Chapter 4

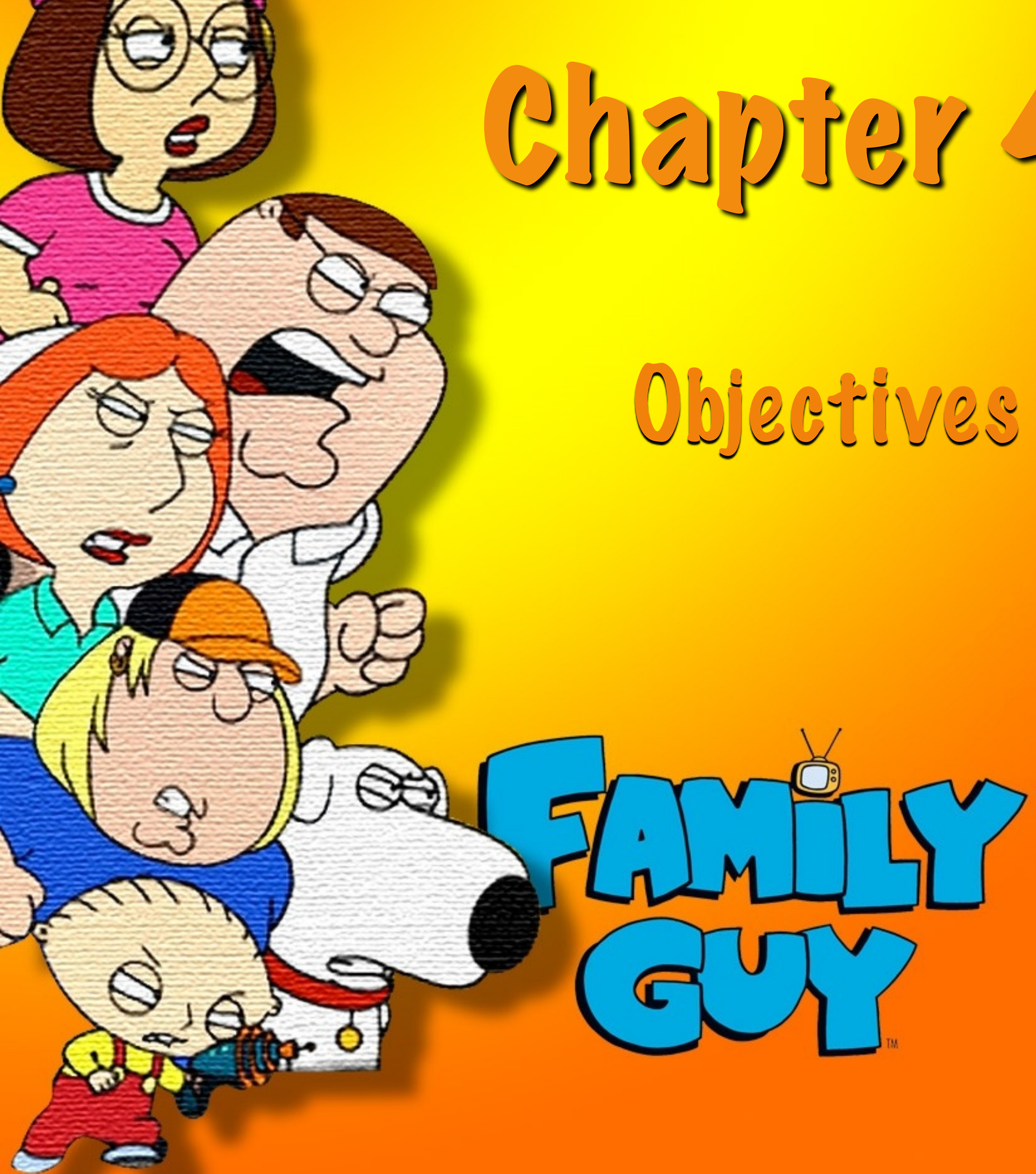
4.6 Graphs of Other Trigonometric Functions

Chapter 4

Homework

4.6 p340 1-75 odd, 79

Chapter 4



Objectives

- Graph $y = \tan x$.
- Graph variations of $y = \tan x$.
- Graph $y = \cot x$.
- Graph variations of $y = \cot x$.
- Graph $y = \csc x$ and $y = \sec x$.
- Graph variations of $y = \csc x$ and sec

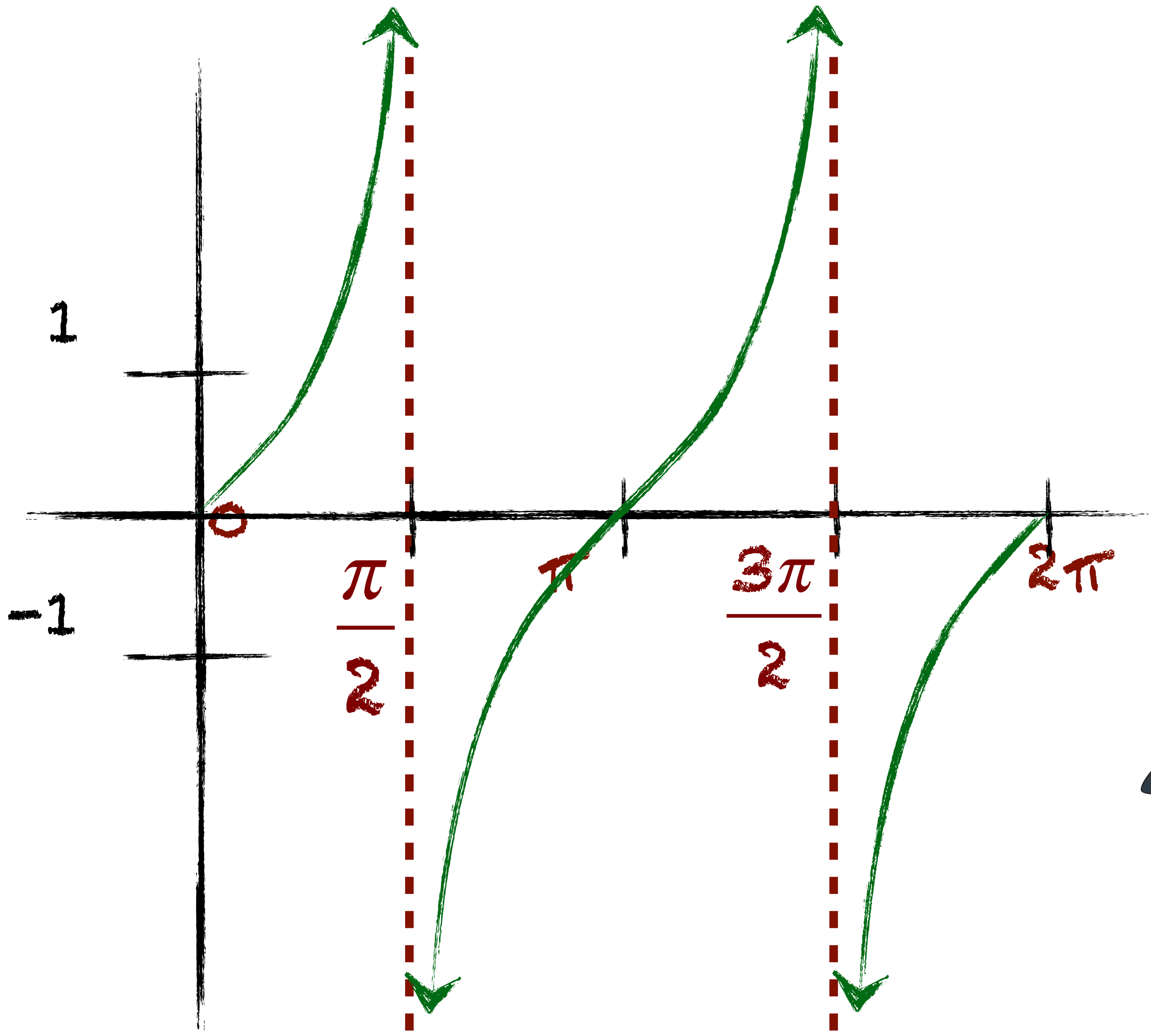
The Graph of y = tan x

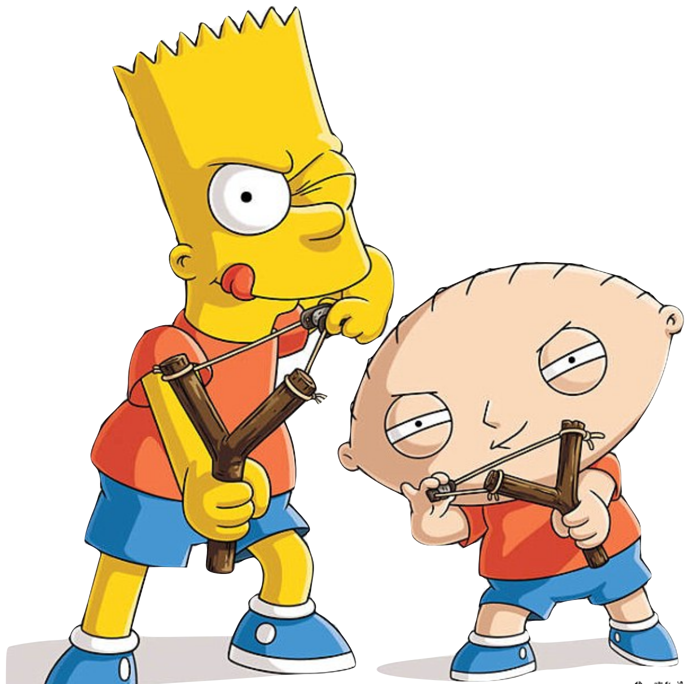
Objective: Students graph tan, cot, sec, csc.

- Complete the table

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
tanx	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0

- The table begins to repeat at π. Period = π.
- tangent is an odd function.
  - tan(-x) = -tanx
- Since cos $\frac{\pi}{2}, \frac{3\pi}{2} = 0$, tan is undefined at $\frac{\pi}{2}, \frac{3\pi}{2}$



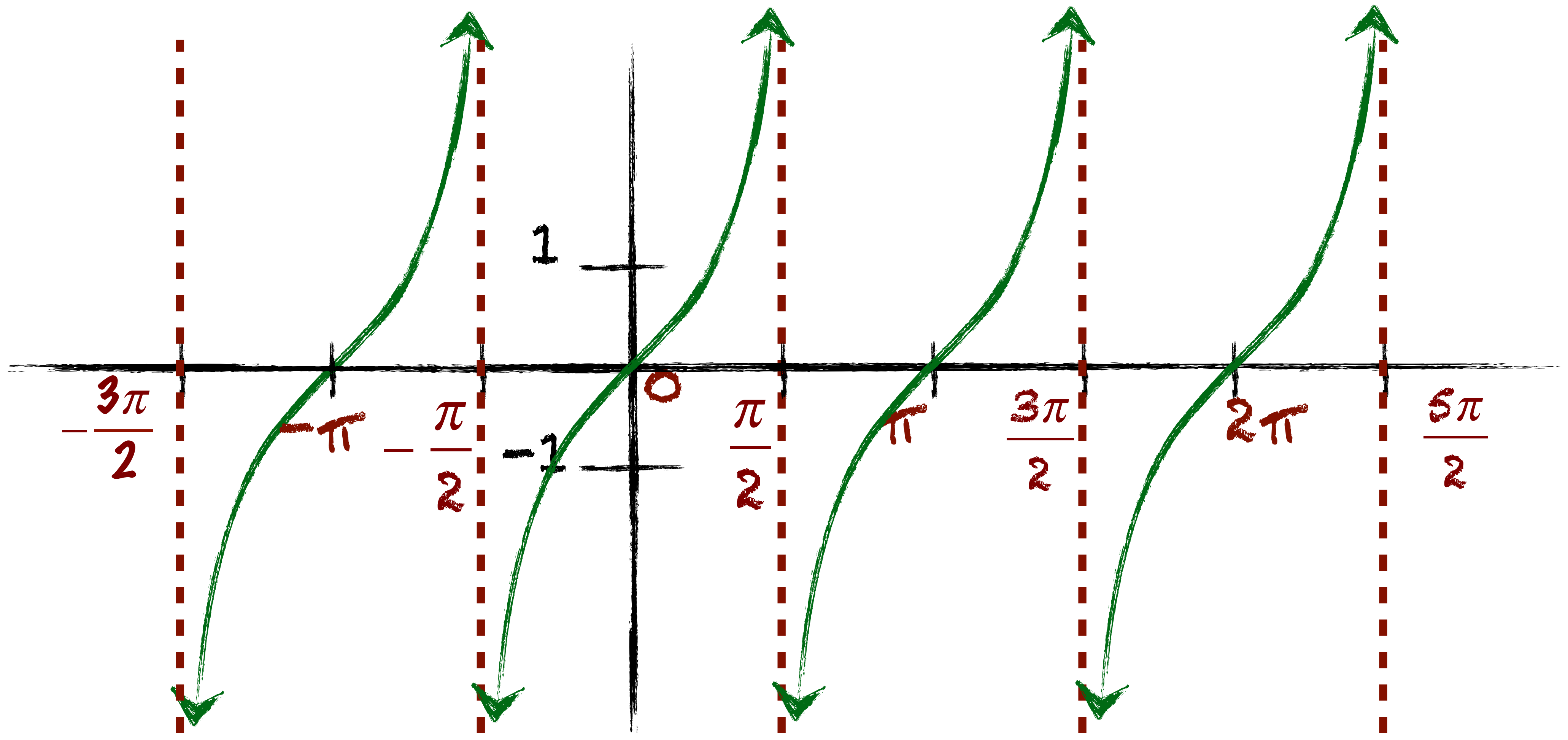


The Tangent curve: f(x) = tanx

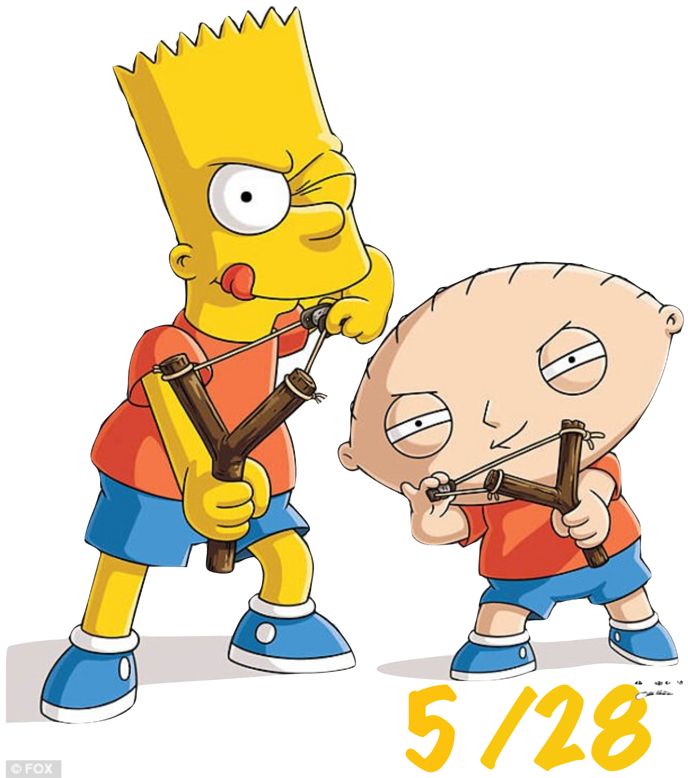
Objective: Students graph tan, cot, sec, csc.

- Period = π
- Domain = all reals except **odd** multiples of $\frac{\pi}{2}$
- Range = all reals
- Vertical **asymptotes** at **odd** multiples of $\frac{\pi}{2}$

$$\left[(2n+1)\frac{\pi}{2}\right]$$



- x-intercepts at multiples of π.
- f(x) = tanx is an odd function with origin symmetry.
- tanx = 1 or -1 at 1/4 intervals.

