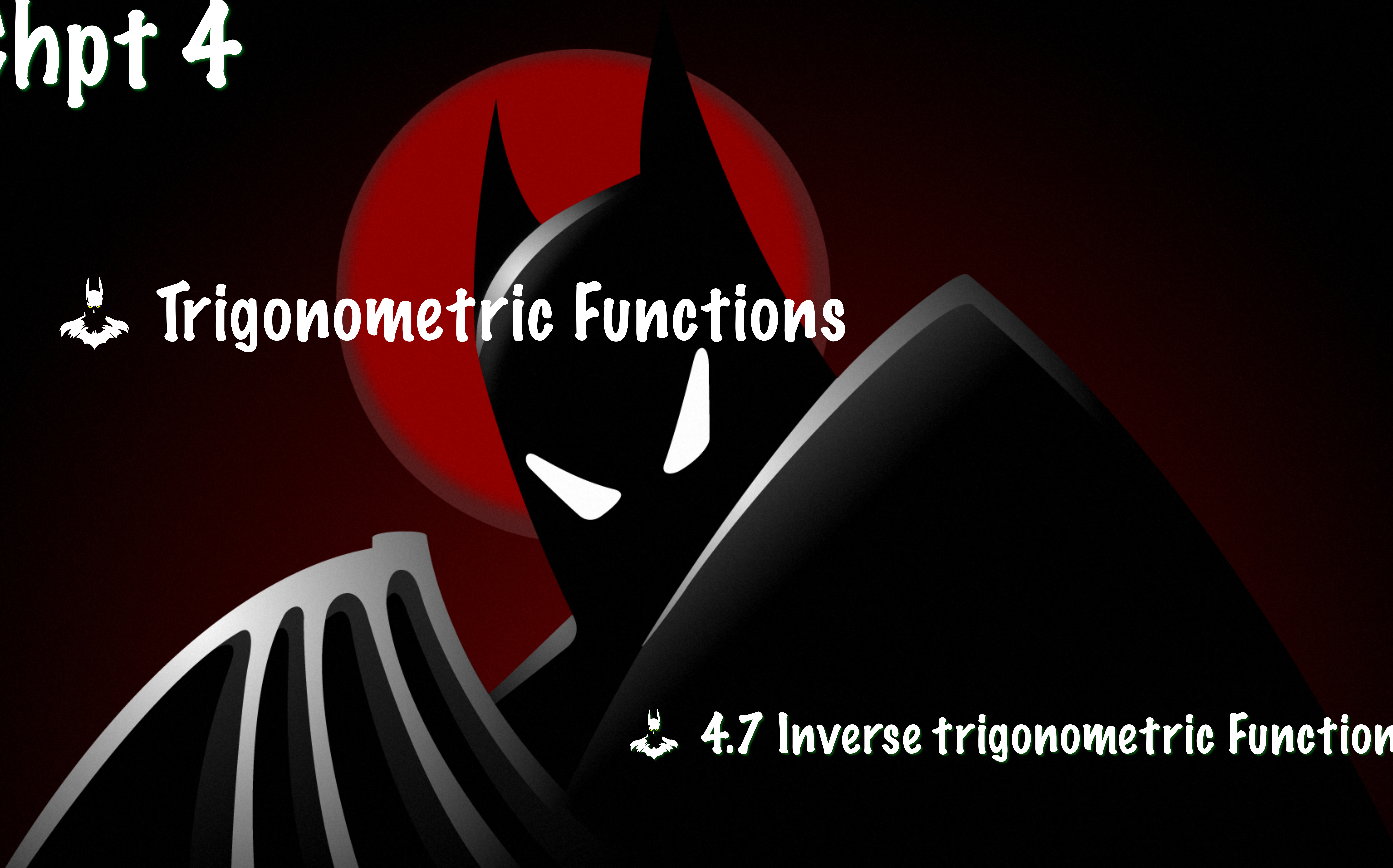
Chpt 4

Trigonometric Functions



4.7 Inverse trigonometric Functions

Chpt 4.7

- Homework
  - 4.7 p349 1-73 odd

Chpt 4.7

Objectives

- Evaluate an inverse sine function.
- Evaluate an inverse cosine function.
- Evaluate an inverse tangent function.
- Use a calculator to evaluate inverse trigonometric functions.
- Graph inverse trigonometric functions.
- Find exact values of composite functions with inverse trigonometric functions.

Inverse Functions - Review

- Here are some things to remember about inverse functions (f^{-1}):

  - **Horizontal Line Test:** If no horizontal line intersects the graph of a function more than once, the function is one-to-one and has an inverse function.

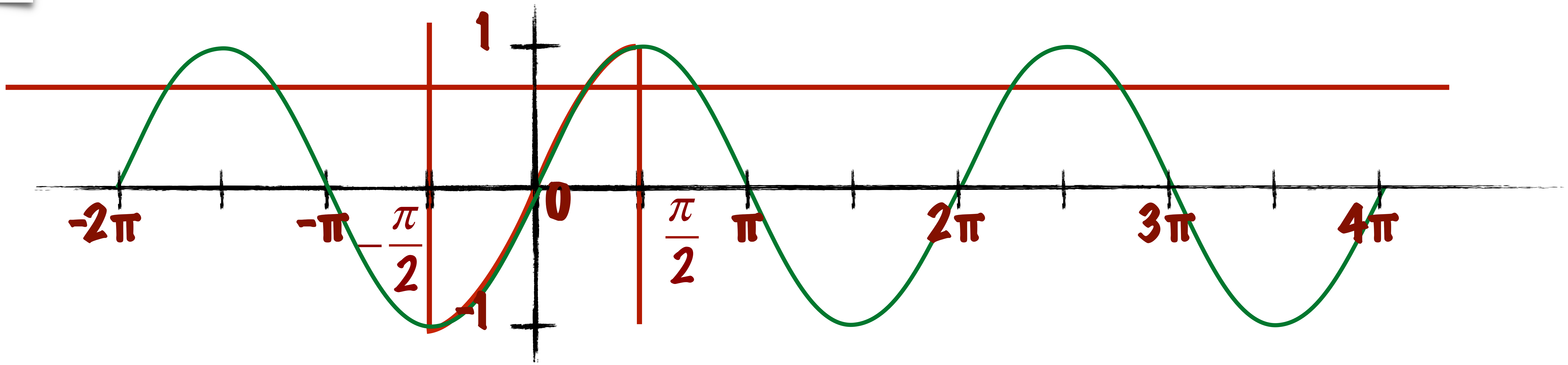
  - **Switch x and y:** If the point (a, b) is on the graph of f, then the point (b, a) is on the graph of the inverse function, f^{-1}.

  - **Reflect across y = x:** The graph of f^{-1} is a reflection of f about the line y = x.

Inverse Sine (Arcsin)

The Inverse Sine Function

y = sinx



- The horizontal line test shows the function y = sinx is **not** a one-to-one function and does not have an inverse.

- So we make a slight adjustment. To allow for an inverse function, we restrict the domain.

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

The Inverse Sine Function

- **Inverse sine function:** $\sin^{-1}$ is the inverse of the restricted sine function $y = \sin x$, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

- $y = \sin^{-1} x$: y equals inverse sine x

- $y = \sin^{-1} x$ means $\sin y = x$ where

$-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

The Inverse Sine Function

Keep in mind which function you are solving.

$$y = \sin x$$

- x is the measure of an **angle**, y is a value between **-1 and 1**

$$y = \sin^{-1} x$$

- x is a value between **-1 and 1**,
- y is the measure of an angle between $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

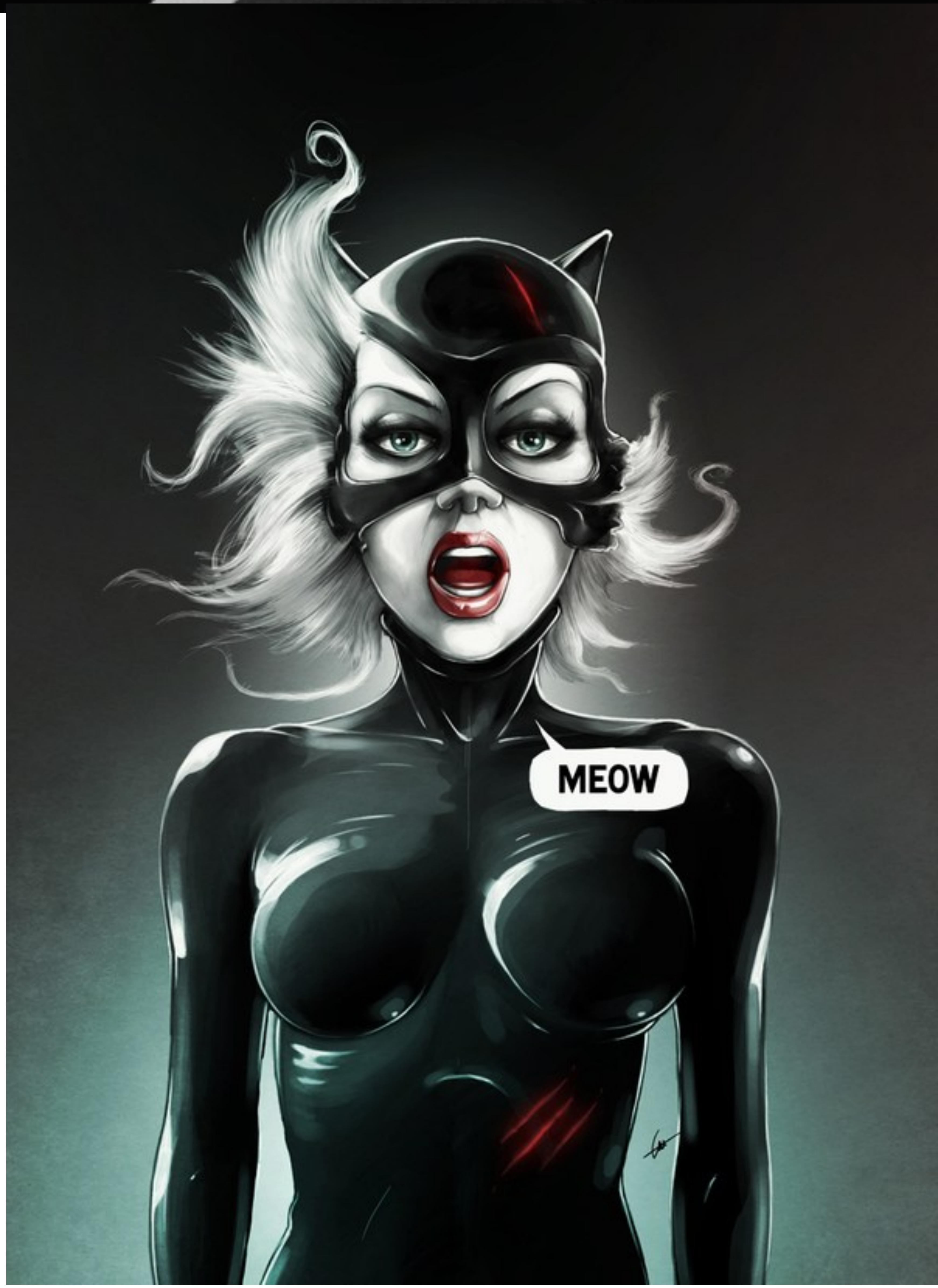
The Inverse Sine Function

- To clarify:
  - We take the sine of an angle to get a ratio.

