Chapter 4.3

Trigonometric Functions

Right Triangle Trigonometry

Chapter 4.3

Homework

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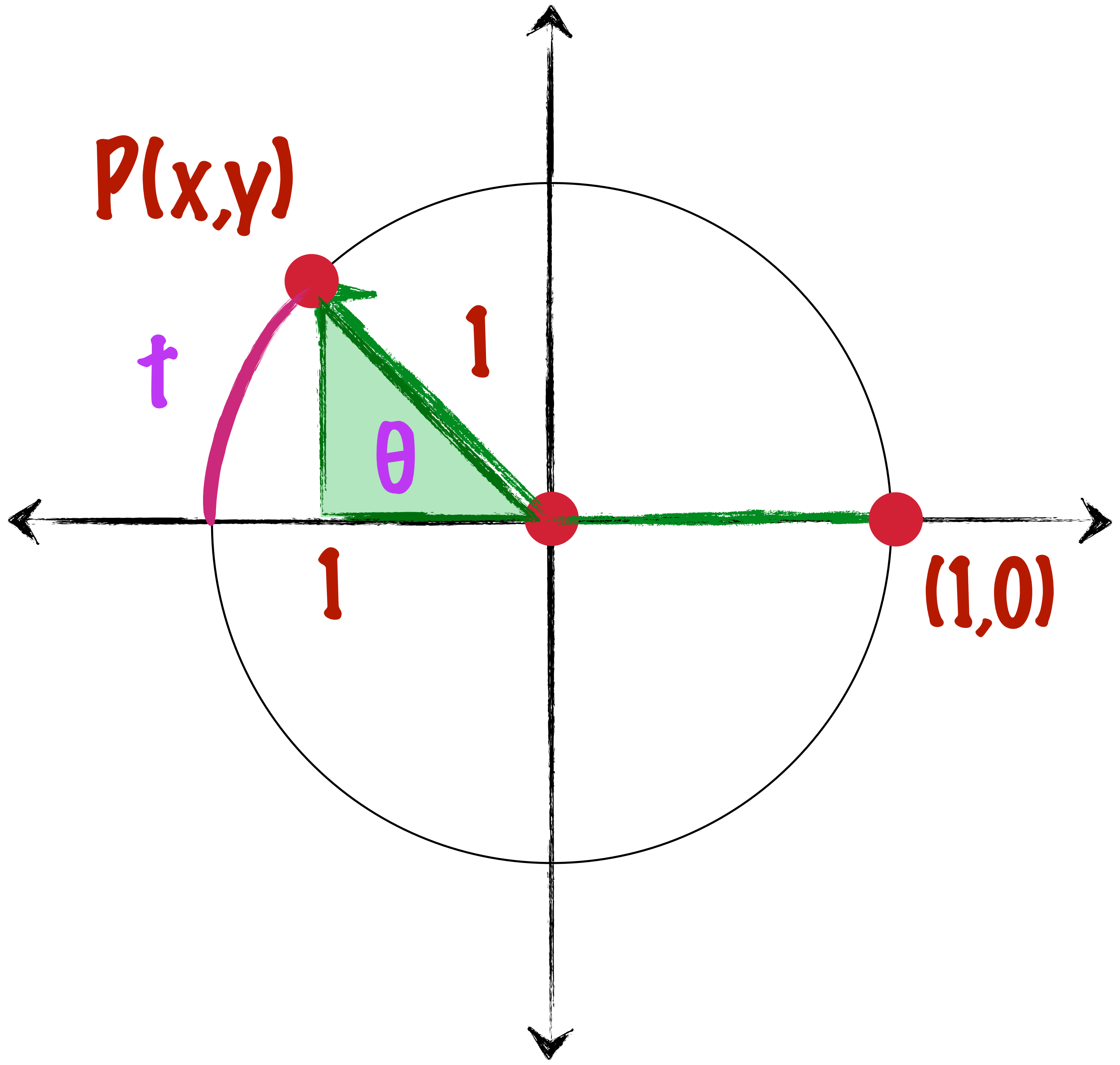
Chapter 4.3

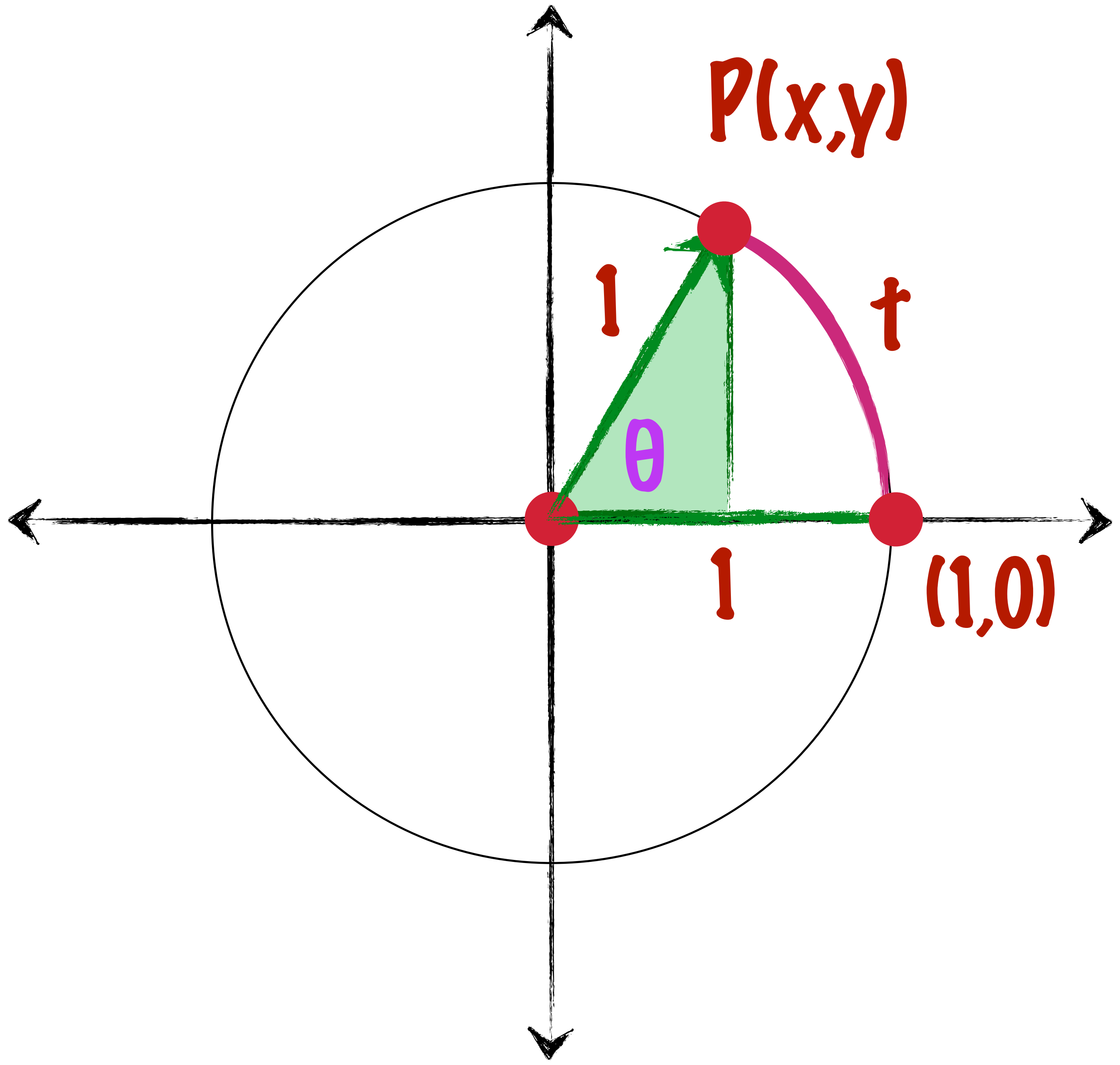
Objectives

- Use right triangles to evaluate trigonometric functions.
- Find function values for $\frac{\pi}{3}(60°); \frac{\pi}{4}(45°); \frac{\pi}{6}(30°)$
- Use equal co-functions of complements
- Use right triangle trigonometry to solve applied problems.

