Chapter 4

Trigonometric Functions

4.5 Graphs of Sine and Cosine Functions

Chapter 4

Homework

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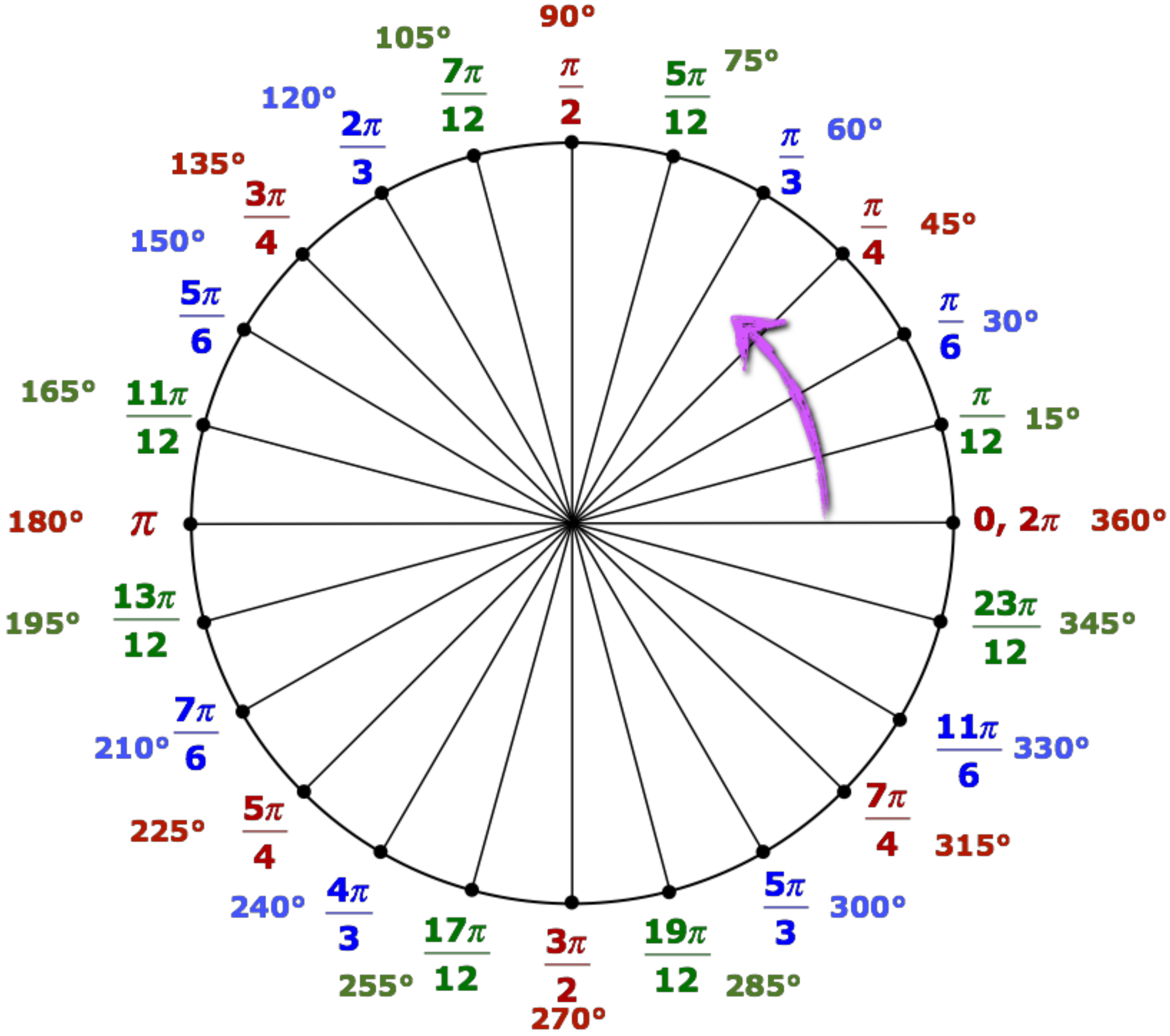
Chapter 4.5

Objectives

- Sketch the graph of $y = \sin x$.
- Sketch the graph of $y = \cos x$.
- Graph transformations of $y = \cos x$.
- Find Amplitude and Period of sine and cosine graphs.
- Graph vertical shifts of sine and cosine curves.
- Model periodic behavior.


90°
$\frac{\pi}{2}$
105°
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75°
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120°
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60°
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135°
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45°
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150°
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30°
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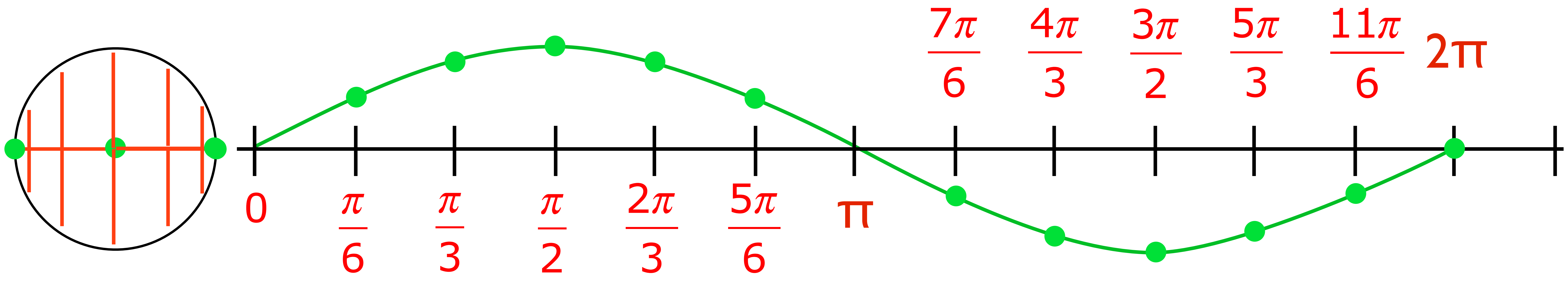
165°
$\frac{11\pi}{12}$
15°
$\frac{\pi}{12}$
180°
π
0, 2π
360°
195°
$\frac{13\pi}{12}$
345°
$\frac{23\pi}{12}$
210°
$\frac{7\pi}{6}$
330°
$\frac{11\pi}{6}$
225°
$\frac{5\pi}{4}$
315°
$\frac{7\pi}{4}$
240°
$\frac{4\pi}{3}$
300°
$\frac{5\pi}{3}$
255°
$\frac{17\pi}{12}$
285°
$\frac{19\pi}{12}$
$\frac{3\pi}{2}$
270°

Graph of Sine Function

The sine function can be graphed by plotting points (x, y) from the unit circle to the coordinate plane.



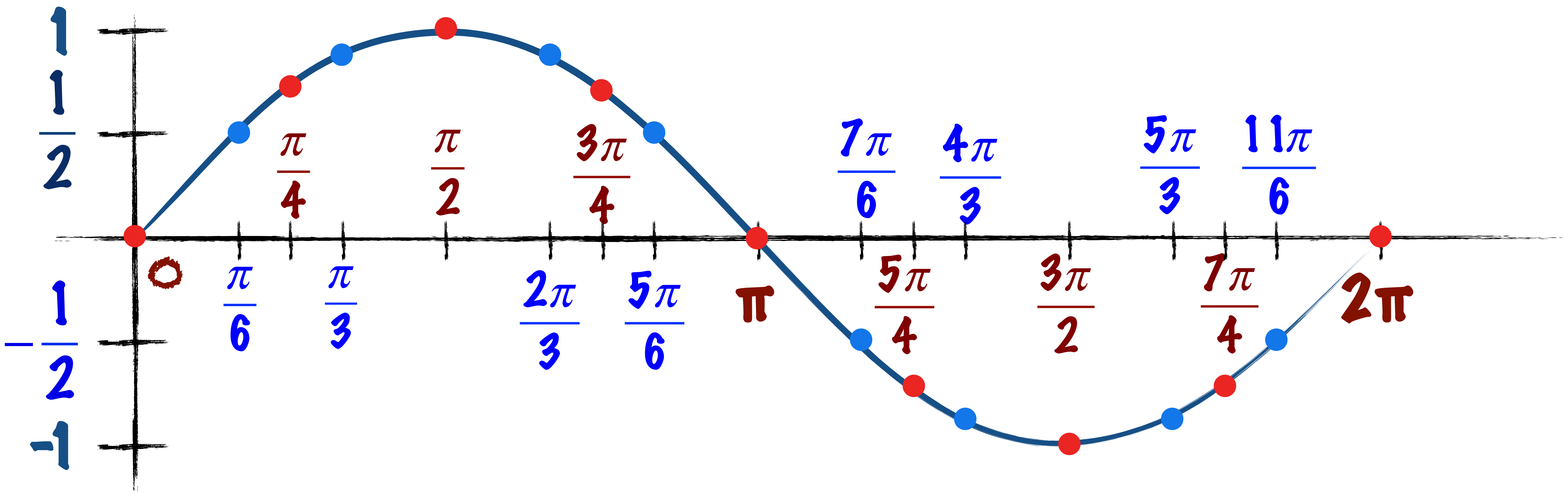
Slide5

The Graph of y = sinx

Complete the table:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Sinx	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

Graph the results:



The Graph of y = sinx

- The sine function is periodic, with a period 2π. That means the graph continues forever in both directions, repeating the pattern every 2π.

- The sine function is an odd function, sin(-x) = -sinx.

- The domain is (-∞, ∞); the range is [-1, 1].

Graphing Variations of $y = \sin x$

- The function $f(x) = \sin x$ is the parent function. The graph of $g(x) = a\sin(bx-c)+d$ transforms like any other function. The rules for transformations (shift, stretch or compress) apply.

  - To graph using values it is necessary to find the period, maximum, and minimum values.

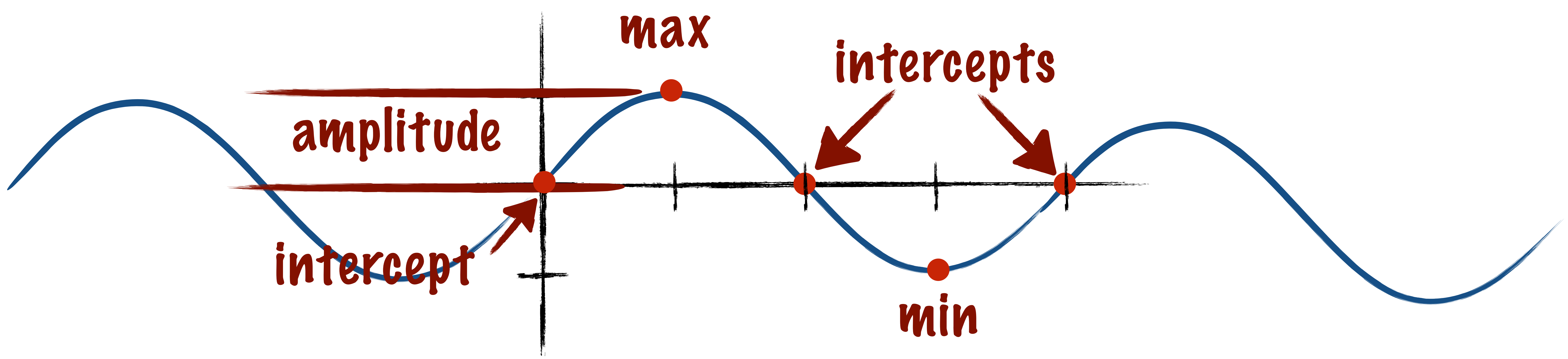
    - The maximum and minimum values come from the **amplitude** of the graph. The **amplitude** is the distance from the extreme values of sine and cosine to the line of equilibrium.

Graphing Variations of y = sinx

- To graph y = asin(bx-c)+d follow the procedure
  - 1. Identify period and amplitude
  - 2. Find 5 key x-values;
    - the x-intercepts (3 values),
    - x-value of maximum f(x),
    - and x-value of minimum f(x).
- To find the 5 x-values, divide the period into 4 sections. The first, middle, and last x are the intercepts. The 2nd x will be the maximum, the 3rd x is the minimum.

Graphing Variations of y = sinx

- Once the x-values have been determined
  - 3. Find y = f(x) for each of those 5 x-values.
    - the x-intercepts (3 values),
    - x-value of maximum f(x),
    - and x-value of minimum f(x).

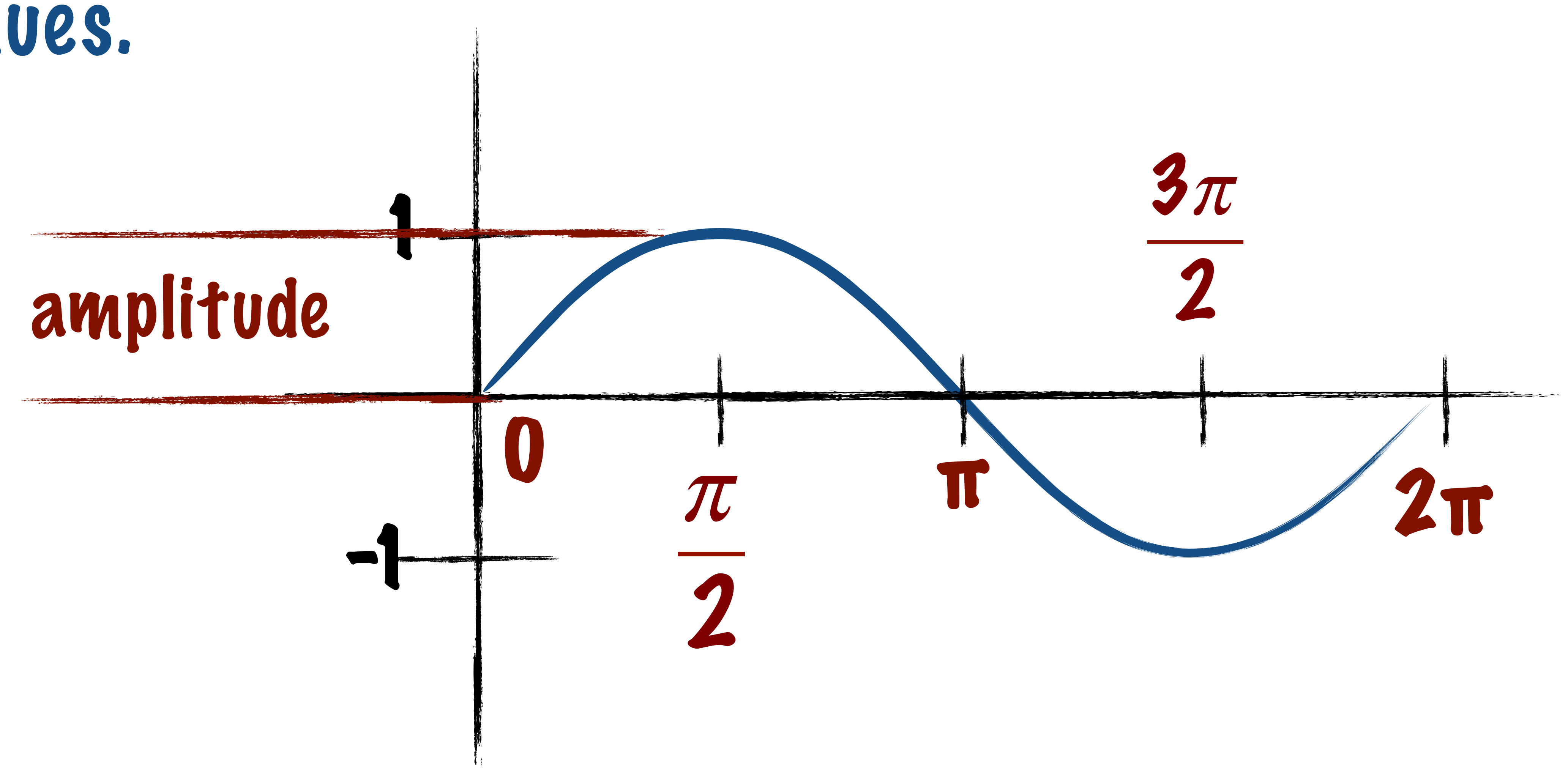


- 4. Draw the sine wave.
- 5. Repeat the sine wave over the desired domain.