$$\sin A = \frac{a}{c}$$

  - We take the inverse sine of a ratio to find an angle.

$$\sin^{-1}\frac{a}{c} = A$$



The Inverse Sine Function

$y = \sin x$

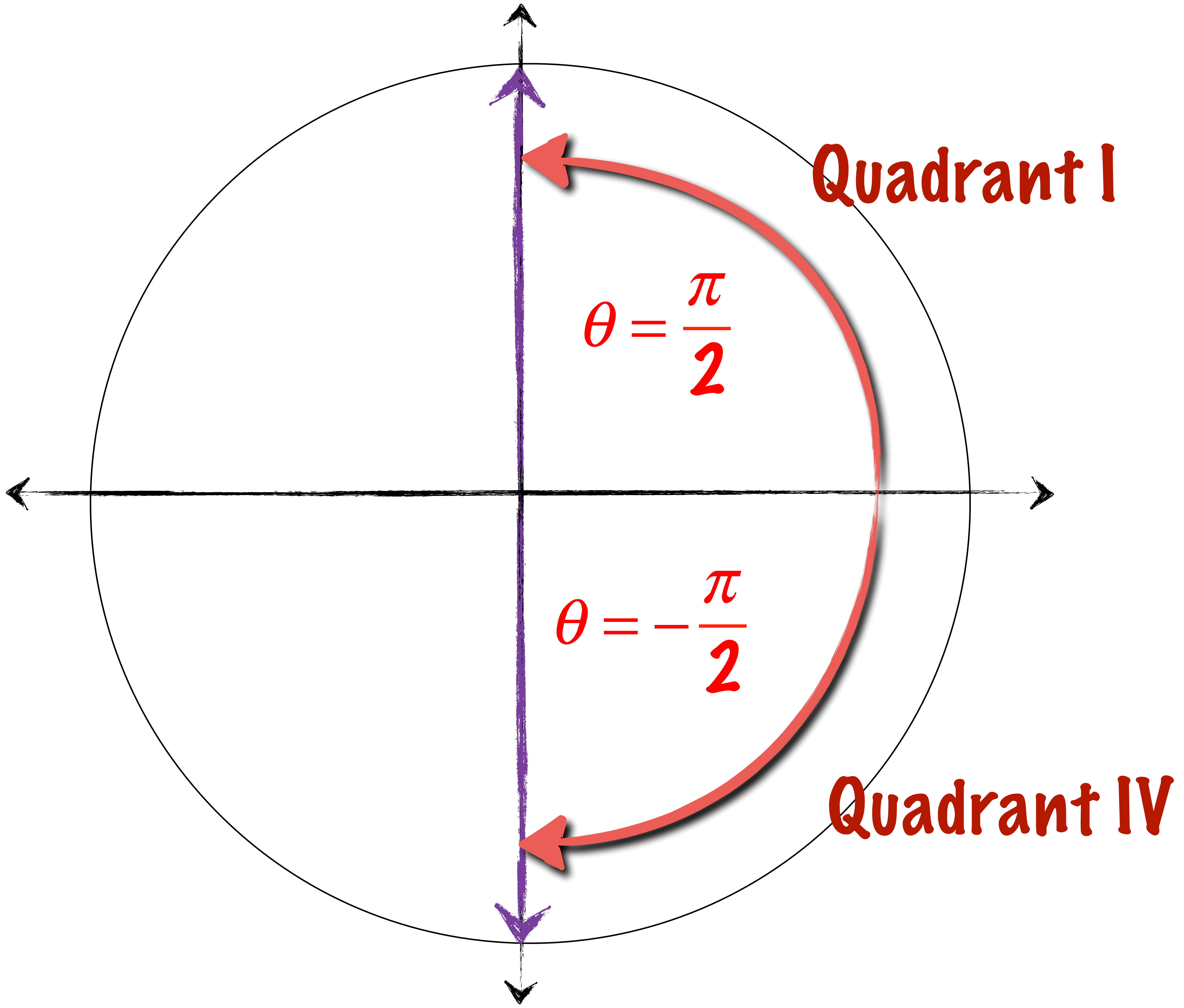
Domain : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range : $[-1, 1]$

$y = \sin^{-1} x$

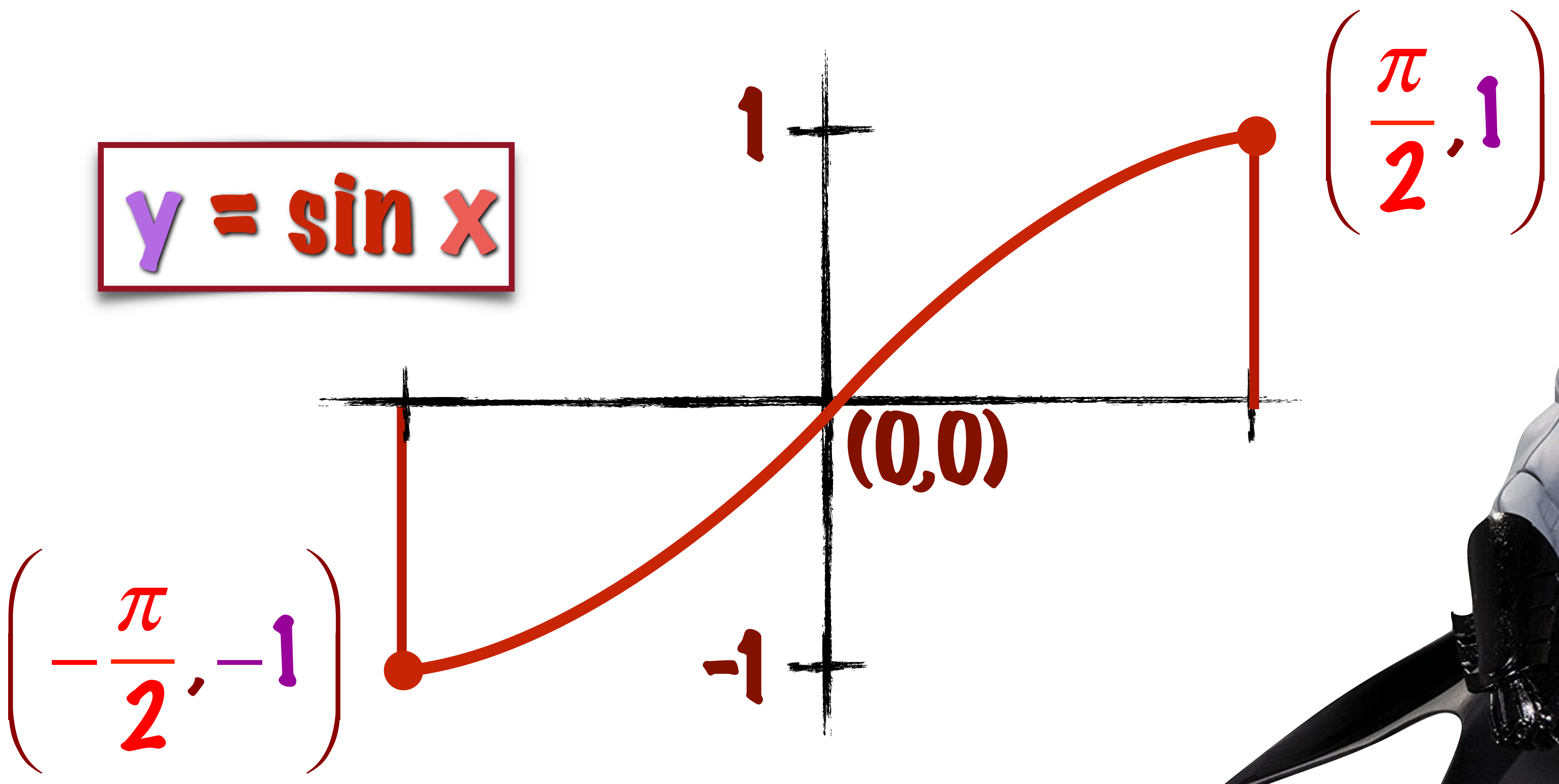
Domain : $[-1, 1]$

Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



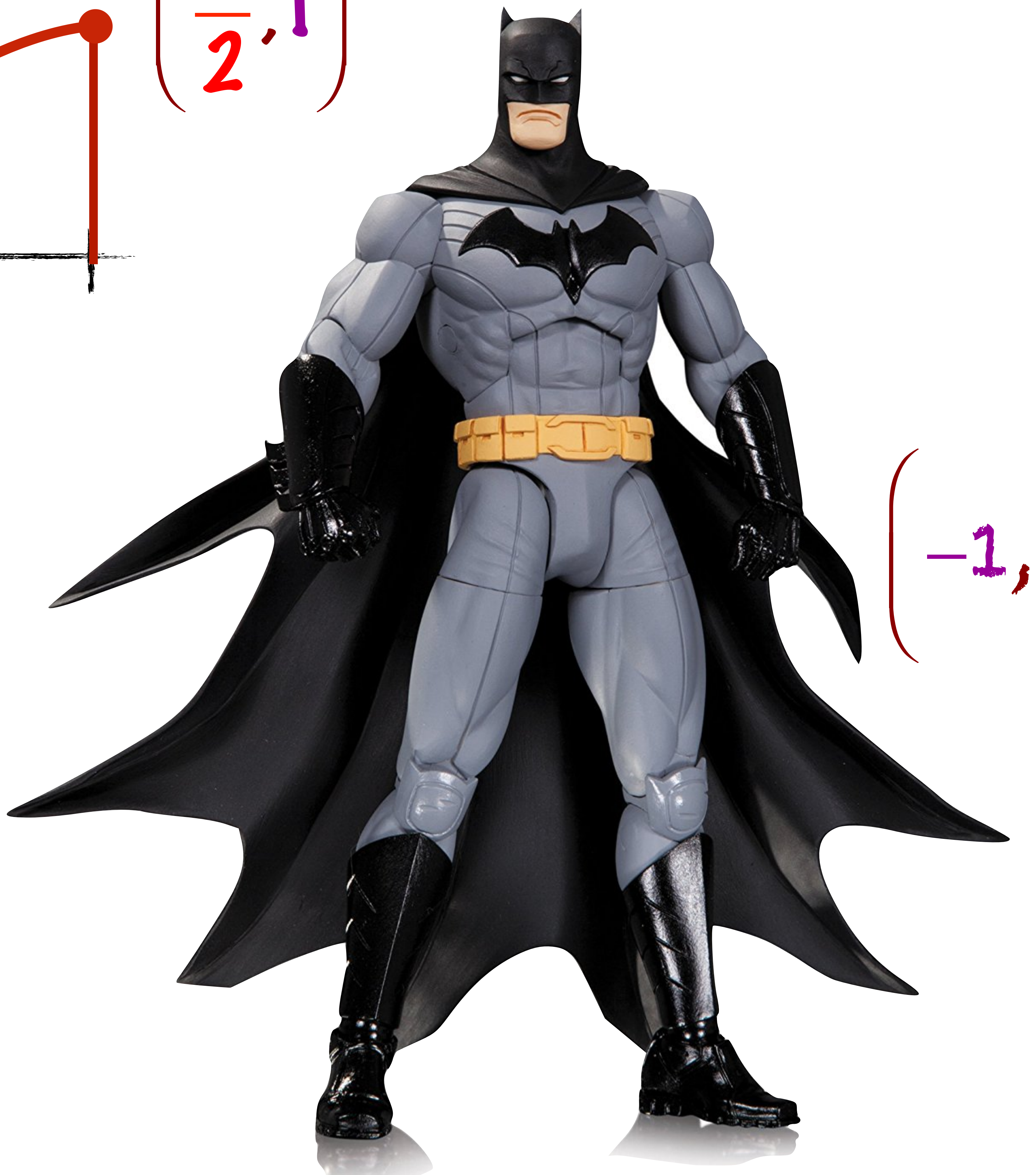
- One way to graph the inverse function $\sin^{-1}$ is to take the points of the function $y = \sin x$ over the restricted domain and switch x and y.