Right Triangle Definitions of Trigonometric Functions

- We have seen how, within the unit circle, we can find right triangles with acute angle θ, to define the **trigonometric functions.**





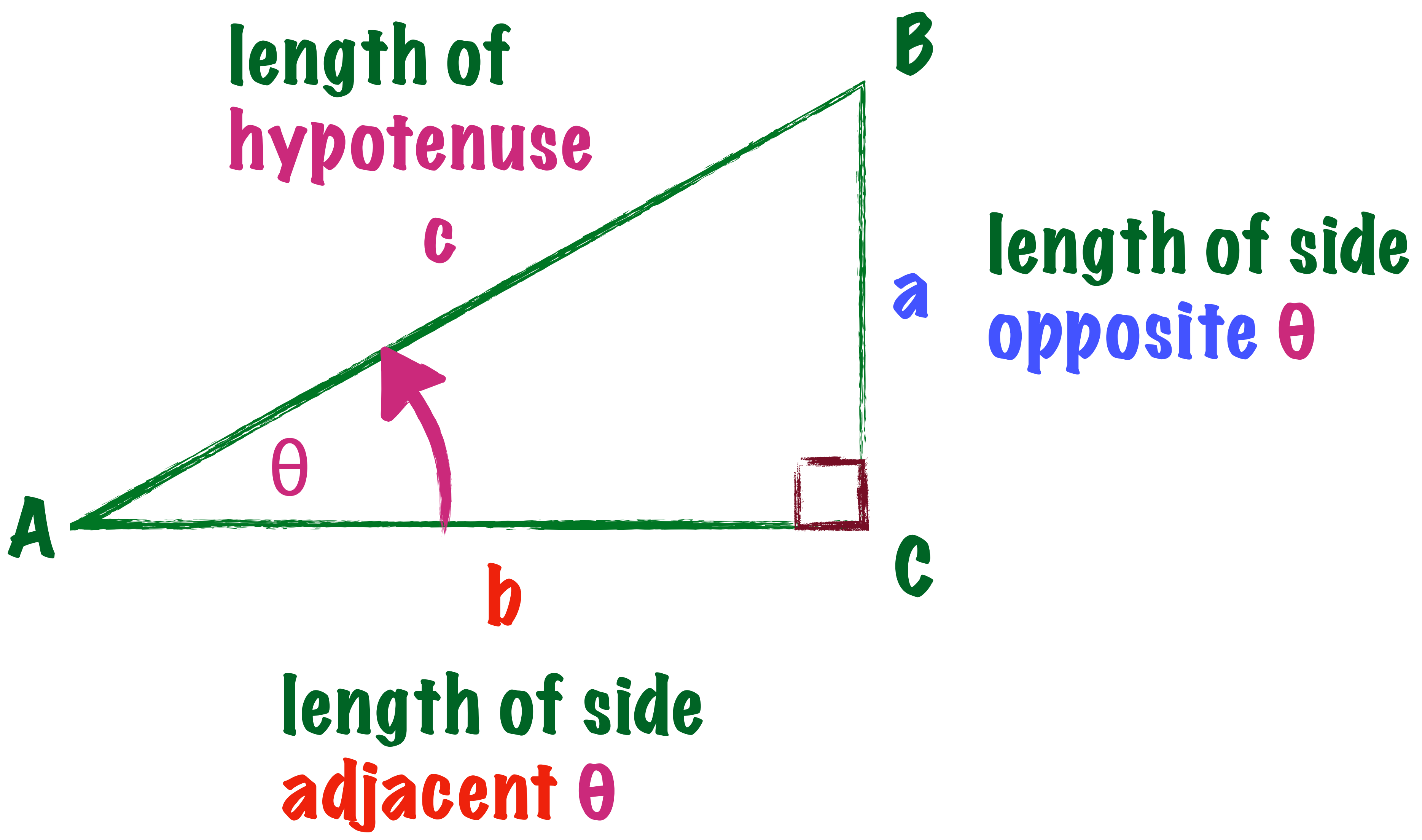
Right Triangle Definitions of Trigonometric Functions

- In general, the trigonometric functions of θ depend only on the size of angle and not on the size of the triangle.

$$\sin\theta = \frac{\text{Length of side Opposite}}{\text{Length of Hypotenuse}} = \frac{a}{c}$$

$$\cos\theta = \frac{\text{Length of side adjacent}}{\text{Length of Hypotenuse}} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{Length of side opposite}}{\text{Length of side adjacent}} = \frac{a}{b}$$



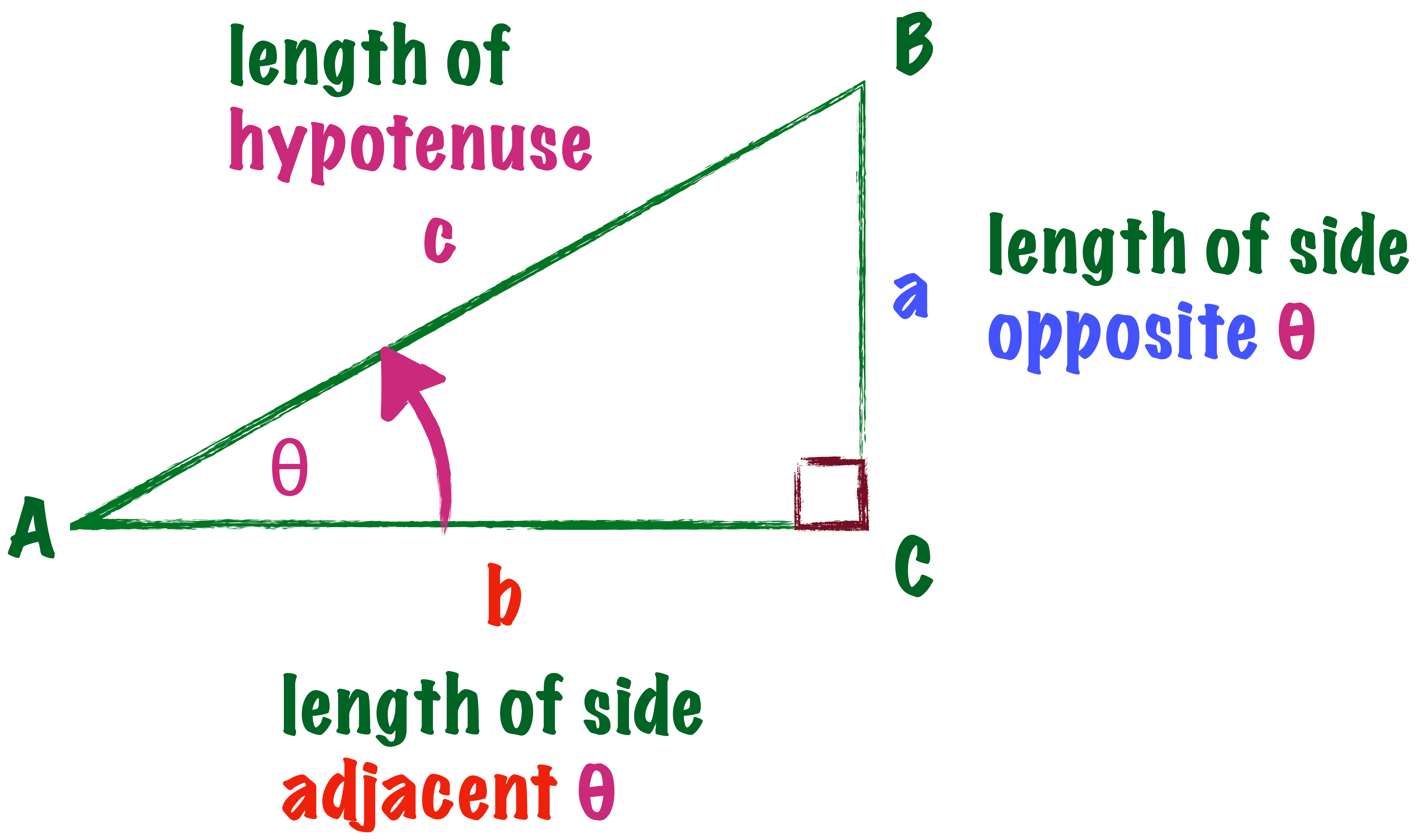
Right Triangle Definitions of Trigonometric Functions