Finding Amplitude

- When we graph y = sinx, the range for y is [-1 1].
  - That means the maximum value for sinx = 1. The amplitude of sinx is 1.
  - To graph y=sinx we find the 5 values for x by dividing the period by 4. $\frac{2\pi}{4}=\frac{\pi}{2}$
  - our 5 values of x are $0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi$
  - 3. Find y = f(x) for each of those 5 x-values.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinx	0	1	0	-1	0

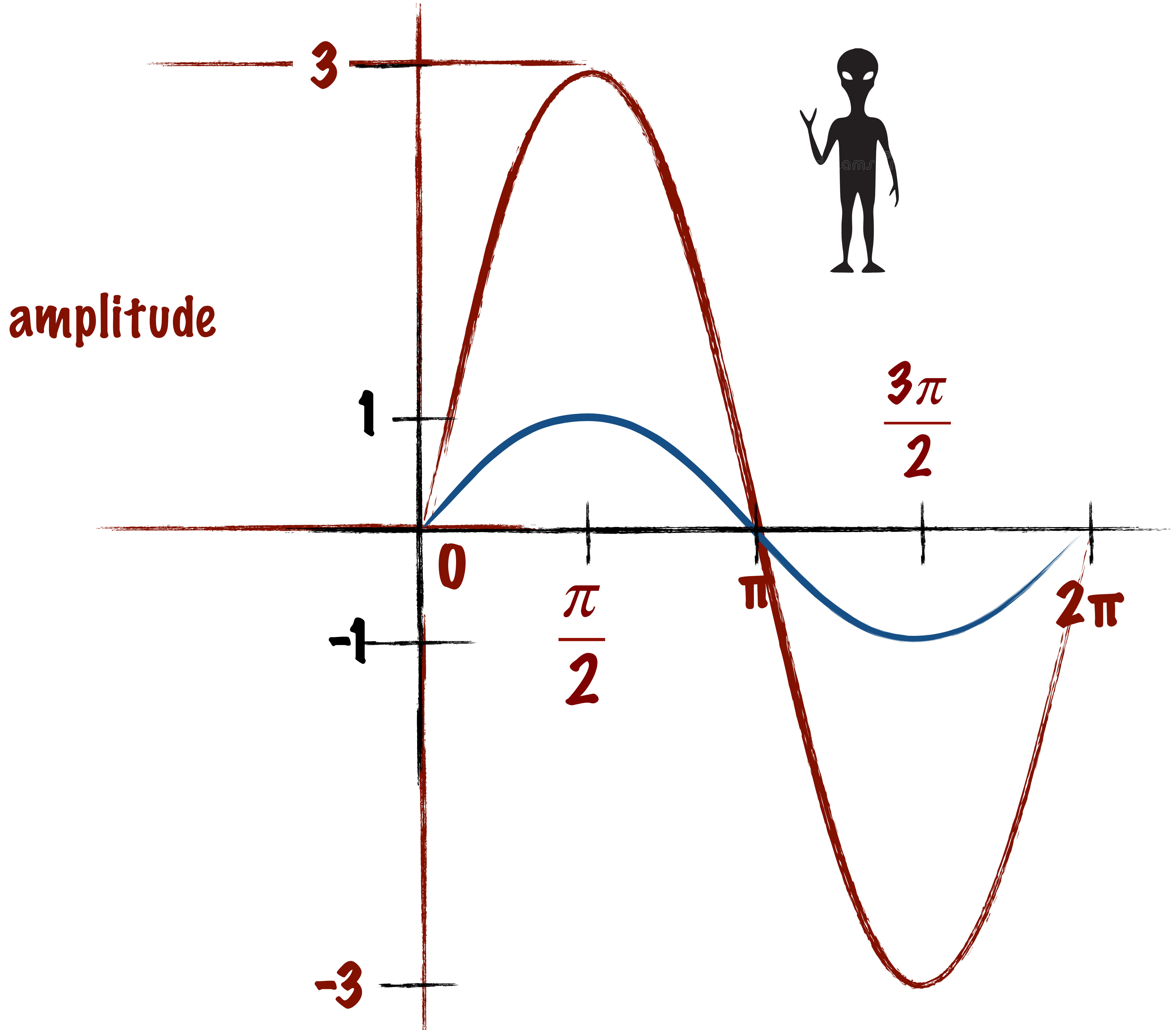


Finding Amplitude

If we graph y = 3sinx, we multiply each f(x) by 3. You should remember that this is a vertical stretch of factor 3. Thus the maximum value of 3sinx = 3(1) = 3. Then the amplitude of y = 3sinx is 3. The period remains 2π.

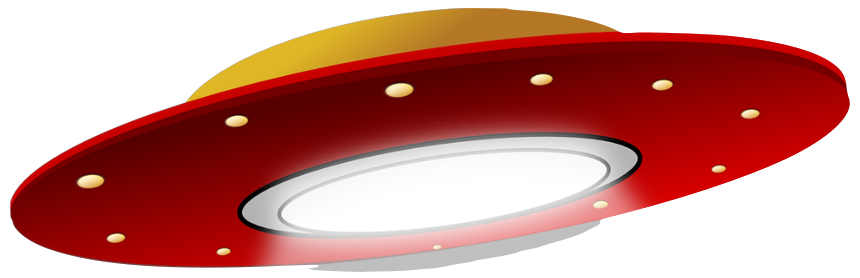
x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinx	0	1	0	−1	0

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
3Sinx	0	3	0	−3	0



Finding Period of sin(2x)

- We know the period of sinx = 2π. But what happens with sin2x?
    - For the moment, let p=2x. We know sin(p) has period 2π, that means the graph begins a new period at 2π.
        - p = 2x, so when 2x = 2π the graph begins a new cycle.
        - Thus, the cycle repeats when x = π. The period of y=sin2x is π.

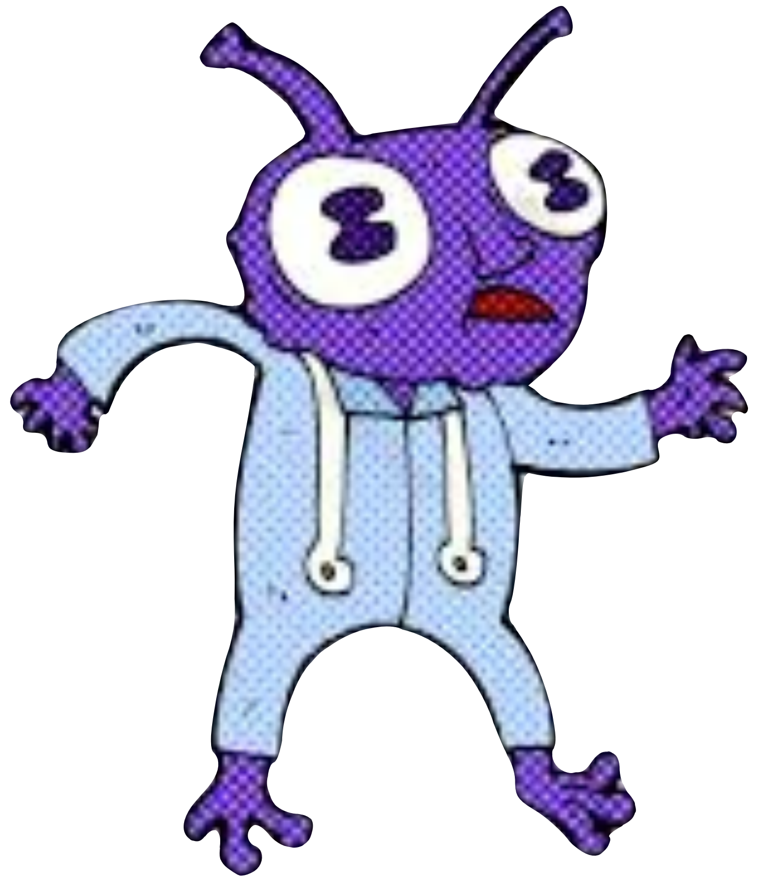


Finding Period of sin(2x)

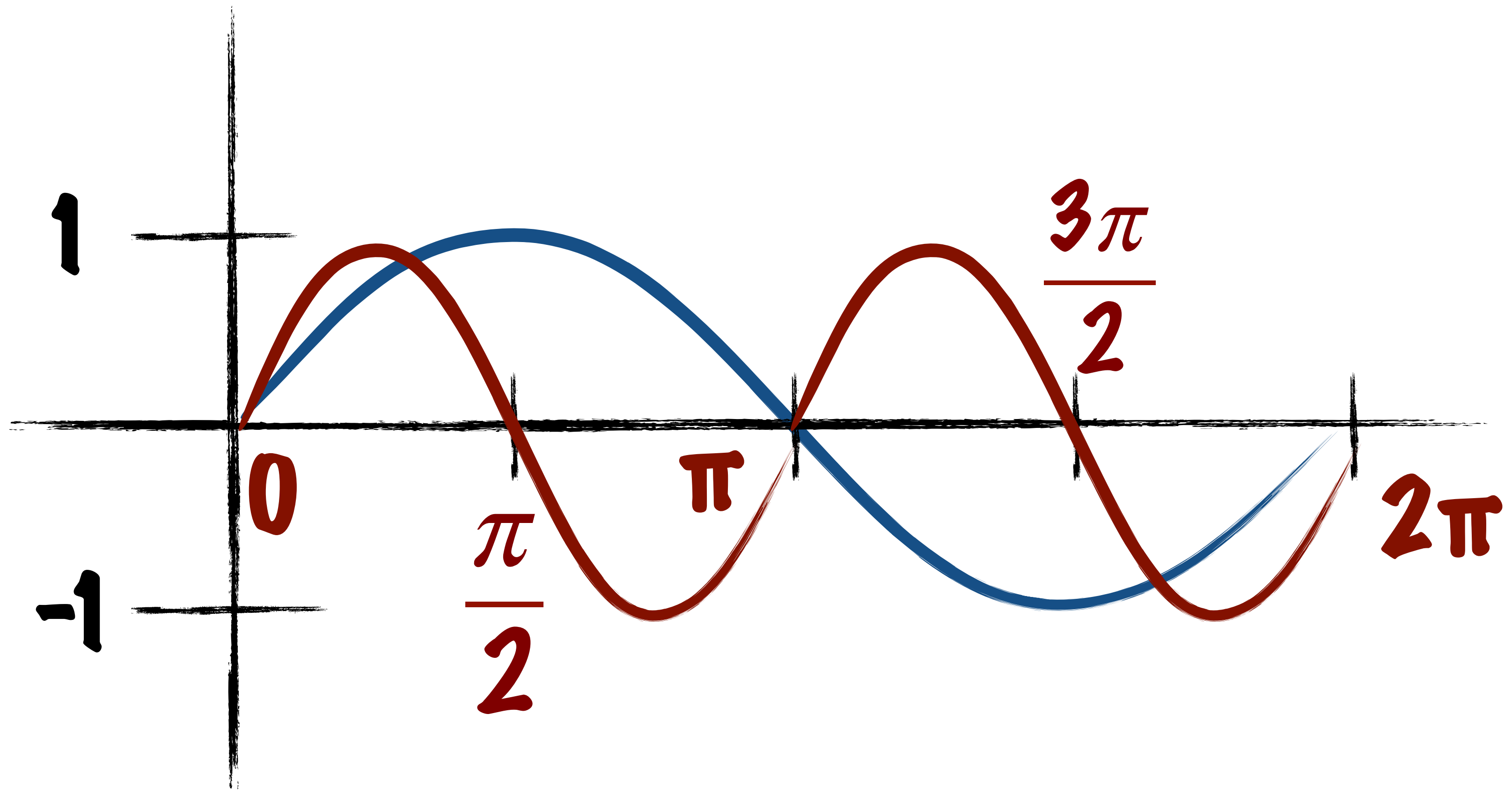
- If we graph y=sin2x we can see the period.

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinx	0	1	0	-1	0

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
2x	0	π	2π	3π	4π
Sin2x	0	0	0	0	0



Uh oh!



Our 5 values work, but we must remember we are working with 2x,
2x = 0, 2x = π/2, 2x = π, 2x = 3π/2, 2x = 2π

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
Sin2x	0	1	0	-1	0	1	0	-1	0

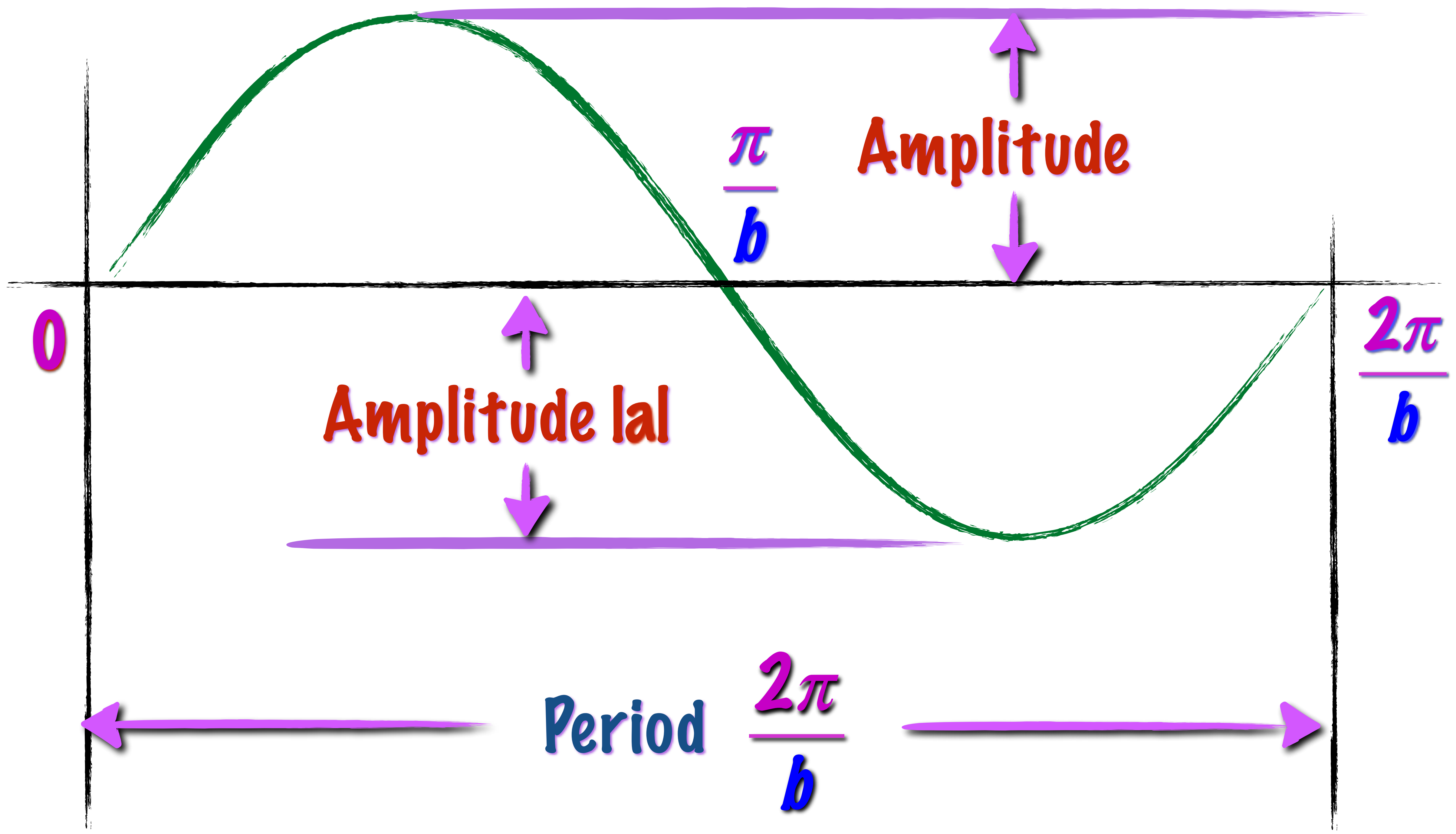
Over the domain [0,2π] the graph of y=sin2x repeats itself.
y=sin2x completes one cycle (period) over the interval [o,π].
The period is π.

- The graph of f(x) = asinbx, where b>0 has:

$$f(x) = a\sin bx$$

$$\text{Amplitude} = |a|$$

$$\text{Period} = \frac{2\pi}{b}$$



Graphing a Function of the Form $y = a\sin bx$

Determine the amplitude and period of $y = 2\sin\frac{1}{2}x$. Then graph the function for $0 \le x \le 8\pi$.

Step 1 Identify the amplitude and the period.