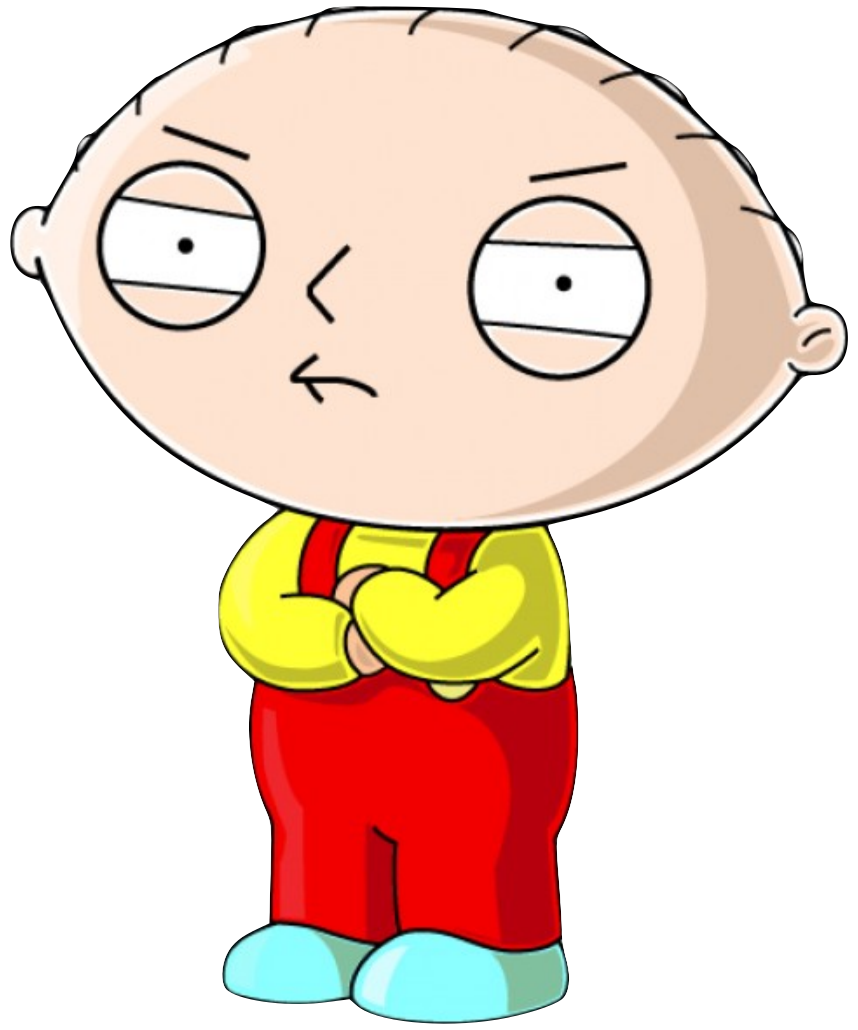


Graphing Variations of y = tanx

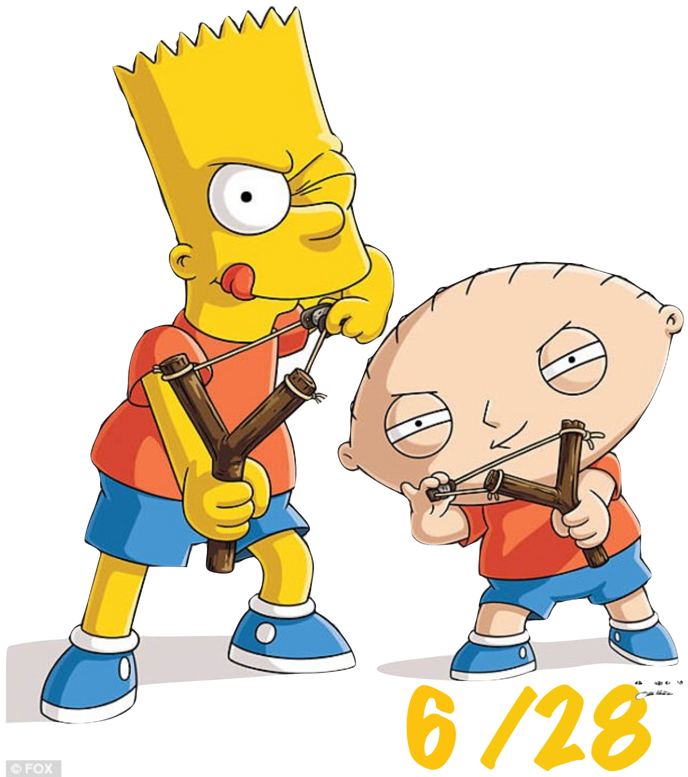
Objective: Students graph tan, cot, sec, csc.

- Graphing y = atan(bx-c), b>0.

1. Find consecutive asymptotes from an interval of one period. $-\frac{\pi}{2} \le bx - c \le \frac{\pi}{2}$
   - The asymptotes are: $bx - c = \frac{\pi}{2}$ and $bx - c = -\frac{\pi}{2}$



2. Find x-intercept midway between asymptotes.
3. Find values of y at 1/4 and 3/4 intervals between asymptotes, these will be y = -a, and y = a.
4. That will be one period, repeat as needed over designated domain.

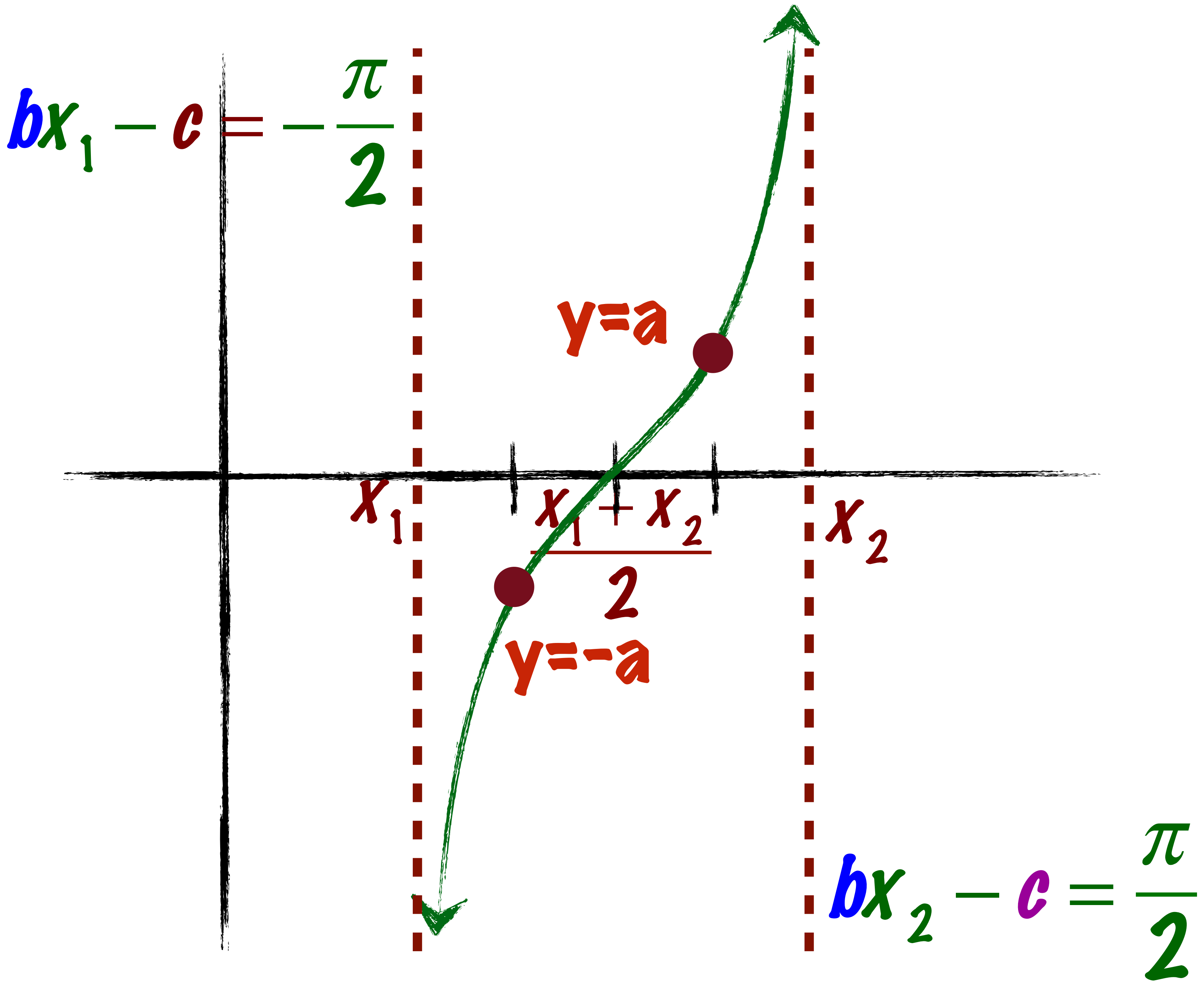


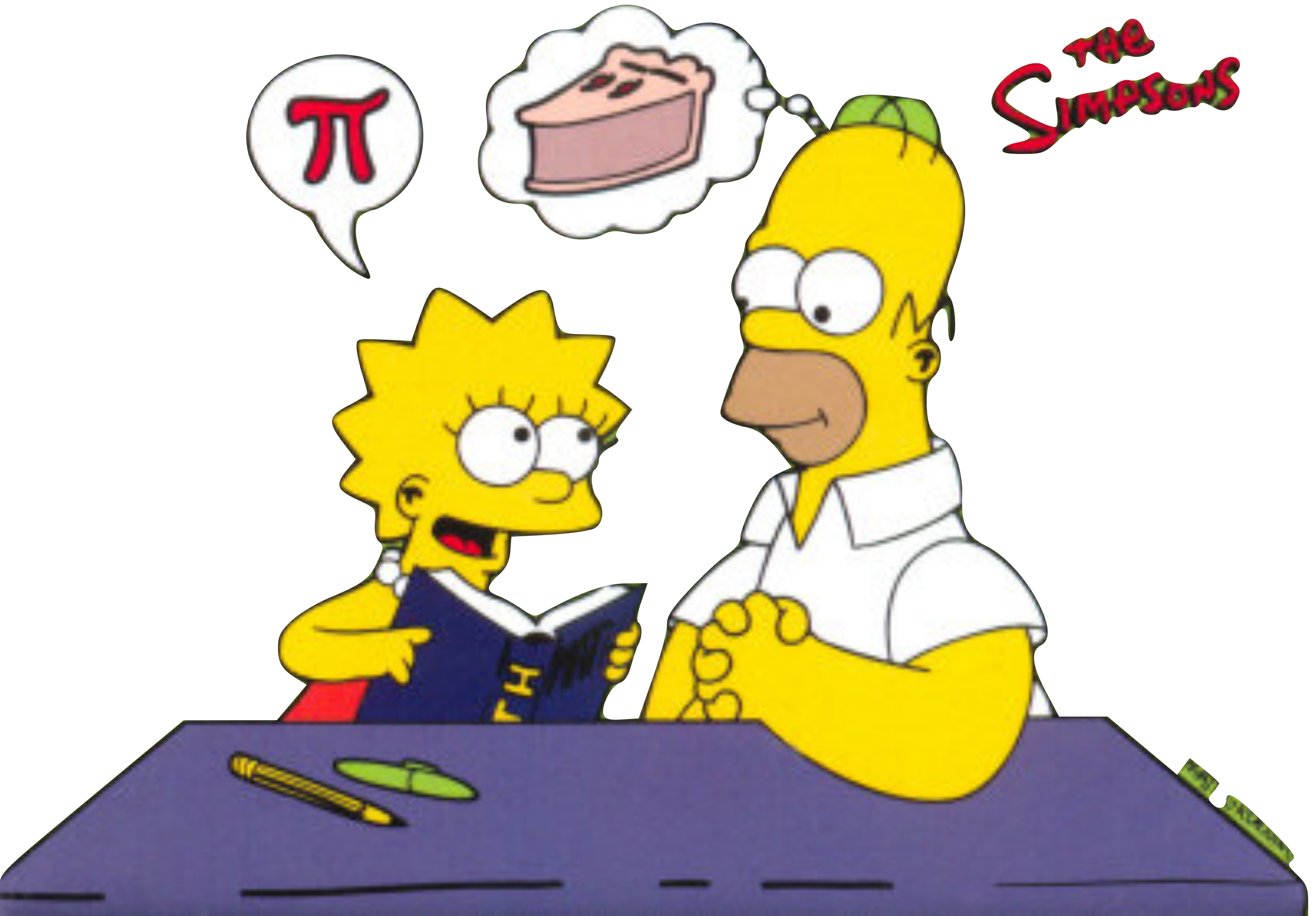
Graphing Variations of y = tan x

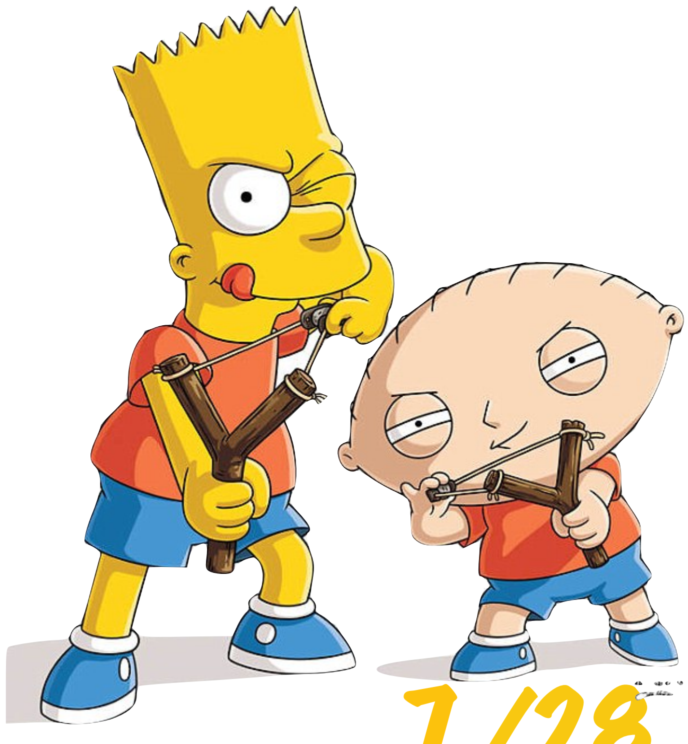
Objective: Students graph tan, cot, sec, csc.

∡ Graphing y = atan(bx-c), b>0. $-\frac{\pi}{2} \le bx - c \le \frac{\pi}{2}$ $bx - c = \pm\frac{\pi}{2}$

1. asymptotes $bx - c = \pm\frac{\pi}{2}$
2. x-intercept $\frac{x_1 + x_2}{2}$
3. y at 1/4 and 3/4 interval, y=±a







Example: Graphing a Tangent Function

Objective: Students graph tan, cot, sec, csc.

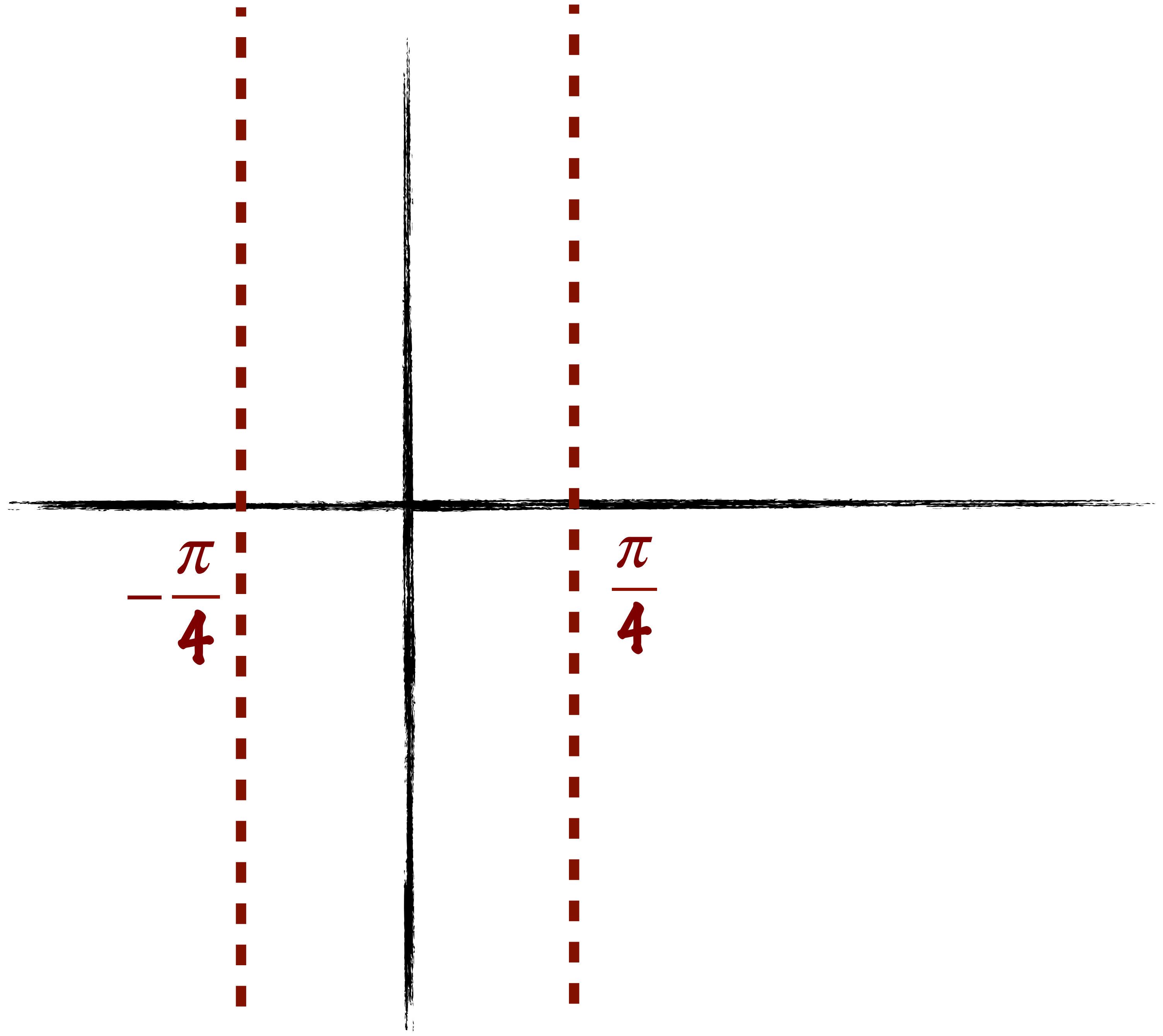
Graph $y = 3\tan 2x$ for $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ $\quad a = 3, b = 2, c = 0$