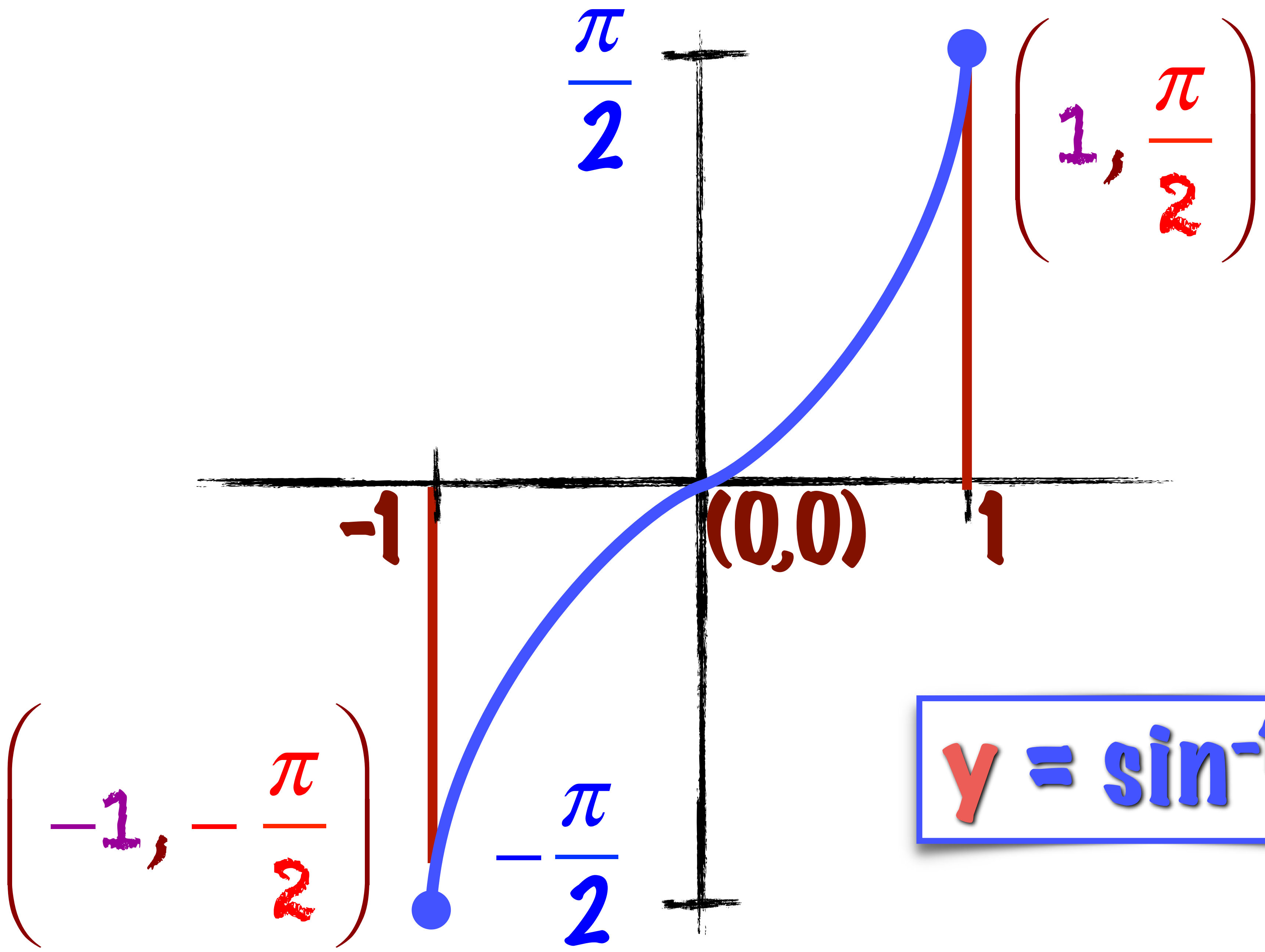
$y = \sin x$



Domain : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range : $[-1, 1]$





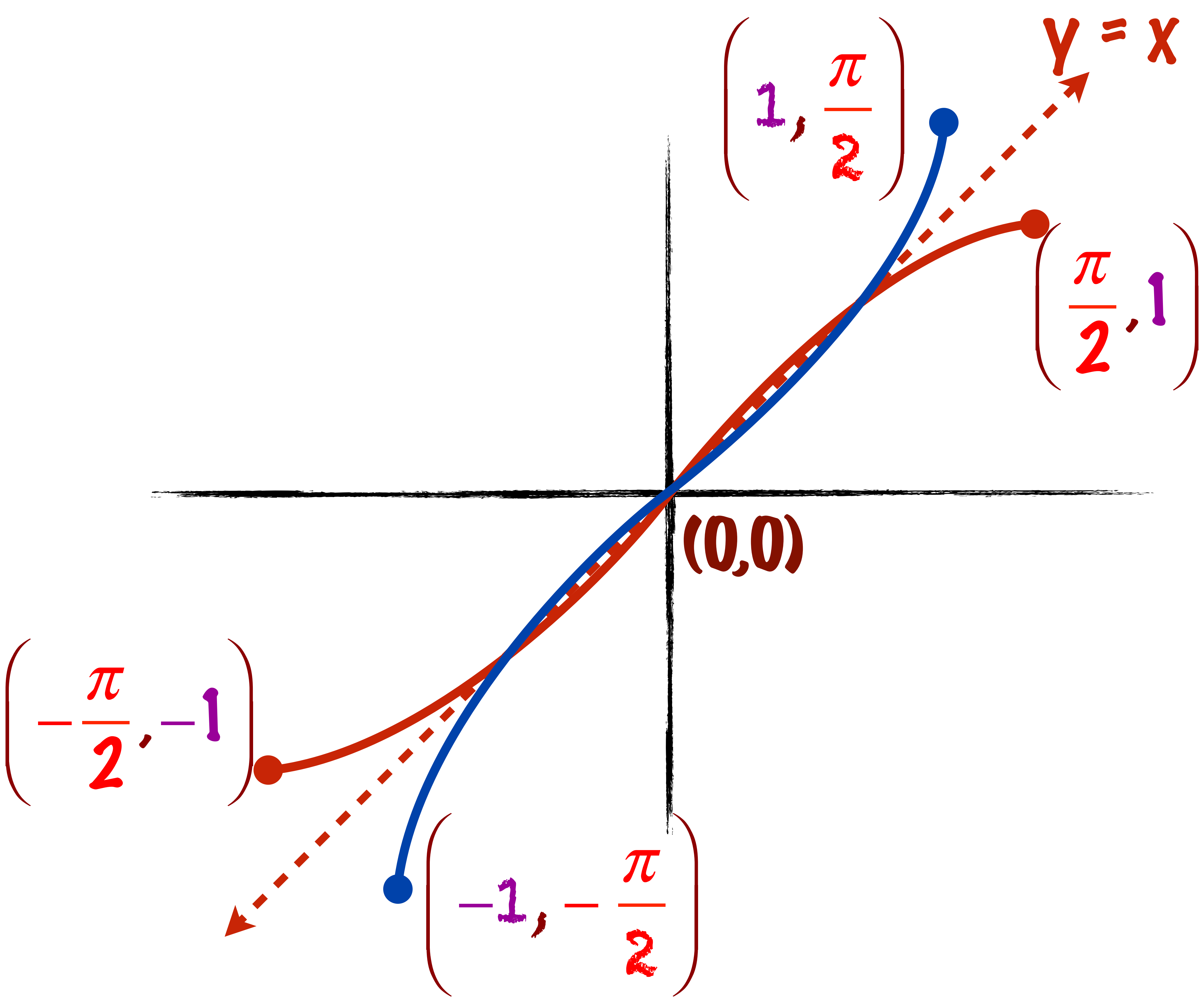
$y = \sin^{-1} x$

Domain : $[-1, 1]$

Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Graphing the Inverse Sine Function

- Also the graph of an inverse is the graph of the function reflected across the line **y = x**. The same is true for $\sin^{-1} x$



Exact values of $\sin^{-1} x$

Finding exact values of $\sin^{-1} x$

1. Let $\theta = \sin^{-1} x$

2. Change to $\sin \theta = x$, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

3. Now we can use the table to find exact values of θ that satisfy $\sin \theta = x$.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin \theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Exact values of $\sin^{-1} x$