- In general, the trigonometric functions of θ depend only on the size of angle and not on the size of the triangle.

$$\csc\theta = \frac{\text{Length of Hypotenuse}}{\text{Length of side Opposite}} = \frac{c}{a}$$

$$\sec\theta = \frac{\text{Length of Hypotenuse}}{\text{Length of side adjacent}} = \frac{c}{b}$$

$$\cot\theta = \frac{\text{Length of side adjacent}}{\text{Length of side opposite}} = \frac{b}{a}$$



Right Triangle Definitions of Trigonometric Functions

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. The six trigonometric functions of the angle θ are defined as follows. (Note that the functions in the second row are the *reciprocals* of the corresponding functions in the first row.)

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations opp, adj, and hyp represent the lengths of the three sides of a right triangle.

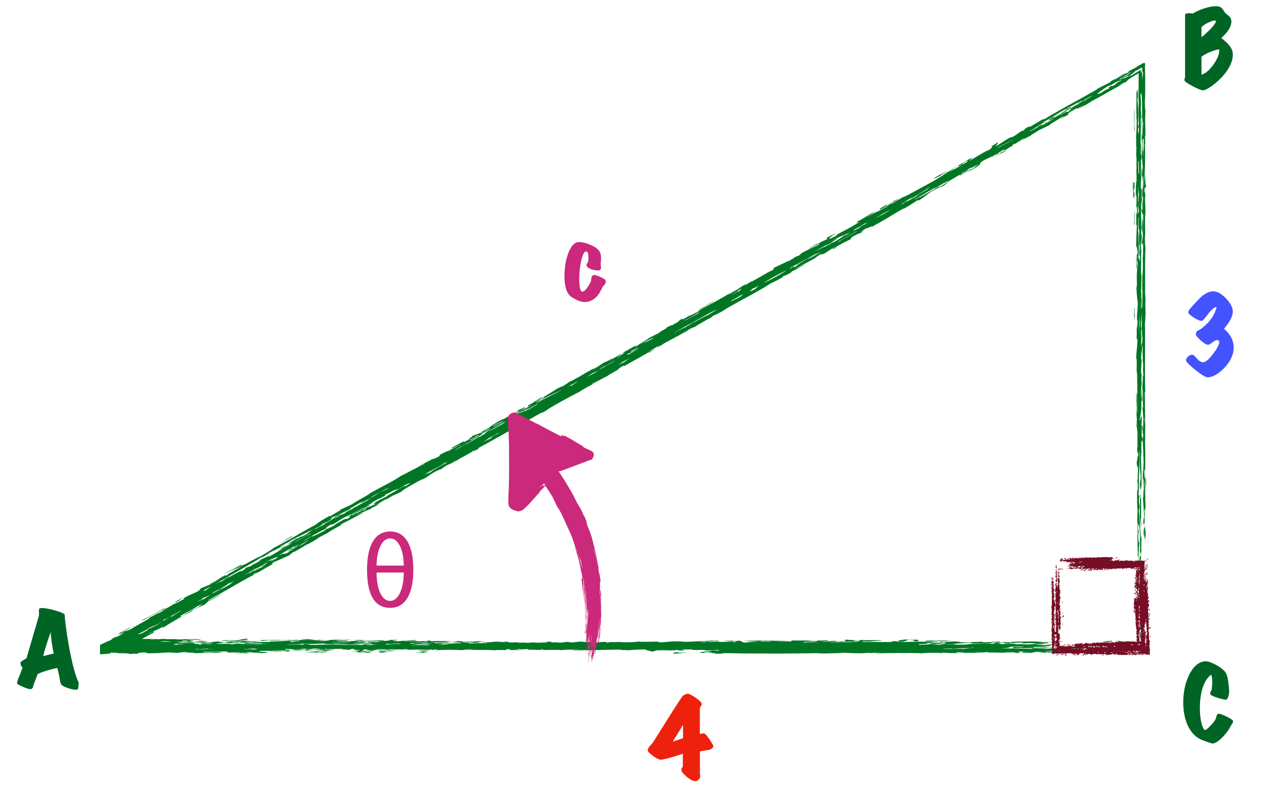
opp $=$ the length of the side *opposite* θ

adj $=$ the length of the side *adjacent to* θ

hyp $=$ the length of the *hypotenuse*

Example: Evaluating Trigonometric Functions

Find the value of the six trigonometric functions in the figure.



Remember Pythagorus?

$$3^2 + 4^2 = c^2$$

$$9 + 16 = c^2$$

$$25 = c^2$$

$$5 = c$$

$$\sin\theta = \frac{3}{5} \qquad \csc\theta = \frac{5}{3}$$

$$\cos\theta = \frac{4}{5} \qquad \sec\theta = \frac{5}{4}$$

$$\tan\theta = \frac{3}{4} \qquad \cot\theta = \frac{4}{3}$$

Function Values for Some Special Angles

- A right triangle with an angle of 45°, or $\frac{\pi}{4}$ radians, is isosceles. The triangle has two sides of equal length.
  - Find the value of the six trigonometric functions in the figure.

$$\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\csc\frac{\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