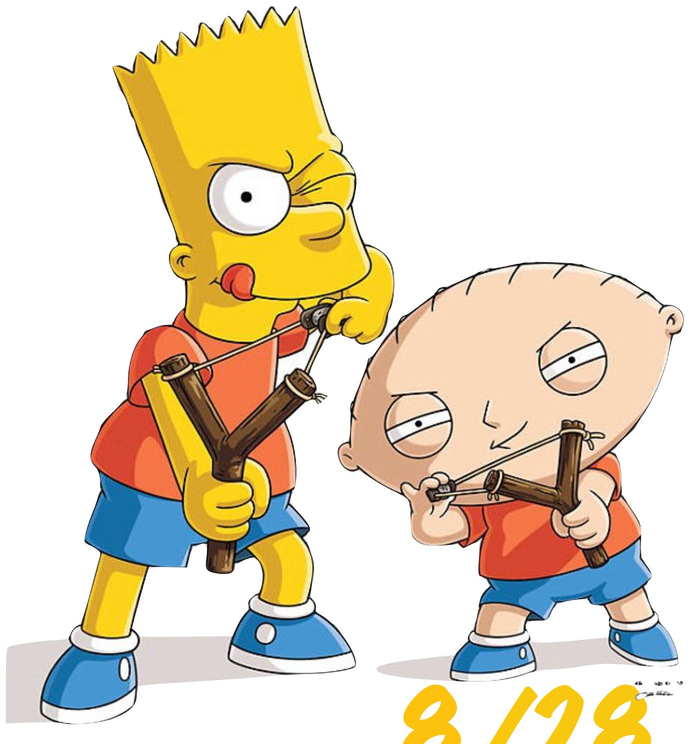
1. asymptotes $-\frac{\pi}{2} \le bx - c \le \frac{\pi}{2}$ $\quad -\frac{\pi}{2} \le 2x - 0 \le \frac{\pi}{2}$ $\quad -\frac{\pi}{4} \le x \le \frac{\pi}{4}$

An interval containing one period is $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

with two consecutive asymptotes at

$$x = -\frac{\pi}{4} \text{ and } x = \frac{\pi}{4}$$





Example: Graphing a Tangent Function

Objective: Students graph tan, cot, sec, csc.

Graph $y = 3\tan 2x$ for $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ a = 3, b = 2, c = 0 $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

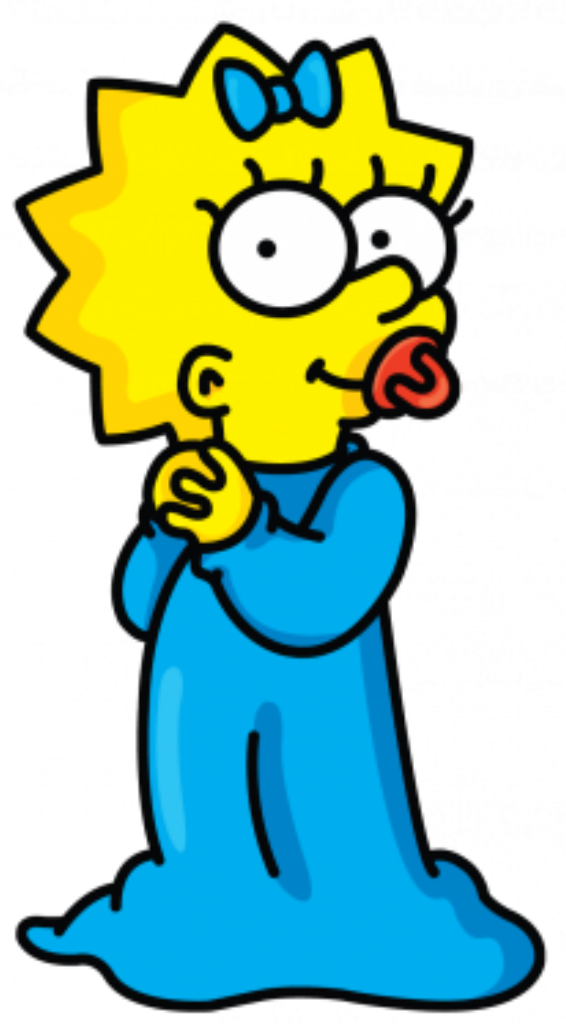
2. x-intercept $x = \dfrac{-\frac{\pi}{4} + \frac{\pi}{4}}{2} = 0$ x-intercept at (0, 0).

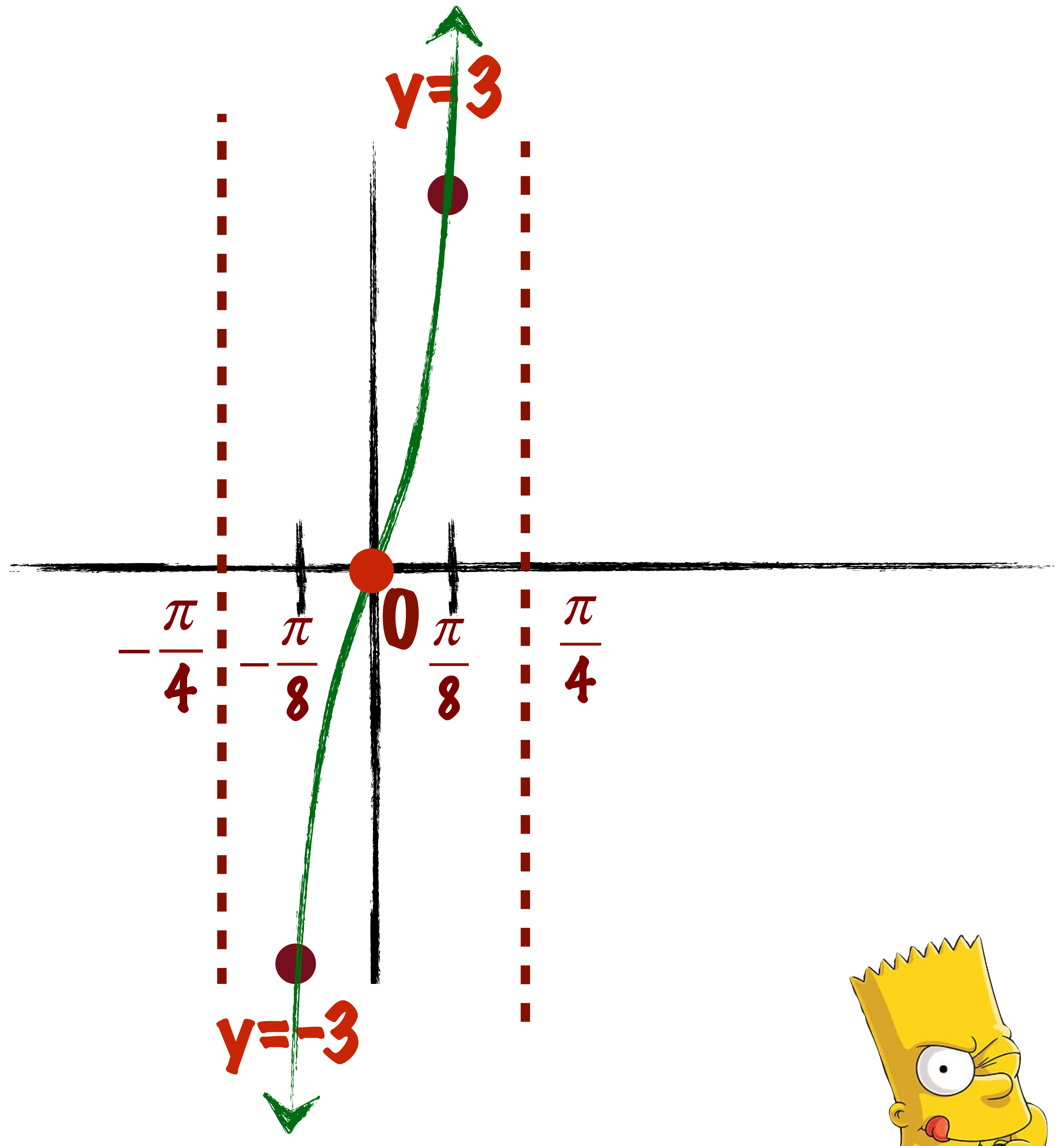
3. y at 1/4 and 3/4 intervals $x = -\frac{\pi}{8}$ $x = \frac{\pi}{8}$

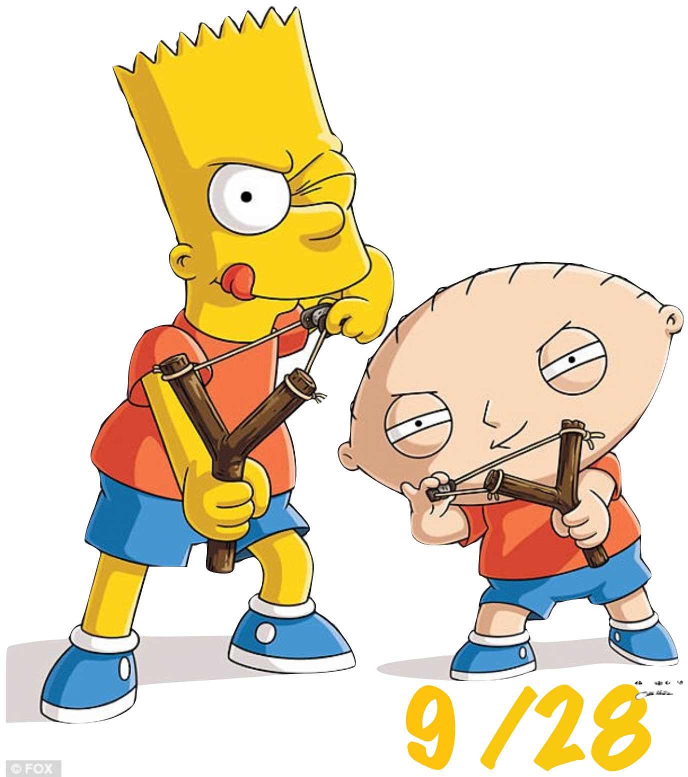
These points have y values of -a and a.

$y = 3\tan 2\left(-\frac{\pi}{8}\right)$ $\left(-\frac{\pi}{8}, -3\right)$

$y = 3\tan 2\left(\frac{\pi}{8}\right)$ $\left(\frac{\pi}{8}, 3\right)$







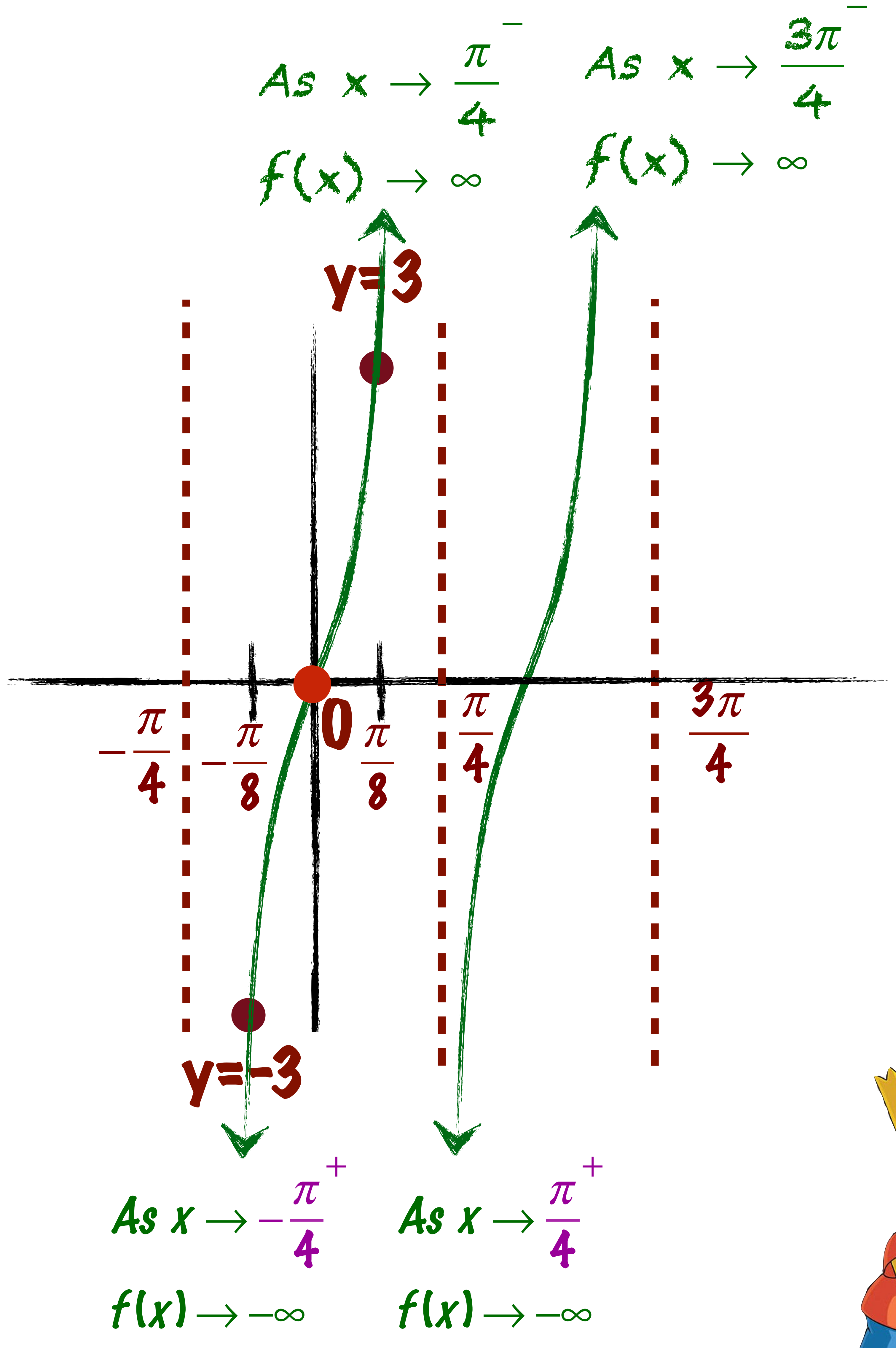
Example: Graphing a Tangent Function

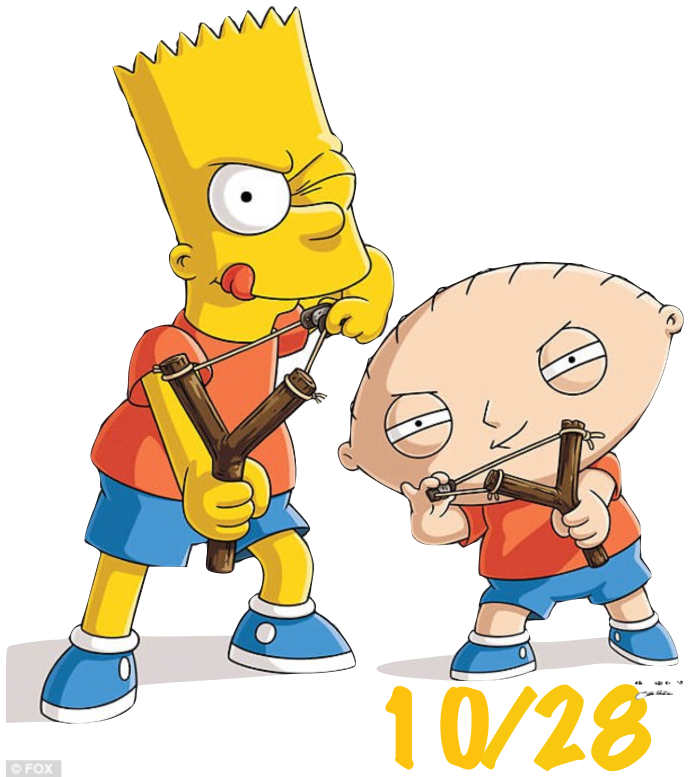
Objective: Students graph tan, cot, sec, csc.