Now we can apply the inverse function to find values for $\sin^{-1}x$

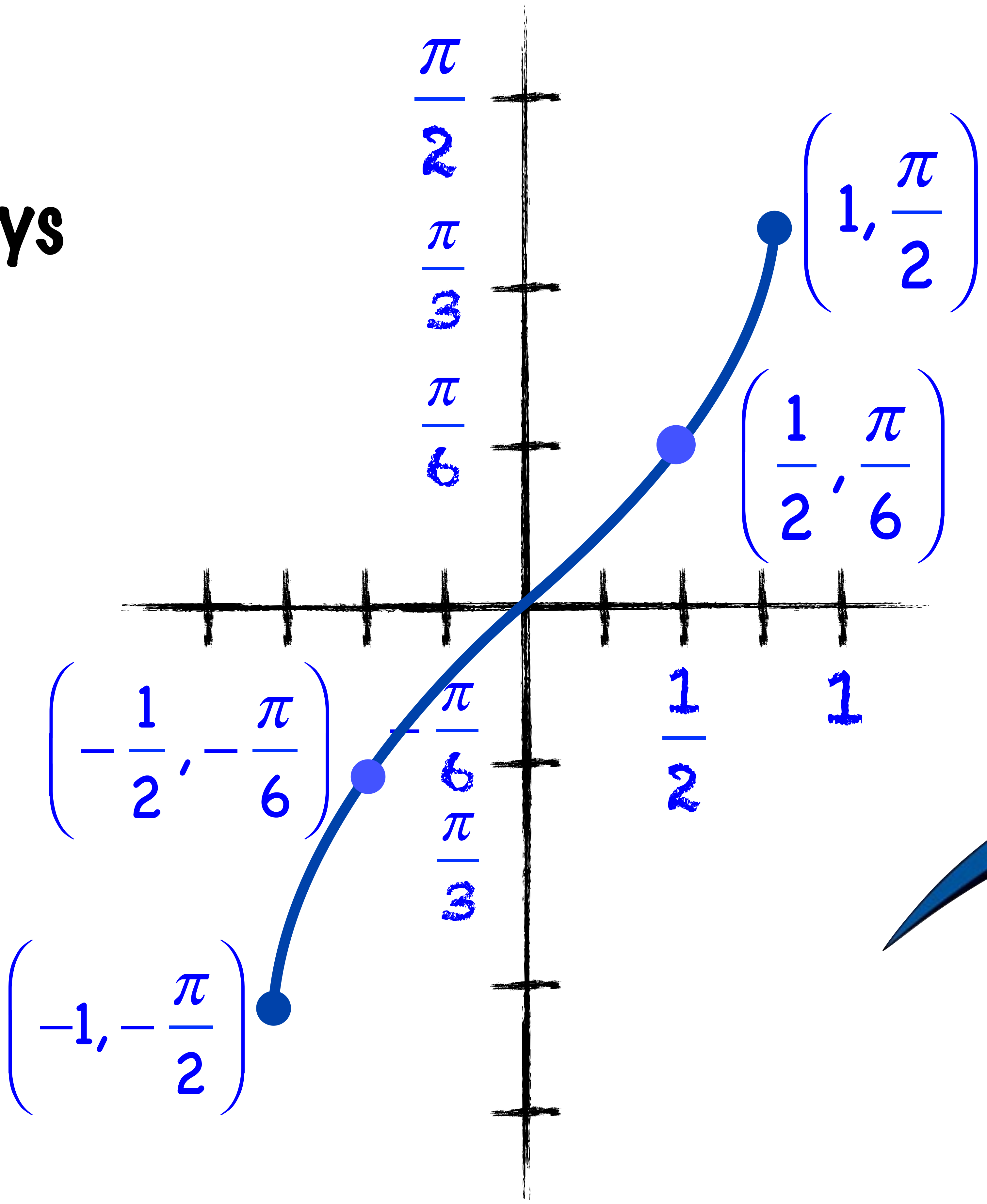
θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

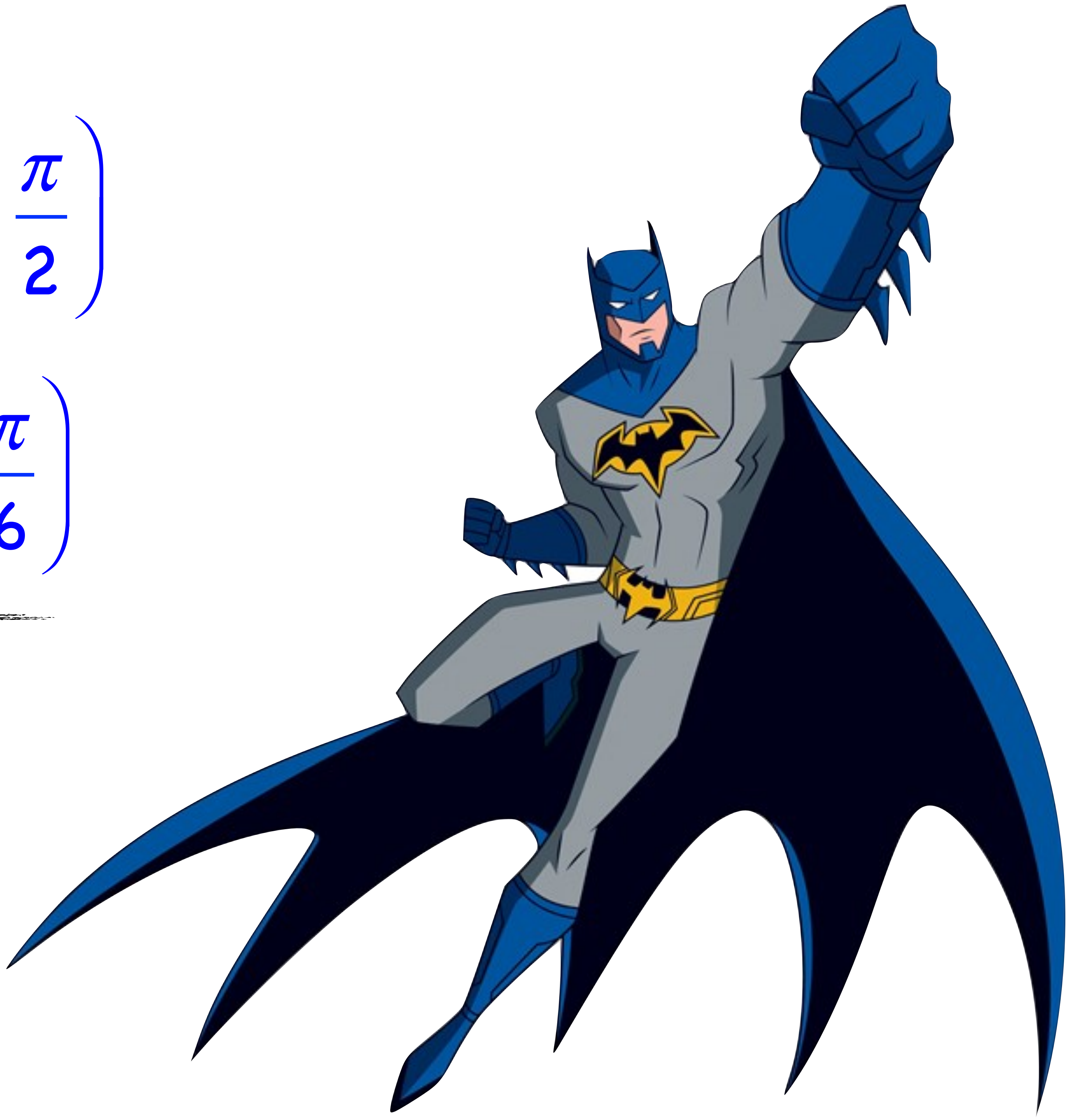
x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\sin^{-1}x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

Graphing

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\sin^{-1}x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

- Now we can graph using the always reliable table of values.





Finding the Exact Value of an Inverse Sine Function

Find the exact value of $\sin^{-1}\frac{\sqrt{3}}{2}$

Step 1 $\theta = \sin^{-1}\frac{\sqrt{3}}{2}$

Step 2 $\sin\theta = \frac{\sqrt{3}}{2},\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin θ	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

Step 3 The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sin is $\frac{\sqrt{3}}{2}$ = $\frac{\pi}{3}$

$$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

Finding the Exact Value of an Inverse Sine Function

Find the exact value of $\sin^{-1}-\frac{\sqrt{2}}{2}$

Step 1 $\theta = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Step 2 $\sin\theta = -\frac{\sqrt{2}}{2}, \ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin θ	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

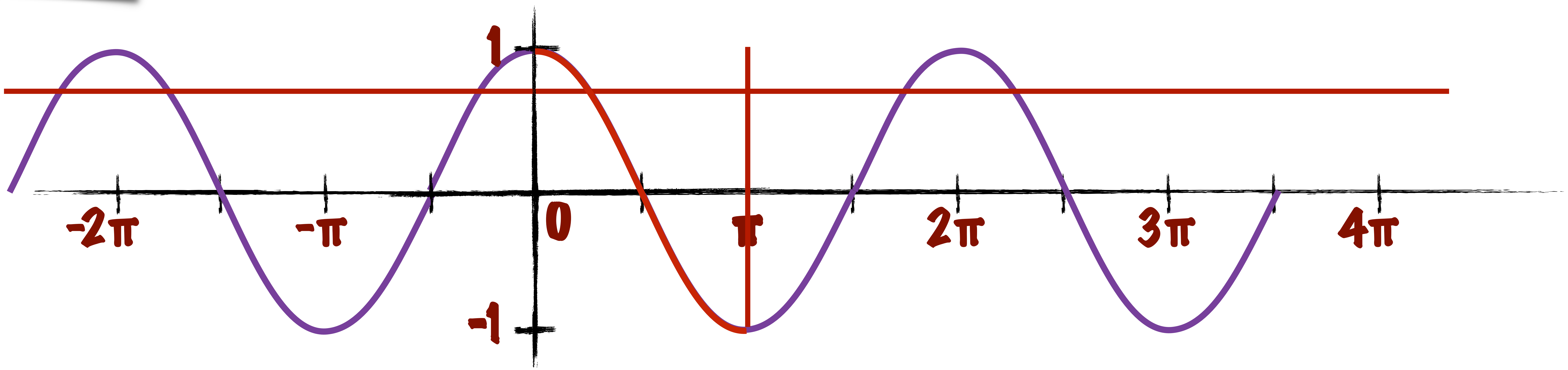
Step 3 The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose sin is $-\frac{\sqrt{2}}{2} = -\frac{\pi}{4}$

$$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

Inverse Cosine (Arccos)