The equation is of the form $y = a \sin bx$

$a = 2, \quad b = \frac{1}{2} \quad$ amplitude $= |2| = 2 \qquad$ period $= \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

The maximum value of y is 2, the minimum value of y is -2, the graph completes one cycle (period) in the interval $[0, 4\pi]$

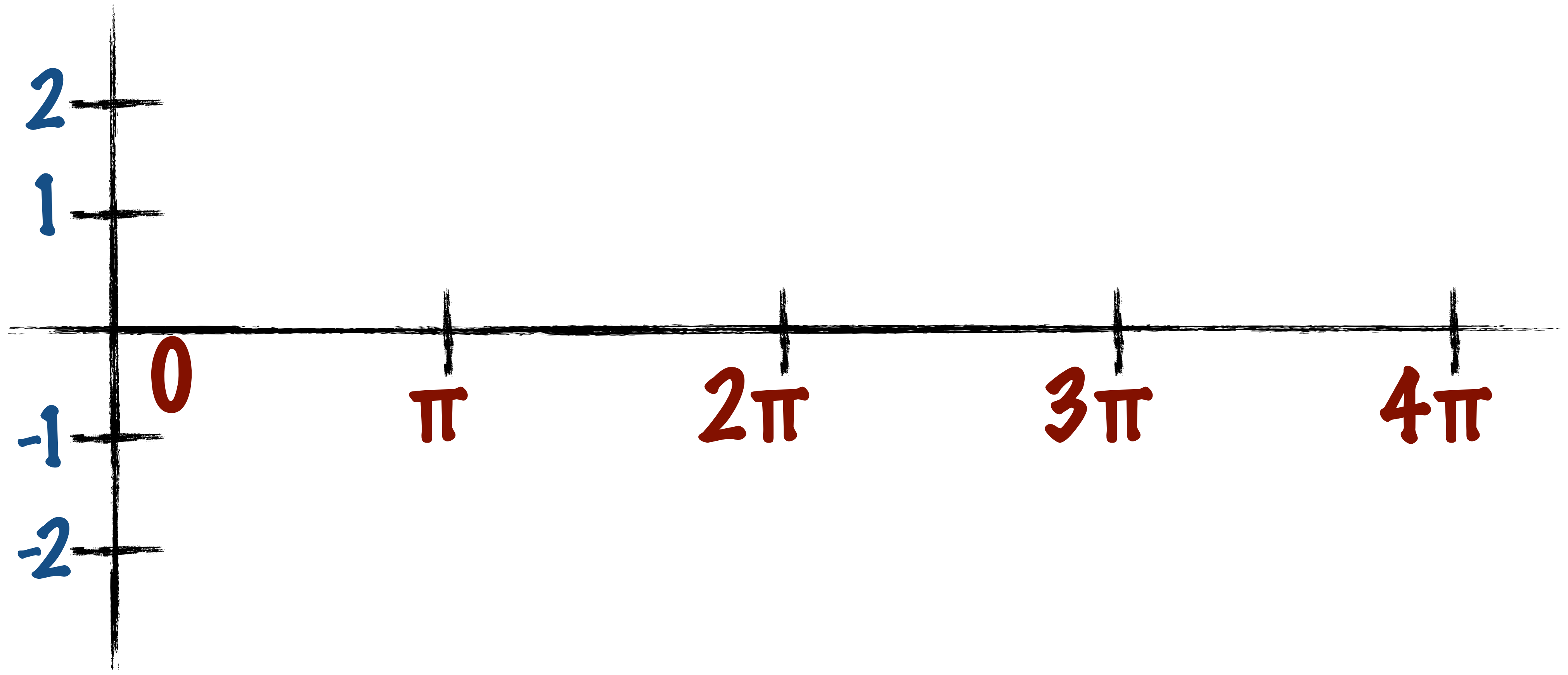
Graphing a Function of the Form y = asinbx

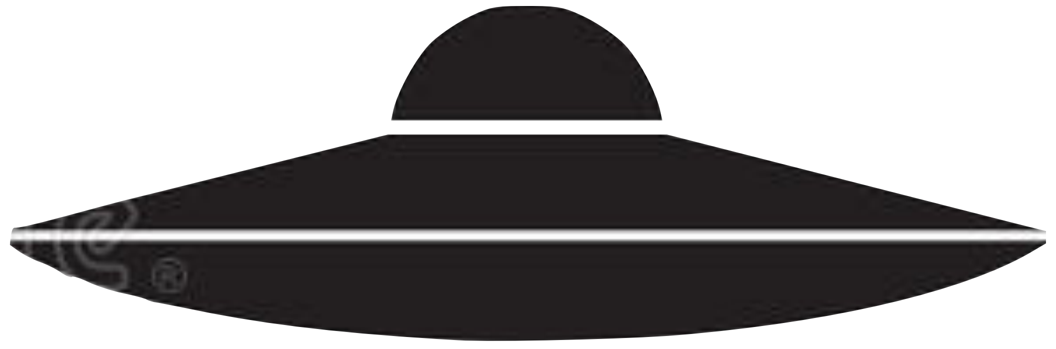
To generate x-values for each of the five key points, divide the period(=4π) by 4. The cycle begins at $x_1 = 0$. We add quarter periods to generate x-values for each of the key points.

$y = 2\sin\frac{1}{2}x$ $a = 2$, $b = \frac{1}{2}$ amplitude = |2| = 2 period = $\dfrac{2\pi}{\frac{1}{2}} = 4\pi$

Step 2 Find the values of x for the five key points.

$\frac{4\pi}{4} = \pi$ The 5 x-values are 0, $0 + \pi = \pi$, $\pi + \pi = 2\pi$, $2\pi + \pi = 3\pi$, $3\pi + \pi = 4\pi$





Graphing a Function of the Form y = asinbx

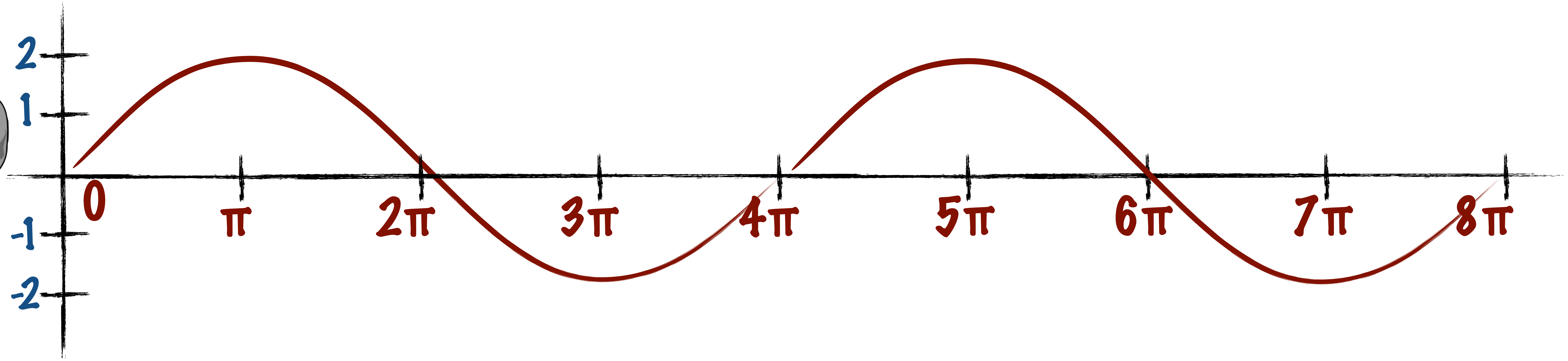
Step 3 Find the values of y for the five key points.

$y = 2\sin\frac{1}{2}x$ $a = 2$, $b = \frac{1}{2}$

amplitude = |2| = 2 period = $\dfrac{2\pi}{\frac{1}{2}} = 4\pi$

x	0	π	2π	3π	4π
$\frac{1}{2}x$	0	$\frac{\pi}{2}$	π	$\frac{3}{2}\pi$	2π
$y = 2\sin\frac{1}{2}x$	0	2	0	-2	0

Step 4 Plot the points and draw the first period.



Step 5 Repeat to cover the interval [0,8π].

Another approach

- Determine the amplitude and period of $y = 2\sin\frac{1}{2}x$. Then graph the function for $0 \le x \le 8\pi$.

  - Let us start with the 5 y-values we know are the critical 5 points for the parent function y = sina.

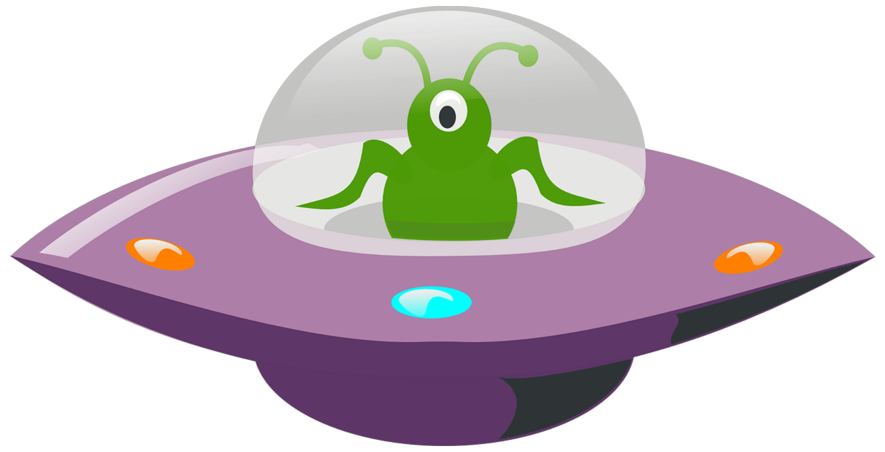
  - We find the x values for those 5 critical points.

a	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sina	0	1	0	-1	0

$$\frac{1}{2}x = 0, x = 0 \qquad \frac{1}{2}x = \frac{\pi}{2}, x = \pi \qquad \frac{1}{2}x = \pi, x = 2\pi$$

$$\frac{1}{2}x = \frac{3\pi}{2}, x = 3\pi \qquad \frac{1}{2}x = 2\pi, x = 4\pi$$

$\frac{1}{2}x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	π	2π	3π	4π
$y = \sin\frac{1}{2}x$	0	1	0	-1	0
$y = 2\sin\frac{1}{2}x$	0	2	0	-2	0

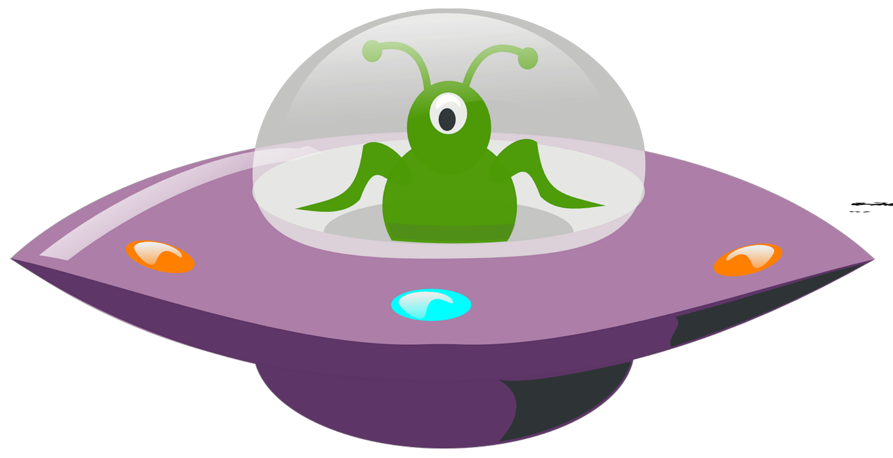


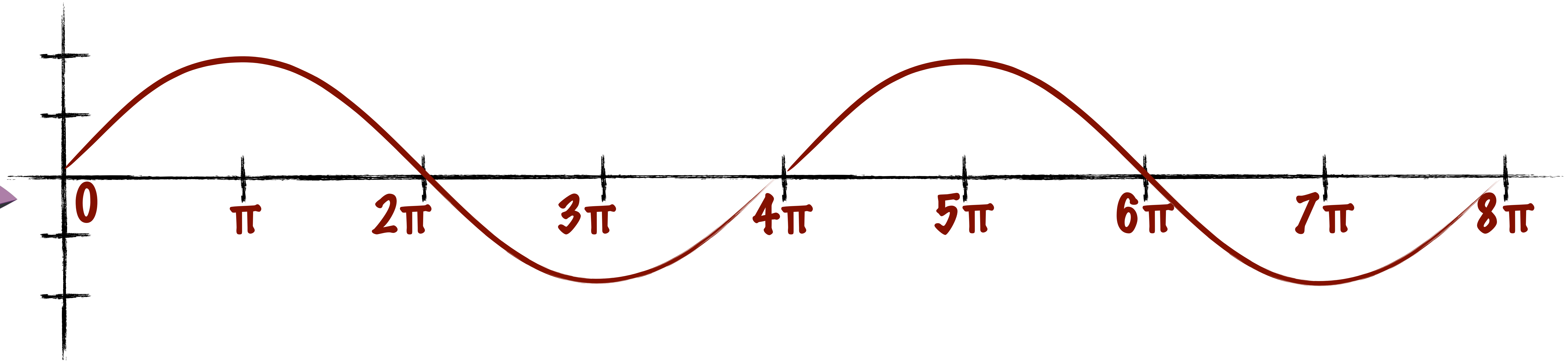
Another approach

Now we have the same table of values

x	0	π	2π	3π	4π
$y = 2\sin\frac{1}{2}x$	0	2	0	-2	0

Step 4: Plot the points and draw the first period.

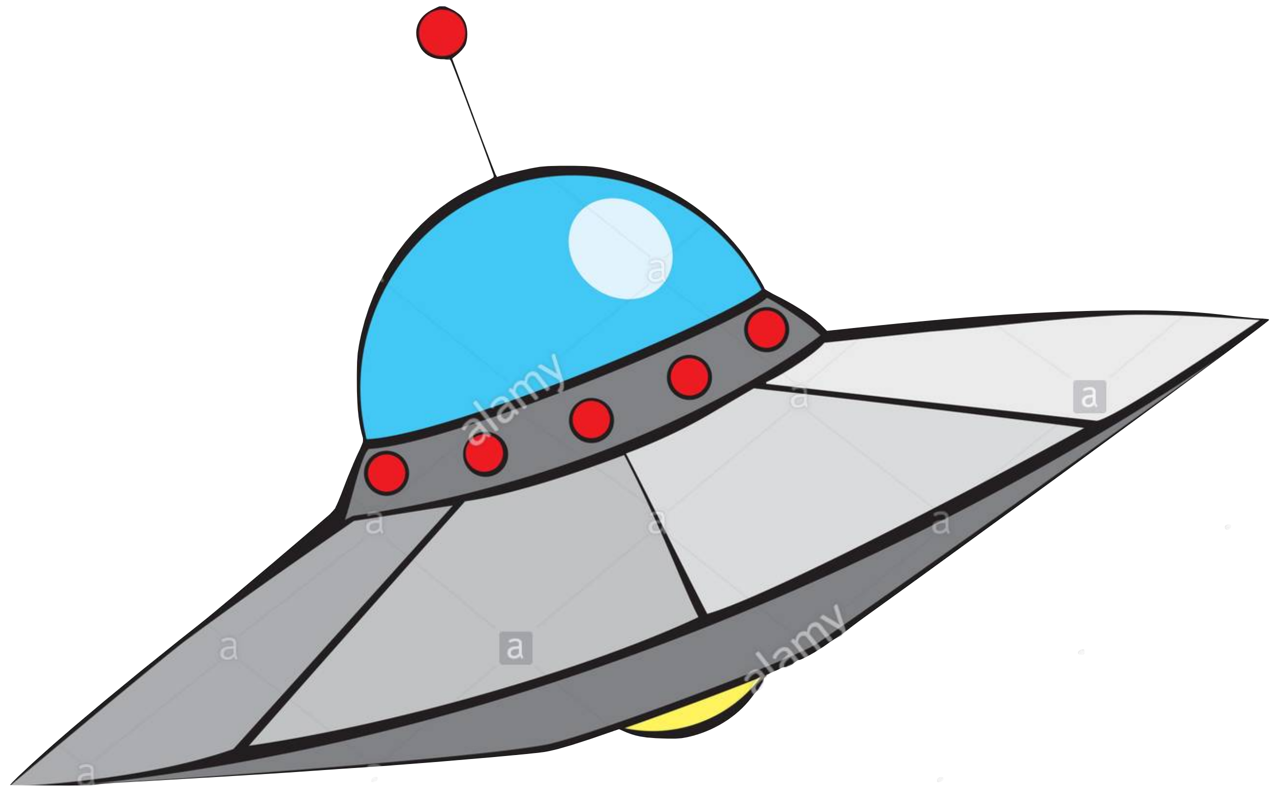




Step 5: Repeat to cover the interval $[0, 8\pi]$.

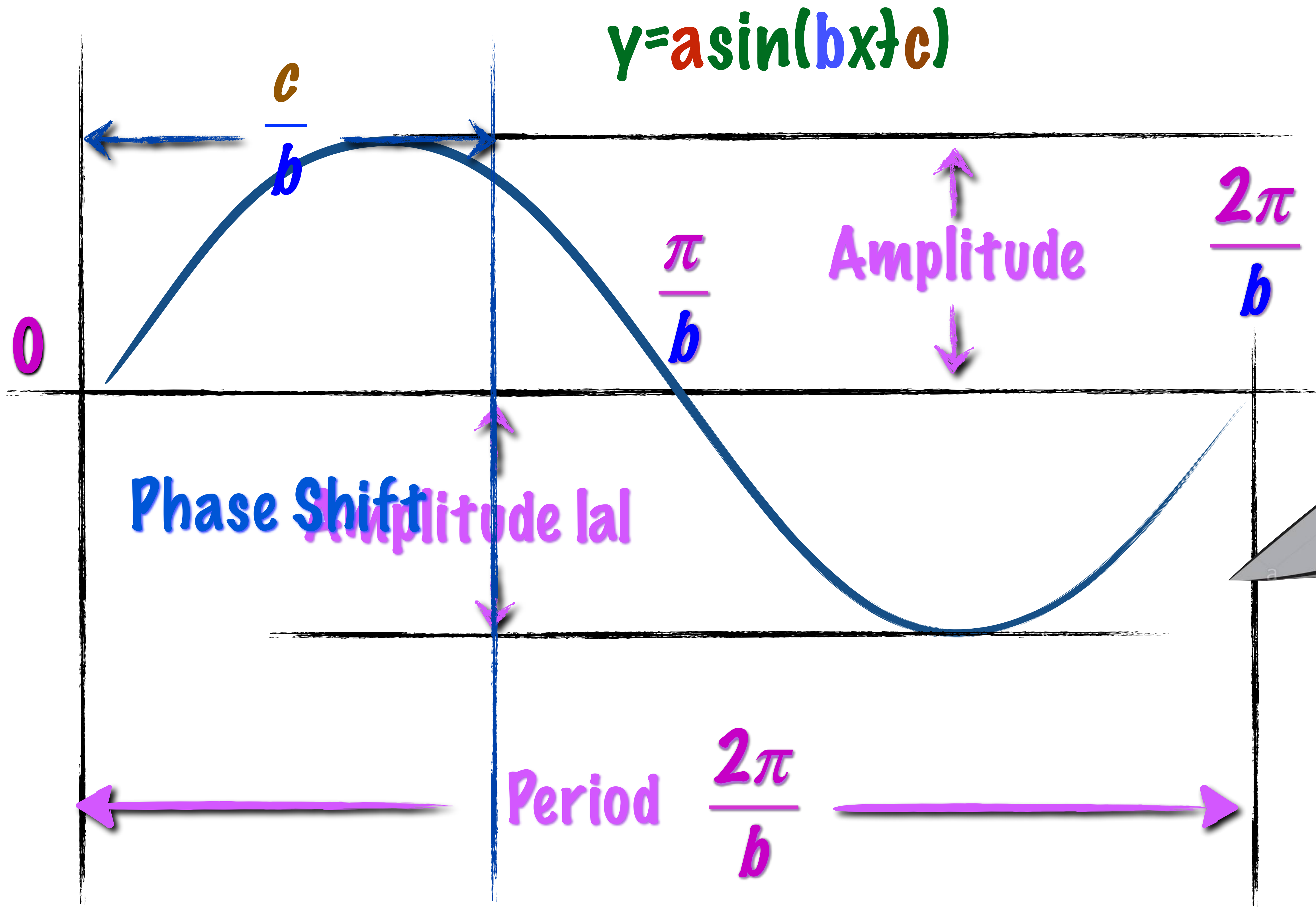
The Graph of y = asin(bx - c)

