solve an equation when the variable is in the exponent by using an **inverse** operation. The inverse of an exponential function is a **logarithmic** function.

🐱 The function given by $f(x) = \log_b x$, where $x > 0$, $b > 0$, and $b \neq 1$, is called the logarithmic function with base b.

🐱 The logarithmic function with base b is the inverse of the exponential function with base b; thus $y = \log_b x$ if and only if $x = b^y$.

exponent (becomes logarithm)

$$y = b^x$$

base

Definition of the Logarithmic Function

🐱 Since exponential functions are one-to-one functions, the inverse of an exponential function is also a function.

🐱 The inverse operation is called finding the logarithm. A **logarithm** is the **exponent** to which a specified **base** is raised to obtain a given value.

🐱 Let me repeat that...

A **logarithm** is the **exponent** to which a specified **base** is raised to obtain a given value (x).

Definition of the Logarithmic Function

🐱 For x > 0 and b > 0, b ≠ 1,

🐱 $y = \log_b x$ is equivalent to $b^y = x$.

The function $f(x) = \log_b x$ is the **logarithmic function** with **base b**.

logarithm is exponent

argument

$$b^y = x \qquad \log_b x = y$$

base stays base

$$b > 0, \ b \neq 1$$
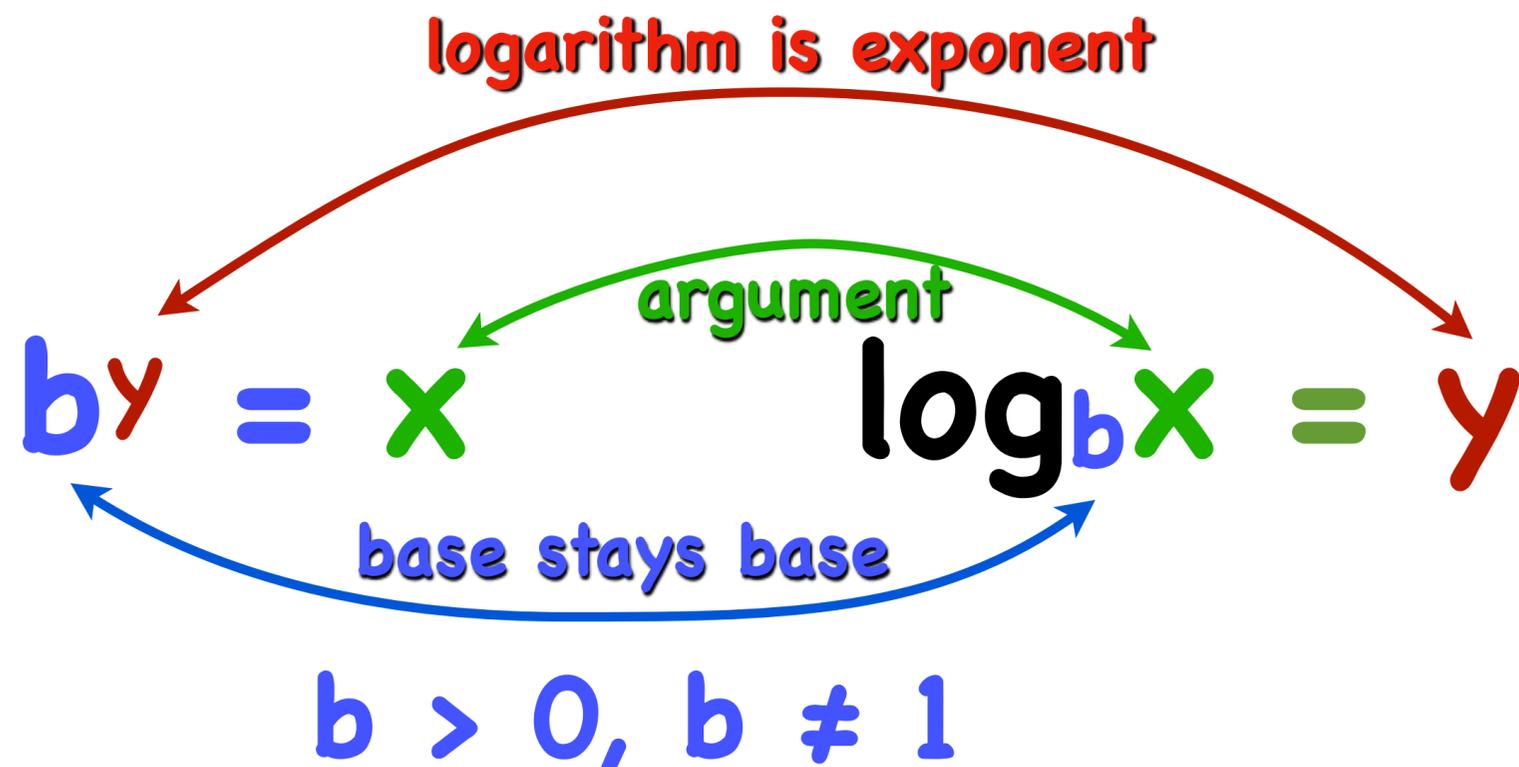
Example: Changing from Logarithmic to Exponential Form