Graph $y = 3\tan 2x$ for $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ $a = 3,\ b = 2,\ c = 0$

4. Repeat over the domain $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$

5. Arrow notations

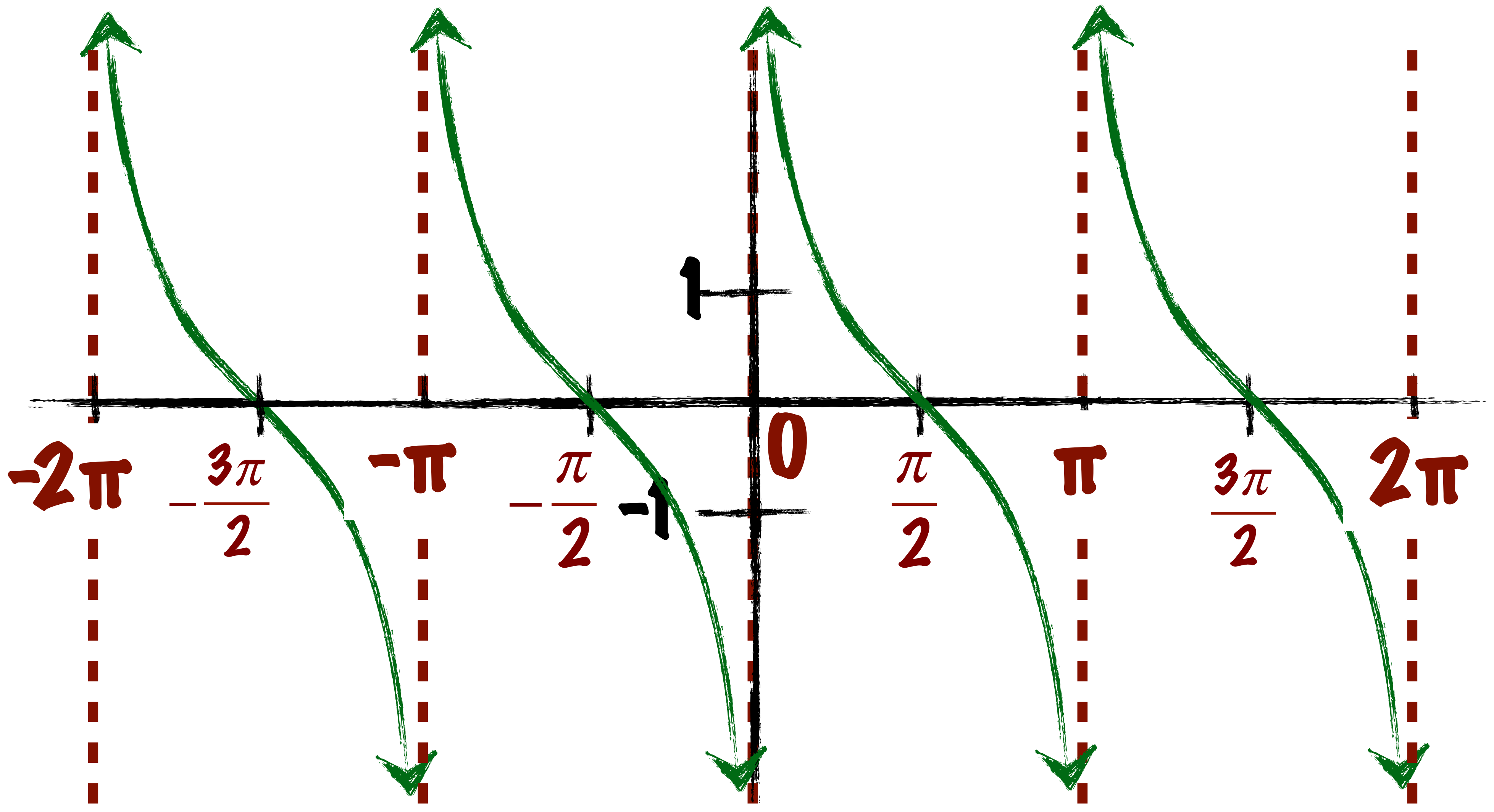


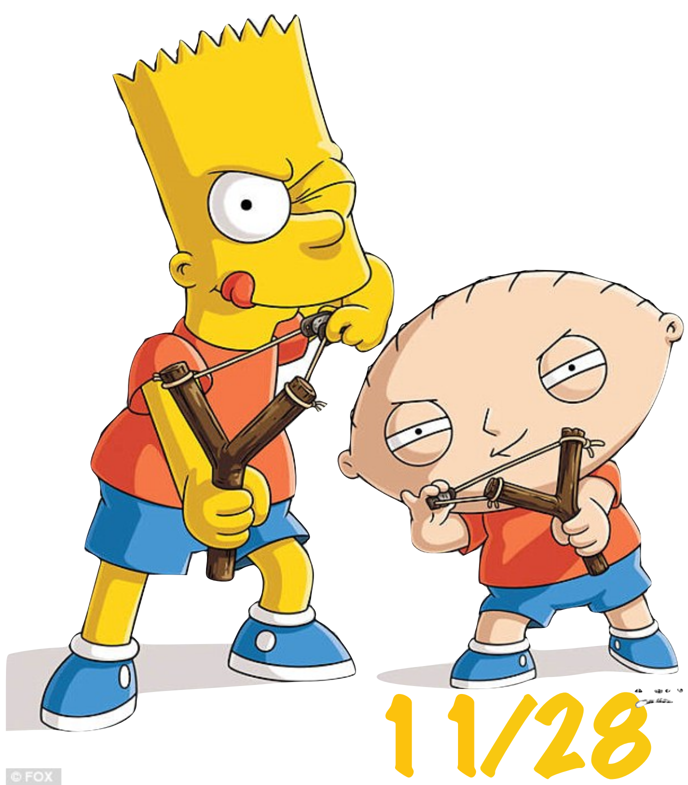


The Cotangent Curve: The Graph of y = cot x

Objective: Students graph tan, cot, sec, csc.

- Graphing y = acot(bx-c), b>0.
- Period = π
- Domain = all reals except integer multiples of π.
- Range = all reals
- Vertical asymptotes at integer multiples of π $(n\pi)$
- x-intercepts at odd multiples of π/2 (midpoint).
- f(x) = cotx is an odd function, origin symmetry.
- cotx = 1 or -1 at 1/4 and 3/4 interval.



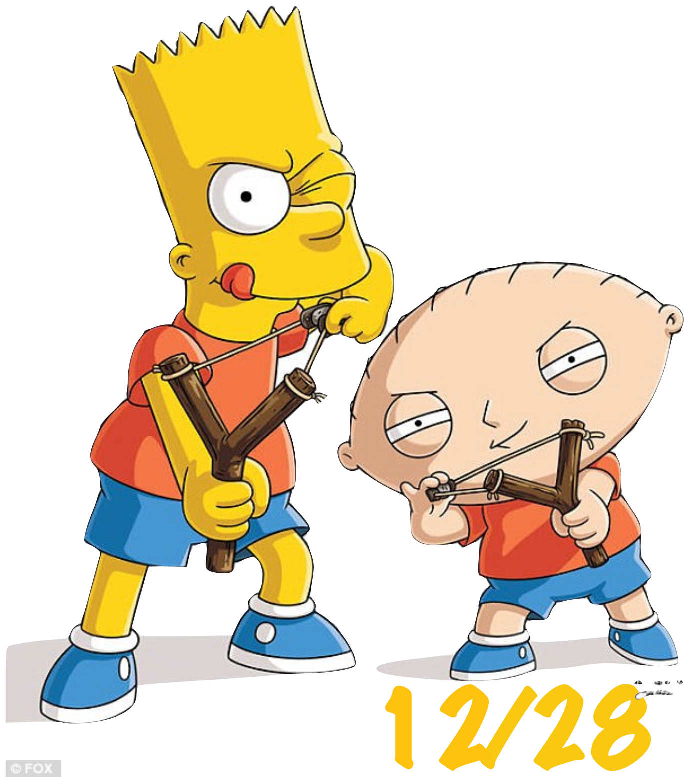


Graphing Variations of y = cotx

Objective: Students graph tan, cot, sec, csc.

- Graphing y = acot(bx-c), b>0.

1. Find consecutive asymptotes from an interval of one period. $0 \le bx - c \le \pi$
   - The asymptotes are: bx-c = 0 and bx-c = π.
2. Find x-intercept midway between asymptotes.
3. Find values of y at 1/4 and 3/4 intervals between asymptotes, these will be y = -a, and y = a.
4. That will be one period, repeat as needed.

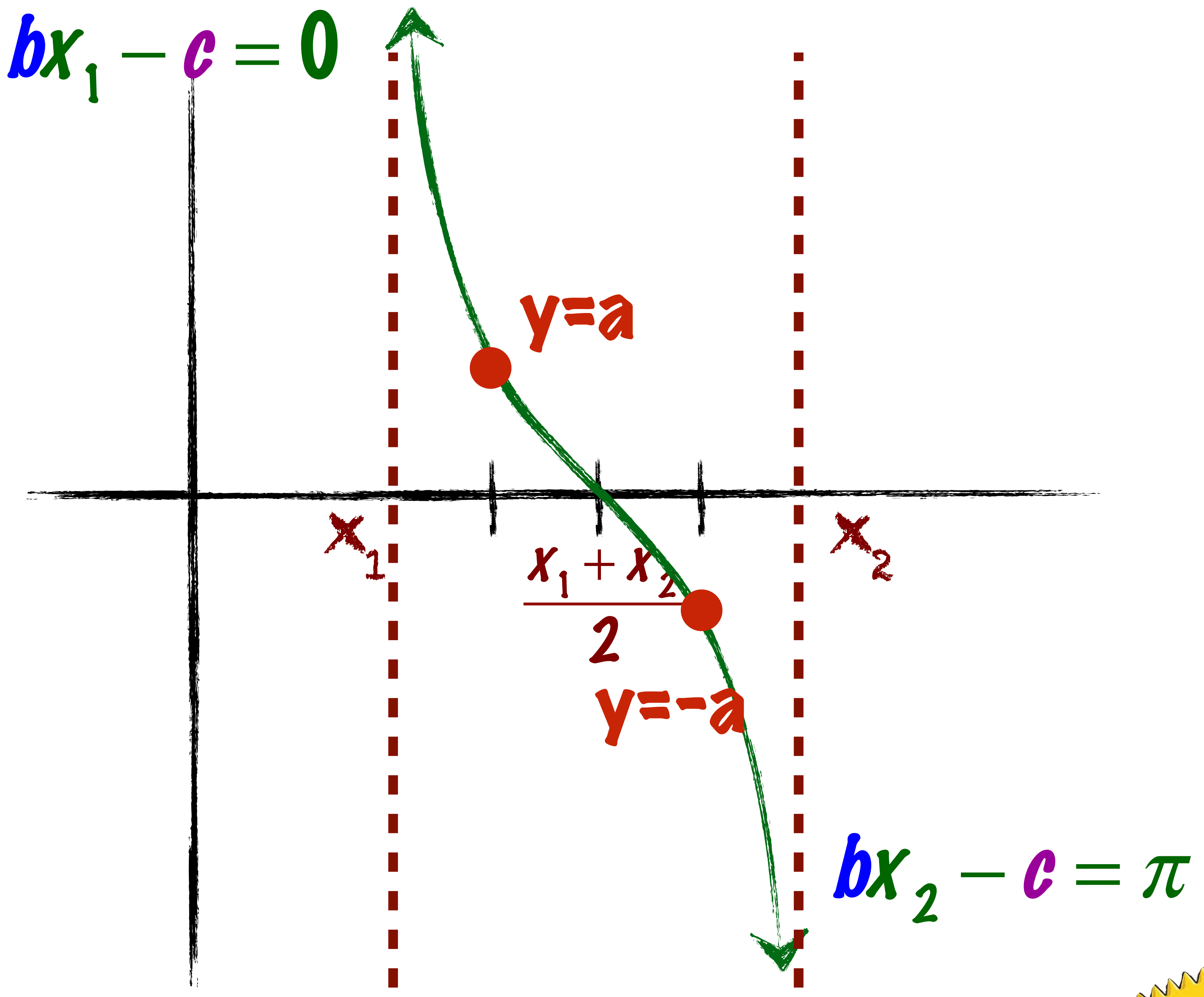


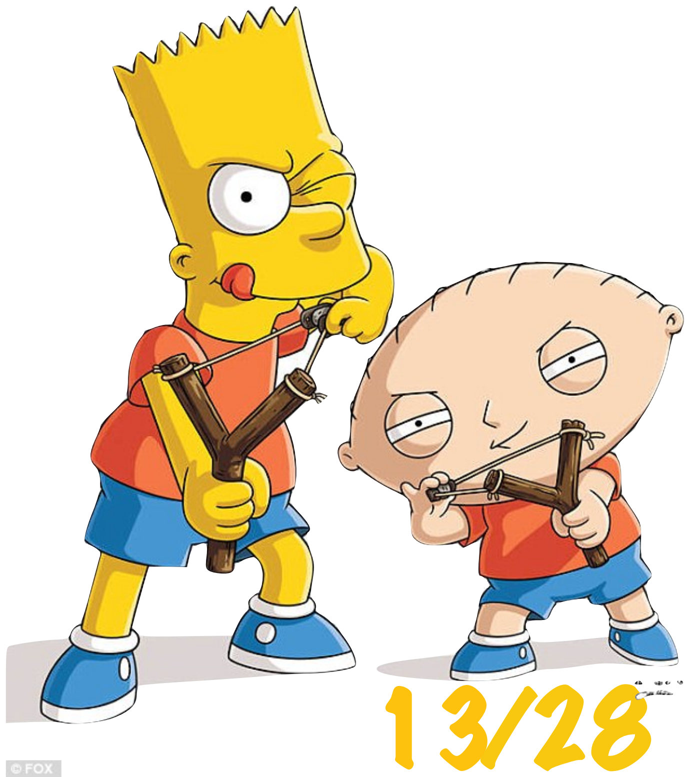
Graphing Variations of y = cotx

Objective: Students graph tan, cot, sec, csc.

- Graphing $y = a\cot(bx - c)$, $b > 0$. $\quad 0 \le bx - c \le \pi \quad bx - c = 0, \pi$

1. asymptotes $\quad bx - c = 0$ and $bx - c = \pi$
2. x-intercept $\quad \dfrac{x_1 + x_2}{2}$
3. y at 1/4 and 3/4 interval, $y = \pm a$





Example: Graphing a Cotangent Function

Objective: Students graph tan, cot, sec, csc.

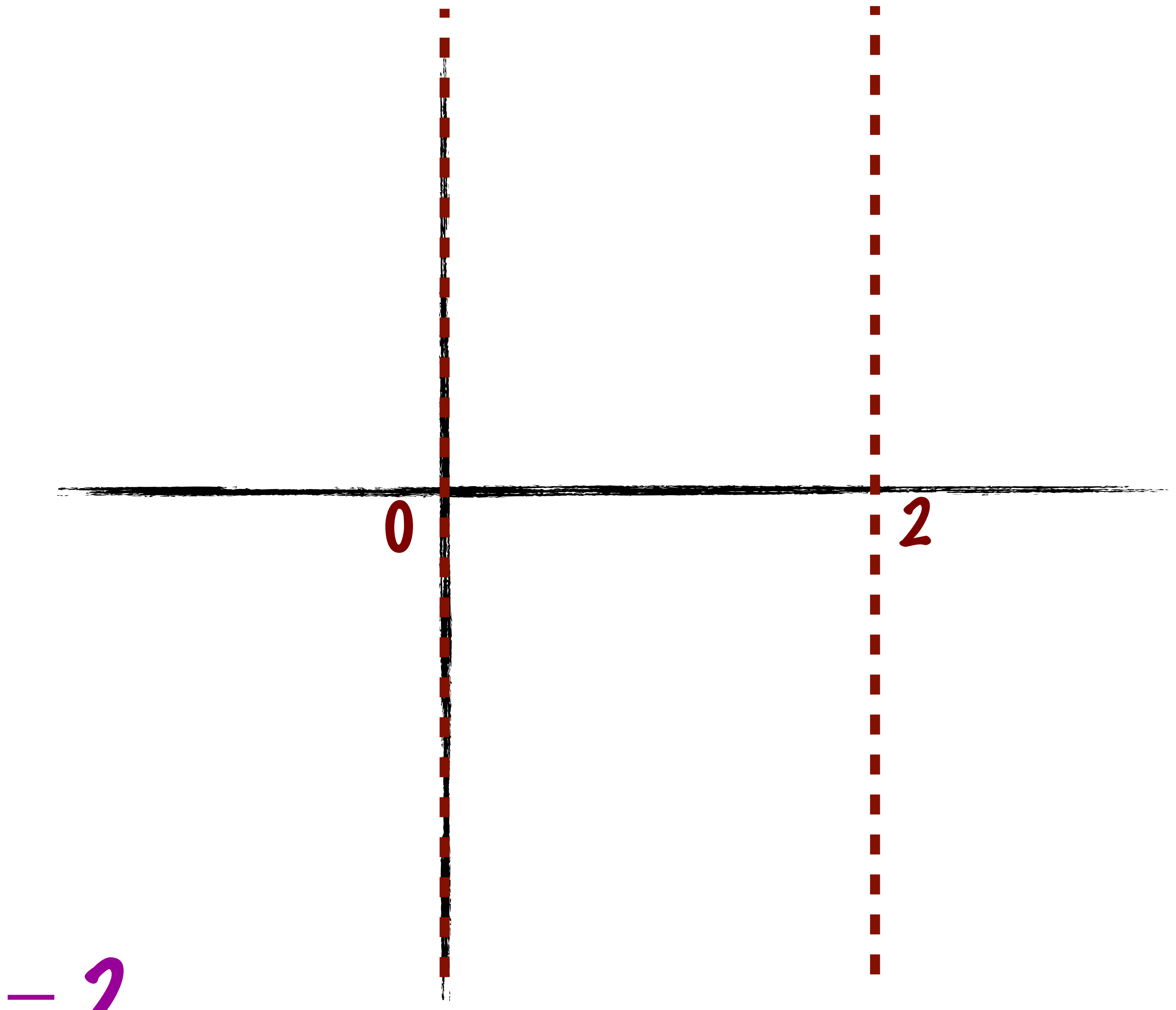
Graph one full period of $y = \frac{1}{2}\cot\frac{\pi}{2}x$ $a = \frac{1}{2}, b = \frac{\pi}{2}, c = 0$