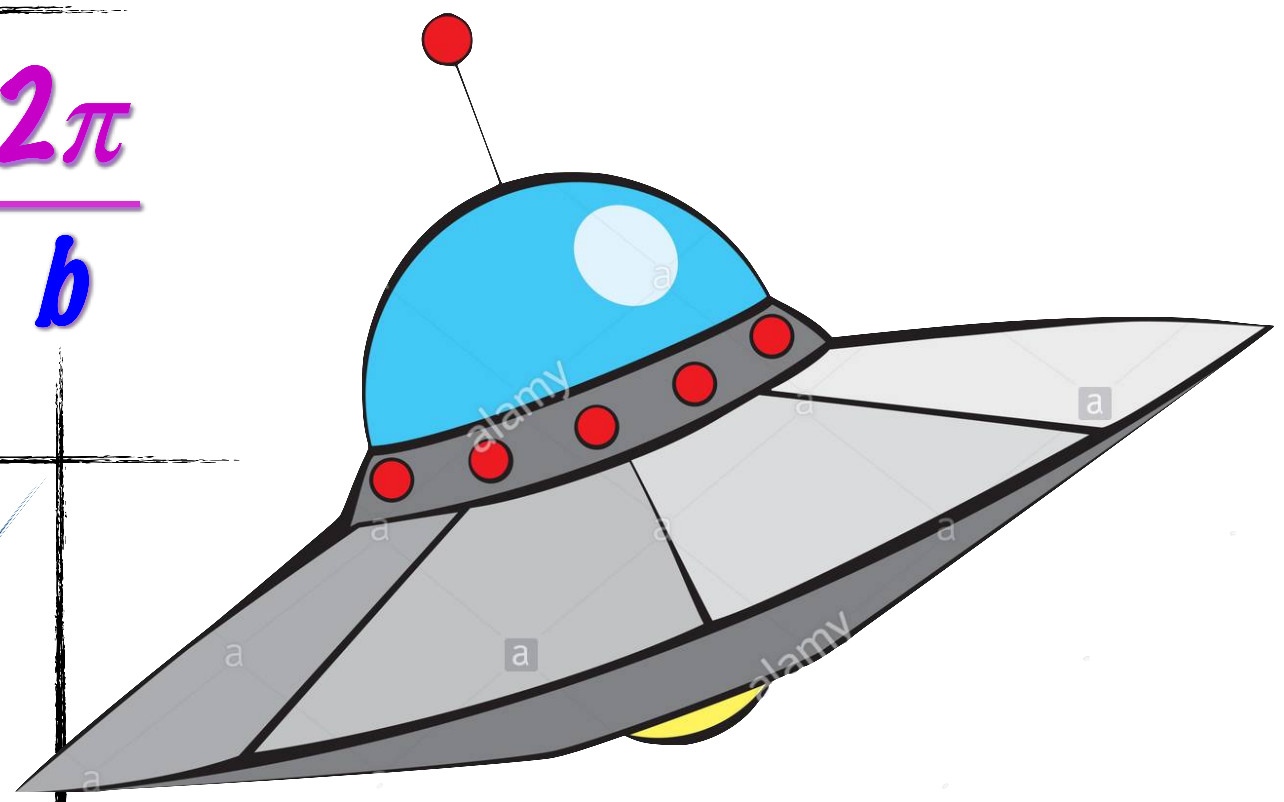
- The graph of y=asin(bx-c) is identical to the graph of y=asinbx, shifted right. (Just like any other function shift.) **The amount of shift is c/b.**
  - Think of y=asin(bx-c) as $y = a\sin\left(b\left(x - \frac{c}{b}\right)\right)$.
    - If c/b>0 shift right (remember x-(c/b)), if c/b<0 shift left.
      - With a periodic function, this is known as a "phase shift" of **c/b**.
- The amplitude remains |a|, and the period remains 2π/b.



The Graph of y = asin(bx - c)

f(x)=asin(bx-c)





Graphing a Function of the Form y = asin(bx - c)

Determine the amplitude, period, and phase shift of $y = 3\sin\left(2x - \frac{\pi}{3}\right)$ then graph one period.

$$y = 3\sin\left(2\left(x - \frac{\pi}{6}\right)\right)$$

$$2x - \frac{\pi}{3} = 0 \quad x = \frac{\pi}{6}$$

Step 1 amplitude, period, and phase shift. $a = 3, b = 2, c = \frac{\pi}{3}$

amplitude: $|a| = |3| = 3$

period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

phase shift: $\frac{c}{b} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$

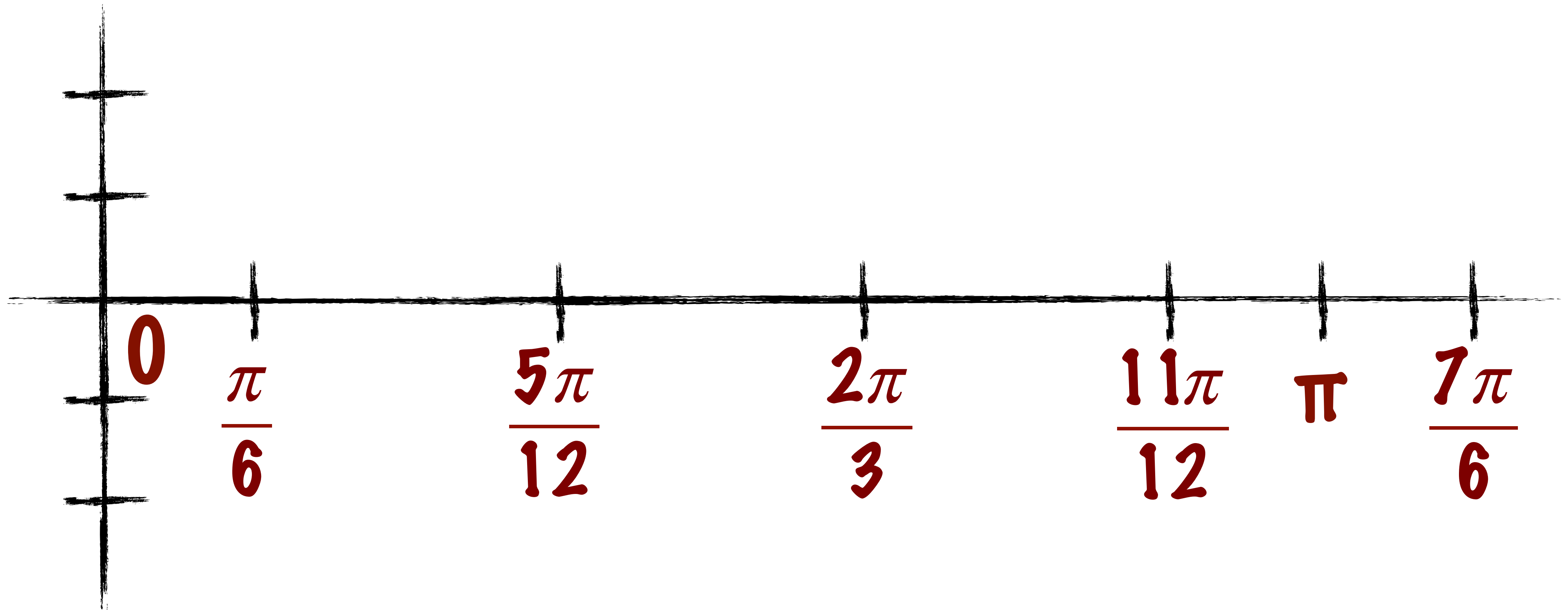
Graphing a Function of the Form y = asin(bx - c)

Step 2 5 key values of x. $y = 3\sin\left(2x - \frac{\pi}{3}\right)$

amplitude: $|a| = |3| = 3$ period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$ phase shift: $\frac{c}{b} = \frac{\frac{\pi}{3}}{2} = \frac{\pi}{6}$

$x_1 = 0 + \frac{\pi}{6}$ $x_2 = \frac{\pi}{6} + \frac{\pi}{4} = \frac{5\pi}{12}$ $x_3 = \frac{5\pi}{12} + \frac{\pi}{4} = \frac{2\pi}{3}$

$x_4 = \frac{2\pi}{3} + \frac{\pi}{4} = \frac{11\pi}{12}$ $x_5 = \frac{11\pi}{12} + \frac{\pi}{4} = \frac{7\pi}{6}$

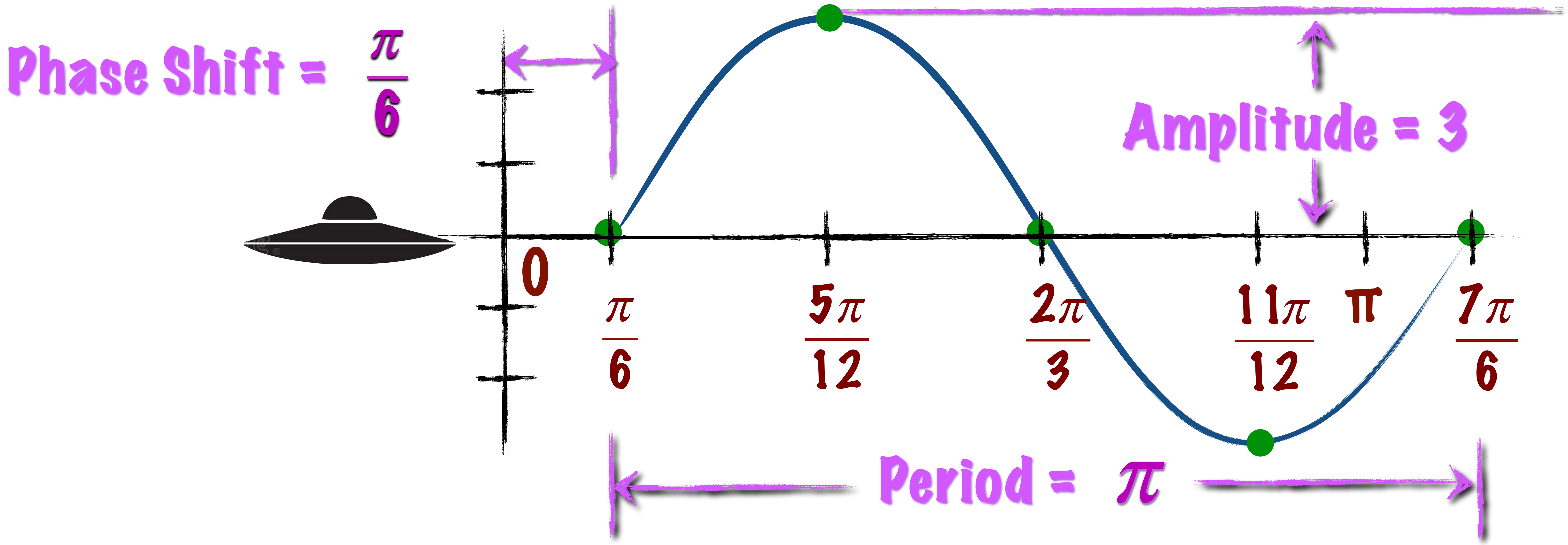


Graphing a Function of the Form y = asin(bx - c)

Step 3 Find the points for the 5 key values of x. $y = 3\sin\left(2x - \frac{\pi}{3}\right)$

x	$\frac{\pi}{6}$	$\frac{5\pi}{12}$	$\frac{2\pi}{3}$	$\frac{11\pi}{12}$	$\frac{7\pi}{6}$
2x	$\frac{\pi}{3}$	$\frac{5\pi}{6}$	$\frac{4\pi}{3}$	$\frac{11\pi}{6}$	$\frac{7\pi}{3}$
$2x - \frac{\pi}{3}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = 3\sin\left(2x - \frac{\pi}{3}\right)$	0	3	0	-3	0

Step 4 Graph one cycle.



Another approach

- Determine the amplitude, period, phase shift, and graph one period of $y = 3\sin\left(2x - \frac{\pi}{3}\right)$.

$$y = 3\sin\left(2\left(x - \frac{\pi}{6}\right)\right)$$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinx	0	1	0	-1	0

- We see the phase shift = $\frac{\pi}{6}$, the period is $\frac{7\pi}{6} - \frac{\pi}{6} = \pi$, and the amplitude is 3.

- We find the x values for the 5 critical points.

$2x - \frac{\pi}{3} = 0, x = \frac{\pi}{6}$ $2x - \frac{\pi}{3} = \frac{\pi}{2}, x = \frac{5\pi}{12}$ $2x - \frac{\pi}{3} = \pi, x = \frac{2\pi}{3}$

$2x - \frac{\pi}{3} = \frac{3\pi}{2}, x = \frac{11\pi}{12}$ $2x - \frac{\pi}{3} = 2\pi, x = \frac{7\pi}{6}$

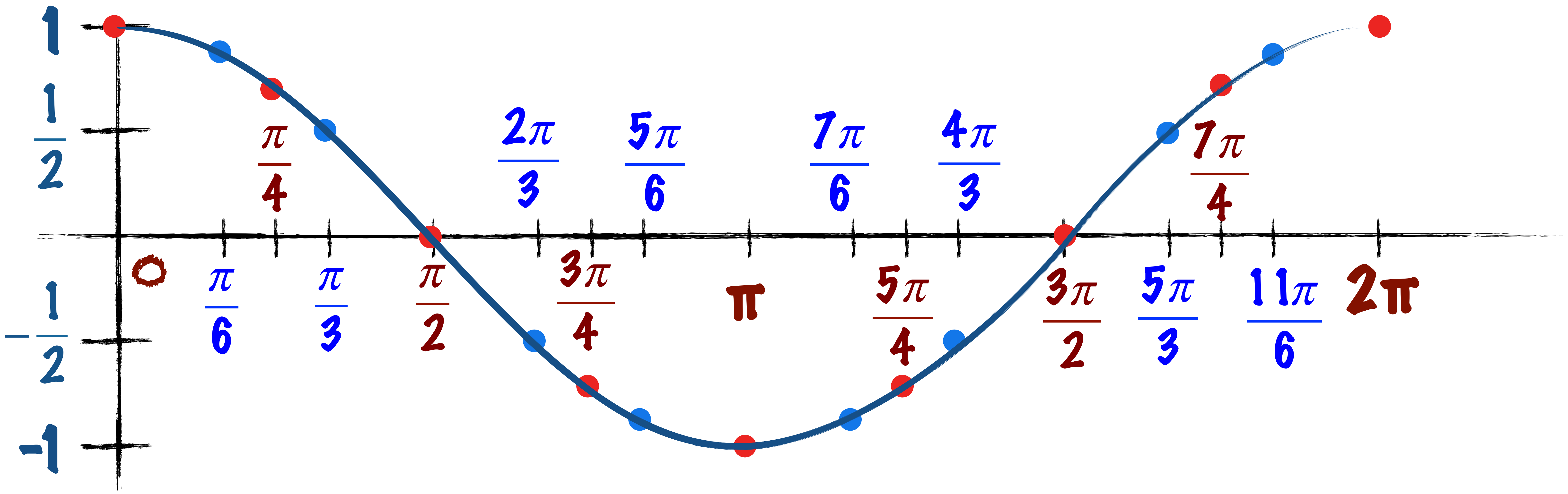
$2x - \frac{\pi}{3}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	$\frac{\pi}{6}$	$\frac{5\pi}{12}$	$\frac{2\pi}{3}$	$\frac{11\pi}{12}$	$\frac{7\pi}{6}$
$y = \sin\left(2x - \frac{\pi}{3}\right)$	0	1	0	-1	0
$y = 3\sin\left(2x - \frac{\pi}{3}\right)$	0	3	0	-3	0

The Graph of y = cosx

Complete the table:

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
Cosx	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Graph the results:



The Graph of y = cosx

- The cosine function is periodic, with a period 2π. That means the graph continues forever in both directions, repeating the pattern every 2π.

- The cosine function is an even function, cos(-x) = cosx.

- The domain is (-∞, ∞); the range is [-1, 1].

Sinusoidal Graphs

The graphs of sine and cosine functions are called sinusoidal graphs.