Inverse Cosine

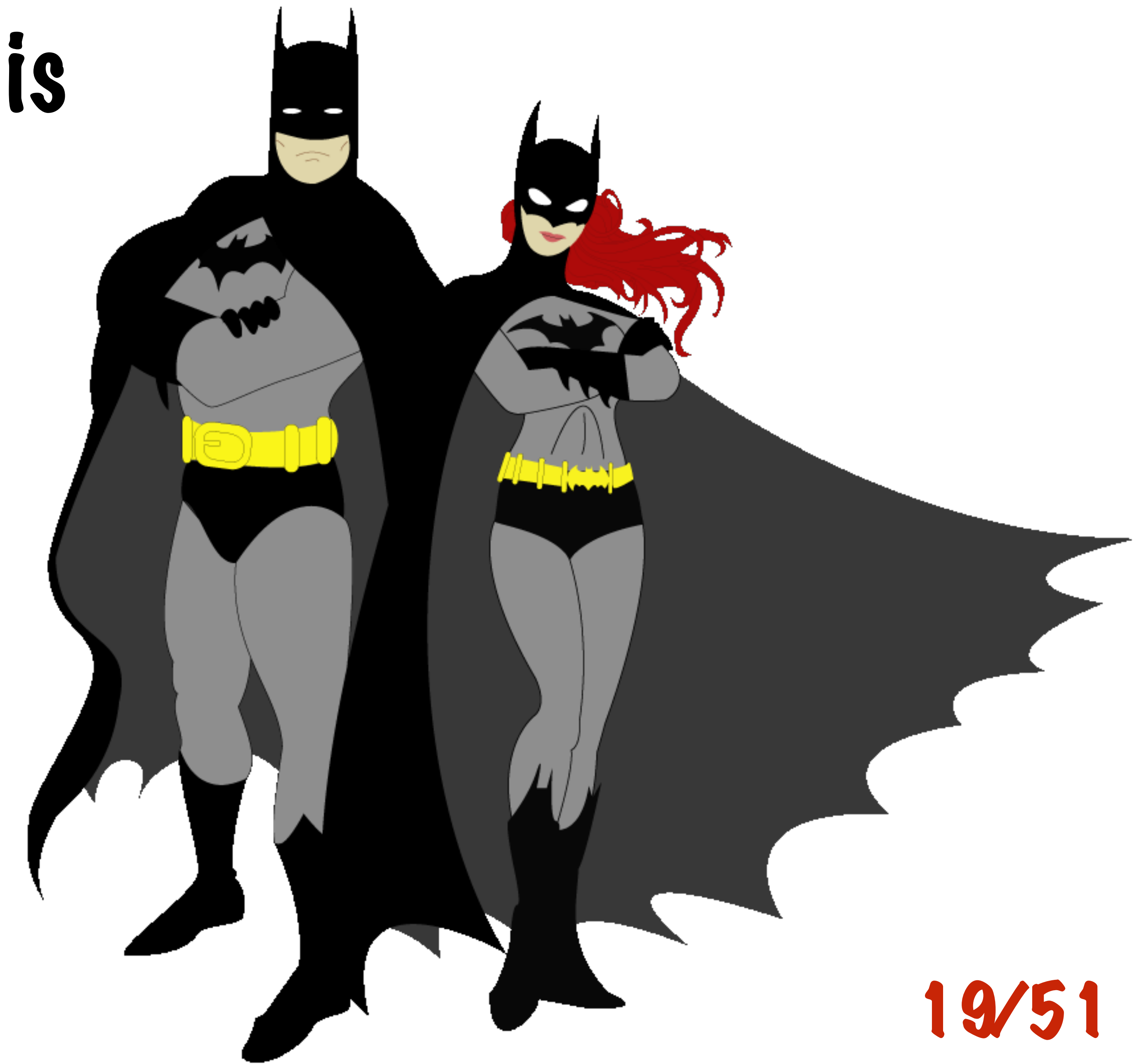
y = cos x



- Once again, the horizontal line test shows the function y = cos x is **not** a one-to-one function and does not have an inverse.

- So we make a slight adjustment. To allow for an inverse function, we restrict the domain.

$$0 \le x \le \pi$$



- **Inverse cosine function:** $\cos^{-1}$ is the inverse of the restricted cosine function $y = \cos x$, $0 \le x \le \pi$

- $y = \cos^{-1} x$: y equals inverse cosine x

- $y = \cos^{-1} x$ means $\cos y = x$ where

$-1 \le x \le 1$ and $0 \le y \le \pi$

The Inverse Cosine Function

Keep in mind which function you are solving.

$$y = \cos x$$

- x is the measure of an angle, y is a value between -1 and 1

$$y = \cos^{-1} x$$

- x is a value between -1 and 1,
- y is the measure of an angle between $0 \le y \le \pi$

Let me repeat that

- To clarify:

  - We take the cosine of an angle to get a ratio.

$$\cos A = \frac{a}{c}$$

  - We take the inverse cosine of a ratio to find an angle.