🐱 Write each equation in its equivalent exponential form:

🐱 $y = \log_b x$ is equivalent to $b^y = x$.

$$3 = \log_7 x \qquad \text{equivalent to} \qquad 7^3 = x$$

$$2 = \log_b 25 \qquad \text{equivalent to} \qquad b^2 = 25$$

$$\log_4 26 = y \qquad \text{equivalent to} \qquad 4^y = 26$$

🐱 Write each equation in its equivalent logarithmic form:

🐱 $y = \log_b x$ is equivalent to $b^y = x$.

$$2^5 = x \qquad \text{equivalent to} \qquad 5 = \log_2 x$$

$$b^3 = 27 \qquad \text{equivalent to} \qquad 3 = \log_b 27$$

$$e^y = 33 \qquad \text{equivalent to} \qquad \log_e 33 = y$$

Example: Evaluating Logarithms

Evaluate: $\log_{10} 100$ means 10 to what power gives 100?

$\log_{10} 100 = y$ $10^y = 100$ $10^2 = 100$ $y = 2$ $\log_{10} 100 = 2$

Evaluate: $\log_{5} \dfrac{1}{125}$ means 5 to what power gives $\dfrac{1}{125}$?

$$\frac{1}{125} = \frac{1}{5^3} = 5^{-3}$$

$\log_{5} \dfrac{1}{125} = y$ $5^y = \dfrac{1}{125} = 5^{-3}$ $\log_{5} \dfrac{1}{125} = -3$

The log is (=) the exponent.

Example: Evaluating Logarithms

Evaluate: $\log_{36} 6$ means 36 to what power equals 6?

$$\log_{36} 6 = y \qquad 36^y = 6 \qquad 6 = \sqrt{36} = 36^{\frac{1}{2}}$$

$$\log_{36} 6 = \log_{36} 36^{\frac{1}{2}} = y$$

$$36^y = 36^{\frac{1}{2}} \qquad y = \frac{1}{2} \qquad \log_{36} 6 = \frac{1}{2}$$

The log is (=) the exponent.

Example: Evaluating Logarithms

Evaluate: $\log_3 \sqrt[7]{3}$ means 3 to what power gives $\sqrt[7]{3}$?

$$\log_3 \sqrt[7]{3} = y \qquad 3^y = \sqrt[7]{3} \qquad \sqrt[7]{3} = 3^{\frac{1}{7}}$$

$$\log_3 \sqrt[7]{3} = \log_3 3^{\frac{1}{7}} = y \qquad 3^y = 3^{\frac{1}{7}} \qquad y = \frac{1}{7}$$

$$\log_3 \sqrt[7]{3} = \frac{1}{7}$$

The log is (=) the exponent.