Sinusoidal Graphs

The graph of y = cosx is the graph of y= sinx with a phase shift of $\frac{\pi}{2}$ left.

$$\cos x = \sin\left(x + \frac{\pi}{2}\right)$$

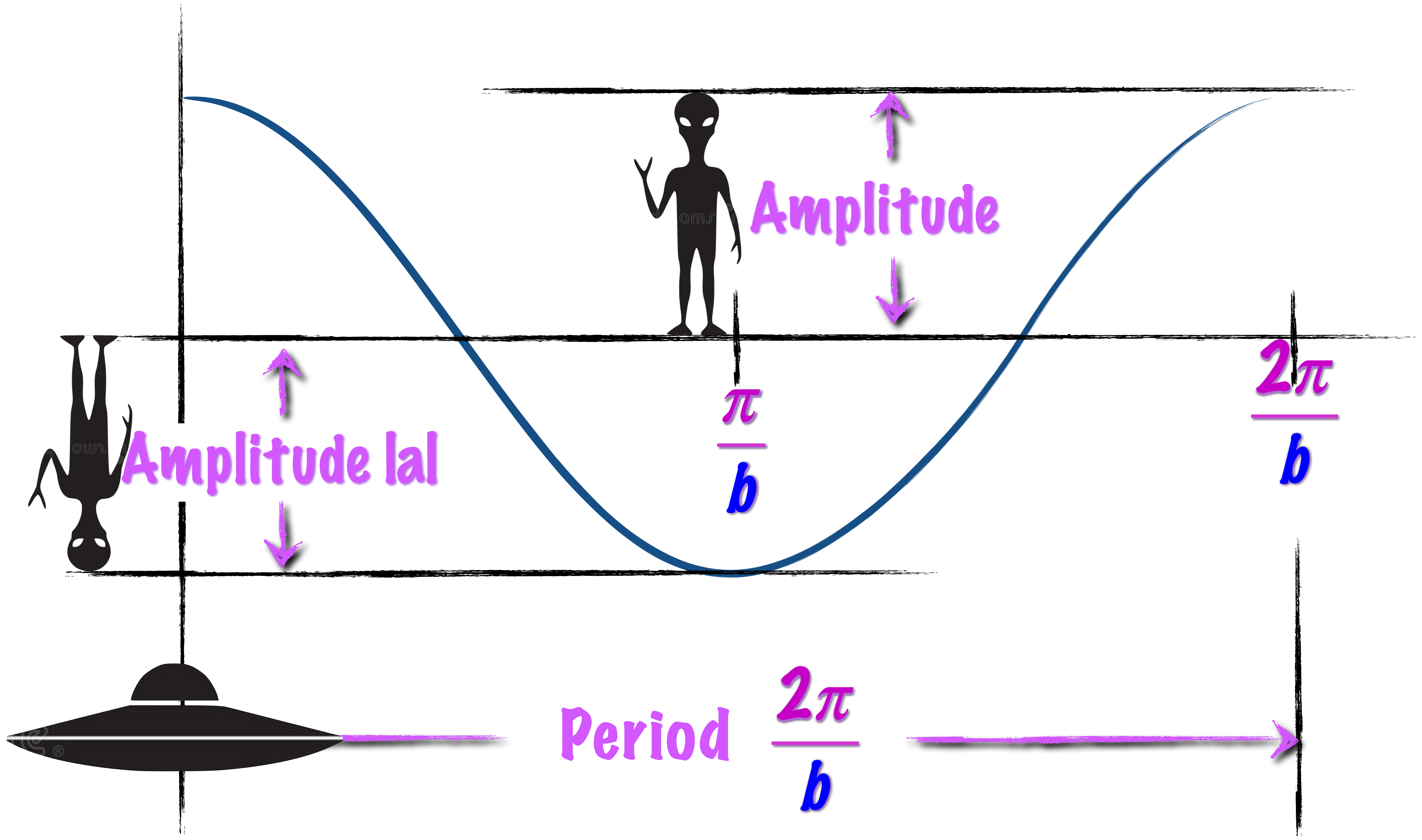
Amplitudes and Periods

- The graph of $f(x) = a\cos bx$, where $b>0$ has:

$$f(x) = a\cos bx$$

Amplitude = $|a|$

Period = $\frac{2\pi}{b}$



Graphing a Function of the Form y = acosbx

Determine the amplitude and period of $y = -4\cos\pi x$, then graph the function for $-2 \leq x \leq 2$.

Step 1 Identify the amplitude and the period.

$$y = -4\cos\pi x$$

The equation is of the form $y = a\cos bx$; $a = -4$, $b = \pi$

amplitude = $|-4| = 4$ period = $\frac{2\pi}{\pi} = 2$

The maximum value of y is 4, the minimum value of y is -4, the graph completes one cycle (period) in the interval [0,2]

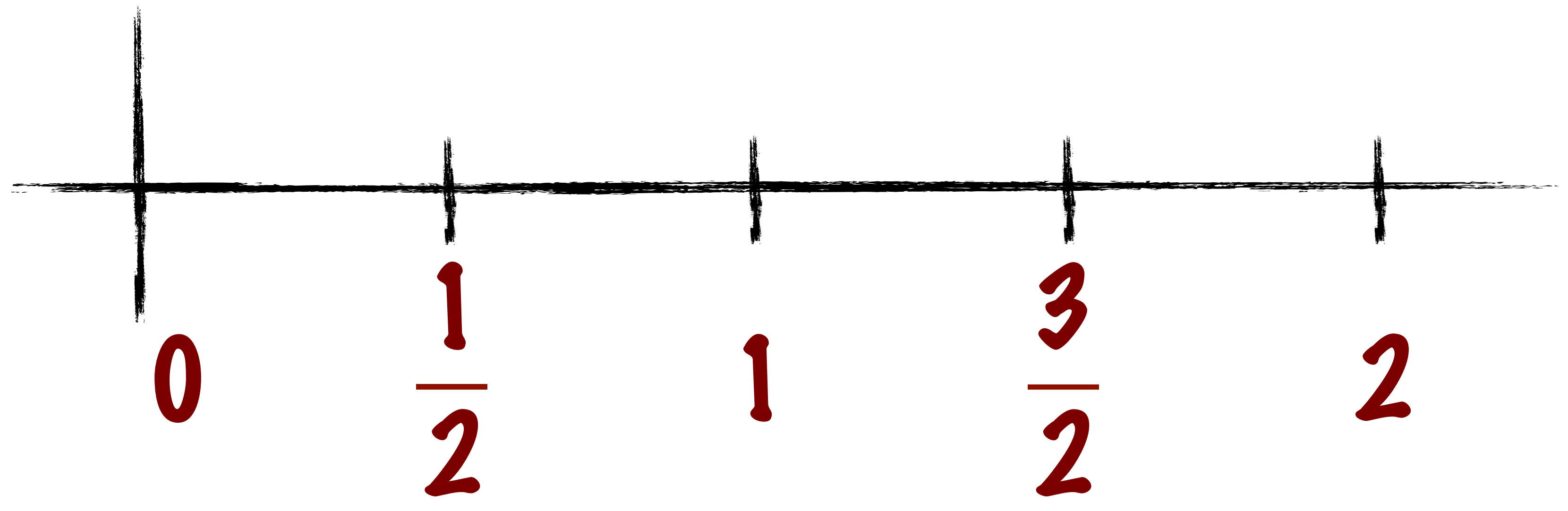
Graphing a Function of the Form $y = -4\cos\pi x$

Step 2 Find the values of x for the five key points. $y = -4\cos\pi x$

- To generate x-values for each of the five key points, divide the period (=2) by 4. The cycle begins at $x_1 = 0$. We add quarter periods to generate x-values for each of the key points.

$$\frac{2}{4} = \frac{1}{2}$$

The 5 x-values are 0, $0 + \frac{1}{2} = \frac{1}{2}$, $\frac{1}{2} + \frac{1}{2} = 1$, $1 + \frac{1}{2} = \frac{3}{2}$, $\frac{1}{2} + \frac{3}{2} = 2$



Graphing a Function of the Form y = -4cosπx

Step 3 Find the values of y for the five key points.

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = -4\cos(\pi x)$	-4	0	4	0	-4

$$y = -4\cos\pi(0) = -4\cos 0 = -4(1) = -4$$

$$y = -4\cos\pi\left(\frac{1}{2}\right) = -4\cos\frac{\pi}{2} = -4(0) = 0$$

$$y = -4\cos\pi(1) = -4\cos\pi = -4(-1) = 4$$

$$y = -4\cos\pi\left(\frac{3}{2}\right) = -4\cos\frac{3\pi}{2} = -4(0) = 0$$

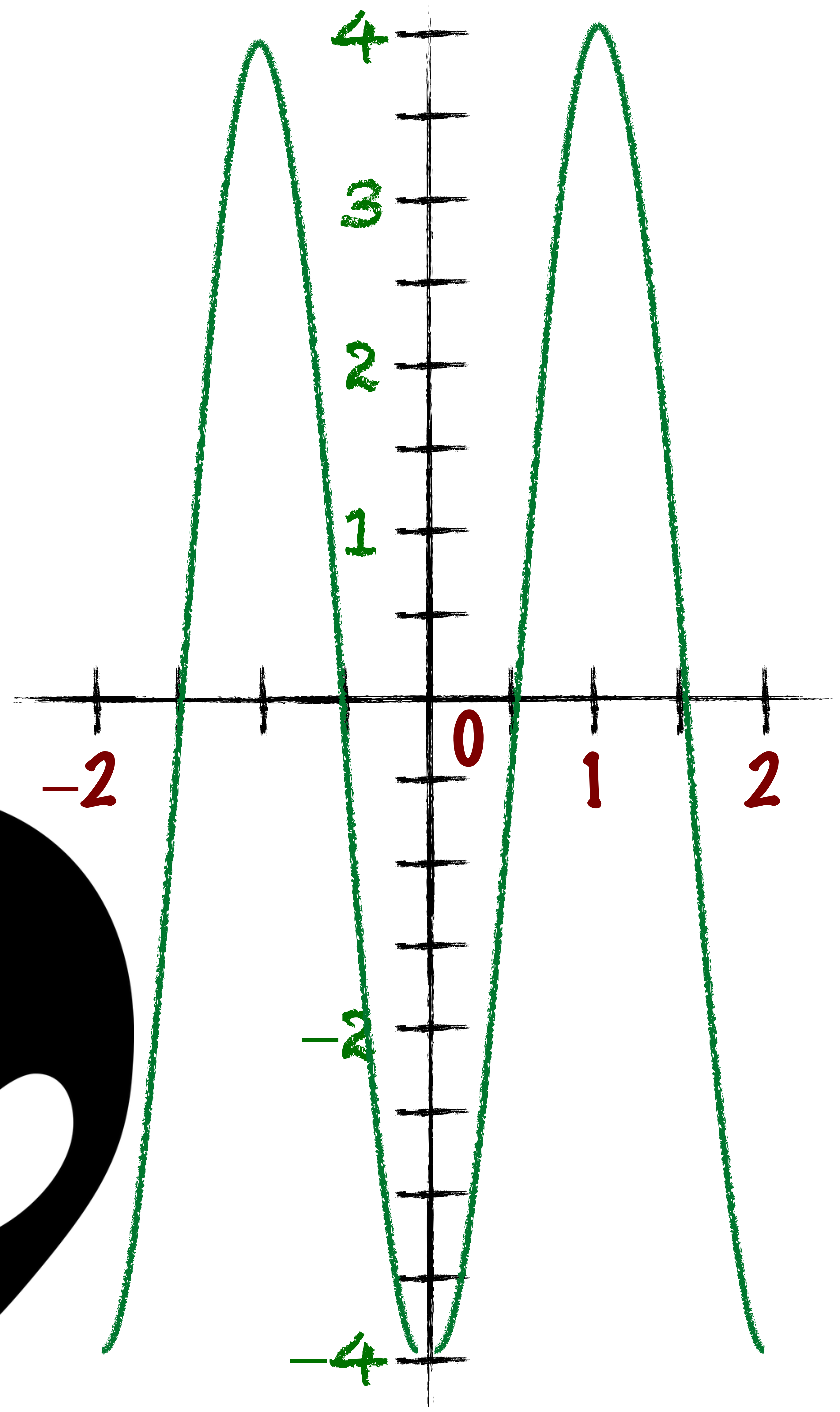
$$y = -4\cos\pi(2) = -4\cos 2\pi = -4(1) = -4$$

Graphing a Function of the Form $y = -4\cos\pi x$

x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = -4\cos(\pi x)$	-4	0	4	0	-4

Step 4 Plot the points and draw the first cycle.

Step 5 Repeat to cover the interval [-2,2].



Another approach

- Determine the amplitude and period of $y = -4\cos \pi x$. Then graph the function for $-2 \le x \le 2$.
  - Let us start with the 5 y-values we know are the critical 5 points for the parent function **y = cosA**.

A	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
CosA	1	0	-1	0	1

  - We find the x values for those 5 critical points.

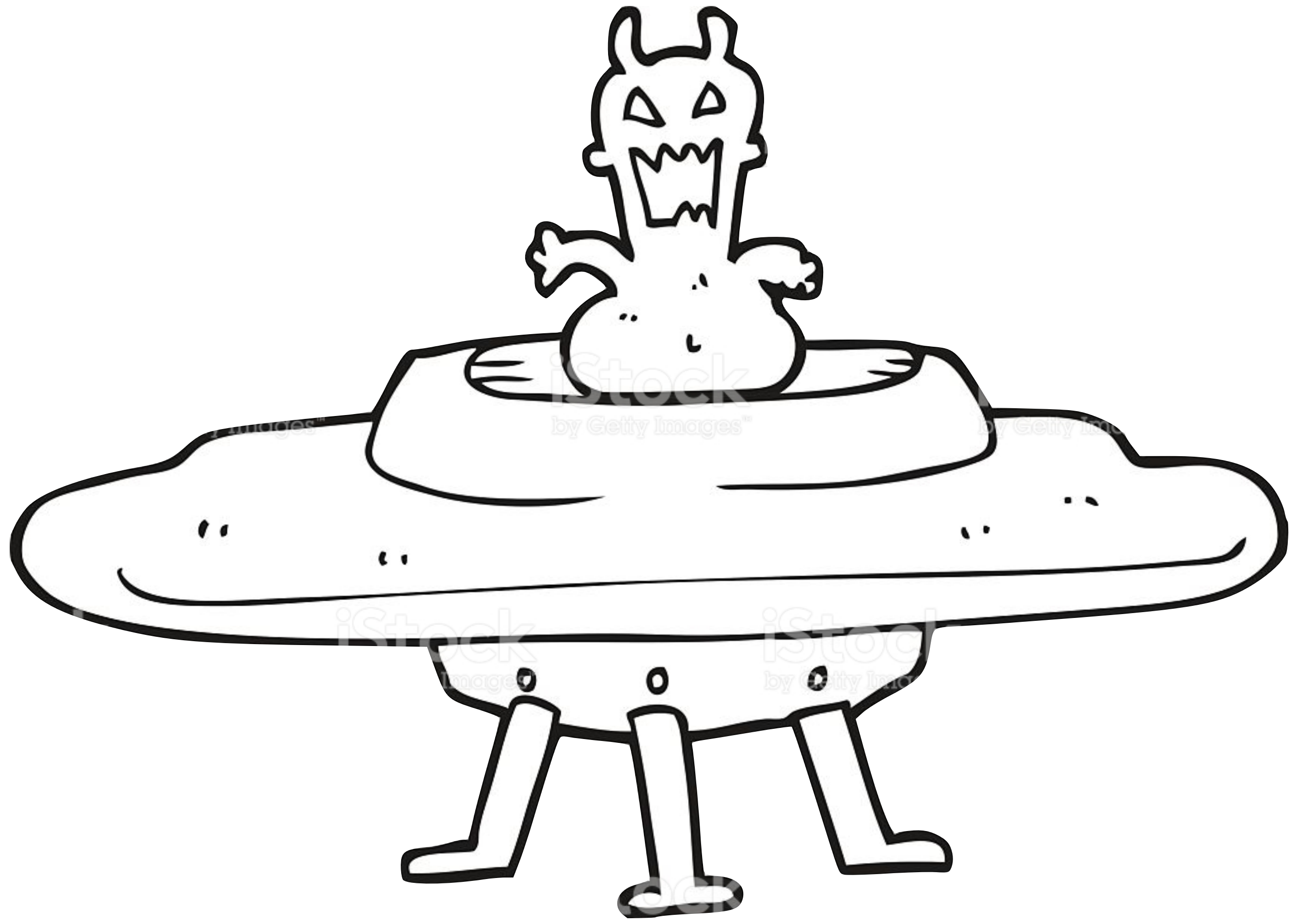
$\pi x = 0, x = 0$ $\quad$ $\pi x = \frac{\pi}{2}, x = \frac{1}{2}$ $\quad$ $\pi x = \pi, x = 1$

$\pi x = \frac{3\pi}{2}, x = \frac{3}{2}$ $\quad$ $\pi x = 2\pi, x = 2$

πx	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$y = \cos \pi x$	1	0	-1	0	1
$y = -4\cos \pi x$	-4	0	4	0	-4