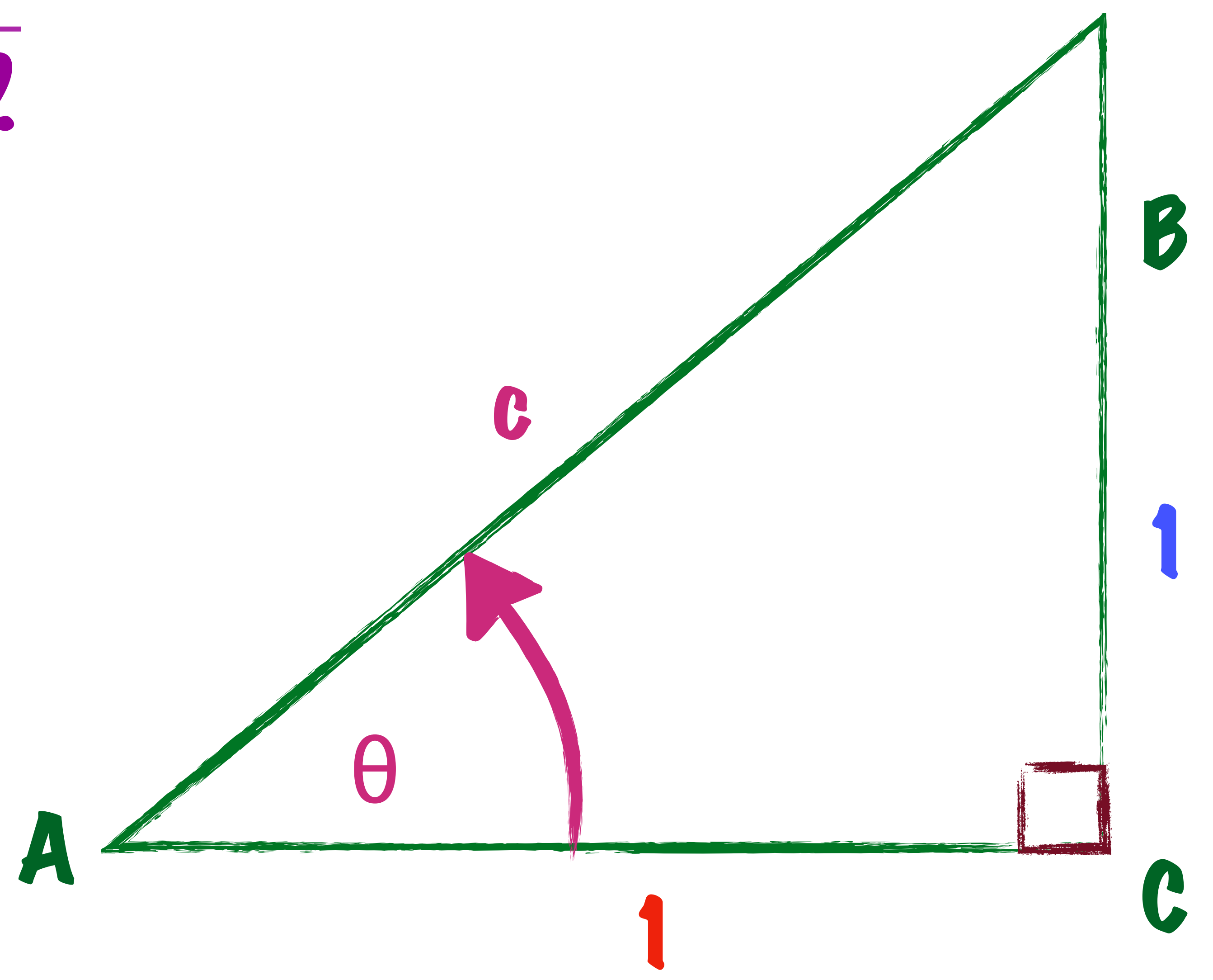
$$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\sec\frac{\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\tan\frac{\pi}{4} = \frac{1}{1} = 1$$

$$\cot\frac{\pi}{4} = \frac{1}{1} = 1$$

$c = \sqrt{2}$



Function Values for Some Special Angles

A right triangle with an of 30°, or $\frac{\pi}{6}$ radians, also has an angle of 60°, or $\frac{\pi}{3}$ radians. In a 30-60-90 triangle, the side opposite the 30° angle is one-half the length of the hypotenuse.

$$\sin\frac{\pi}{6}=\frac{\frac{1}{2}}{1}=\frac{1}{2}=\cos\frac{\pi}{3}$$

$$\cos\frac{\pi}{6}=\frac{\frac{\sqrt{3}}{2}}{1}=\frac{\sqrt{3}}{2}=\sin\frac{\pi}{3}$$

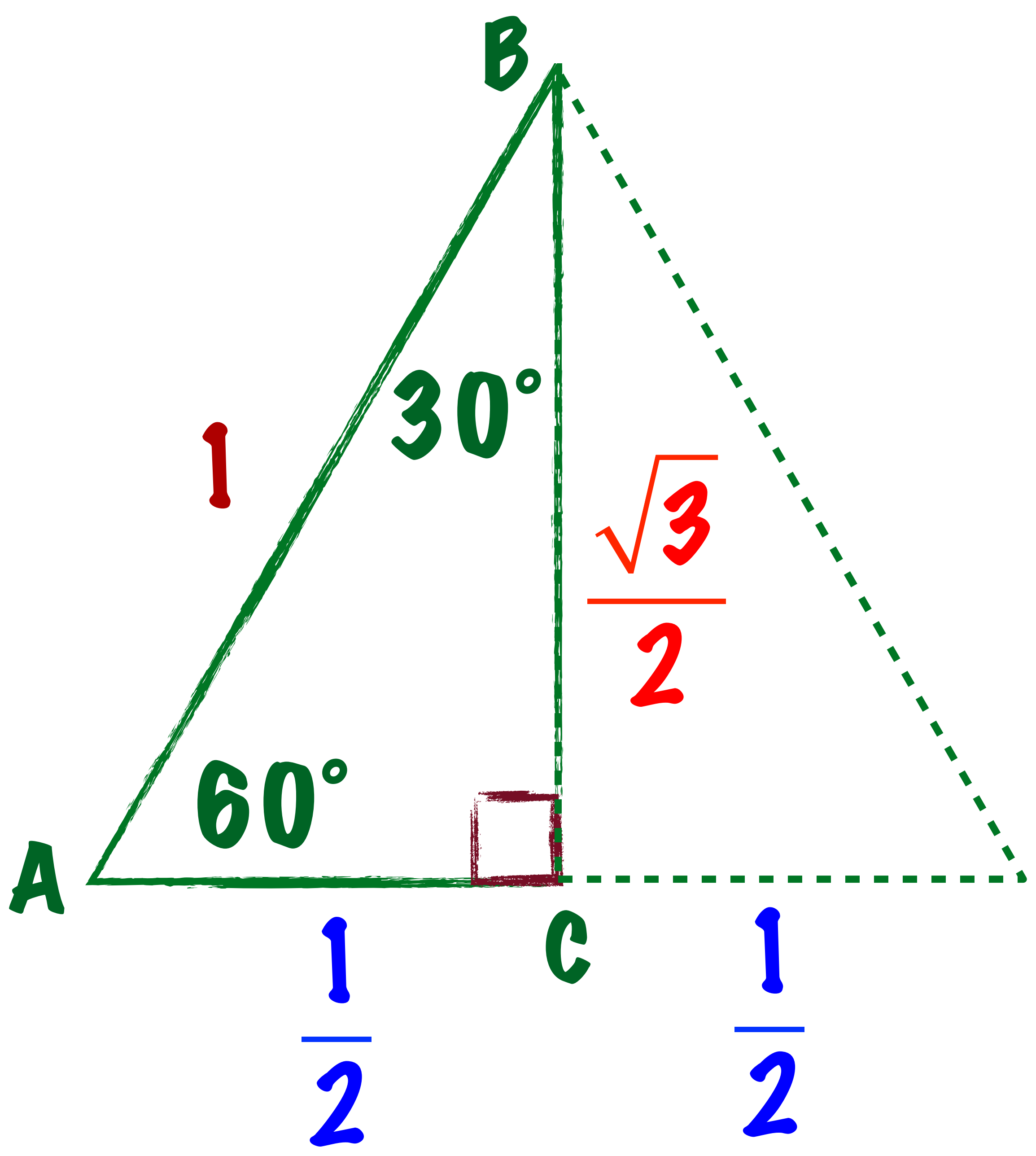
$$\tan\frac{\pi}{6}=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}} \qquad \tan\frac{\pi}{3}=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$$