Basic Log Properties Involving One

1. $\log_b b = 1$ because 1 is the exponent to which b must be raised to obtain b. $(b^1 = b)$

2. $\log_b 1 = 0$ because 0 is the exponent to which b must be raised to obtain 1. $(b^0 = 1)$

Evaluate: $\log_9 9$ Because $\log_b b = 1$, we conclude $\log_9 9 = 1$

Evaluate: $\log_8 1$ Because $\log_b 1 = 0$, we conclude $\log_8 1 = 0$.

3. $\log_b x = \log_b y$ iff $x = y$.

Inverse Properties of Logarithms

For b > 0 and b ≠ 1,

$$\log_b b^x = x \quad \text{and} \quad b^{\log_b x} = x$$

Evaluate: $\log_7 7^8$ $\log_7 7^8 = 8$

Evaluate: $3^{\log_3 17}$ $3^{\log_3 17} = 17$

Example: Graphs of Exponential and Logarithmic Functions

Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ in the same rectangular coordinate system.

We can first set up a table of values for $f(x) = 3^x$.

x	f(x)=3^x
–2	1/9
–1	1/3
0	1
1	3
2	9

Reverse the coordinates for the inverse function $g(x) = \log_3 x$

x	g(x)=log₃x
1/9	–2
1/3	–1
1	0
3	1
9	2

Example: Graphs of Exponential and Logarithmic Functions

We now graph $f(x) = 3^x$ and $g(x) = \log_3 x$ the same rectangular coordinate system.

x	f(x)=3^x
–2	1/9
–1	1/3
0	1
1	3
2	9

x	g(x)=$\log_3 x$
1/9	–2
1/3	–1
1	0
3	1
9	2

$g(x) = \log_3 x$ is $f(x) = 3^x$

reflected across y = x.

$f(x) = 3^x$

y = x

$g(x) = \log_3 x$
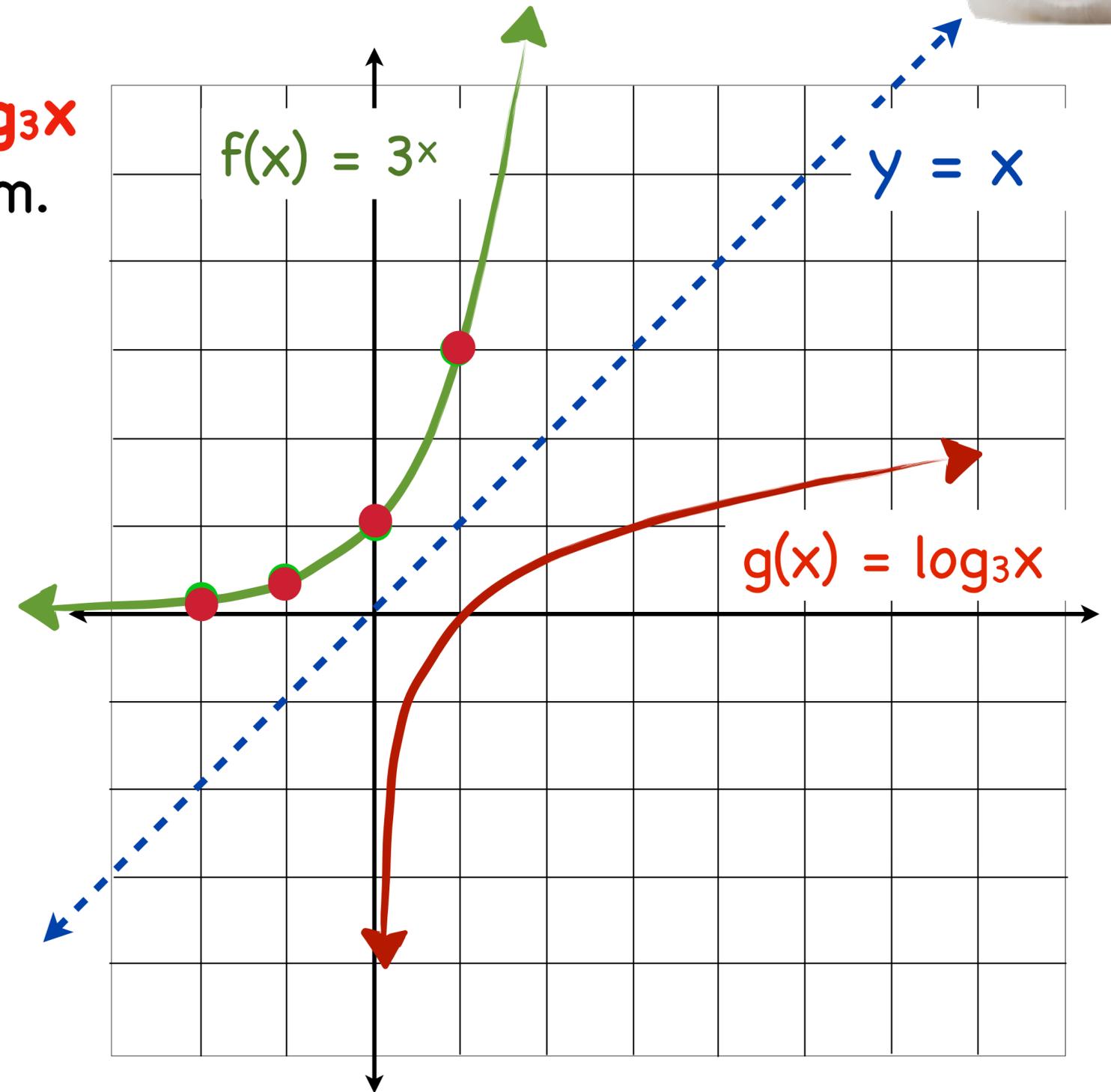
Characteristics of Logarithmic functions of the Form $f(x) = \log_b x$