$$\cos^{-1}\frac{a}{c} = A$$

y = cos x

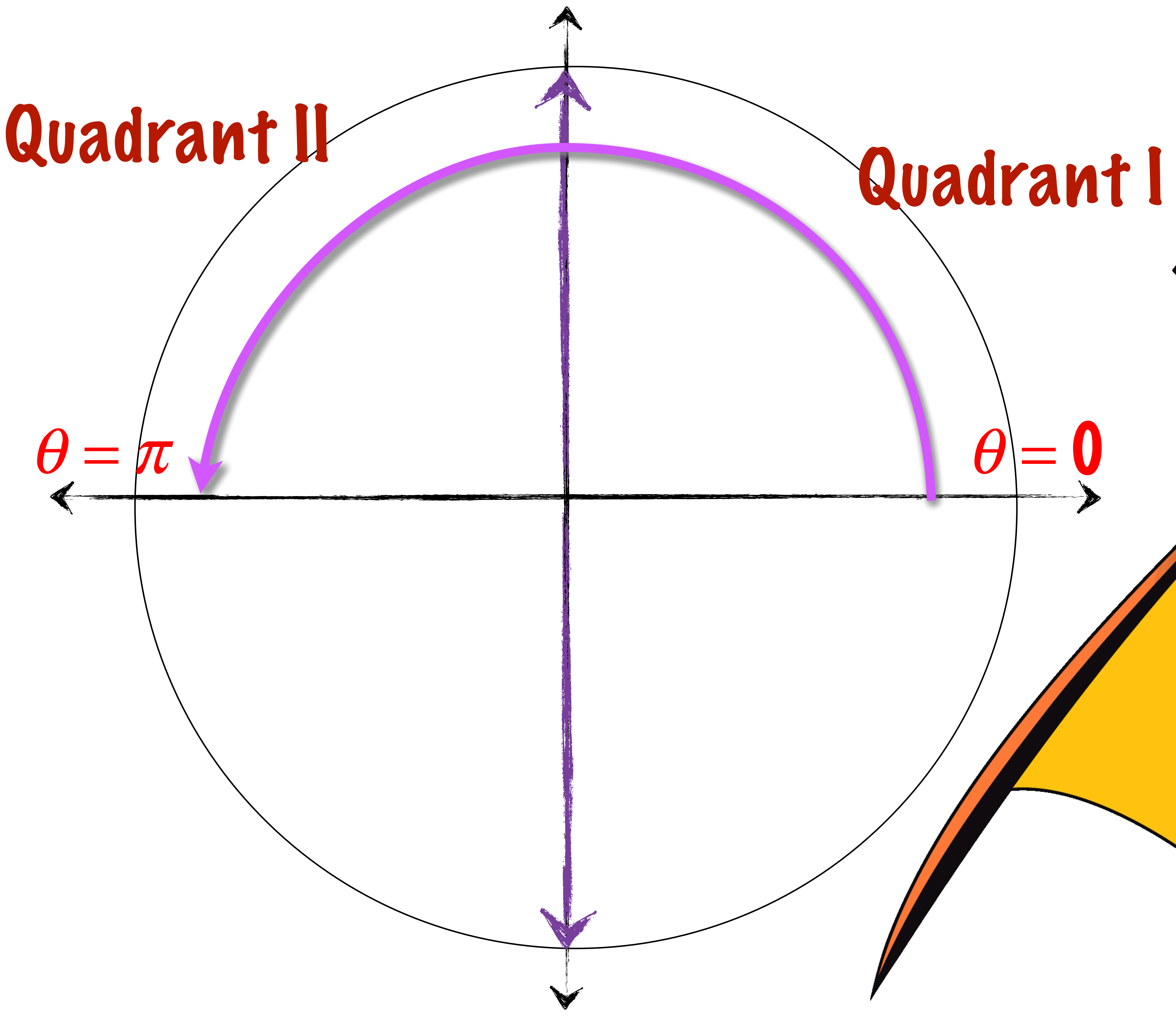
Domain : $[0,\pi]$

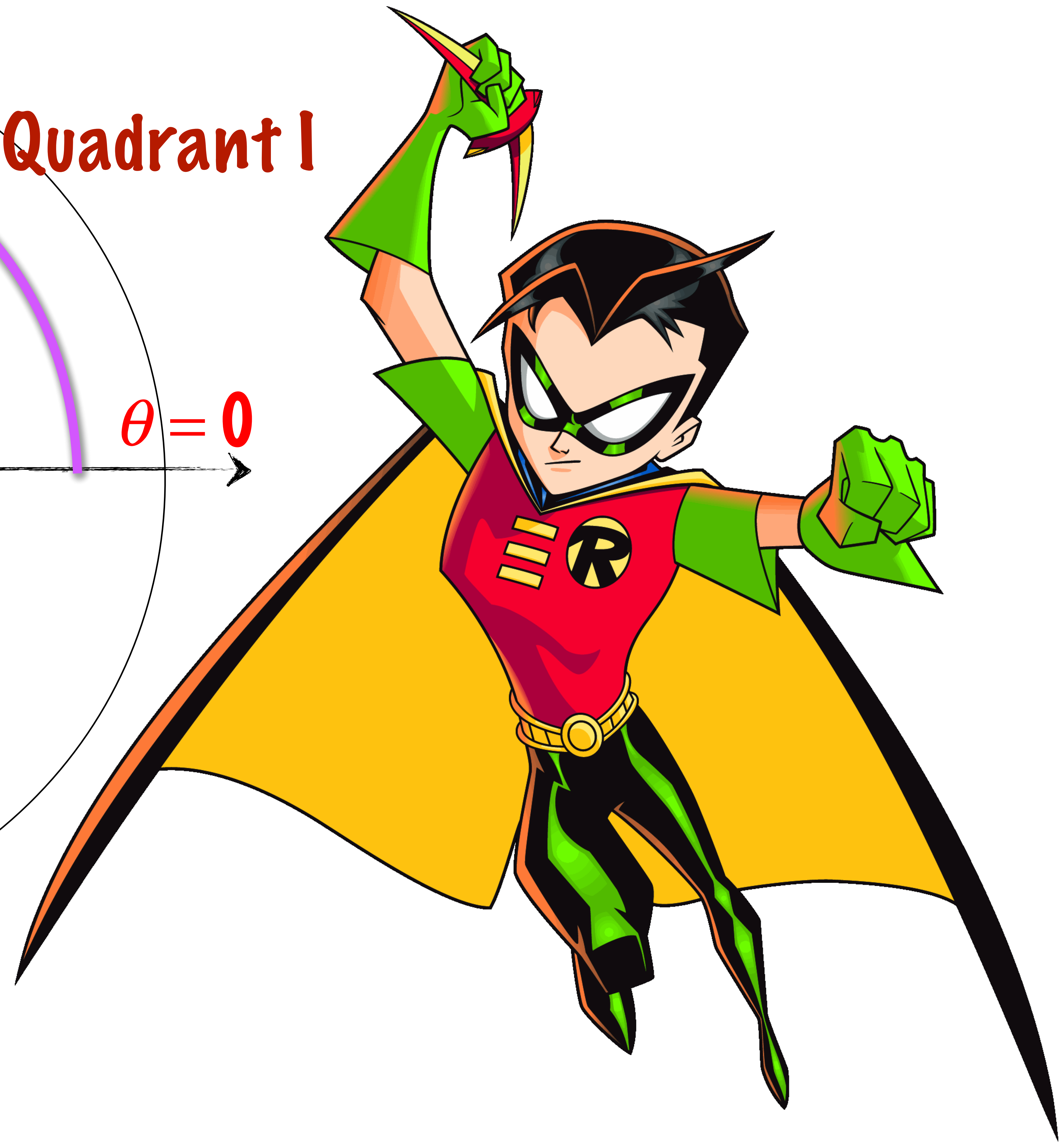
Range : $[-1,1]$

y = sin⁻¹ x

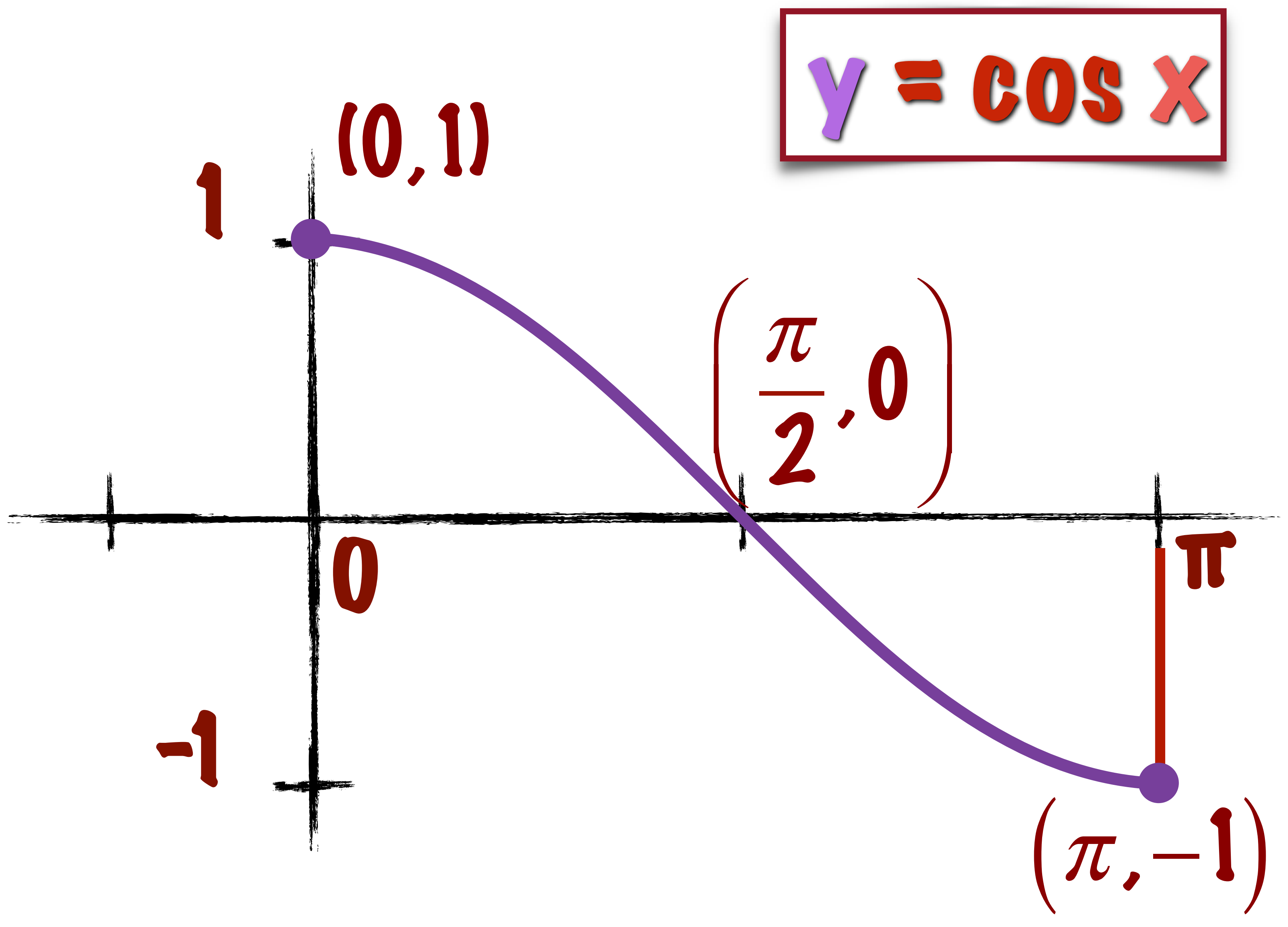
Domain : $[-1,1]$

Range : $[0,\pi]$



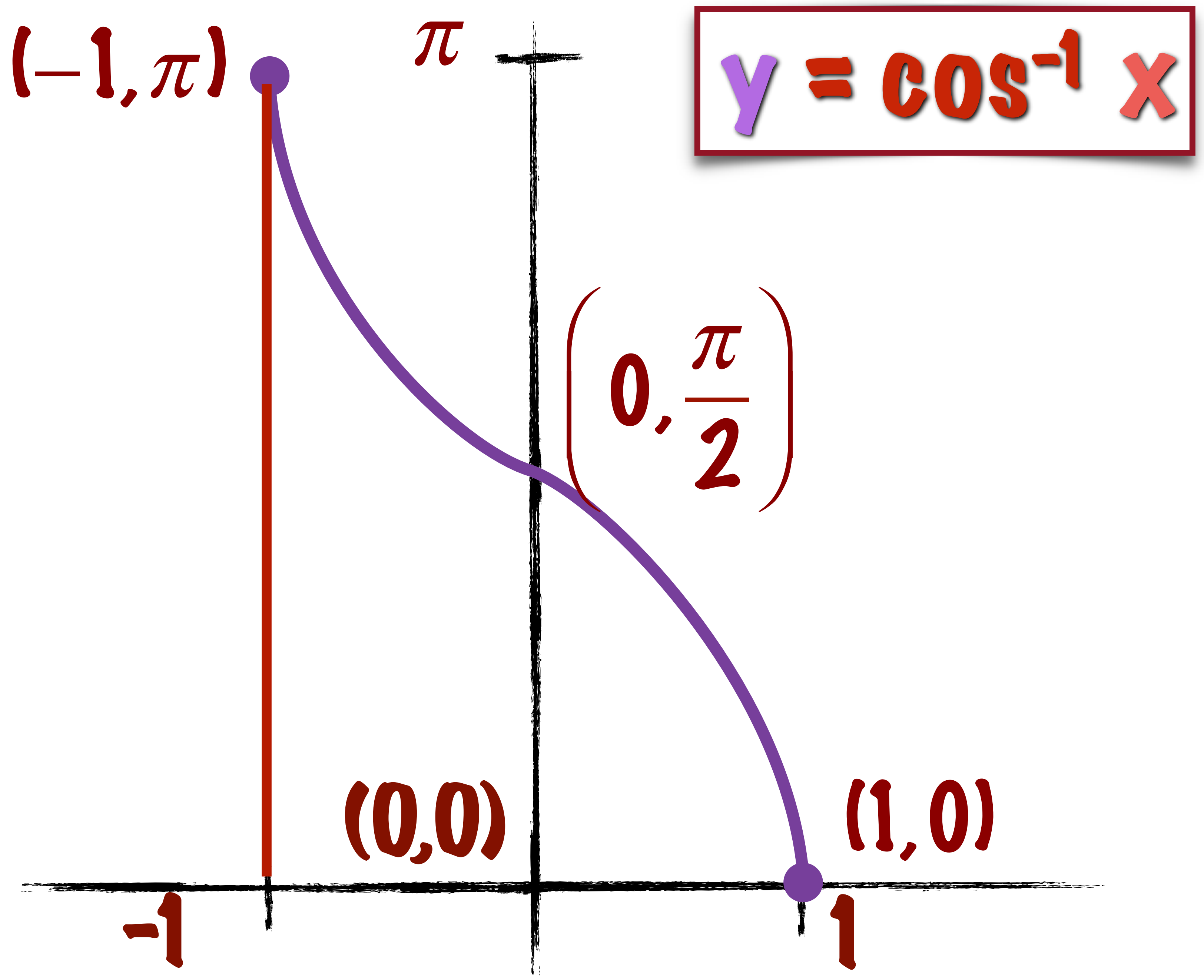


- To graph the inverse function $\cos^{-1}$ we can take the points of the function y = sin x over the restricted domain and switch x and y.



Domain : $[0,\pi]$

Range : $[-1,1]$



Domain : $[-1,1]$

Range : $[0,\pi]$

Graphing the Inverse Cosine Function

Finding exact values of $\cos^{-1} x$

1. Let $\theta = \cos^{-1} x$

2. Change to $\cos \theta = x$, where $0 \leq \theta \leq \pi$

3. Now we can use the table to find exact values of θ that satisfy $\cos \theta = x$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

Find the exact value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Step 1 $\theta = \cos^{-1}\left(-\frac{1}{2}\right)$

Step 2 $\cos\theta = -\frac{1}{2},\ 0 \le \theta \le \pi$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

Step 3 The angle between 0 and π whose sin is $-\frac{1}{2} = \frac{2\pi}{3}$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

Finding Exact Values of $\cos^{-1} x$

Find the exact value of $\cos^{-1}\frac{\sqrt{3}}{2}$

Step 1 $\theta = \cos^{-1}\frac{\sqrt{3}}{2}$

Step 2 $\cos\theta = \frac{\sqrt{3}}{2},\ 0 \le \theta \le \pi$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
Cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

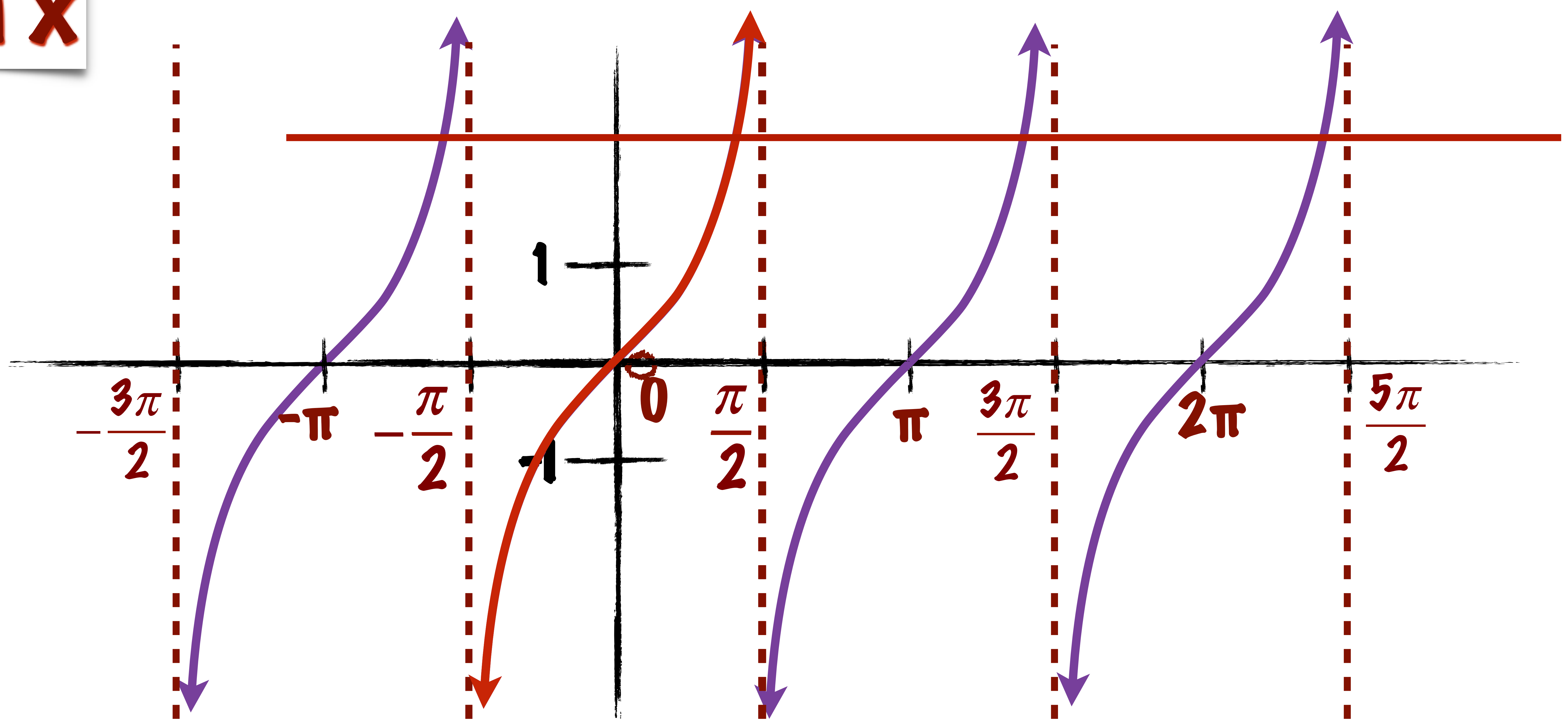
Step 3 The angle between 0 and π whose cos is $\frac{\sqrt{3}}{2} = \frac{\pi}{6}$