$$\left(\frac{1}{2}\right)^2+a^2=1^2$$

$$\frac{1}{4}+a^2=1$$

$$a^2=\frac{3}{4}$$

$$a=\frac{\sqrt{3}}{2}$$



Special Angles

- If you are asked to find these ratios, provide the exact values. Do not find the calculator approximations unless specifically asked to do so.

Sines, Cosines, and Tangents of Special Angles

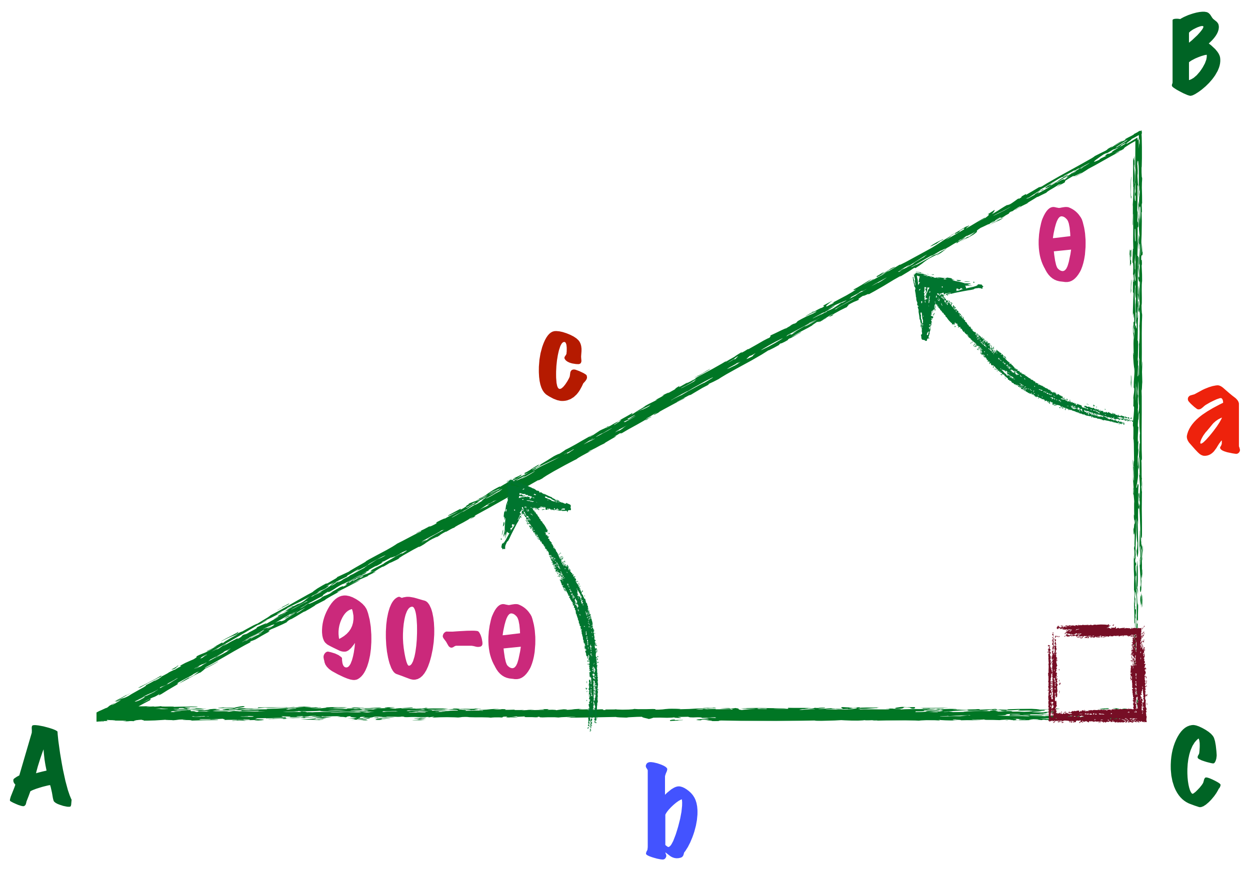
$$\sin 30° = \sin \frac{\pi}{6} = \frac{1}{2} \qquad \cos 30° = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \qquad \tan 30° = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\sin 45° = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \cos 45° = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \qquad \tan 45° = \tan \frac{\pi}{4} = 1$$

$$\sin 60° = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \qquad \cos 60° = \cos \frac{\pi}{3} = \frac{1}{2} \qquad \tan 60° = \tan \frac{\pi}{3} = \sqrt{3}$$

Trigonometric Functions and Complements

- Two positive angles are **complements** if their sum is 90° or $\frac{\pi}{2}$.
    - Any pair of trigonometric functions **f** and **g** for which $g(\theta) = f(90^\circ - \theta)$ and $f(\theta) = g(90^\circ - \theta)$ are called **co-functions**.