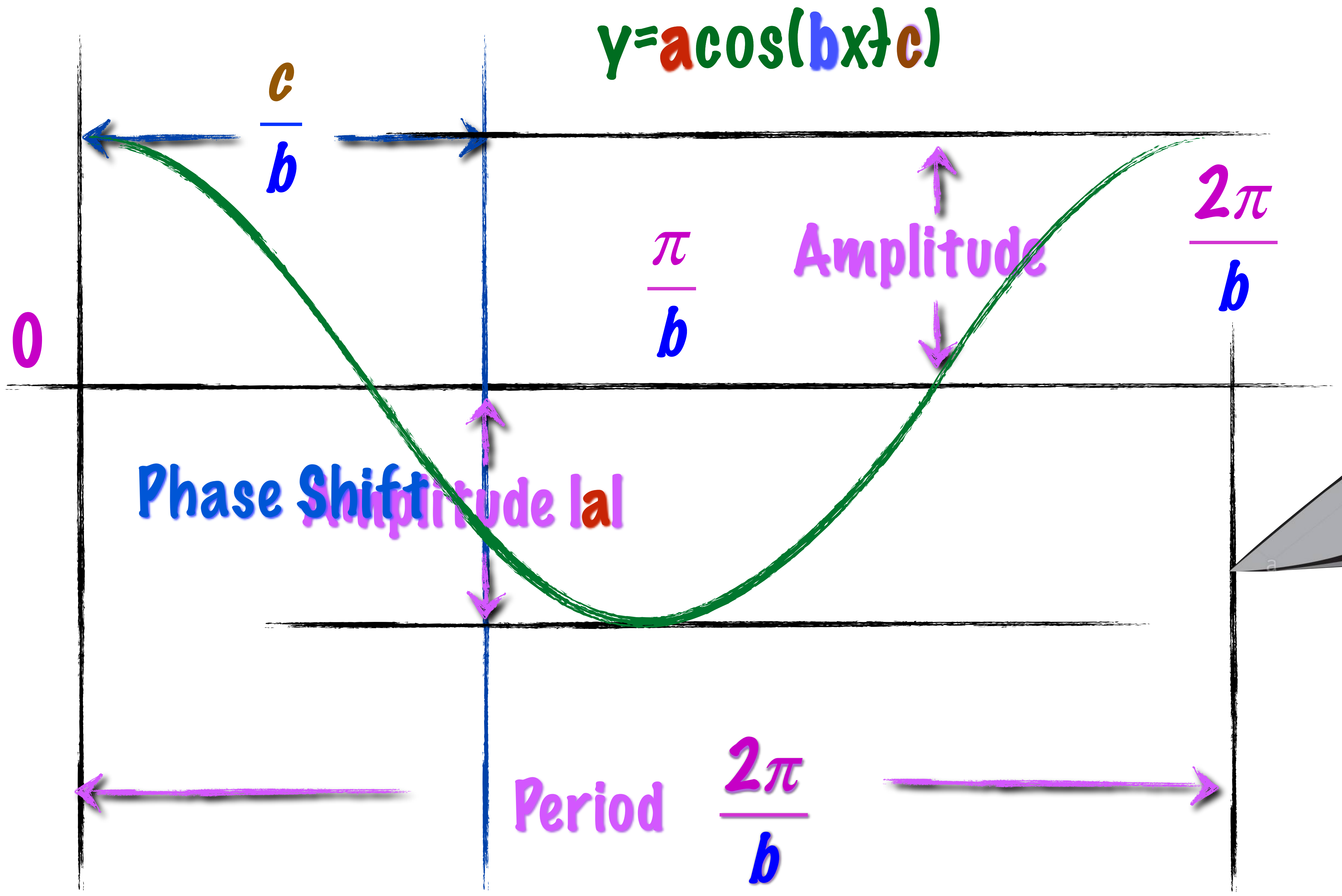
The Graph of y = acos(bx - c)

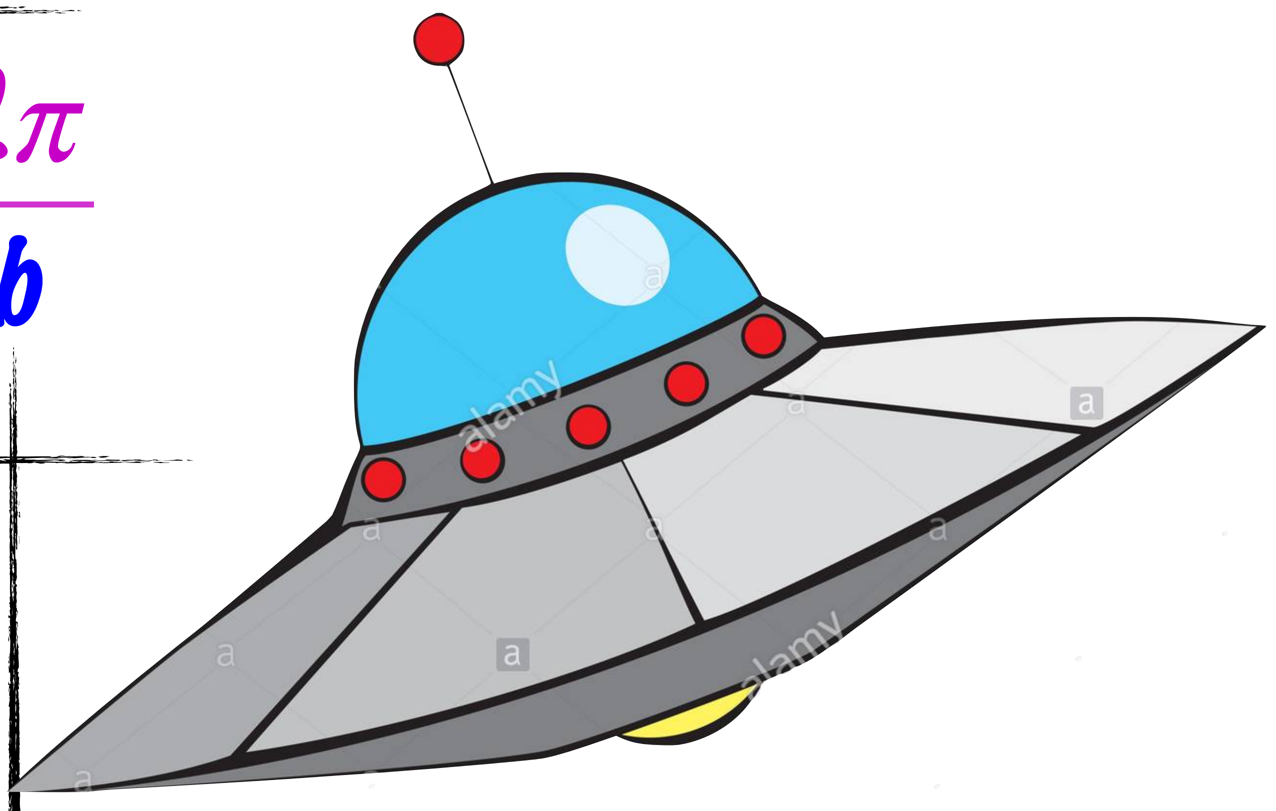
- The graph of $y = a\cos(bx - c)$ is identical to the graph of $y = a\cos bx$, shifted right. (Just like any other function shift.) **The amount of shift is c/b.**
  - Think of $y = a\cos(bx - c)$ as $y = a\cos[b(x - c/b)]$. Or think $bx - c = 0$, $x = c/b$
    - If $c/b > 0$ shift right $(bx - c)$, if $c/b < 0$ shift left.
    - This is also a "**phase shift**" of c/b.
  - The amplitude remains $|a|$, and the period remains $2\pi/b$.



The Graph of y = acos(bx - c)

$f(x)=a\cos(bx-c)$





Graphing a Function of the Form y = acos(bx - c)

Determine the amplitude, period, and phase shift of $y=\frac{3}{2}\cos\left(2x+\pi\right)$ then graph one period.

Step 1 amplitude, period, and phase shift.

y = acos(bx-c)

$a=\frac{3}{2}, b=2, c=\pi$

amplitude: $\left|\frac{3}{2}\right|=\frac{3}{2}$

phase shift: $\frac{c}{b}=\frac{-\pi}{2}=-\frac{\pi}{2}$

period: $\frac{2\pi}{b}=\frac{2\pi}{2}=\pi$

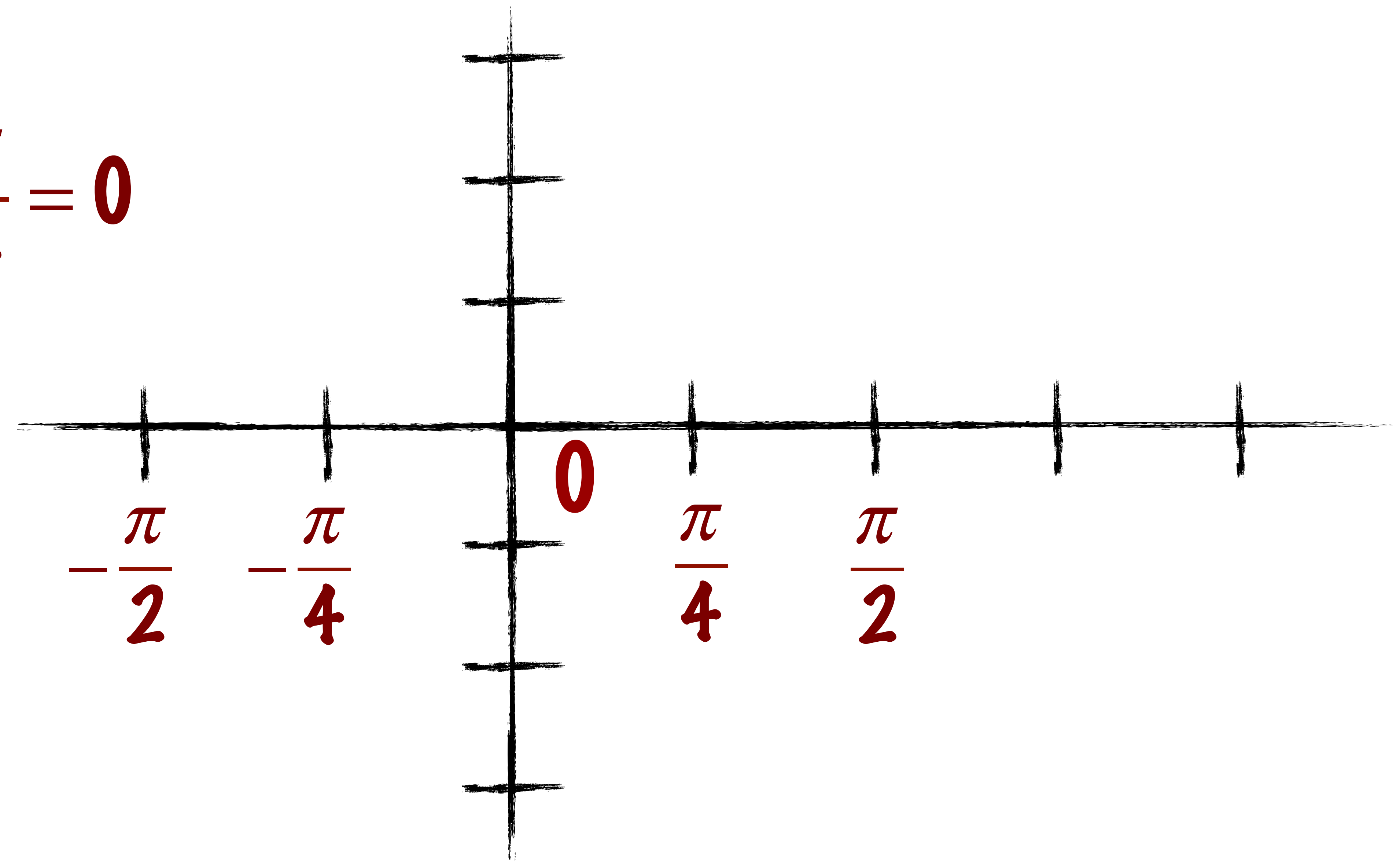
Graphing a Function of the Form $y = a\cos(bx - c)$

Step 2 5 key values of x. amplitude: $\left|\frac{3}{2}\right| = \frac{3}{2}$ phase shift: $\frac{c}{b} = \frac{-\pi}{2} = -\frac{\pi}{2}$ period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

$$y = \frac{3}{2}\cos\left(2x + \pi\right)$$

$x_1 = 0 + -\frac{\pi}{2} = -\frac{\pi}{2}$ $x_2 = -\frac{\pi}{2} + \frac{\pi}{4} = -\frac{\pi}{4}$ $x_3 = -\frac{\pi}{4} + \frac{\pi}{4} = 0$

$x_4 = 0 + \frac{\pi}{4} = \frac{\pi}{4}$ $x_5 = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

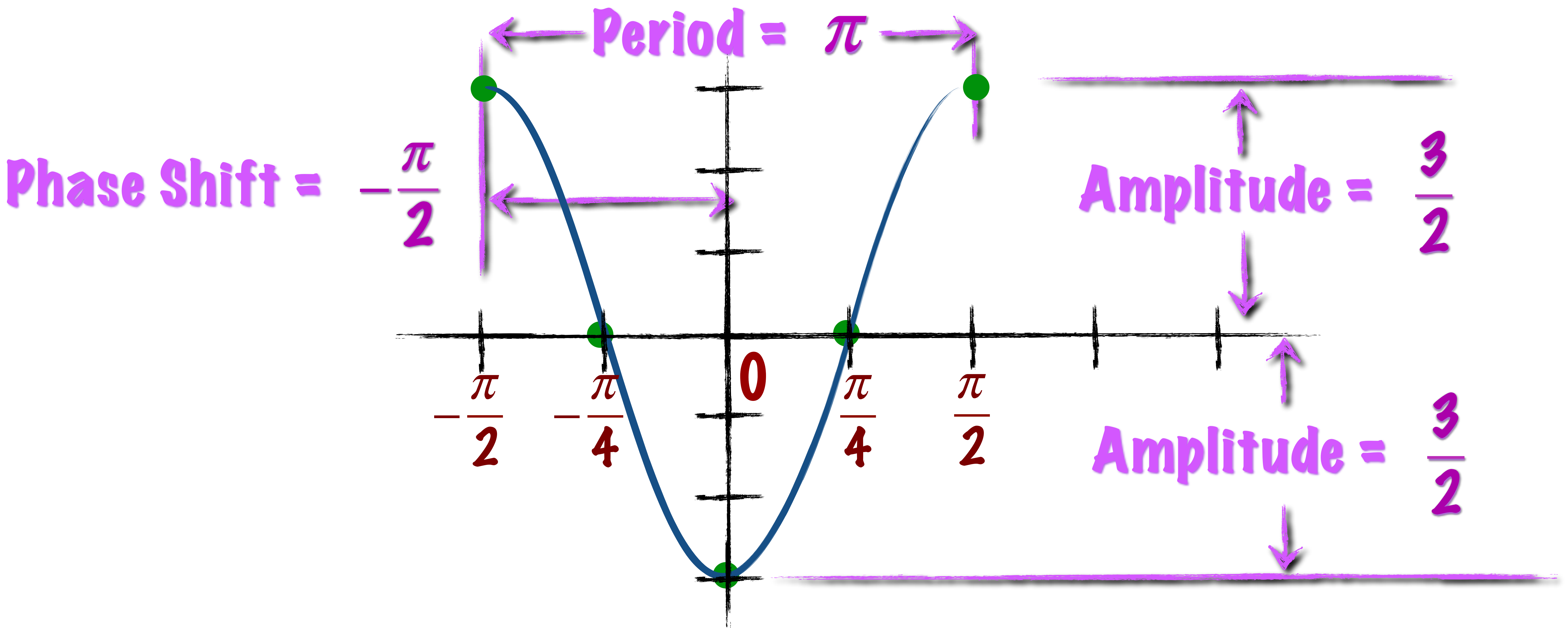


Step 3 Find the points for the 5 key values of x.

$$y = \frac{3}{2}\cos\left(2x + \pi\right)$$

x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y = \frac{3}{2}\cos\left(2x + \pi\right)$	$\frac{3}{2}$	0	$-\frac{3}{2}$	0	$\frac{3}{2}$

Step 4 Graph one cycle.



Another approach

- Determine the amplitude, period, and phase shift of $y=\frac{3}{2}\cos\left(2x+\pi\right)$ then graph one period.

a	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Cosa	1	0	-1	0	1

$$y=\frac{3}{2}\cos\left(2\left(x-\left(-\frac{\pi}{2}\right)\right)\right)$$

- We see the phase shift = $-\frac{\pi}{2}$, the period is $\frac{\pi}{2}--\frac{\pi}{2}=\pi$, and the amplitude is 3/2.

- We find the x values for the 5 critical points.

$2x+\pi=0, x=-\frac{\pi}{2}$ $\quad$ $2x+\pi=\frac{\pi}{2}, x=-\frac{\pi}{4}$ $\quad$ $2x+\pi=\pi, x=0$

$2x+\pi=\frac{3\pi}{2}, x=\frac{\pi}{4}$ $\quad$ $2x+\pi=2\pi, x=\frac{\pi}{2}$

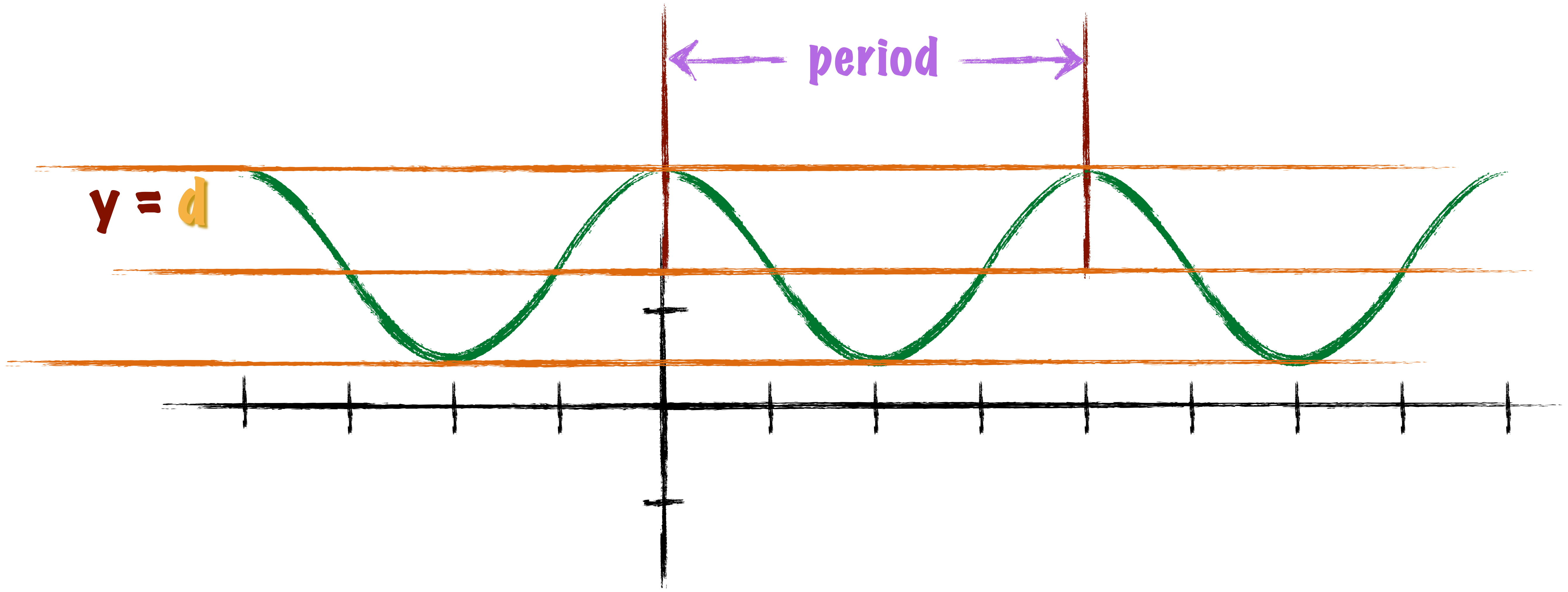
$2x+\pi$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
x	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$y=\cos\left(2x+\pi\right)$	1	0	-1	0	1
$y=\frac{3}{2}\cos\left(2x+\pi\right)$	$\frac{3}{2}$	0	$-\frac{3}{2}$	0	$\frac{3}{2}$

Vertical Shifts of Sinusoidal Graphs y=asin(bx-c)+d

- For sinusoidal graphs of the form y=asin(bx-c)+d and y=acos(bx-c)+d the constant d causes a vertical shift in the graph.
  - These vertical shifts result in sinusoidals oscillating about the horizontal line y = d (equilibrium) rather than about the x-axis.