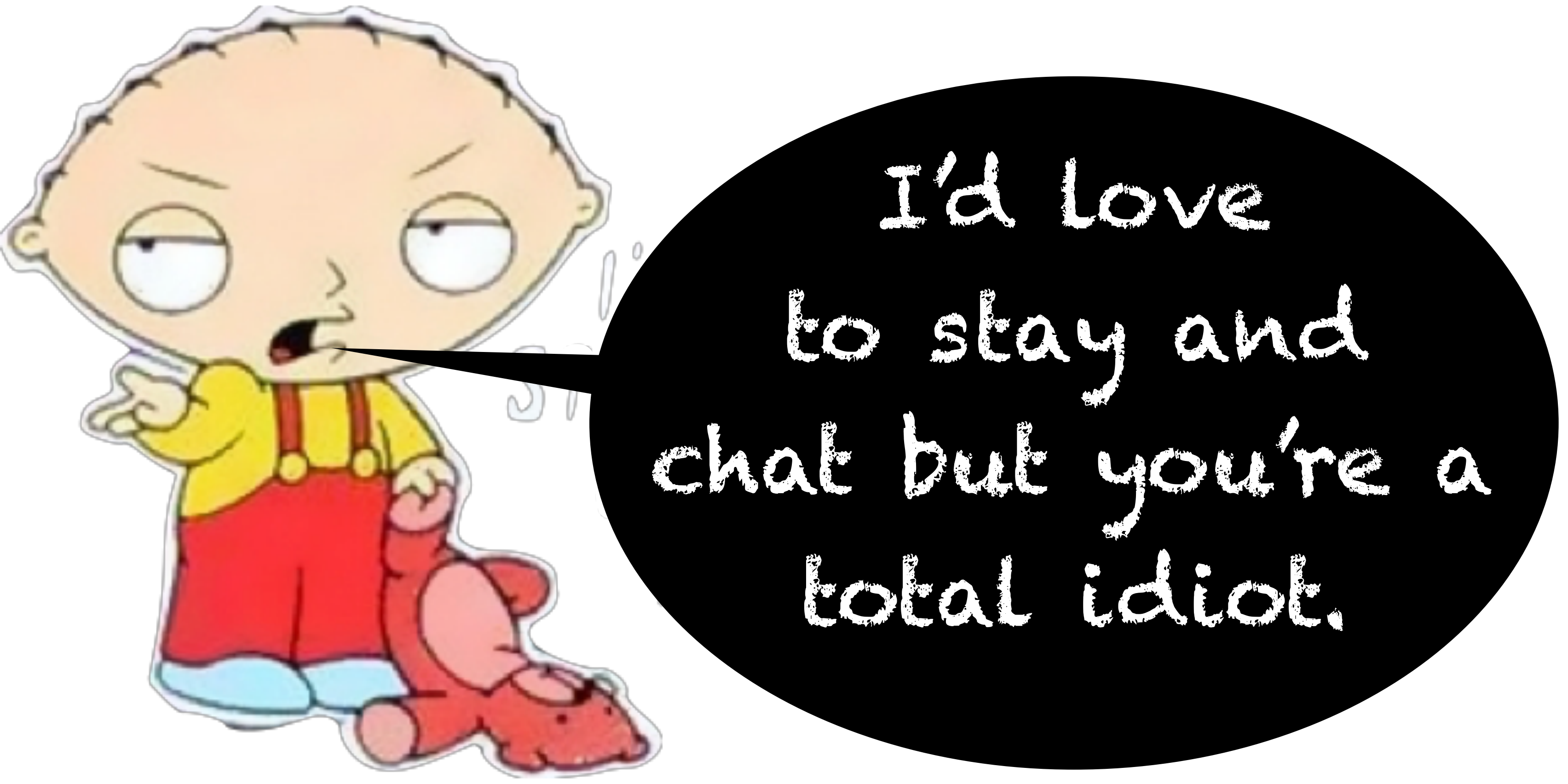
1. asymptotes $bx - c = 0$ *and* $bx - c = \pi$

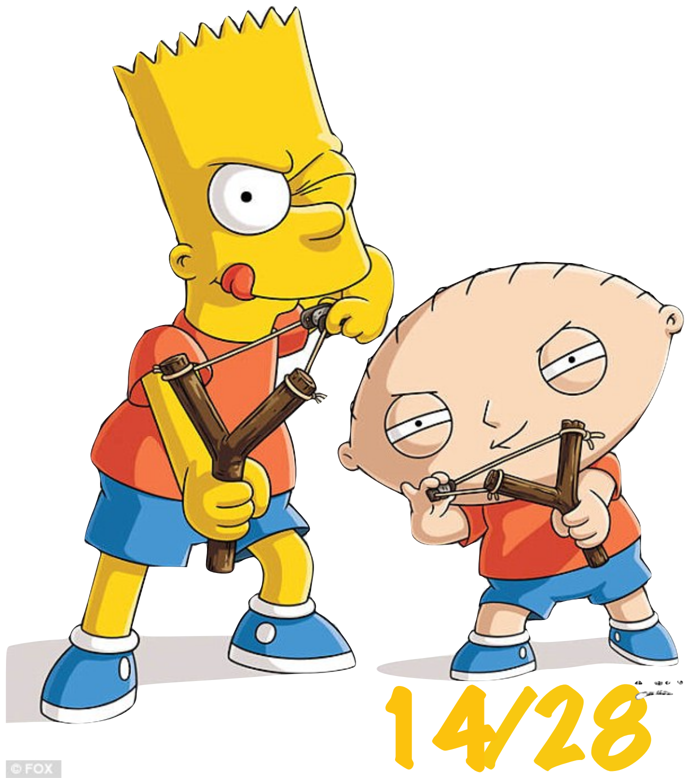
   $\frac{\pi}{2}x - 0 = 0,\ \frac{\pi}{2}x - 0 = \pi$ $x = 0,\ x = 2$

   An interval containing one period is $(0, 2)$

   with two consecutive asymptotes at $x = 0$ *and* $x = 2$







Example: Graphing a Cotangent Function

Objective: Students graph tan, cot, sec, csc.

Graph one full period of $y = \frac{1}{2}\cot\frac{\pi}{2}x$ $\quad a = \frac{1}{2}, b = \frac{\pi}{2}, c = 0$

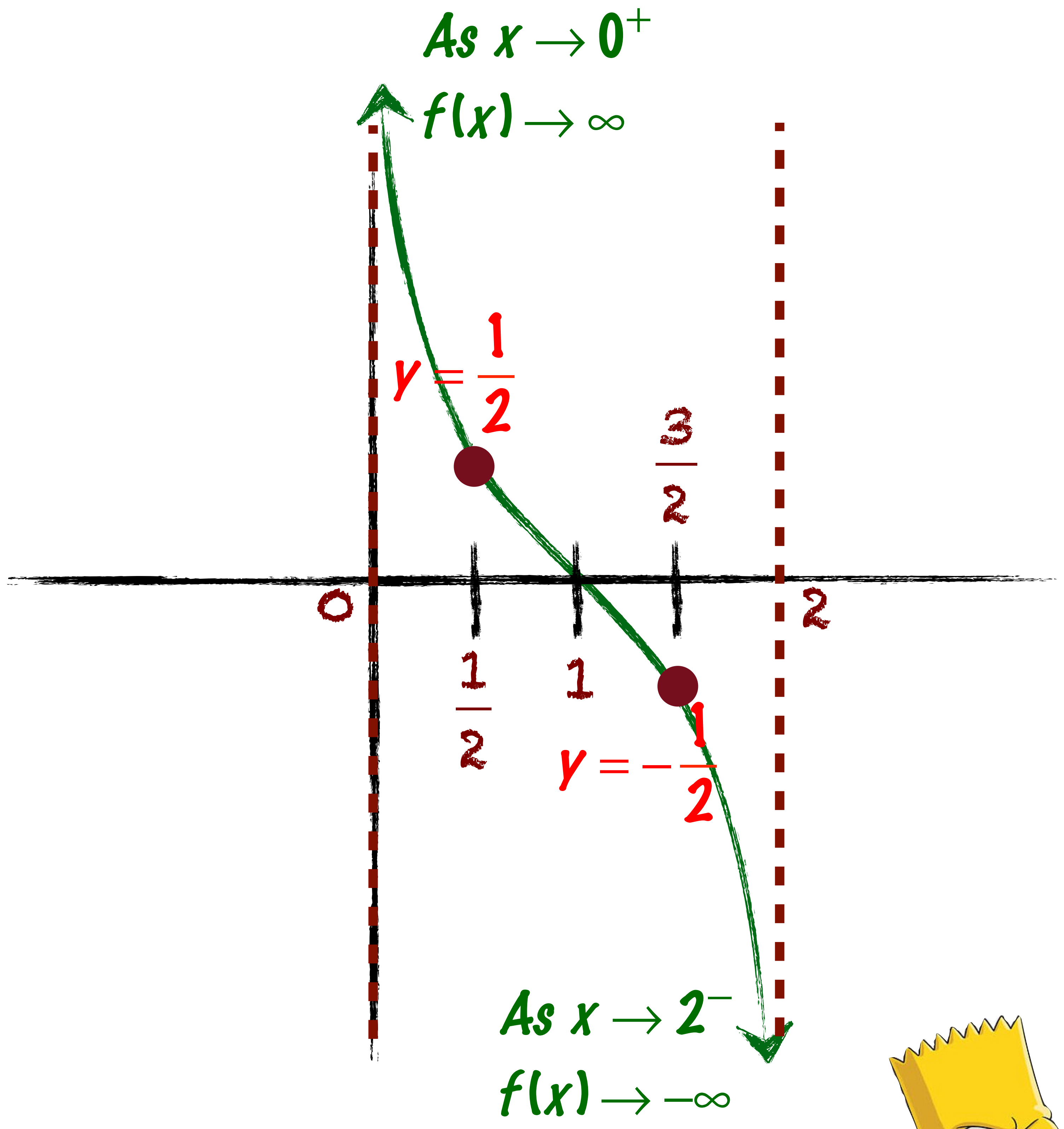
2. x-intercept $\quad x = \frac{0+2}{2} = 1 \quad$ x-intercept at (1, 0).

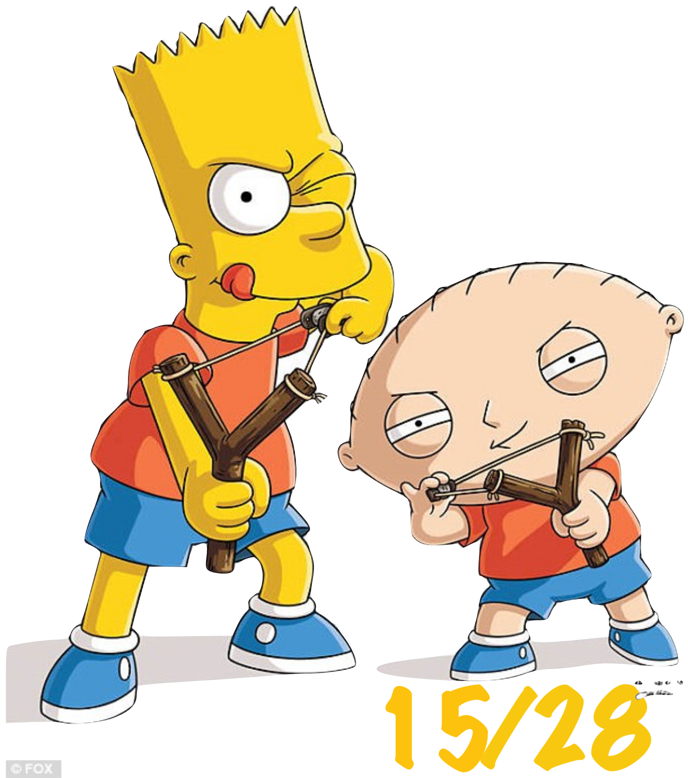
3. y at 1/4 and 3/4 intervals

$$x = \frac{1}{2} \qquad x = \frac{3}{2}$$

These have y values of a and -a.

$$\left(\frac{1}{2}, \frac{1}{2}\right) \qquad \left(\frac{3}{2}, -\frac{1}{2}\right)$$





The graphs of y = csc x and y = sec x

Objective: Students graph tan, cot, sec, csc.

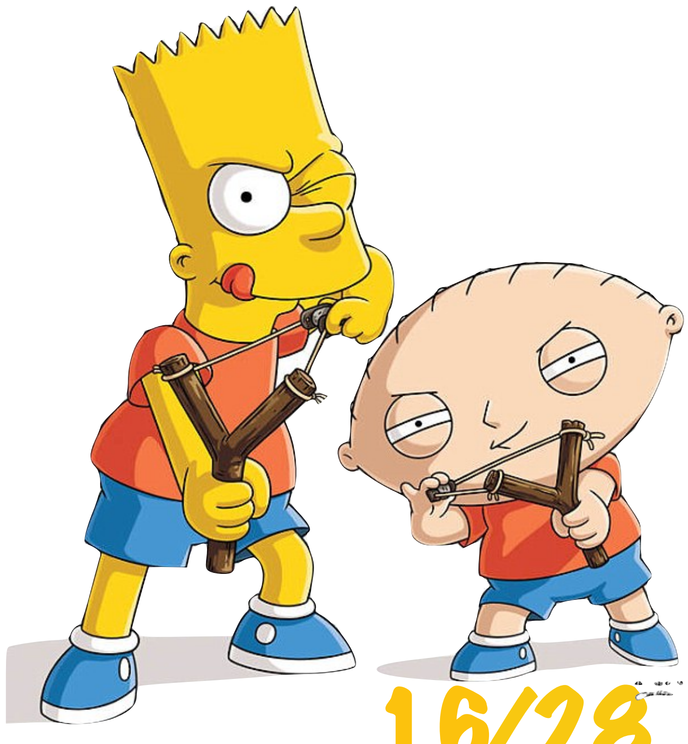
We can obtain the graphs of the cosecant and secant curves by using the reciprocal identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

We obtain the graph of y = csc x by taking reciprocals of the y-values of y = sin x.
The vertical asymptotes of y = csc x occur at the x-intercepts of y = sin x.