$$\sin\theta = \frac{b}{c} = \cos(90-\theta)$$

$$\cos\theta = \frac{a}{c} = \sin(90-\theta)$$

$$\tan\theta = \frac{b}{a} = \cot(90-\theta)$$

Cofunction Identities

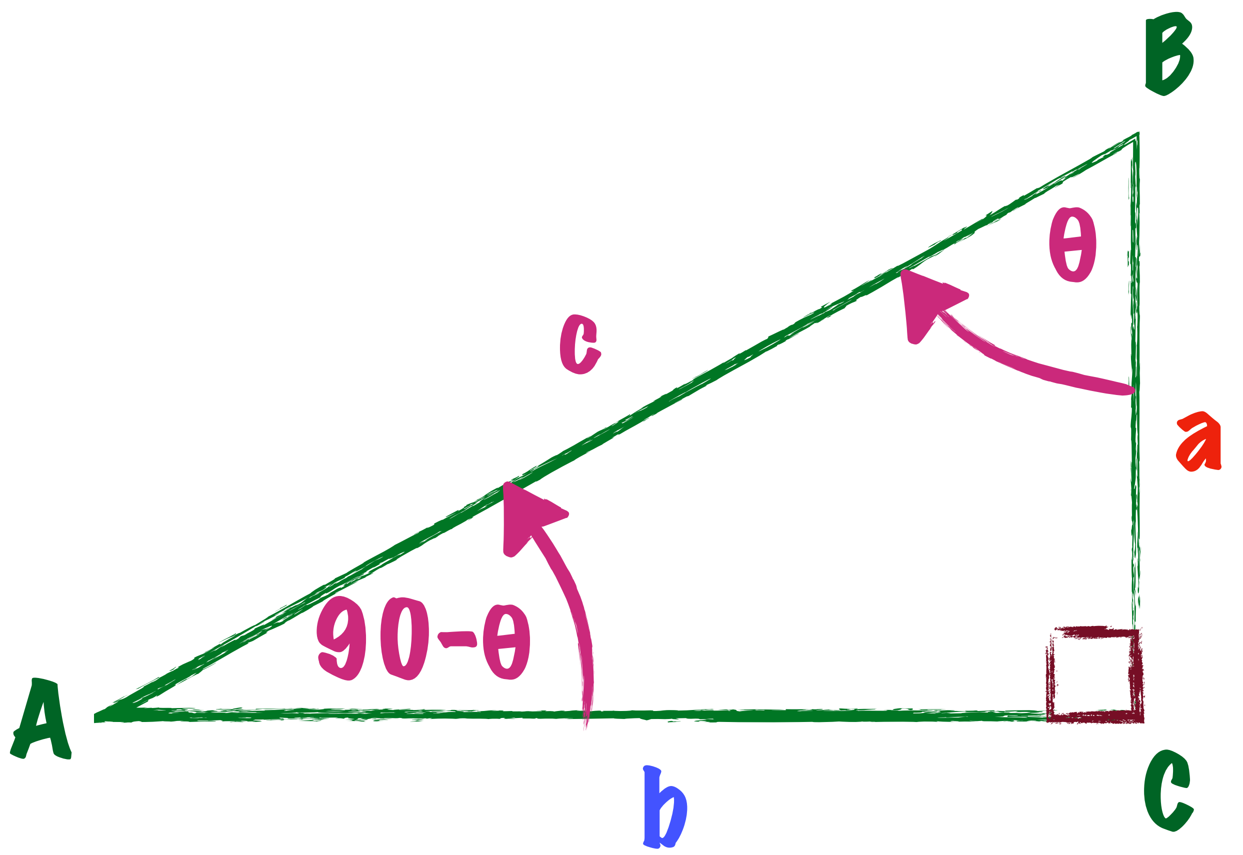
The value of a trigonometric function of θ is equal to the co-function of the complement of θ (90°-θ). Co-functions of complementary angles are equal.

If θ is measured in degrees.

$\cos\theta = \sin(90^\circ - \theta)$ $\quad$ $\sec\theta = \csc(90^\circ - \theta)$

$\sin\theta = \cos(90^\circ - \theta)$ $\quad$ $\csc\theta = \sec(90^\circ - \theta)$

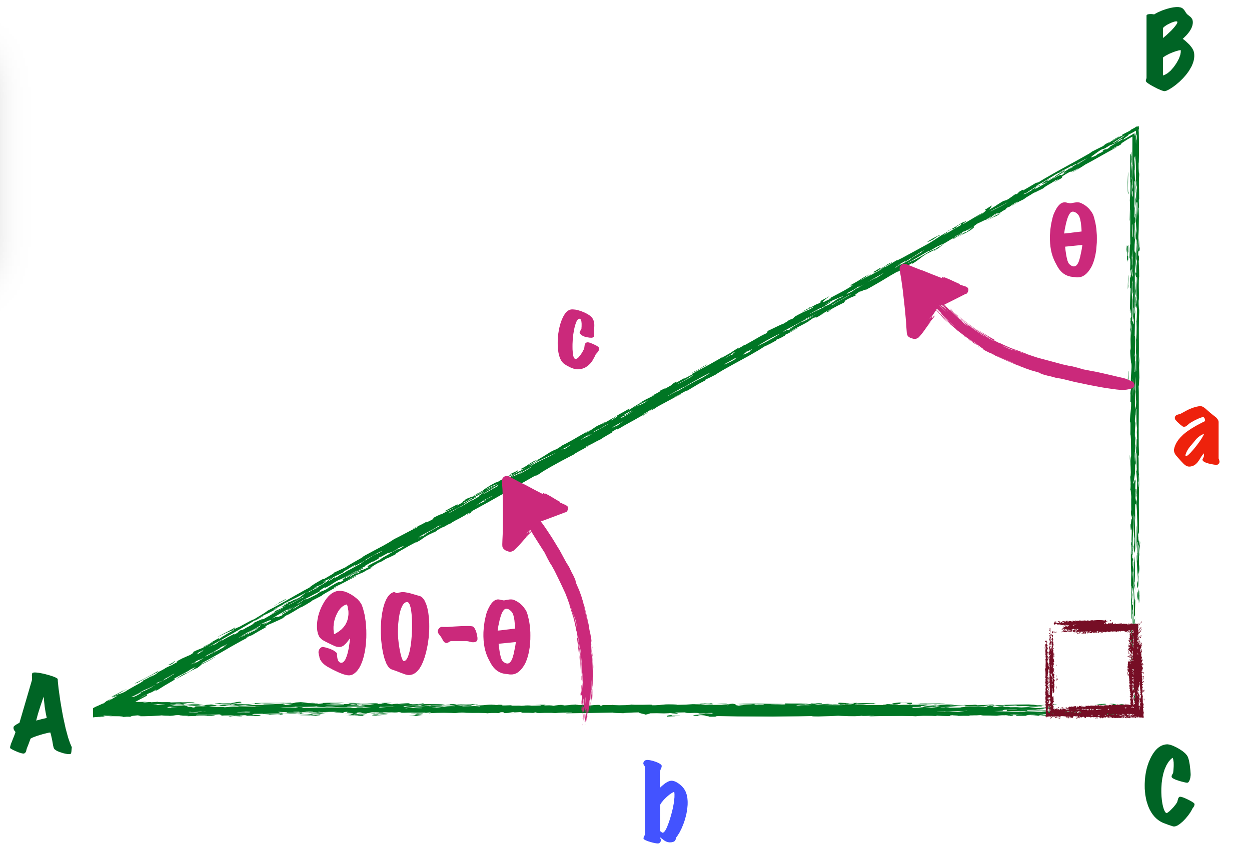
$\tan\theta = \cot(90^\circ - \theta)$ $\quad$ $\cot\theta = \tan(90^\circ - \theta)$



Cofunction Identities

The value of a trigonometric function of θ is equal to the co-function of the complement of θ ($\frac{\pi}{2}$-θ). Co-functions of complementary angles are equal.

If θ is measured in radians.



$$\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right) \qquad \csc\theta = \sec\left(\frac{\pi}{2}-\theta\right) \qquad \tan\theta = \cot\left(\frac{\pi}{2}-\theta\right)$$

$$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right) \qquad \sec\theta = \csc\left(\frac{\pi}{2}-\theta\right) \qquad \cot\theta = \tan\left(\frac{\pi}{2}-\theta\right)$$

Using Cofunction Identities

Find a cofunction with the same value as the given expression:

a. $\sin 46^\circ = \cos(90^\circ - 46^\circ) = \cos 44^\circ$

b. $\cot \frac{\pi}{12} = \tan\left(\frac{\pi}{2} - \frac{\pi}{12}\right) = \tan \frac{5\pi}{12}$

Trig Identities

We have seen these identities and you can use them to find the values of all the trigonometric ratios.