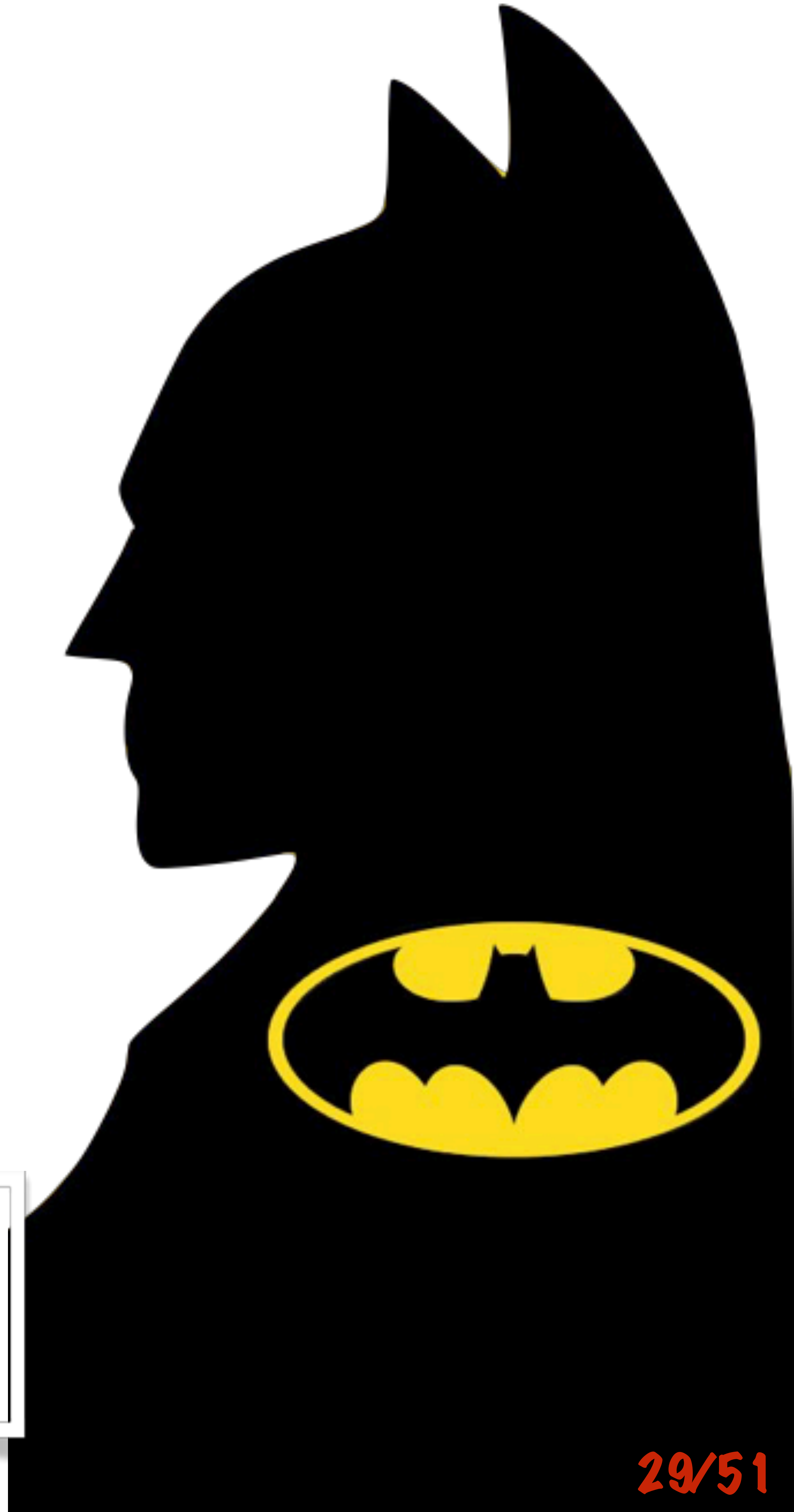
$$\cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

Inverse Tangent (arctan)

Inverse Tangent

y = tan x





- Yet again, the horizontal line test shows the function y = tan x is **not** a one-to-one function and does not have an inverse.
- So we make a slight adjustment. To allow for an inverse function, we restrict the domain.

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

Inverse Tangent

- **Inverse tangent function:** $\tan^{-1}$ is the inverse of the restricted tangent function y = tan x, $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

  - $y = \tan^{-1} x$: y equals inverse tangent x

  - $y = \tan^{-1} x$ means $\tan y = x$ where

    $-\infty \le x \le \infty$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Keep in mind which function you are solving.

$$y = \tan x$$

- x is the measure of an angle, y is a value between $-\infty$ and ∞

$$y = \tan^{-1} x$$

- x is a value between $-\infty$ and ∞,
- y is the measure of an angle between $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

The Inverse Tangent Function

$y = \tan x$

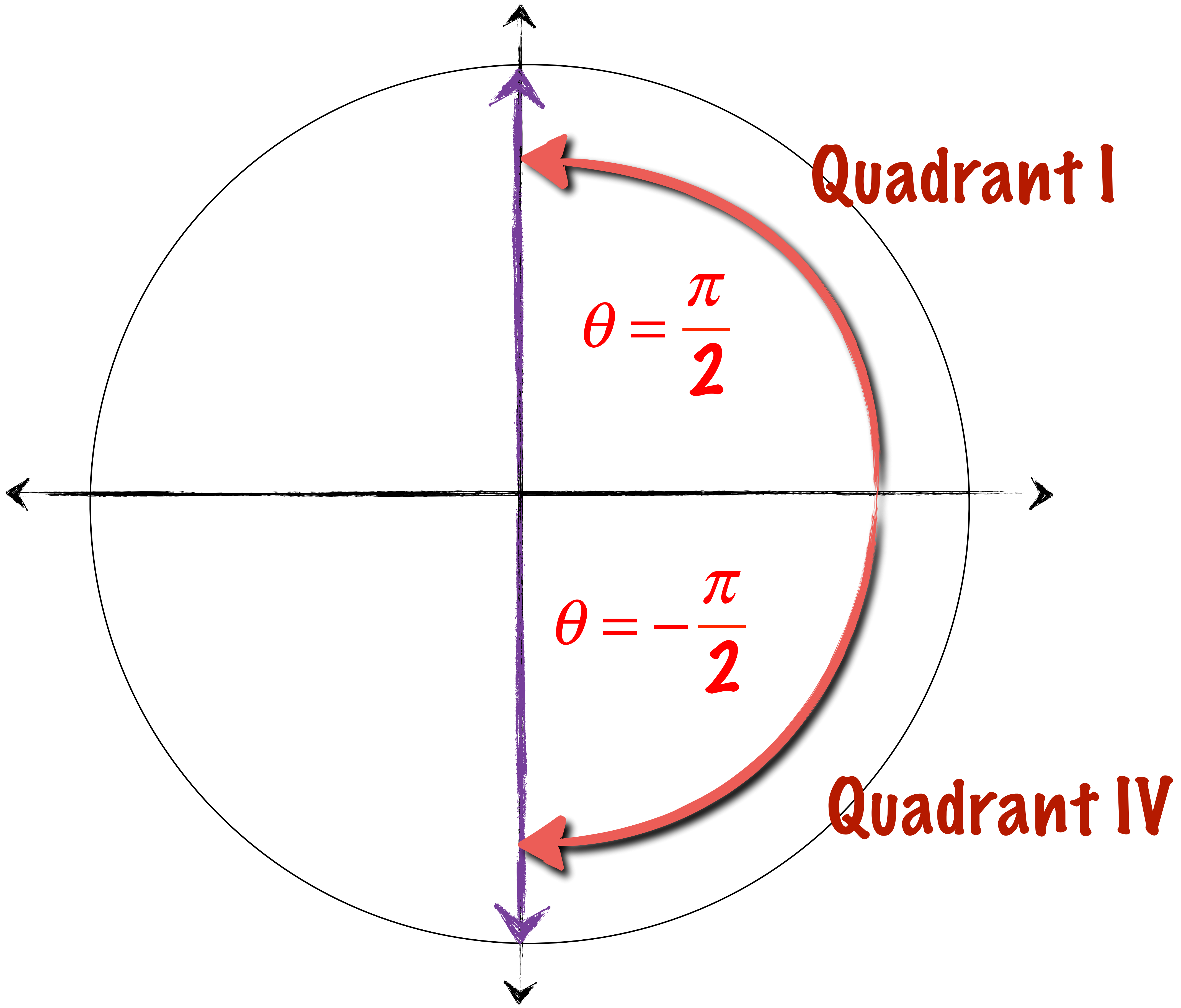
Domain : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range : $[-\infty, \infty]$