🐱 The domain of $f(x) = \log_b x$ consists of all **positive** real numbers: $(0, \infty)$

🐱 The range of $f(x) = \log_b x$ consists of all real numbers: $(-\infty, \infty)$

🐱 The graphs of all log functions of the form $f(x) = \log_b x$ pass through the point $(1,0)$.

🐱 If $b > 1$, $f(x) = \log_b x$ is an increasing function and has a graph that goes up to the right

🐱 If $0 < b < 1$, $f(x) = \log_b x$ is a decreasing function and has a graph that goes down to the right

🐱 The graph of $f(x) = \log_b x$ is asymptotic to the y-axis. (The y-axis is an asymptote of $f(x) = \log_b x$).

Table Repeat

Properties of Logarithms

1. $\log_a 1 = 0$ because $a^0 = 1$.

2. $\log_a a = 1$ because $a^1 = a$.

3. $\log_a a^x = x$ and $a^{\log_a x} = x$ Inverse Properties

4. If $\log_a x = \log_a y$, then $x = y$.   One-to-One Property

Transformations of Log Function

Table 4.4   Transformations Involving Logarithmic Functions

In each case, c represents a positive real number.

Transformation	Equation	Description
Vertical translation	$g(x) = \log_b x + c$	• Shifts the graph of $f(x) = \log_b x$ upward c units.
	$g(x) = \log_b x - c$	• Shifts the graph of $f(x) = \log_b x$ downward c units.
Horizontal translation	$g(x) = \log_b(x + c)$	• Shifts the graph of $f(x) = \log_b x$ to the left c units. Vertical asymptote: $x = -c$
	$g(x) = \log_b(x - c)$	• Shifts the graph of $f(x) = \log_b x$ to the right c units. Vertical asymptote: $x = c$
Reflection	$g(x) = -\log_b x$	• Reflects the graph of $f(x) = \log_b x$ about the x-axis.
	$g(x) = \log_b(-x)$	• Reflects the graph of $f(x) = \log_b x$ about the y-axis.
Vertical stretching or shrinking	$g(x) = c \log_b x$	• Vertically stretches the graph of $f(x) = \log_b x$ if $c > 1$. • Vertically shrinks the graph of $f(x) = \log_b x$ if $0 < c < 1$.
Horizontal stretching or shrinking	$g(x) = \log_b(cx)$	• Horizontally shrinks the graph of $f(x) = \log_b x$ if $c > 1$. • Horizontally stretches the graph of $f(x) = \log_b x$ if $0 < c < 1$.