Reciprocal Identities

$$\csc t = \frac{1}{\sin t} \qquad \sec t = \frac{1}{\cos t} \qquad \cot t = \frac{1}{\tan t}$$

Quotient Identities

$$\tan t = \frac{\sin t}{\cos t}$$

$$\cot t = \frac{\cos t}{\sin t}$$

Pythagorean Identities

$$\sin^2 t + \cos^2 t = 1$$

$$\cot^2 t + 1 = \csc^2 t$$

$$1 + \tan^2 t = \sec^2 t$$

STUDY TIP

You can also use the reciprocal identities for sine, cosine, and tangent to evaluate the cosecant, secant, and cotangent functions with a calculator. For instance, you could use the following keystroke sequence to evaluate sec 28°.

1 [÷] [COS] 28 [ENTER]

The calculator should display 1.1325701.

Using Trig Identities

If θ is an acute angle such that cos θ = 0.3, find:

$\cos\theta = 0.3 = \frac{3}{10}$

$\sin\theta = \frac{\sqrt{91}}{10}$

$\sin^2 t + \cos^2 t = 1$

$\sin^2 t + 0.3^2 = 1$

$\sin^2 t + 0.09 = 1$

$\sin^2 t = 0.91 = 91/100$

$\sin t = \sqrt{\frac{91}{100}} = \frac{\sqrt{91}}{10}$

$\tan\theta = \frac{\sqrt{91}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\tan\theta = \frac{\frac{\sqrt{91}}{10}}{\frac{3}{10}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{10}{3}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{10}{\sqrt{91}}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{3}{\sqrt{91}}$

These are exact values, not calculator approximations.

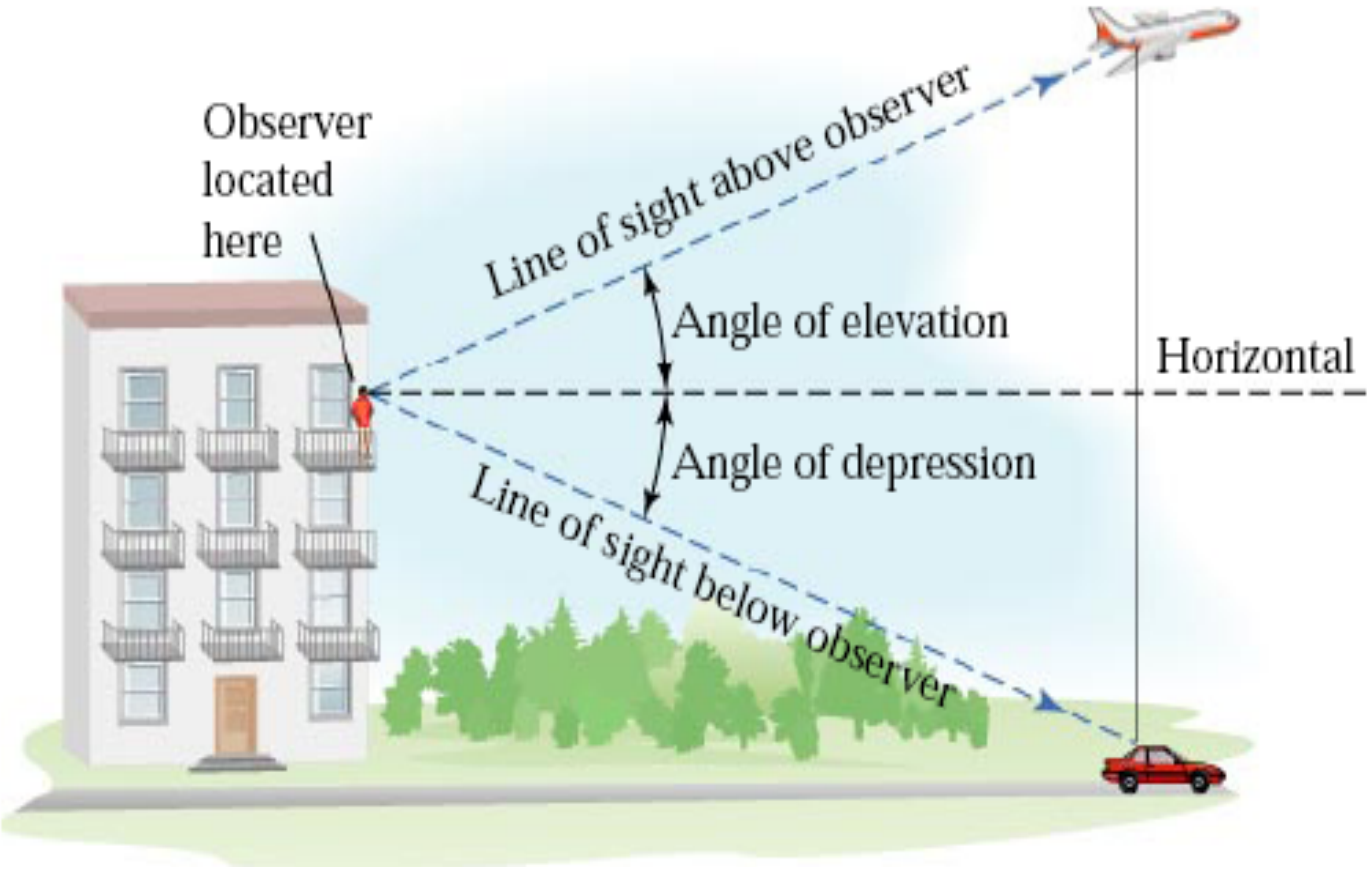
TI-84

Using the TI-84 is simple, but you **MUST MAKE CERTAIN THE CALCULATOR IS IN THE CORRECT MODE!**

- Set the calculator to degrees MODE ∨ RADIAN **DEGREE**
  - Find cos 15.3° COS 1 5 . 3 ENTER = .9645574185
- Set the calculator to radians MODE ∨ **RADIAN** DEGREE
  - Find cos 15.3 COS 1 5 . 3 ENTER = –.9179307804
  - Quite a difference.

Angle of Elevation and Angle of Depression

- An angle formed by a horizontal line and the line of sight to an object that is **above** the horizontal line is called the **angle of elevation**. The angle formed by the horizontal line and the line of sight to an object that is **below** the horizontal line is called the **angle of depression**.