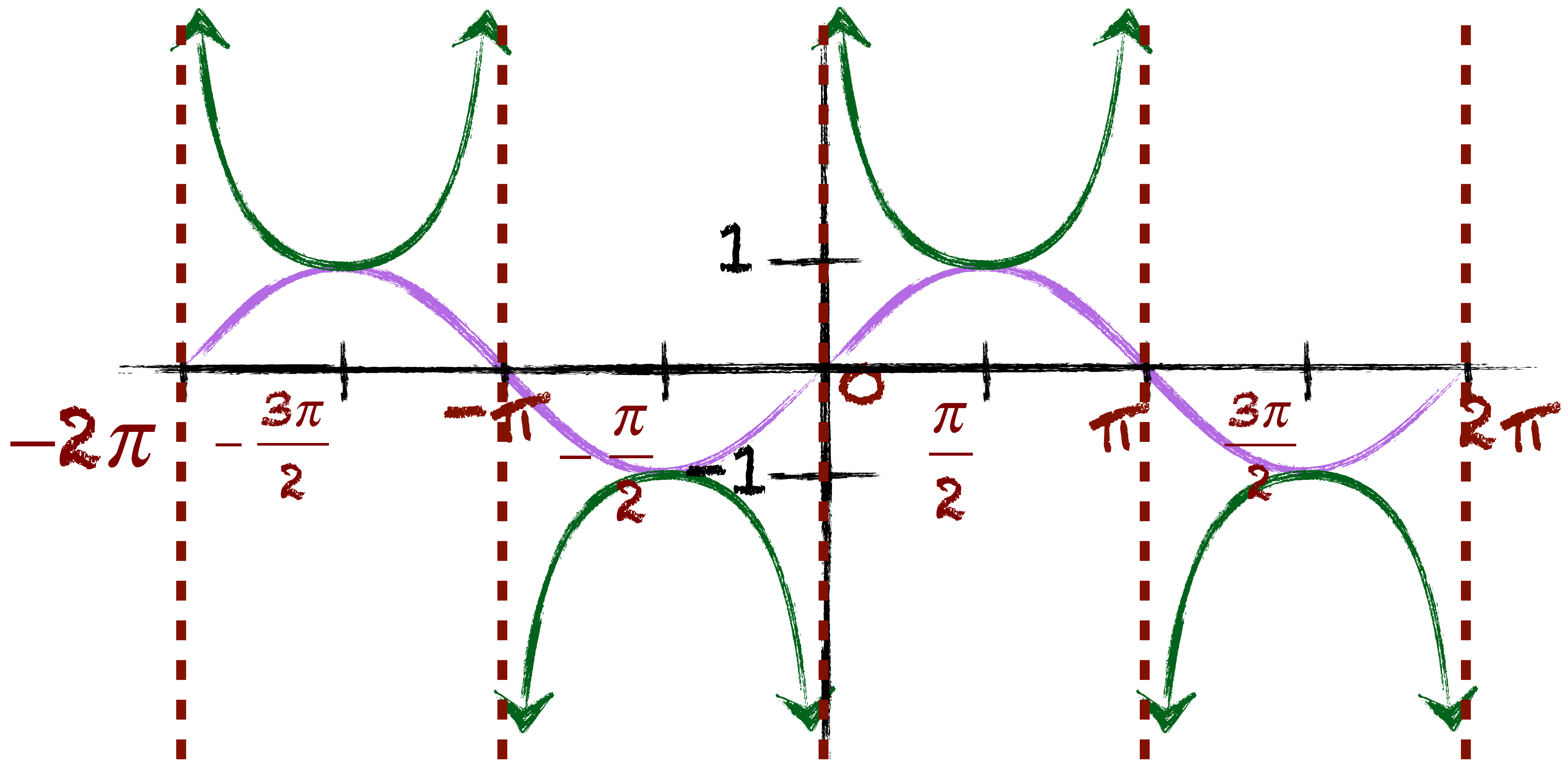
We obtain the graph of y = sec x by taking reciprocals of the y-values of y = cos x.
The vertical asymptotes of y = sec x occur at the x-intercepts of y = cos x.

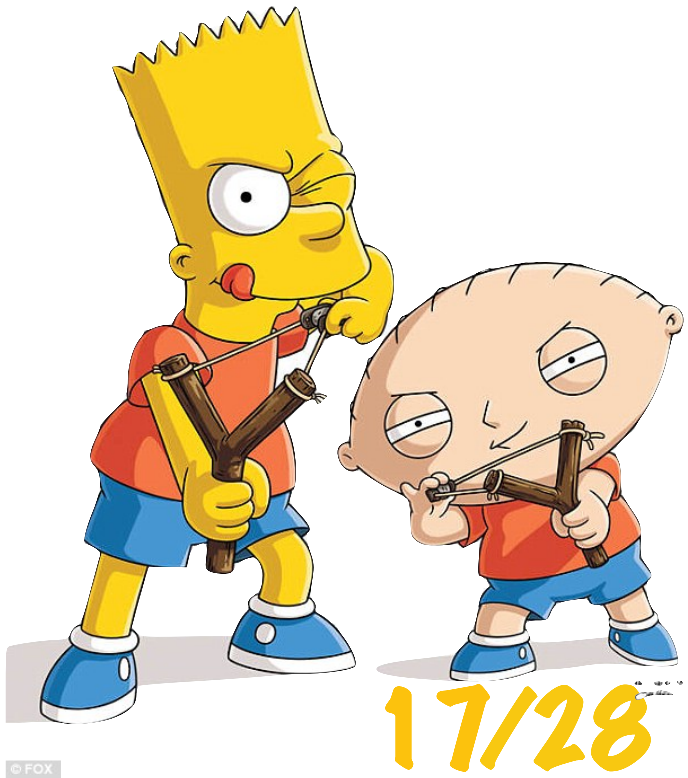


The Cosecant Curve: The Graph of y = cscx

Objective: Students graph tan, cot, sec, csc.

- Graphing y = csc(x)
- Period = 2π
- Domain = all reals except integer multiples of π.
- Range = (-∞,-1] U [1,∞)
- Vertical asymptotes at integer multiples of π $(n\pi)$
- no x-intercepts.
- f(x) = cscx is an odd function, origin symmetry.
- cscx = 1 or -1 at 1/4 and 3/4 periods.





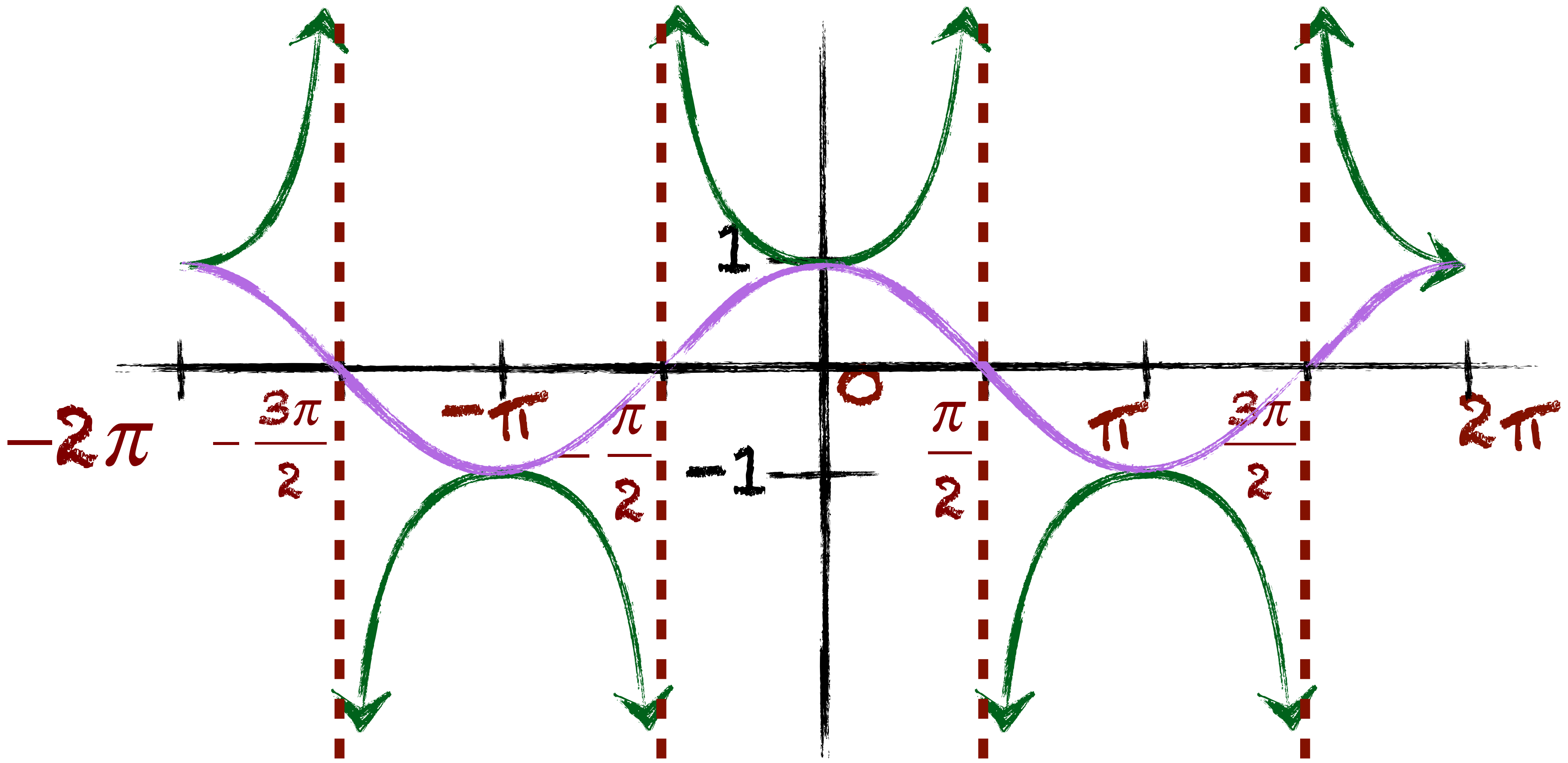
The Secant Curve: The Graph of y = secx

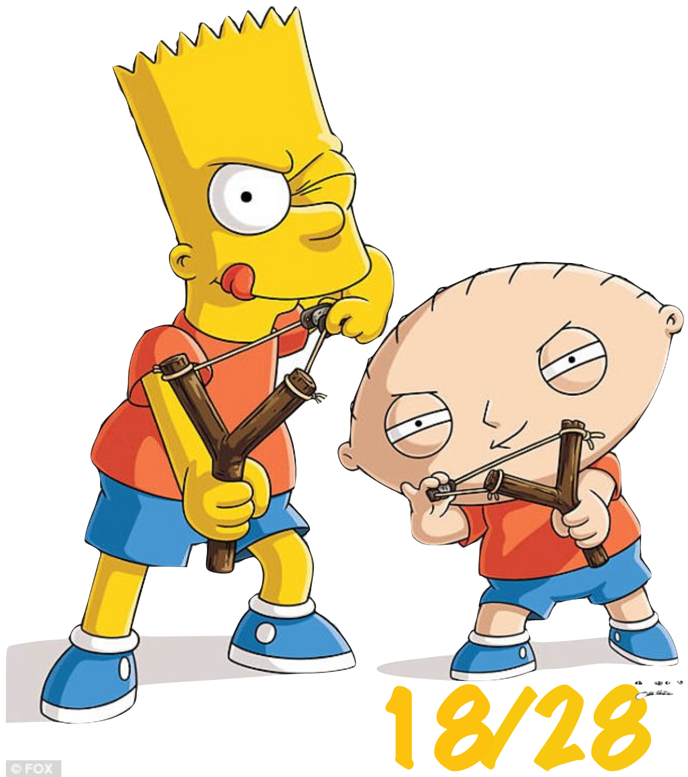
Objective: Students graph tan, cot, sec, csc.

- Graphing y = sec(x)
- Period = 2π
- Domain = all reals except odd multiples of π/2.
- Range = (-∞,-1] U [1,∞)
- Vertical asymptotes at odd multiples of π/2

$$\left[(2n-1)\frac{\pi}{2}\right]$$

- no x-intercepts.
- f(x) = secx is an even function, y-axis symmetry.
- secx = 1 or -1 at 1/4 and 3/4 period.



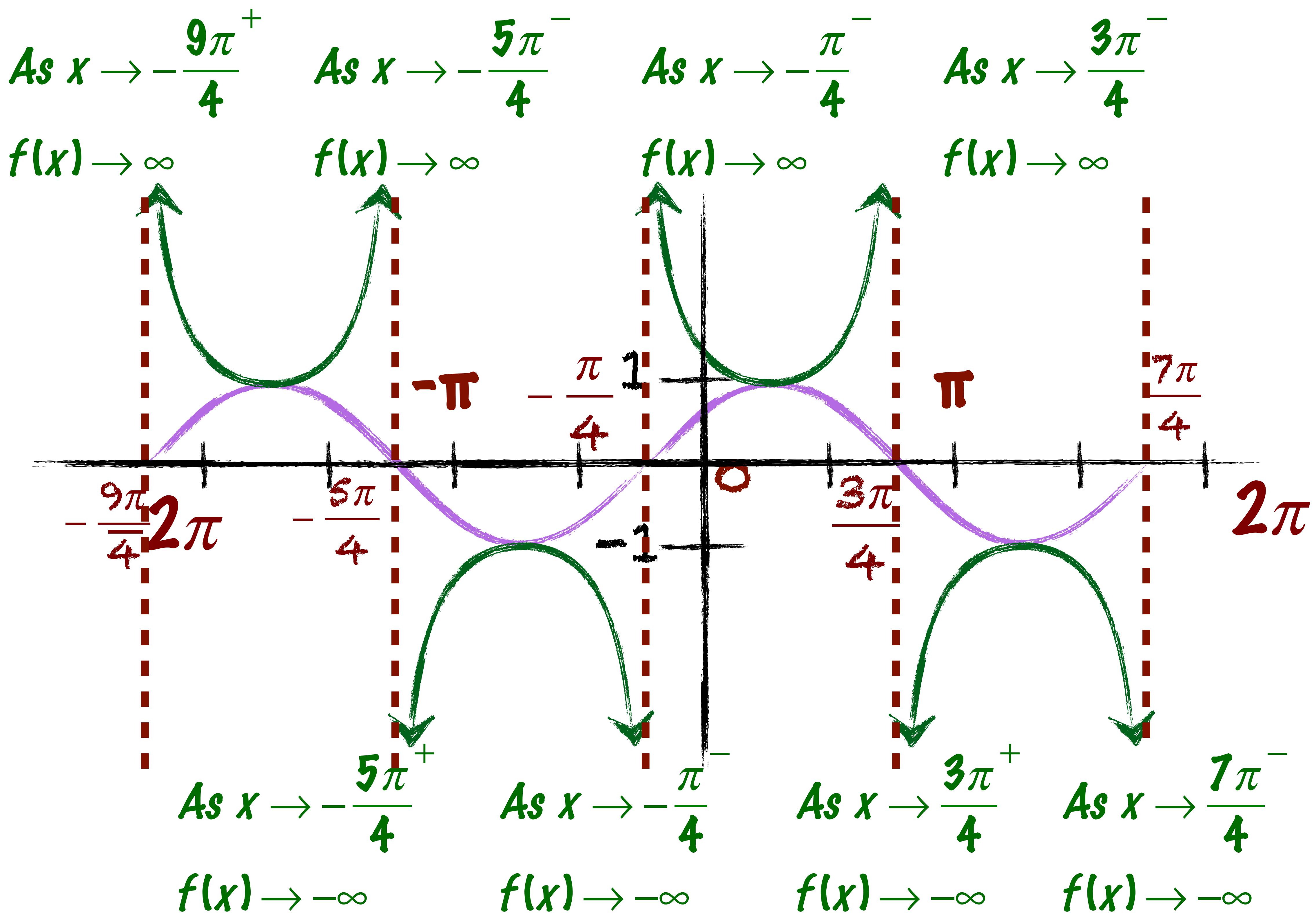


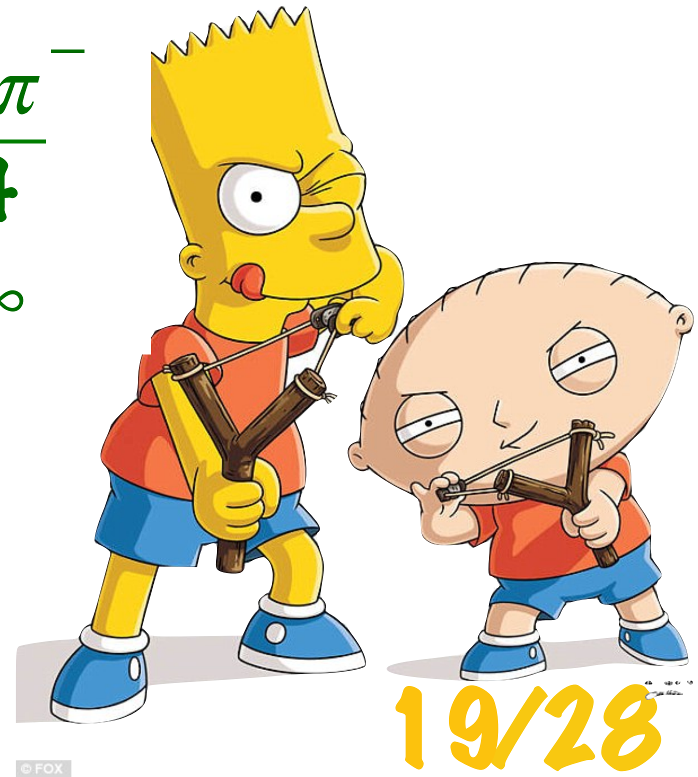
Using a Sine Curve to Obtain a Cosecant Curve

Objective: Students graph tan, cot, sec, csc.

Use the graph of $y = \sin\left(x + \frac{\pi}{4}\right)$ to obtain the graph of $y = \csc\left(x + \frac{\pi}{4}\right)$

- The sin graph is shifted left π/4.
- The x-intercepts of the sine graph correspond to the vertical asymptotes of the cosecant graph.





Example: Graphing a Secant Function

Objective: Students graph tan, cot, sec, csc.