The maximum value of y is d + |a|.

The minimum value of y is d - |a|.



A Vertical Shift y=2cosx+1

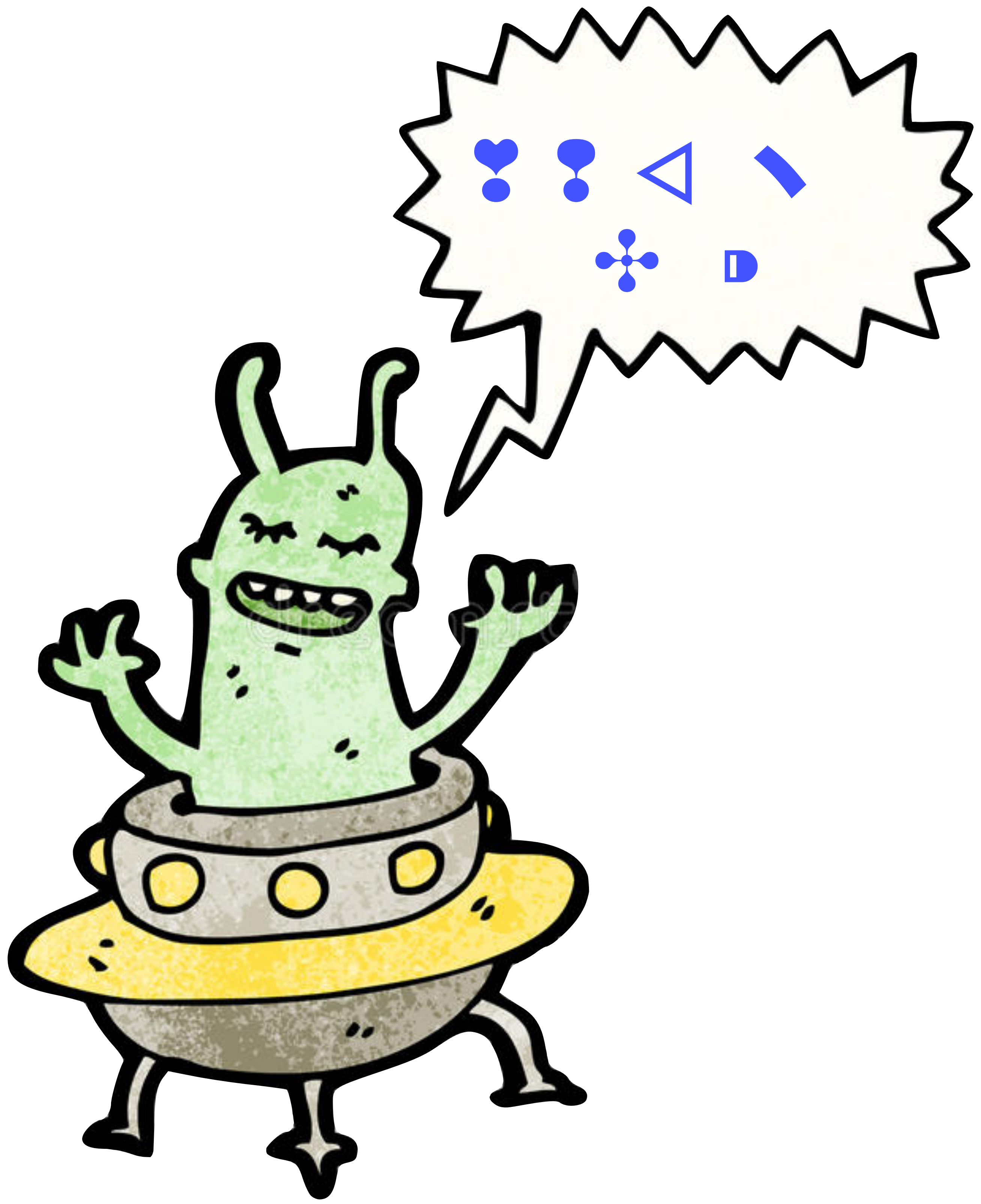
Graph one period of the function y=2cosx+1.

Step 1 amplitude, period, and phase shift.

y=2cosx+1 y = acos(bx-c)+d $a=2, b=1, c=0, d=1$

amplitude: $|2|=2$ phase shift: $\frac{c}{b}=\frac{0}{1}=0$

period: $\frac{2\pi}{1}=2\pi$ vertical shift: $d=+1$



A Vertical Shift $y=2\cos x+1$

Step 2 5 key values of x.

$y=2\cos x+1$

amplitude: $|2| = 2$

period: $\frac{2\pi}{1} = 2\pi$

phase shift: $\frac{c}{b} = \frac{0}{1} = 0$

vertical shift: $d = +1$

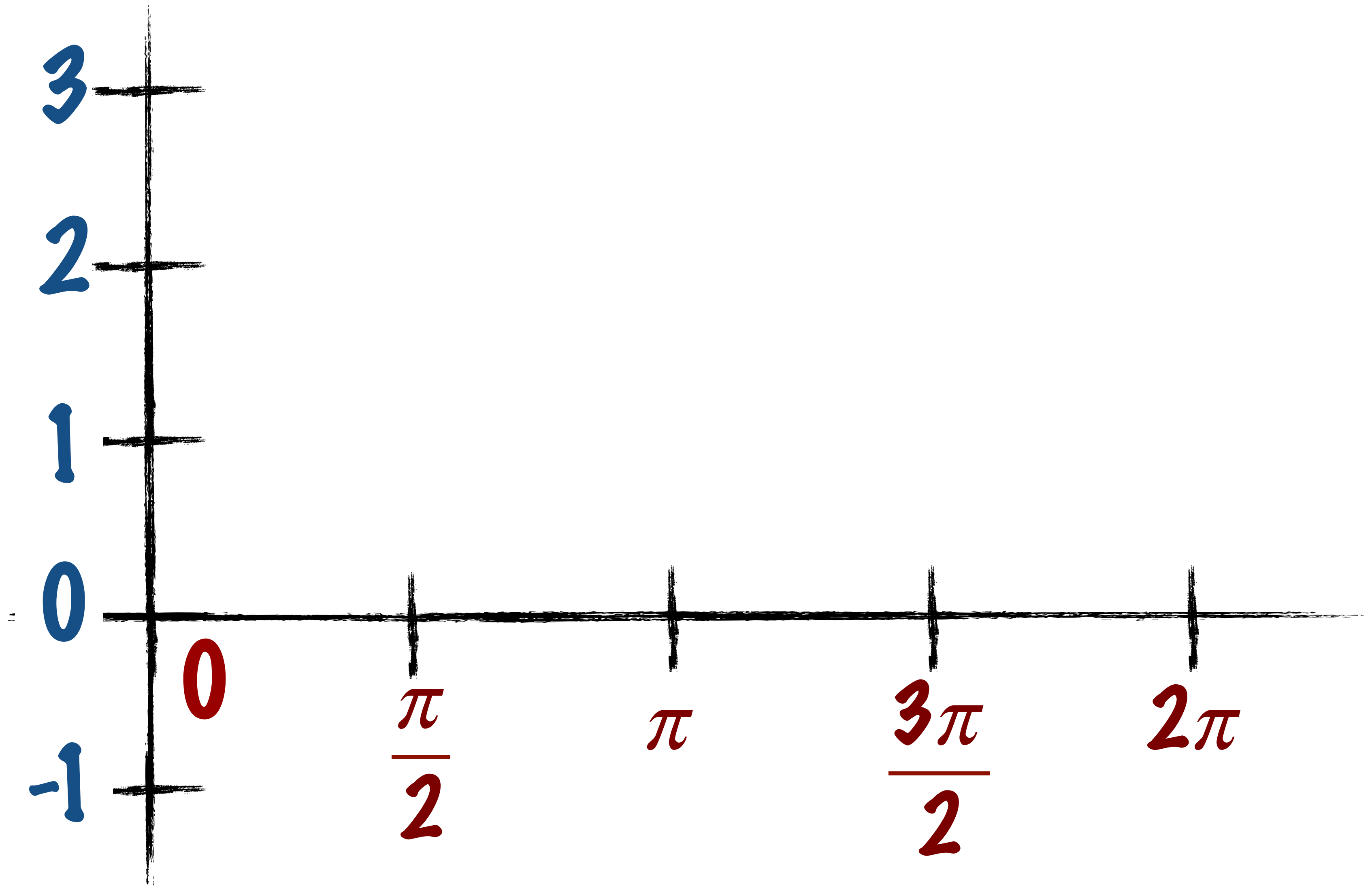
$\frac{2\pi}{4} = \frac{\pi}{2}$ $x_1 = 0$

$x_2 = 0 + \frac{\pi}{2} = \frac{\pi}{2}$

$x_3 = \frac{\pi}{2} + \frac{\pi}{2} = \pi$

$x_4 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$x_5 = \frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$



A Vertical Shift y=2cosx+1

Step 3 Find the points for the 5 key values of x.

$$y = 2\cos 0 + 1 = 3$$

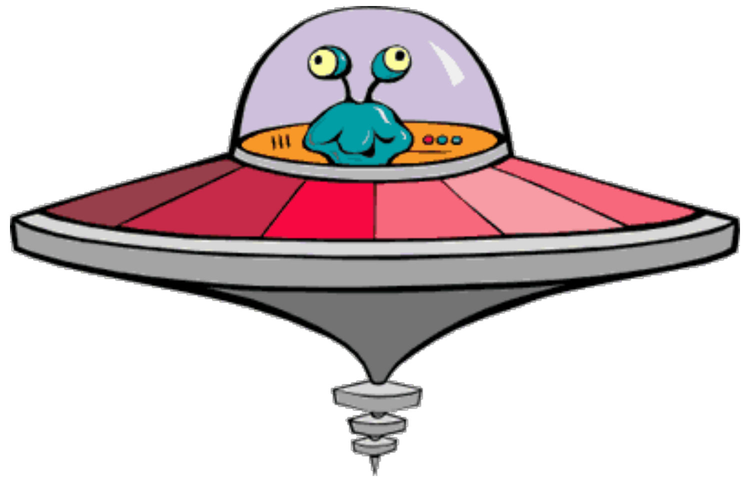
$$y = 2\cos\frac{\pi}{2} + 1 = 1$$

$$y = 2\cos\pi + 1 = -1$$

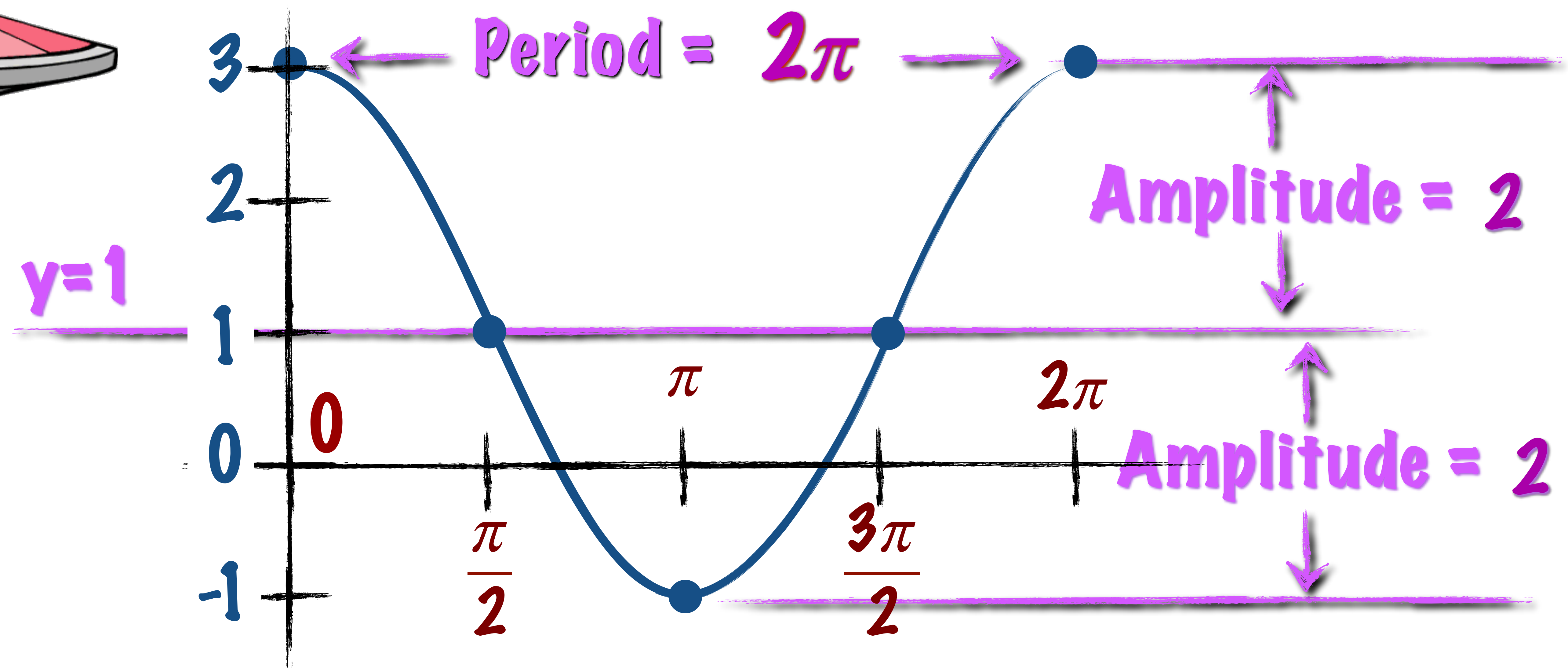
$$y = 2\cos\frac{3\pi}{2} + 1 = 1$$

$$y = 2\cos 2\pi + 1 = 3$$

x	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = 2\cos(x) + 1$	3	1	-1	1	3



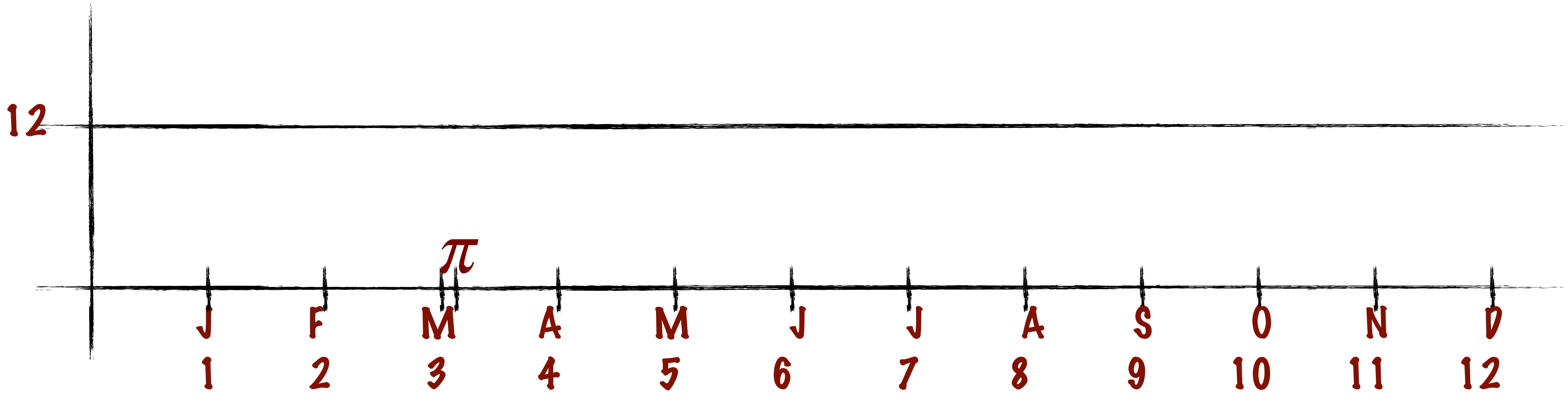
Step 4 Graph one cycle.



Modeling Periodic Behavior

- A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y=a\cos(bx-c)+d$ to model the hours of daylight.