The Domain of a Logarithmic Function

The domain of an exponential function of the form $f(x) = b^x$ includes all real numbers and its range is the set of positive real numbers.

Because the logarithmic function is the inverse of the exponential function, the logarithmic function reverses the domain and the range of the exponential function,

The domain of a logarithmic function of the form $f(x) = \log_b x$ is the set of all positive real numbers. In general, **the domain of $f(x) = \log_b[g(x)]$ consists of all x for which $g(x) > 0$**.

Example: Finding the Domain of a Logarithmic Function

Find the domain of $f(x) = \log_4(x - 5)$.

The domain of f consists of all x for which $x - 5 > 0$.

Thus, the domain of f is (5, ∞)

If we graph $f(x) = \log_4(x - 5)$ we see the domain.

$f(x) = \log_4(x - 5)$ is $g(x) = \log_4 x$ shifted 5 units right.

All point of $f(x) = \log_4(x-5)$ have x coordinates > 5.

vertical asymptote

x = 5

$f(x) = \log_4(x-5)$

$g(x) = \log_4 x$
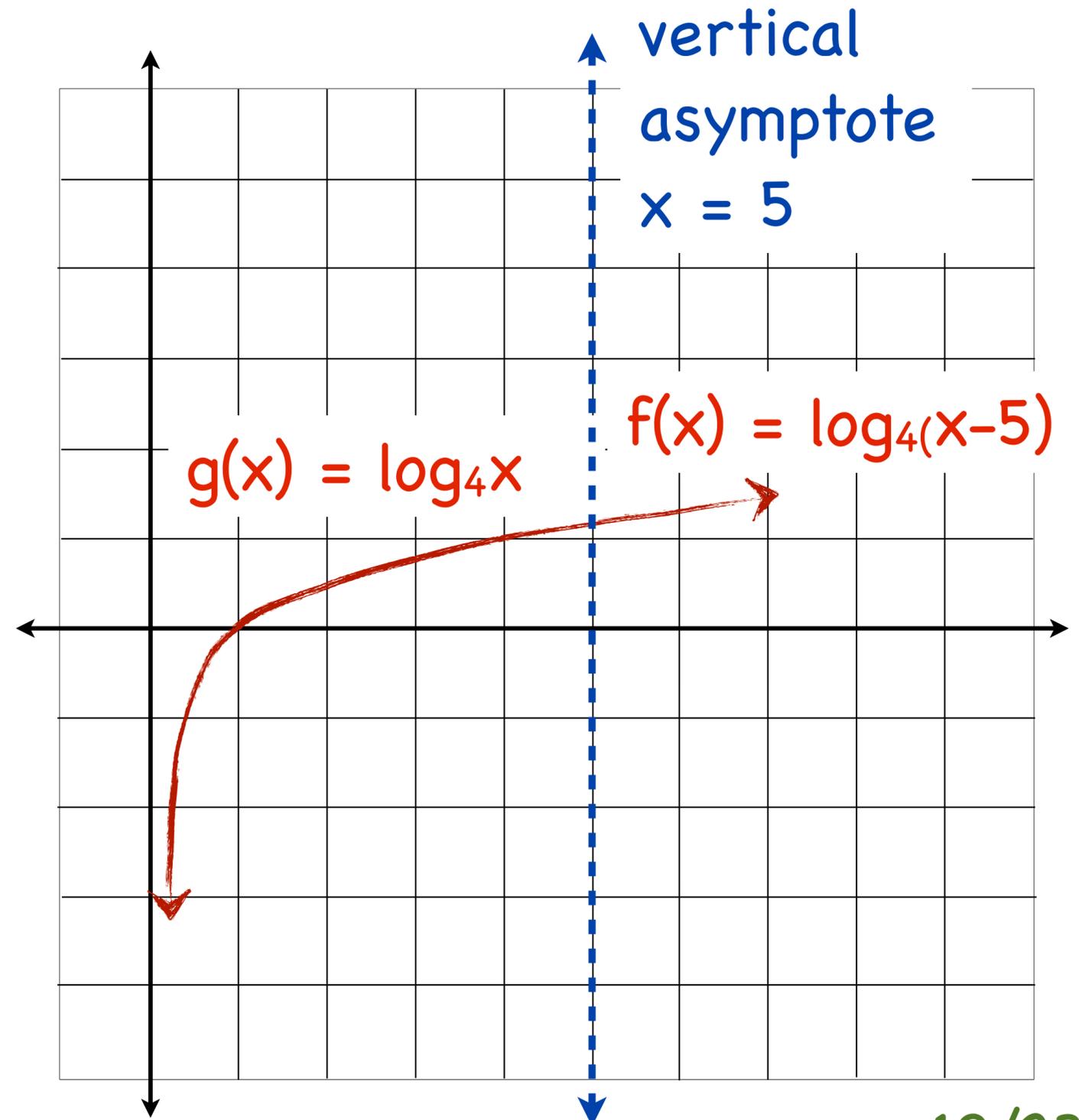
Common Logarithms

The logarithmic function with **base 10** is called the **common logarithmic** function.

The function $f(x) = \log_{10} x$ is usually expressed **f(x) = logx**.

The TI–84 has a `LOG` button for **common log**. You should not need instruction on how to calculate the value of a **common log** on the TI.

Example: Application

The percentage of adult height attained by a boy **x years old** can be modeled by **f(x) = 29 + 48.8log(x + 1)** where **x** represents the boy's age (from 5 to 15) and **f(x)** represents the percentage of his adult height. Approximately what percentage of his adult height has a boy attained at age ten?