Graph y = 2sec2x for $-\frac{3\pi}{4} < x < \frac{3\pi}{4}$

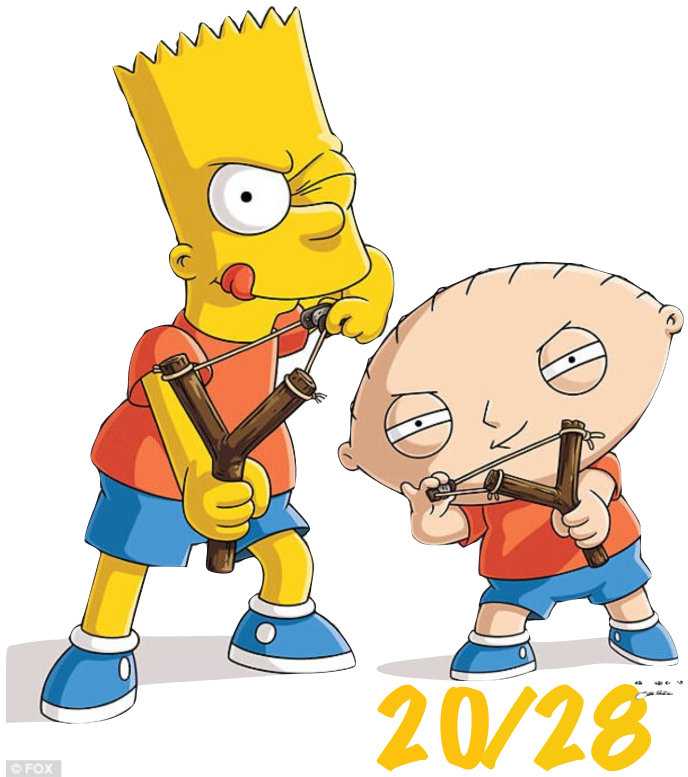
Begin by graphing the reciprocal function, y = 2cos2x.
This equation is of the form y = acosbx, with a=2, b=2.

amplitude: |a|=|2|=2

period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

∡ We will use quarter-periods to find x-values for the five key points.

∡ The key points are: $0 \quad \frac{\pi}{4} \quad \frac{\pi}{2} \quad \frac{3\pi}{4} \quad \pi$



Example: Graphing a Secant Function

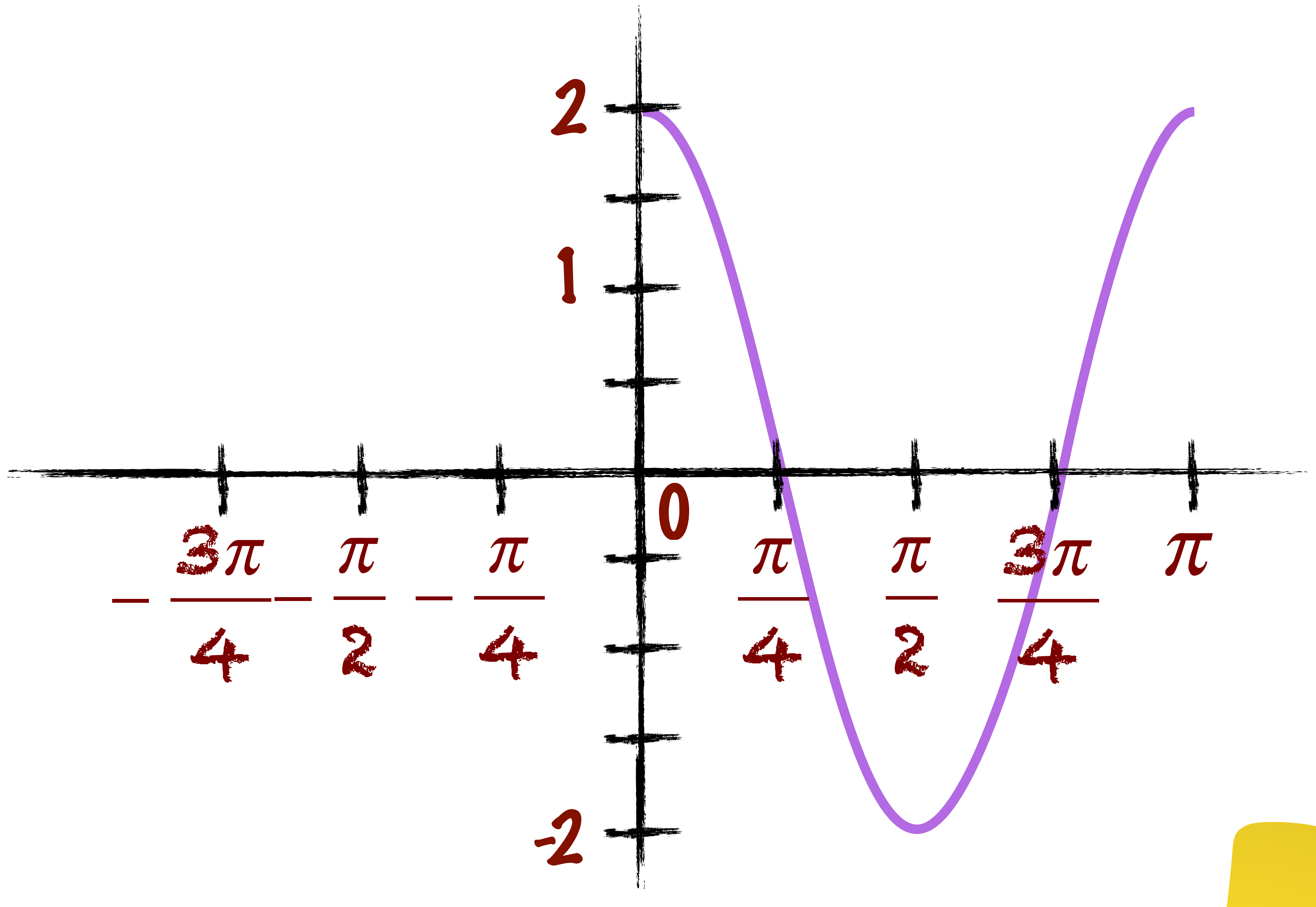
Objective: Students graph tan, cot, sec, csc.

Graph y = 2sec2x for $-\frac{3\pi}{4} < x < \frac{3\pi}{4}$

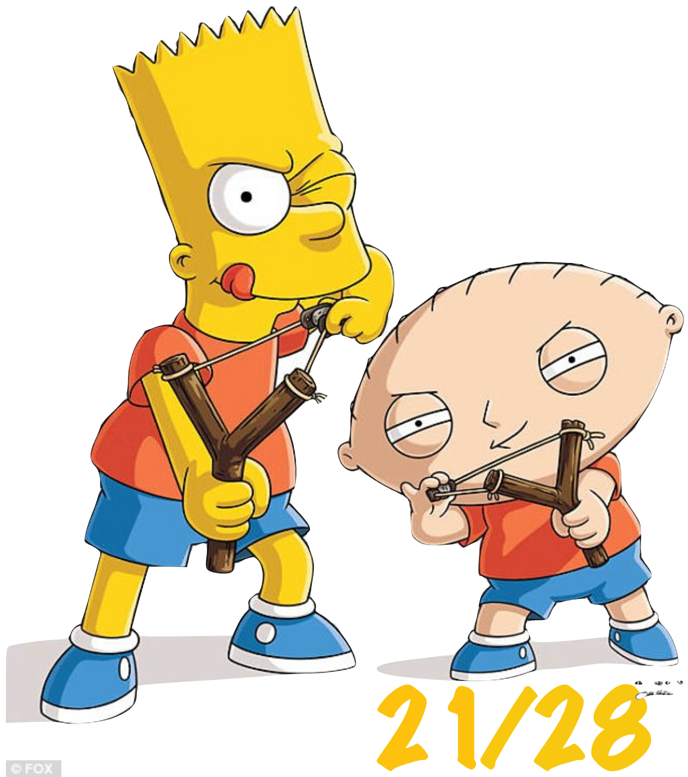
Begin by graphing y = 2cos2x

amplitude: |a|=|2|=2

period: $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$



x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
2Cos2x	2	0	-2	0	2



Example: Graphing a Secant Function

Objective: Students graph tan, cot, sec, csc.

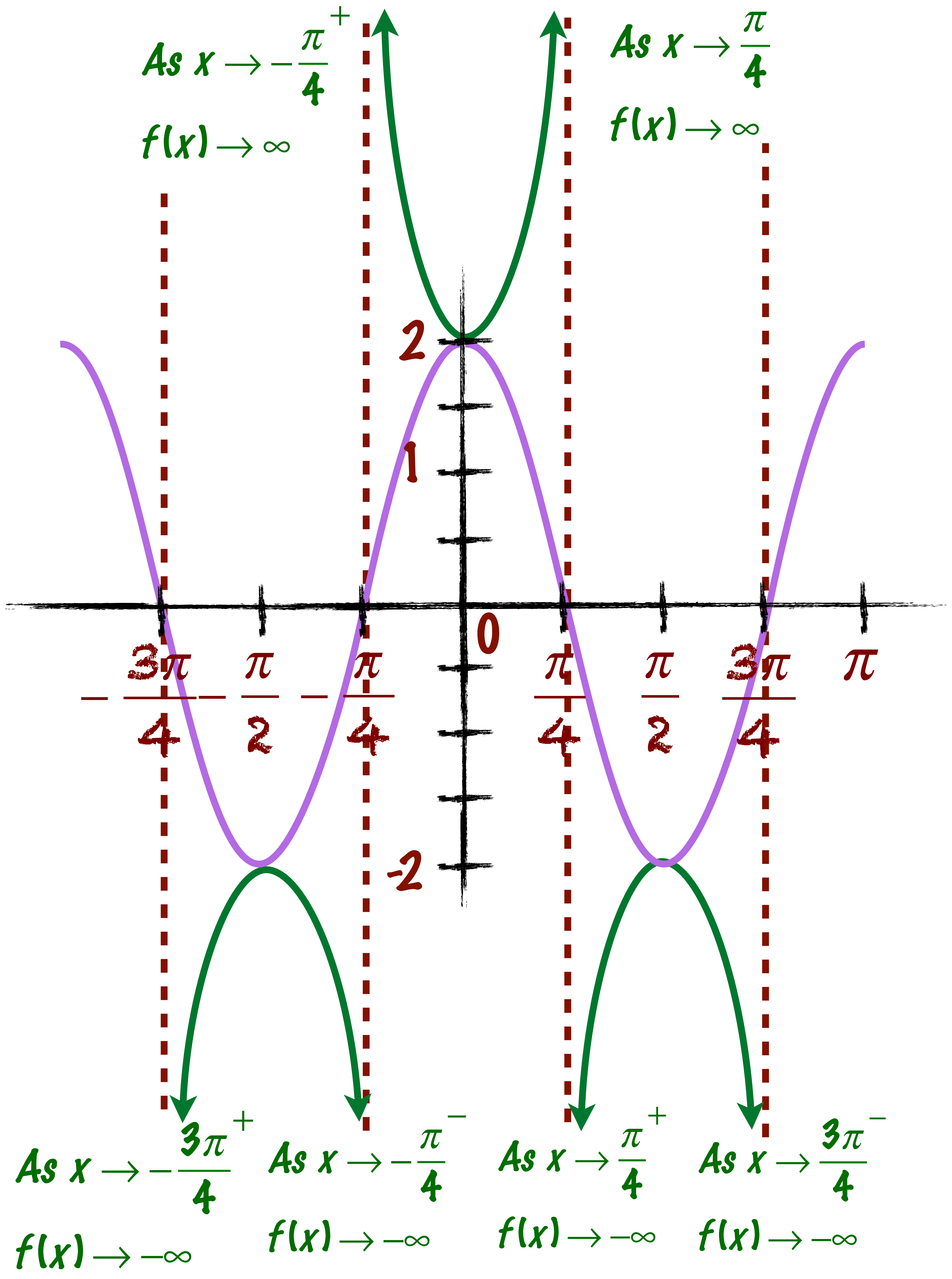
Graph y = 2sec2x for $-\frac{3\pi}{4} < x < \frac{3\pi}{4}$

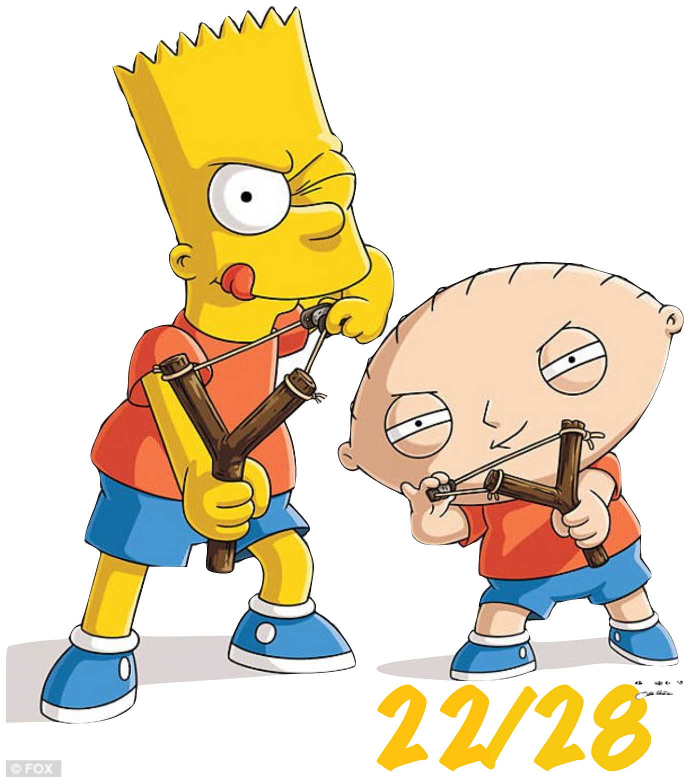
Repeat over the domain.

Draw asymptotes through the x-intercepts.

Draw the graph of the inverse y = 2sec2x

Do not forget arrow notation.





The Six Curves of Trigonometry

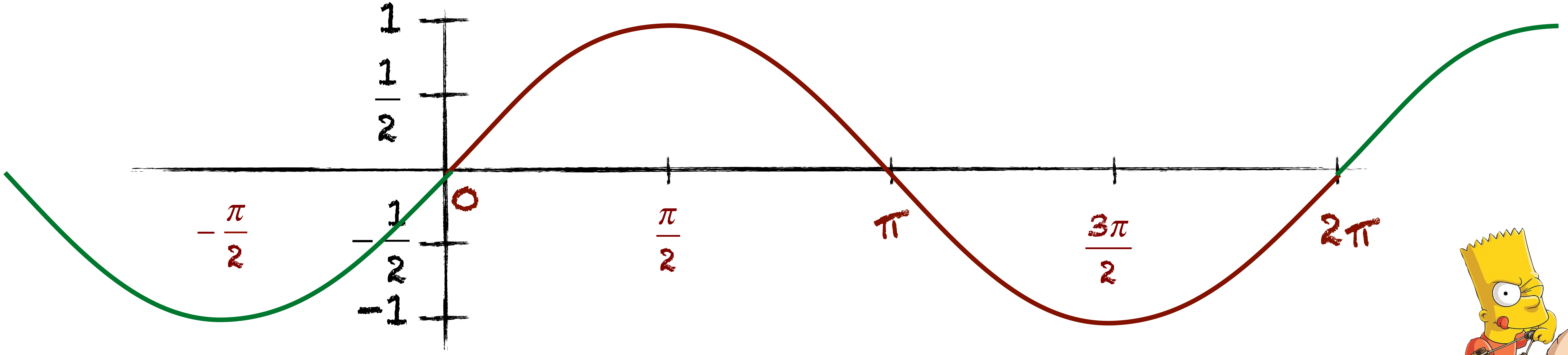
Objective: Students graph tan, cot, sec, csc.