  - Because the hours of daylight range from a minimum of 10 to a maximum of 14, the curve oscillates about the middle value, 12 hours. Thus, $d = 12$.



Modeling Periodic Behavior

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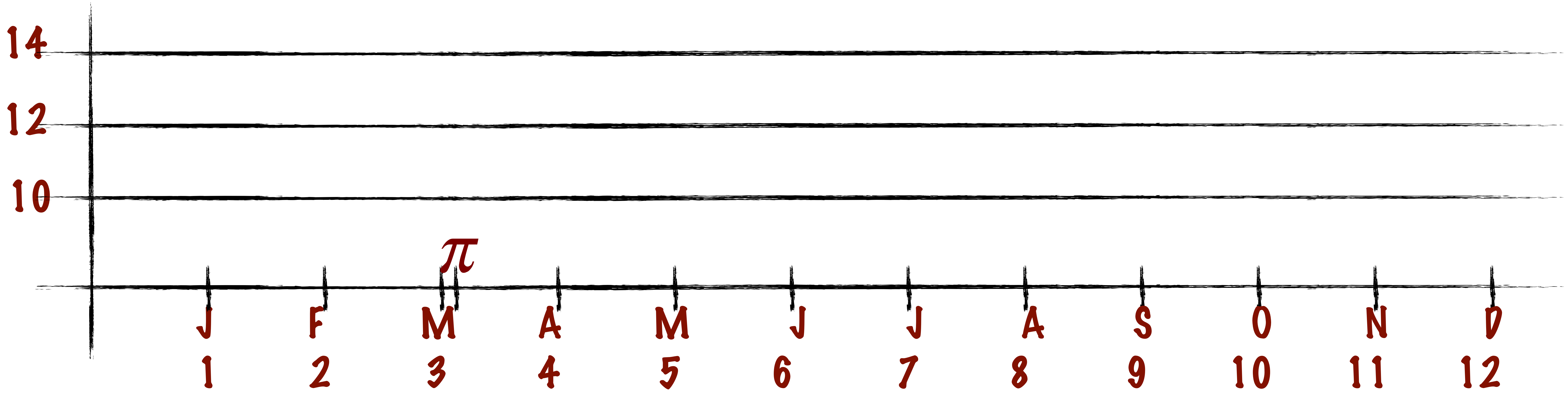
  - The maximum number of hours of daylight is 14, which is 2 hours more than 12 hours. Thus, a, the amplitude, is 2; a = 2.

d = 12



Modeling Periodic Behavior

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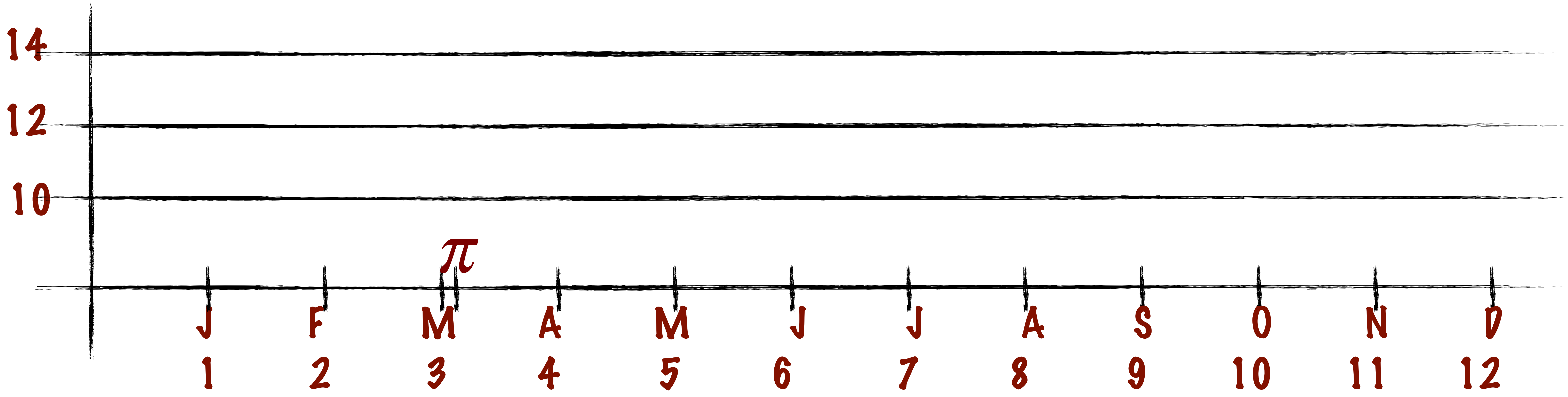
- The complete cycle occurs over a period of 12 months.

period = 12mo = $\frac{2\pi}{b}$ $\quad b=\frac{\pi}{6}$

a = 2 d = 12

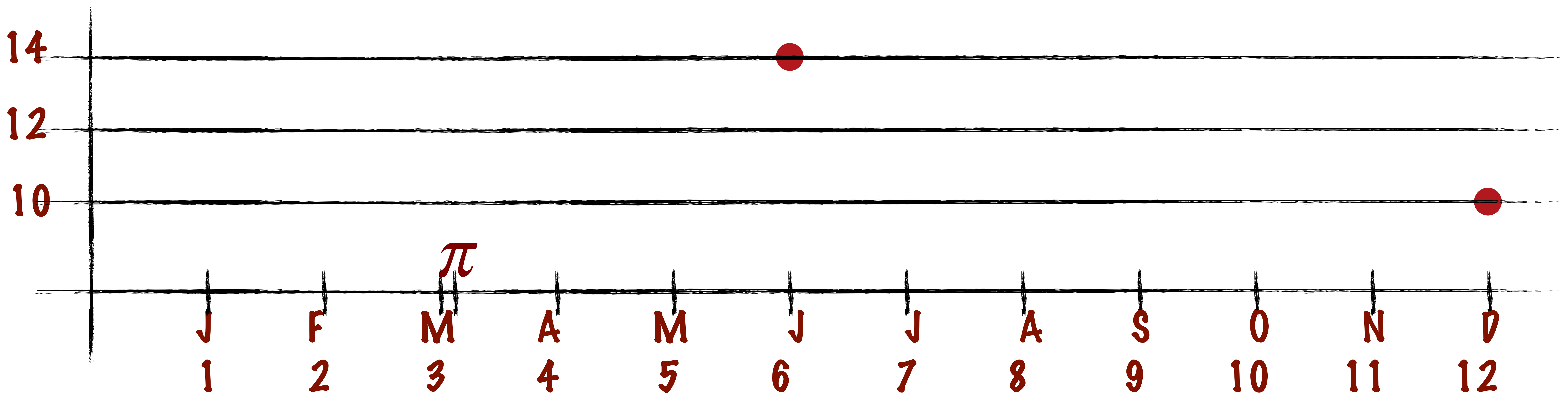


Modeling Periodic Behavior

- A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form $y = a\cos(bx - c) + d$ to model the hours of daylight.

  - The maximum number of hours of daylight occur in June, the minimum occurs in December.

$a = 2 \quad d = 12 \quad b = \frac{\pi}{6}$



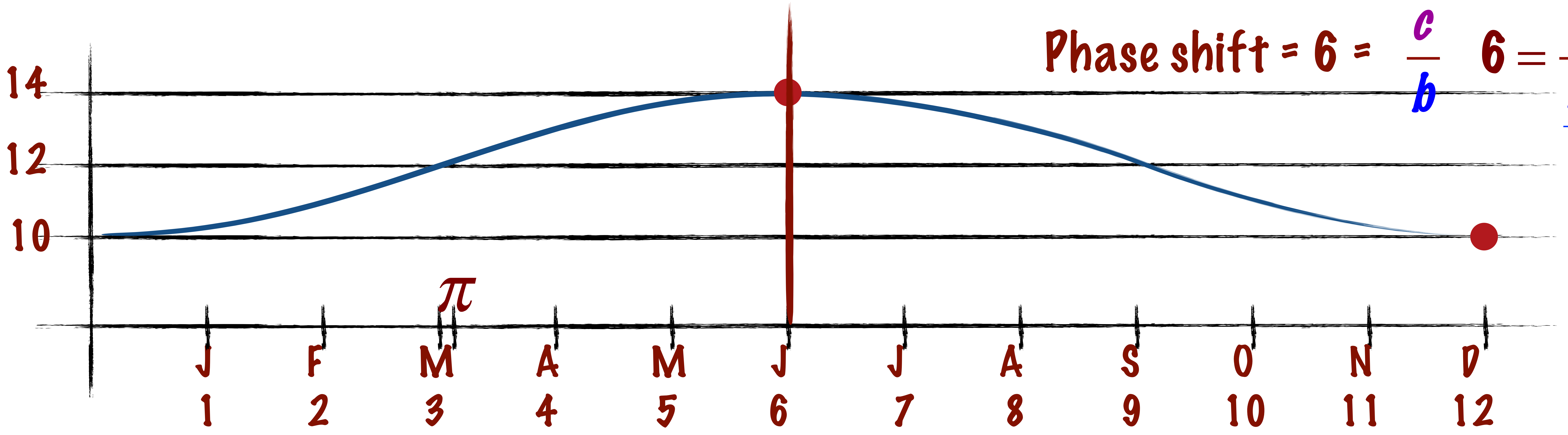
Modeling Periodic Behavior

- A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form y=acos(bx-c)+d to model the hours of daylight.
  - We lay a sine wave on top of the points:

    a = 2 d = 12 $b = \frac{\pi}{6}$

    - The starting point of the cycle is March (x=6) for a cosine function.

Phase shift = 6 = $\frac{c}{b}$ $6 = \frac{c}{\frac{\pi}{6}}$ $c = \pi$



Modeling Periodic Behavior

- The maximum number of hours of daylight is 14, which is 2 hours more than 12 hours. Thus, a, the amplitude, is 2; a = 2.

a = 2

- The complete cycle occurs over a period of 12 months.

period = 12mo = $\frac{2\pi}{b}$ $b = \frac{\pi}{6}$

$b = \frac{\pi}{6}$

- The starting point of the cycle is March (x=6) for a cosine function.

Phase shift = 6 = $\frac{c}{b}$ $6 = \frac{c}{\frac{\pi}{6}}$ $c = \pi$ $c = \pi$

- Because the hours of daylight range from a minimum of 10 to a maximum of 14, the curve oscillates about the middle value, 12 hours. Thus, d = 12.

d = 12

Modeling Periodic Behavior

- A region that is 30° north of the Equator averages a minimum of 10 hours of daylight in December. Hours of daylight are at a maximum of 14 hours in June. Let x represent the month of the year, with 1 for January, 2 for February, 3 for March, and 12 for December. If y represents the number of hours of daylight in month x, use a sine function of the form y=acos(bx-c)+d to model the hours of daylight.

$a = 2 \quad b = \frac{\pi}{6} \quad c = \pi \quad d = 12$

$$y = 2\cos\left(\frac{\pi}{6}x - \pi\right) + 12$$

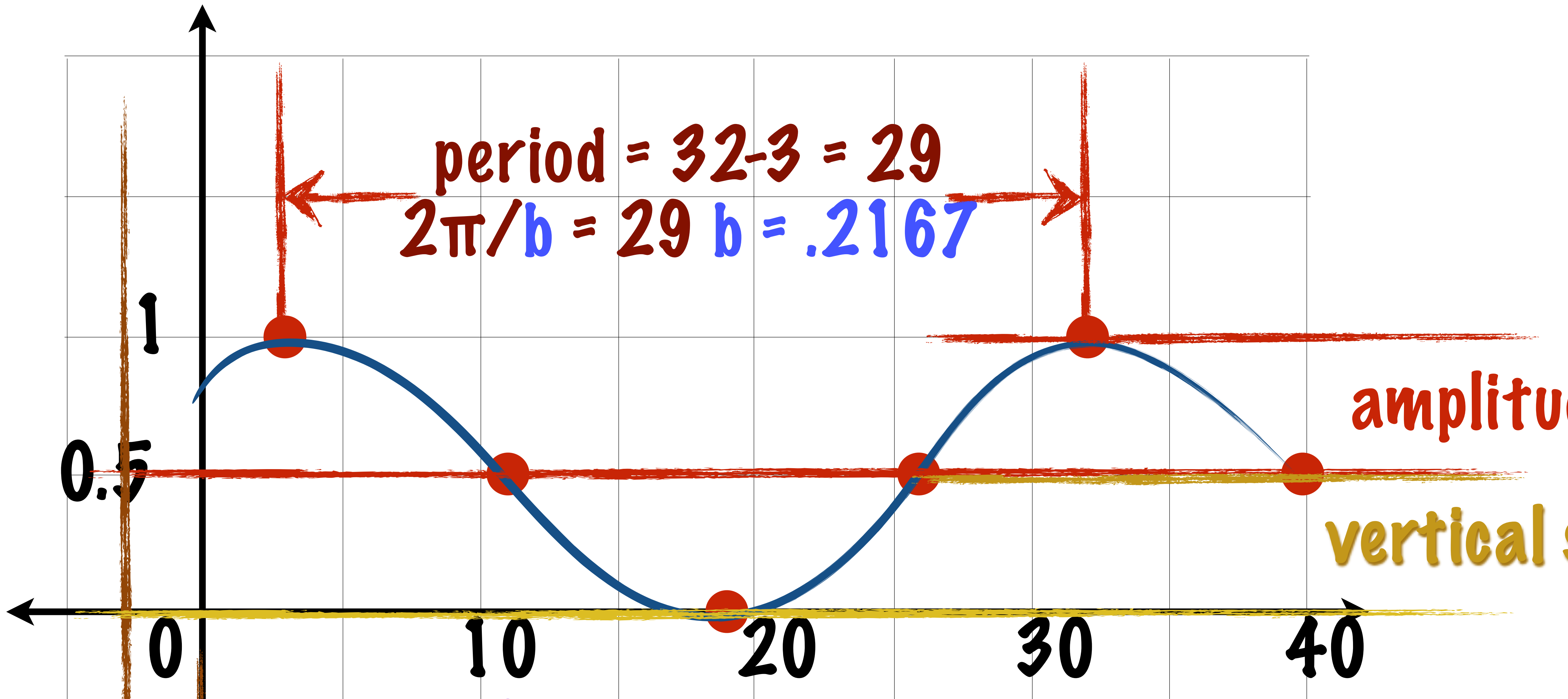
- This model the hours of daylight for each day of the year 30° north of the equator (30th parallel passing through Houston and New Orleans).

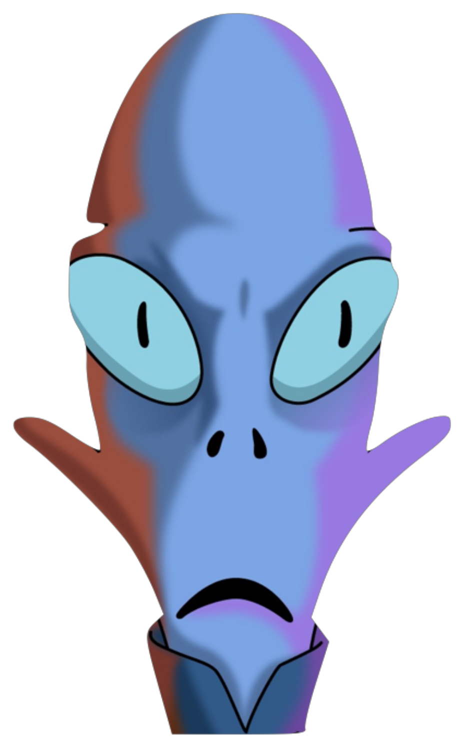
Modeling Sinusoidal Behavior

- **Data Analysis: Astronomy** The percent of the moon's face that is illuminated on day of the year 2007, where x = 1 represents January 1, is shown in the table.
  - (a) Create a scatter plot of the data.
  - (b) Find a trigonometric model that fits the data.
  - (c) Add the graph of your model in part (b) to the scatter plot. How well does the model fit the data?
  - (d) What is the period of the model?
  - (e) Estimate the moon's percent illumination for March 12, 2007.

x	y
3	1.0
11	0.5
19	0.0
26	0.5
32	1.0
40	0.5

Looks like a sine wave, $y = a\sin(bx-c)+d$





x	y
3	1.0
11	0.5
19	0.0
26	0.5
32	1.0
40	0.5

phase shift =

$1 = .5\sin(.2167(3)-c) + .5$

$.5 = .5\sin(.2167(3)-c)$

$1 = \sin(.2167(3)-c)$

$.2167(3)-c = \pi/2$

$c = -.9207$

$y = a\sin(bx-c)+d$

$y = .5\sin(.2167x+.92)+.5$

Mar 3 = 71

$y = .5\sin(.2167(71)+.921)+.5$

$y = .2182$

We could also estimate the point at which the curve comes back to equilibrium (3 - 29/4) = -4.25

$-4.25-c/.2167=0$ (sin(0) = 0)

On March 3 there is about 22% of the moon showing.

Modeling Sinusoidal Behavior with TI-84

- Let us see if TI agrees with us.
- Enter the data into two lists
- Now we will do a sine regression

$y = .5\sin(.2167x+.92)+.5$

$y = .5111\sin(.2164x+.7258)+.4883$

- Pretty close!

STAT ➢ **CALC** ⋏ **C:SinReg** ENTER

Iterations: 3

Xlist: L_1 2nd 1

Ylist: L_2 2nd 2

Period: 29

Store RegEQ: Y_1 **VARS** ➢ **Y-VARS** 1 1:Function ENTER ENTER

Calculate ENTER

y=a*sin(bx+c)=d

a= .5111434882

b= .2163933129

c= .7258071718

d= .488303521