f(x) = 29 + 48.8log(x + 1)

f(10) = 29 + 48.8log(10 + 1)

f(10) = 29 + 48.8log(11)

f(10) ≈ 29 + 48.8(1.04139)

f(10) ≈ 79.82

A ten-year old boy has attained approximately 79.8% of his adult height.

Natural Logarithms

The logarithmic function with **base e** is called the **natural logarithmic function**.

The function **f(x) = log$_e$x** is usually expressed **f(x) = lnx**.

The properties of the natural logarithm are identical to the properties for logarithms of any other base.

$$\text{lne = 1} \quad (e^1 = e) \qquad \text{ln1 = 0} \quad (e^0 = 1)$$

🐱 The domain of f(x) = lnx consists of all positive real numbers: (0, ∞)

🐱 The range of f(x) = lnx consists of all real numbers: (-∞, ∞)

🐱 The graph of f(x) = lnx is asymptotic to the y-axis.

🐱 The graph of f(x) = lnx passes through the point (1,0).

The TI-84 has a `LN` button for **natural logs**. You should not need instruction on how to calculate the value of a **natural log** on the TI.

Example: Application

When the outside air temperature is anywhere from 72° to 96° Fahrenheit, the temperature in an enclosed vehicle climbs by 43° in the first hour. The function

$$f(x) = 13.4\ln x - 11.6$$

models the temperature increase, f(x), in degrees Fahrenheit, after x minutes. Use the function to find the temperature increase, to the nearest degree, after 30 minutes.

$f(x) = 13.4\ln x - 11.6$

$f(30) = 13.4\ln 30 - 11.6$

$f(30) \approx 13.4(3.4012) - 11.6$

$f(30) \approx 33.98°$

The temperature will increase by approximately 34° after 30 minutes.