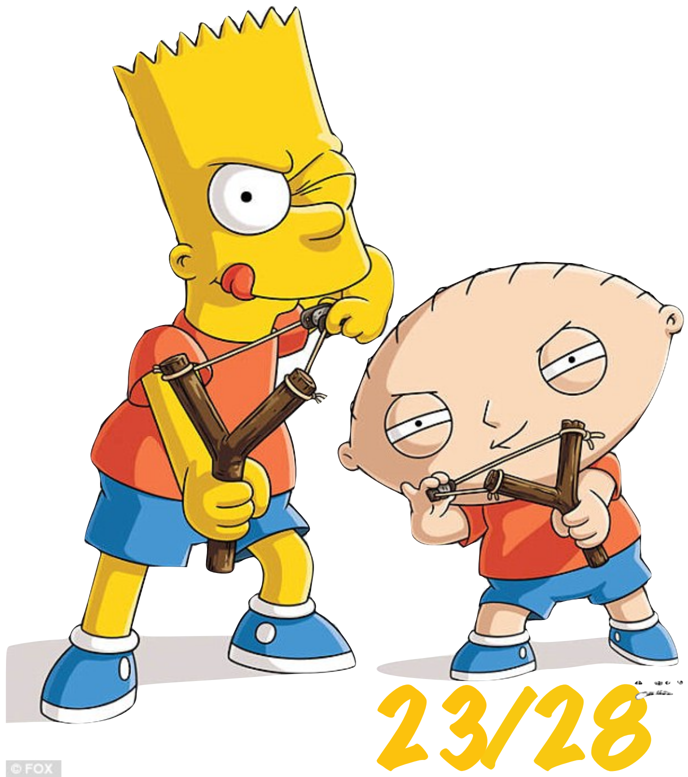
$f(x) = \sin x$

Period = 2π

Domain = all reals

Range = $[-1, 1]$





The Six Curves of Trigonometry

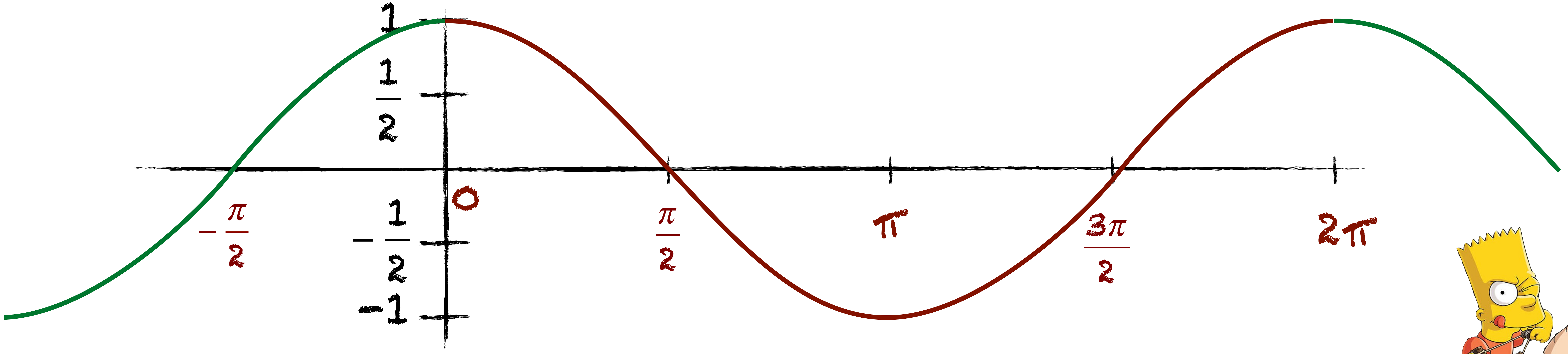
Objective: Students graph tan, cot, sec, csc.

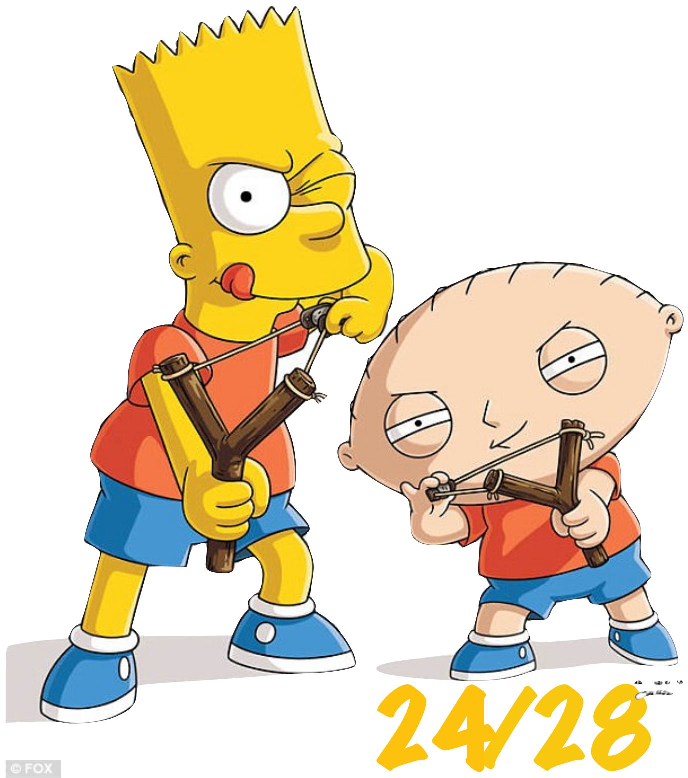
$f(x) = \cos x$

Period = 2π

Domain = all reals

Range = $[-1, 1]$





The Six Curves of Trigonometry

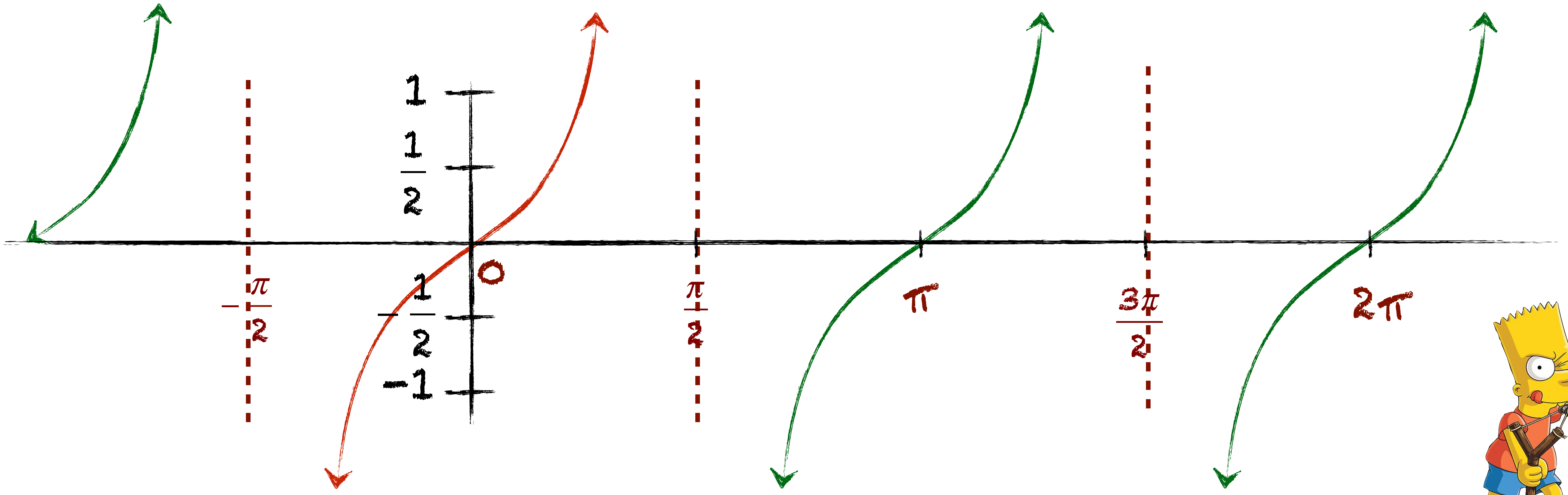
Objective: Students graph tan, cot, sec, csc.

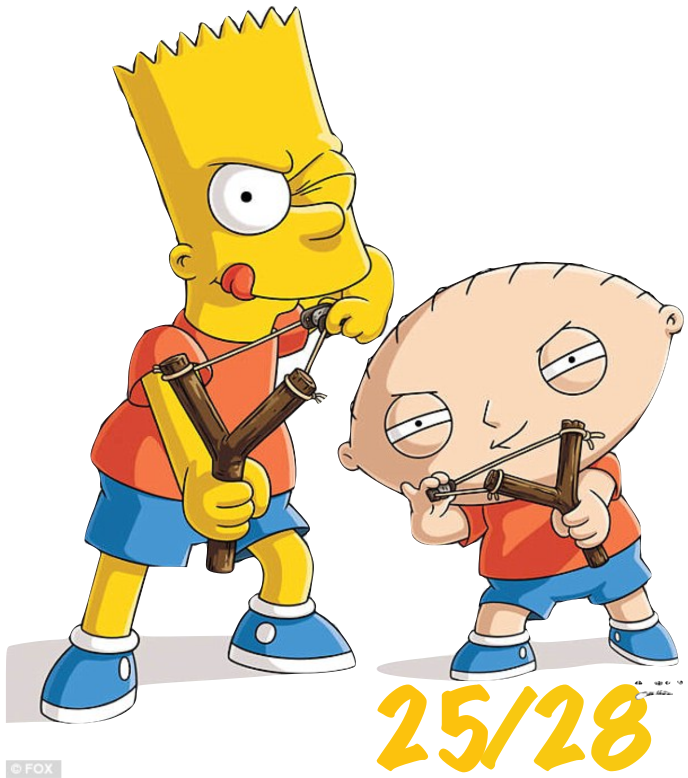
$f(x) = \tan x$

Period = π

Domain = all reals except odd multiples of $\frac{\pi}{2}$

Range = all reals





The Six Curves of Trigonometry

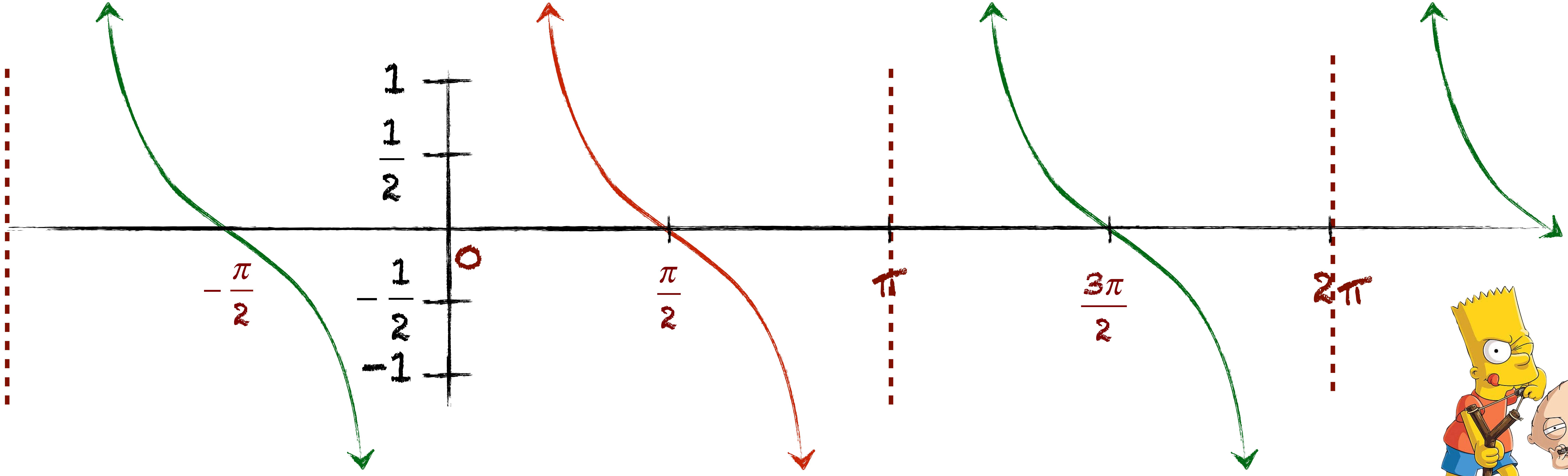
Objective: Students graph tan, cot, sec, csc.

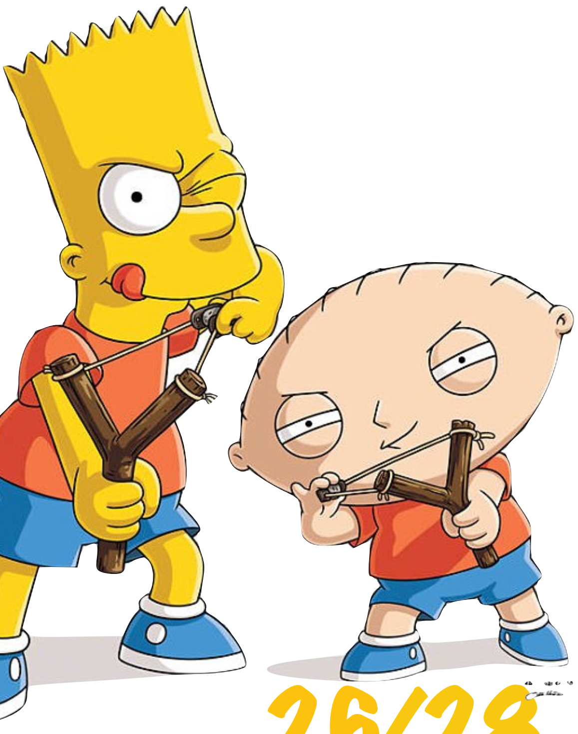
$f(x) = \cot x$

Period = π

Domain = all reals except integer multiples of π

Range = all reals



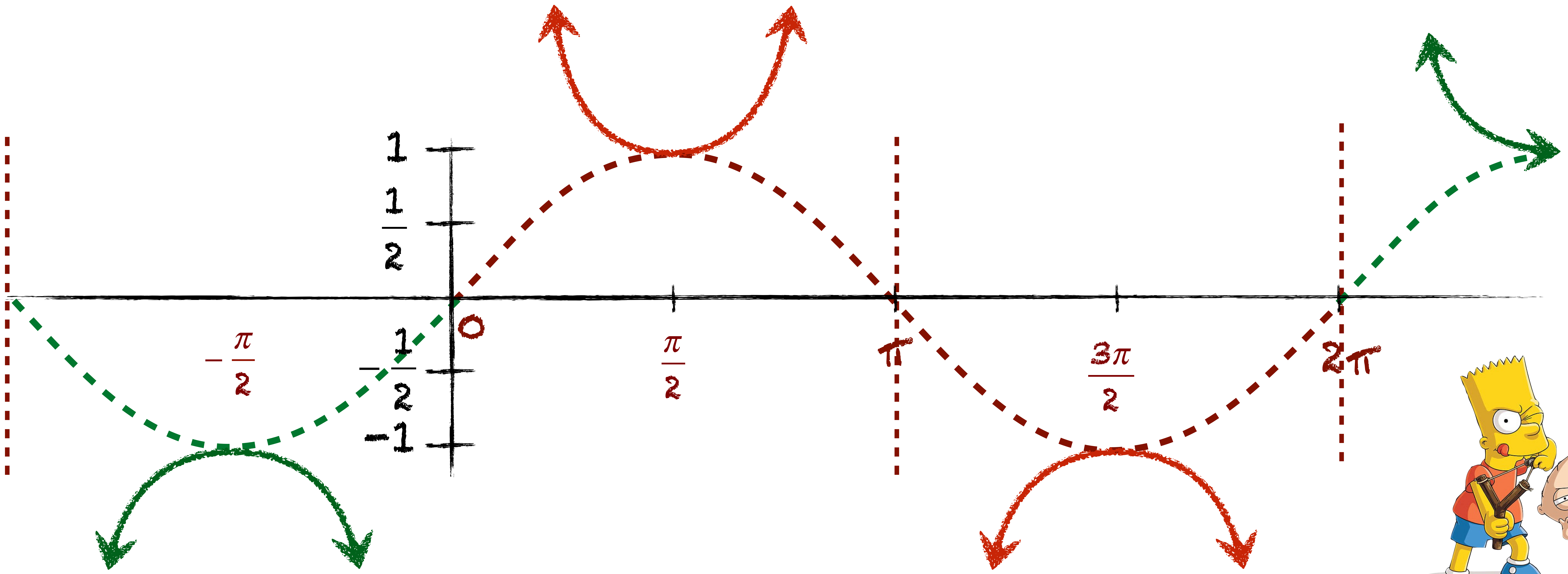


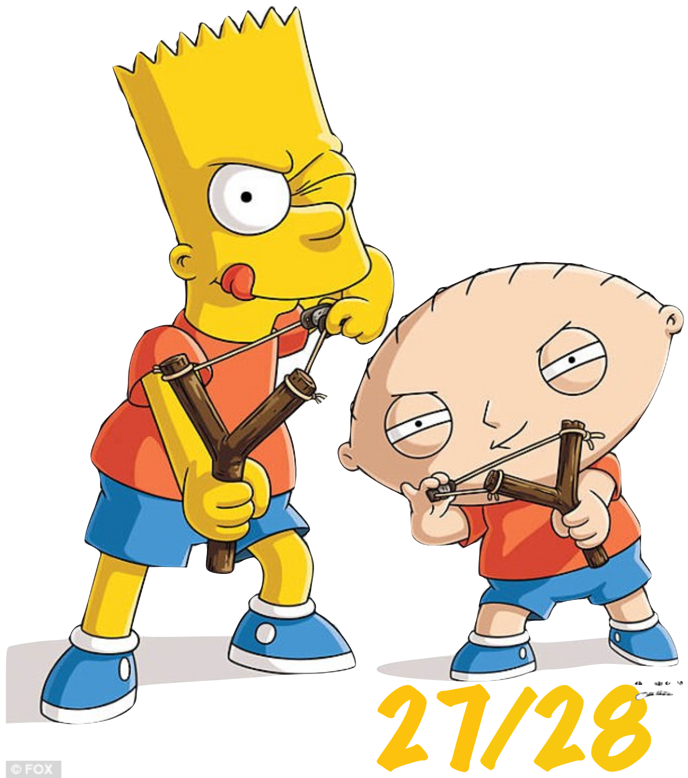
The Six Curves of Trigonometry

Objective: Students graph tan, cot, sec, csc.

f(x) = cscx

Period = 2π   Domain = all reals except integer multiples of π.   Range = (-∞, -1] U [1, ∞)





The Six Curves of Trigonometry

Objective: Students graph tan, cot, sec, csc.

f(x) = secx

Period = 2π Domain = all reals except odd multiples of π/2. Range = (-∞, -1] U [1, ∞)