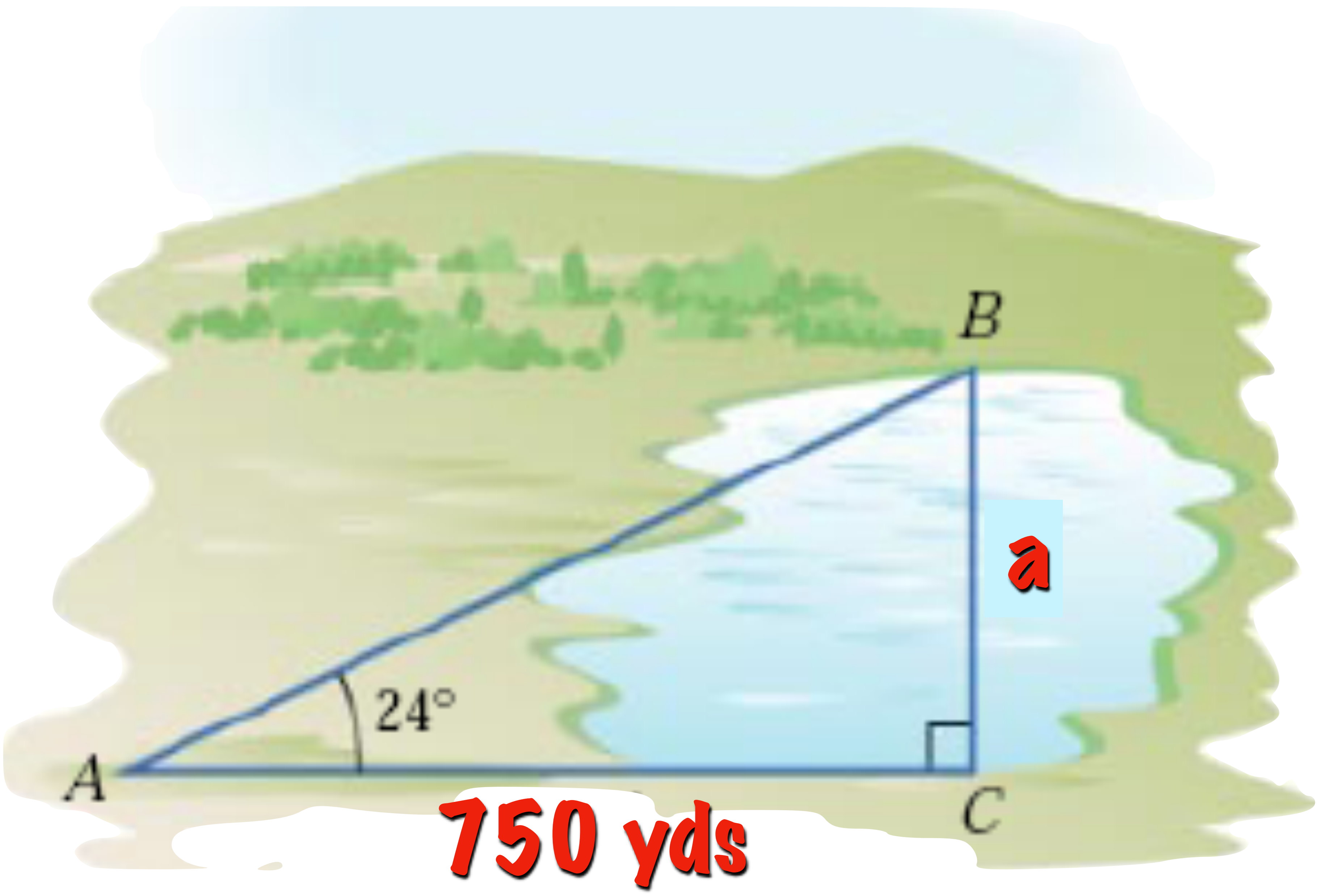
Problem Solving Using an Angle of Elevation

- The irregular blue shape in the figure represents a lake. The distance across the lake, **a**, is unknown. To find this distance, a surveyor took the measurements shown in the figure. What is the distance across the lake?

$$\tan 24^\circ = \frac{a}{750yd}$$

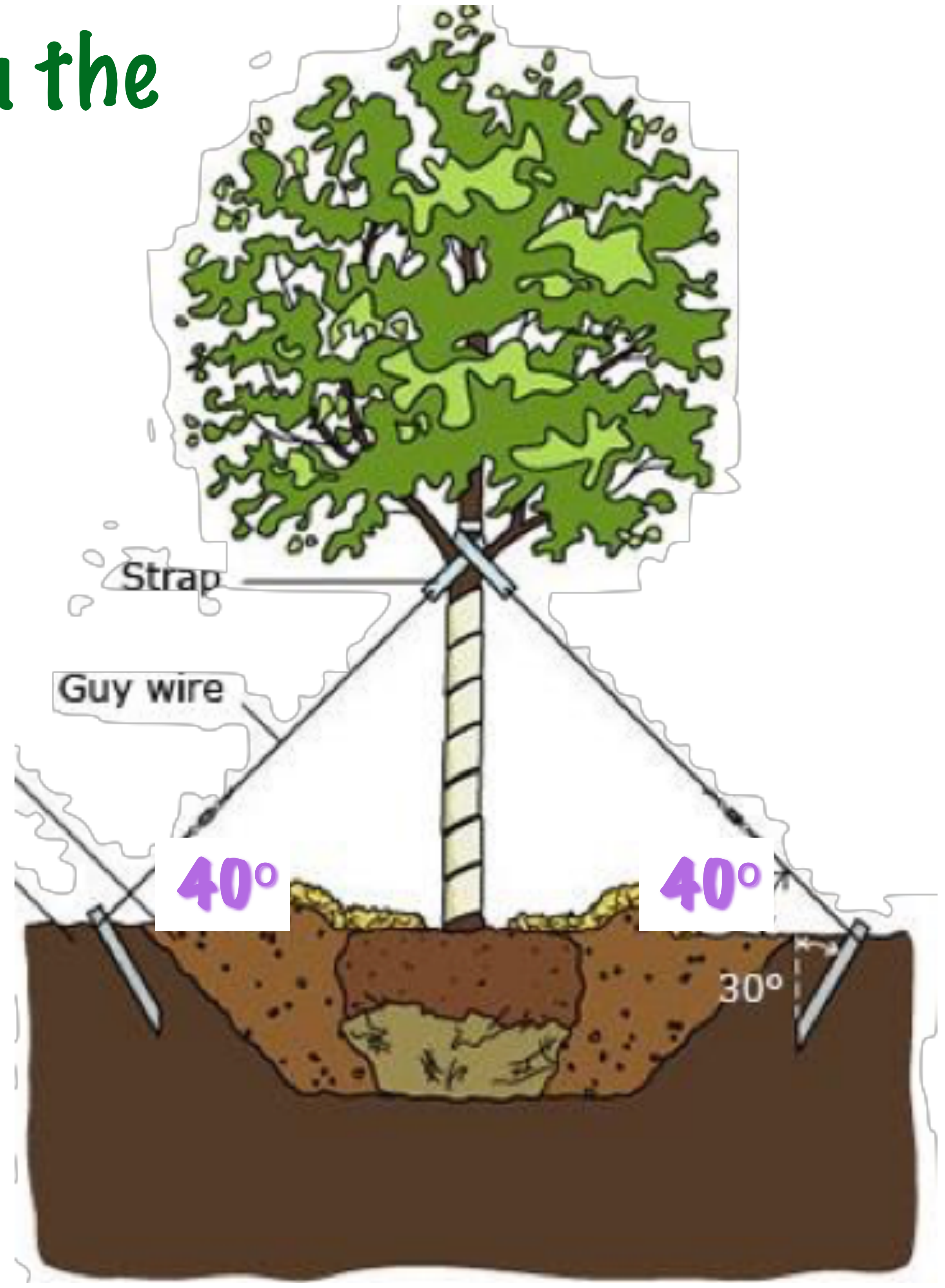
$$a = 750\tan 24 = 750(.4452)$$

The distance across the lake is approximately 333.9 yards.



Example

- You have a new Eucalyptus tree that is growing tall and skinny. Lately the wind has been bending the tree more than your spouse would like so you decide to support the tree with guy ropes. Plus the ropes will trip up the neighbor kid who keeps cutting through your yard, so no downside.
  - If you have 20 feet of rope at a 40° angle of elevation, how far away from the tree must you stake the ropes?



$$\cos 40^\circ = \frac{\text{Length of side adjacent}}{\text{Length of hypotenuse}} = \frac{s}{20}$$

$$\cos 40^\circ = .7660444431 = \frac{s}{20}$$

$$s \approx 15.32 \text{ feet}$$