$y = \tan^{-1} x$

Domain : $[-\infty, \infty]$

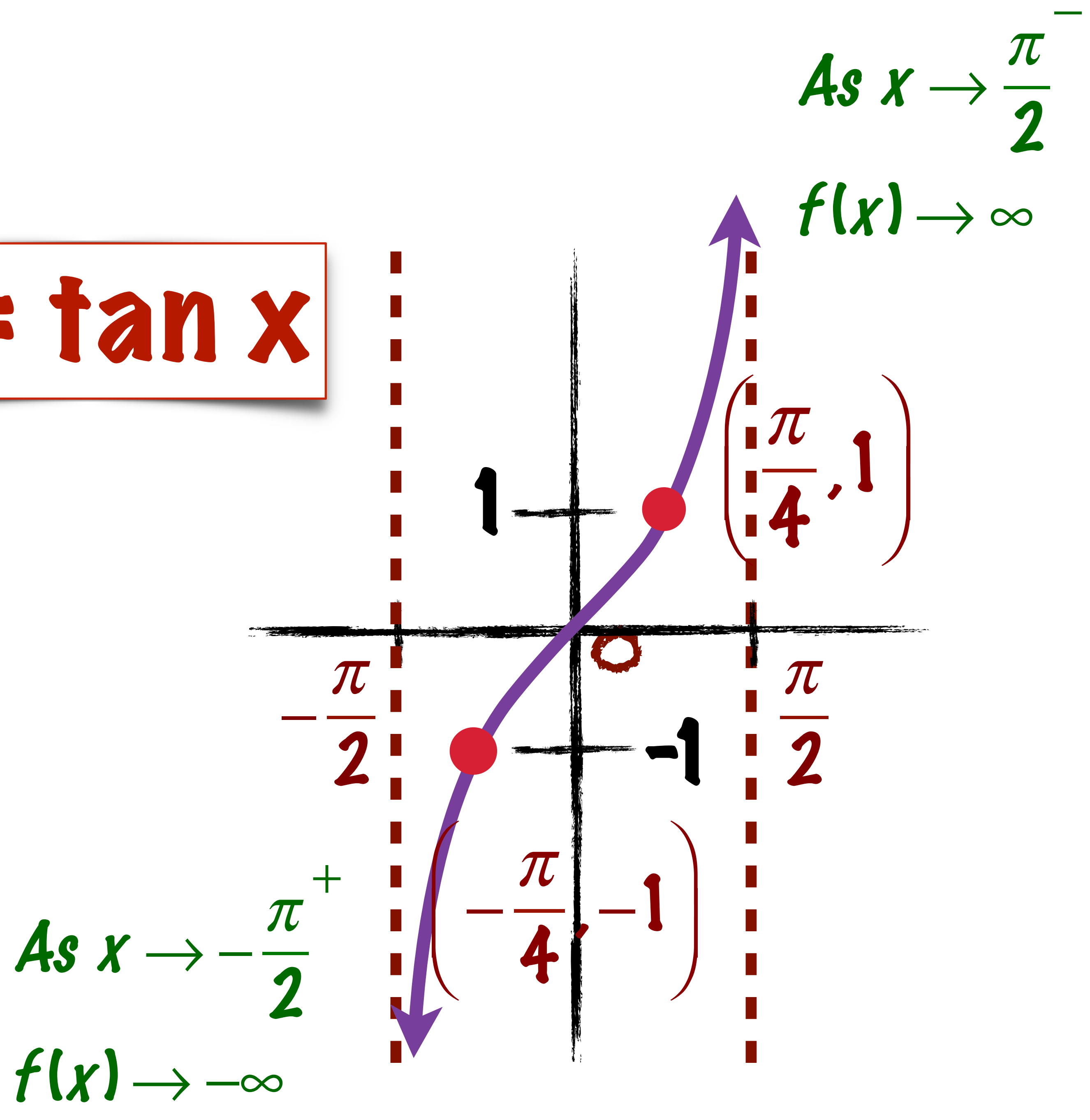
Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



The Inverse Tangent Function

- And we can graph the inverse function $\tan^{-1}$ by taking the points of the function $y = \tan x$ over the **restricted** domain and switching x and y.

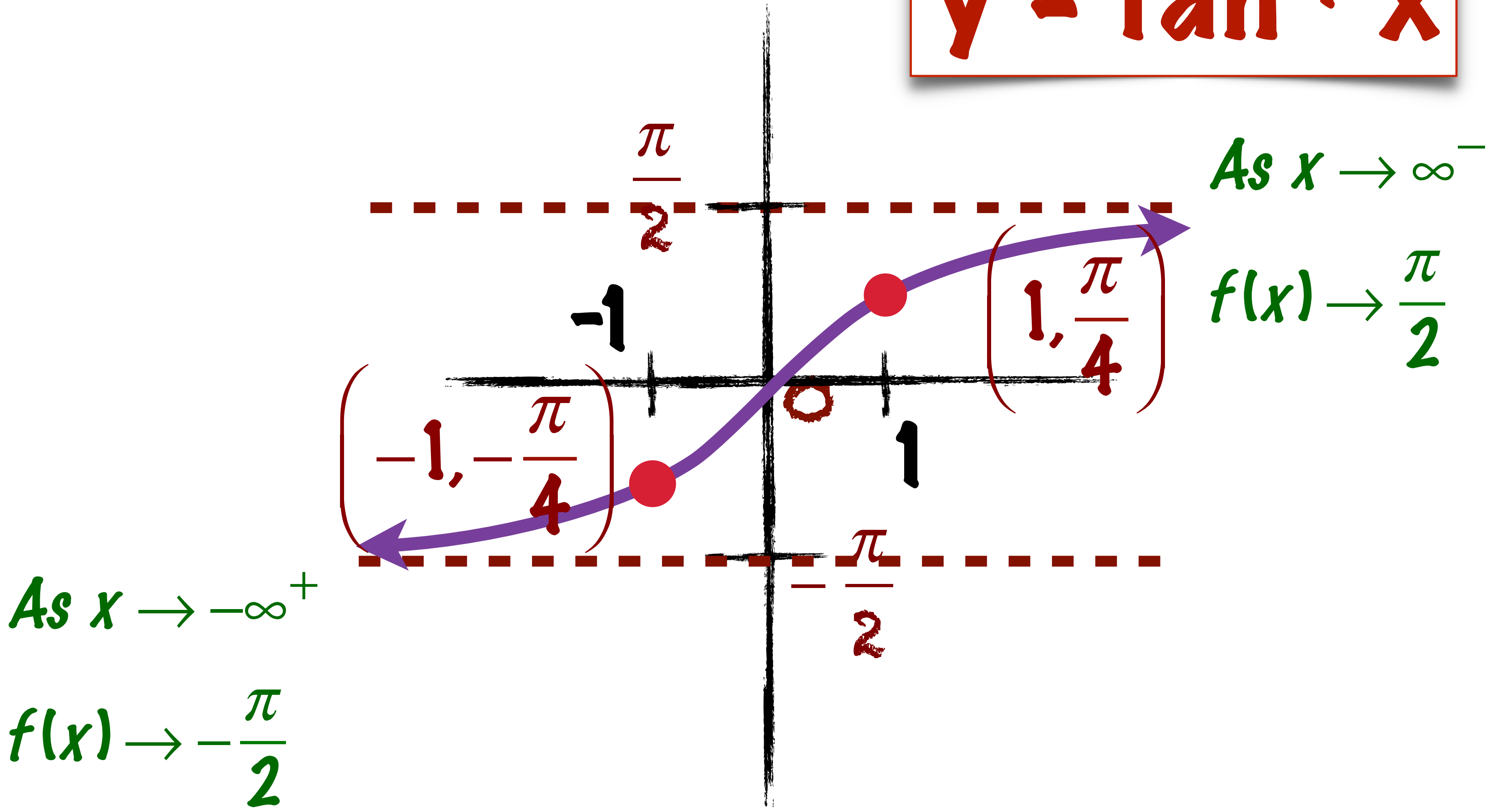
$y = \tan x$



Domain : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range : $[-\infty, \infty]$

$y = \tan^{-1} x$



Domain : $[-\infty, \infty]$

Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Finding exact values of $\tan^{-1} x$

1. Let θ = $\tan^{-1} x$

2. Change to tan θ = x, where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

3. Now we can use the table to find exact values of θ that satisfy tan θ = x.

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
tan θ	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und

Finding Exact Values of $\tan^{-1} x$

Find the exact value of $\tan^{-1}(-1)$

Step 1 $\theta = \cos^{-1}(-1)$

Step 2 $\tan\theta = -1, -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan\theta$	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und

Step 3 The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tan is -1 = $-\frac{\pi}{4}$

$$\tan^{-1}(-1) = -\frac{\pi}{4}$$

Ex: Finding the Exact Value of an Inverse Tangent

Find the exact value of $\tan^{-1}\sqrt{3}$

Step 1 $\theta = \tan^{-1}\sqrt{3}$

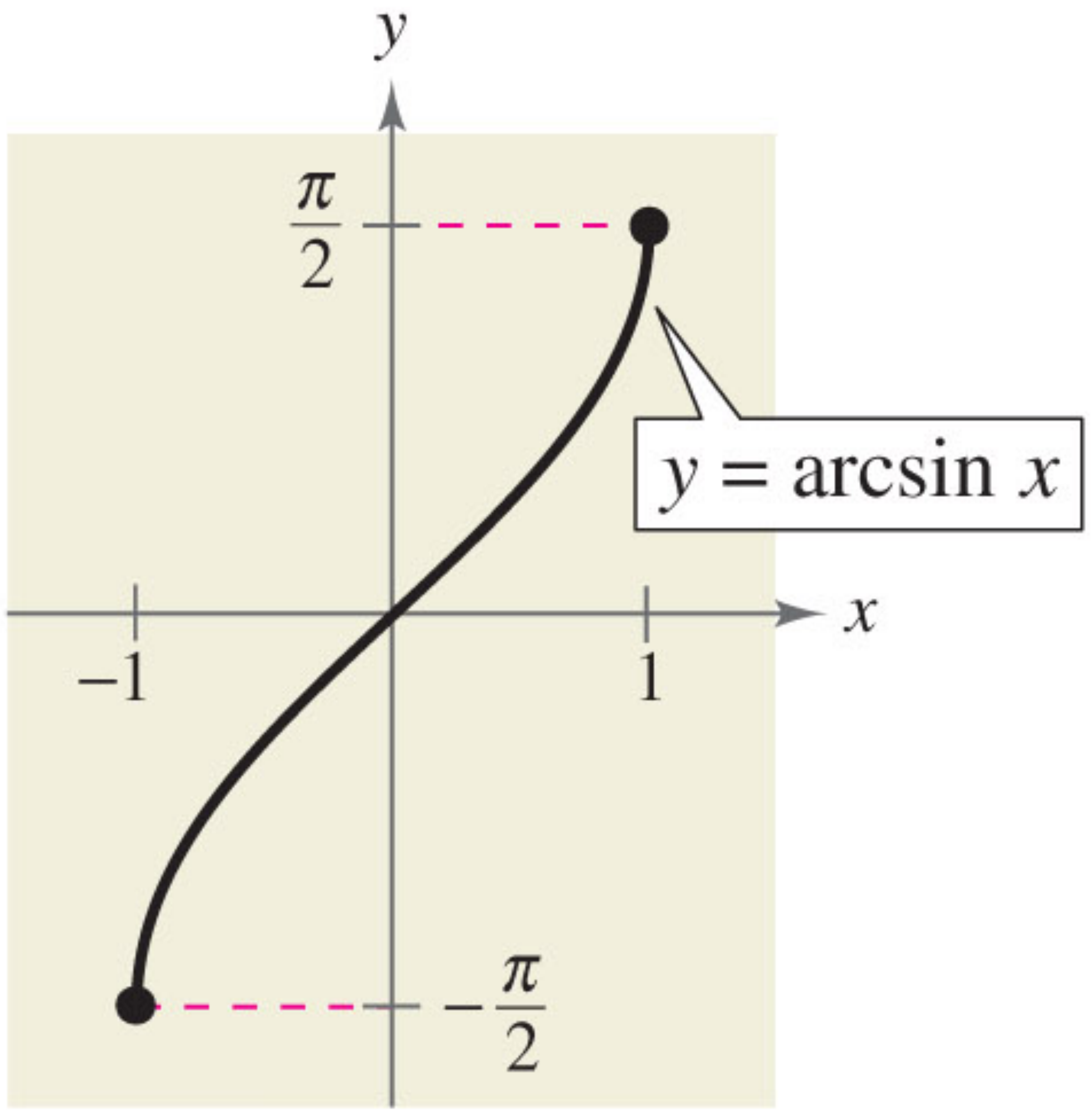
Step 2 $\tan\theta = \sqrt{3},\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

θ	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan\theta$	und	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und

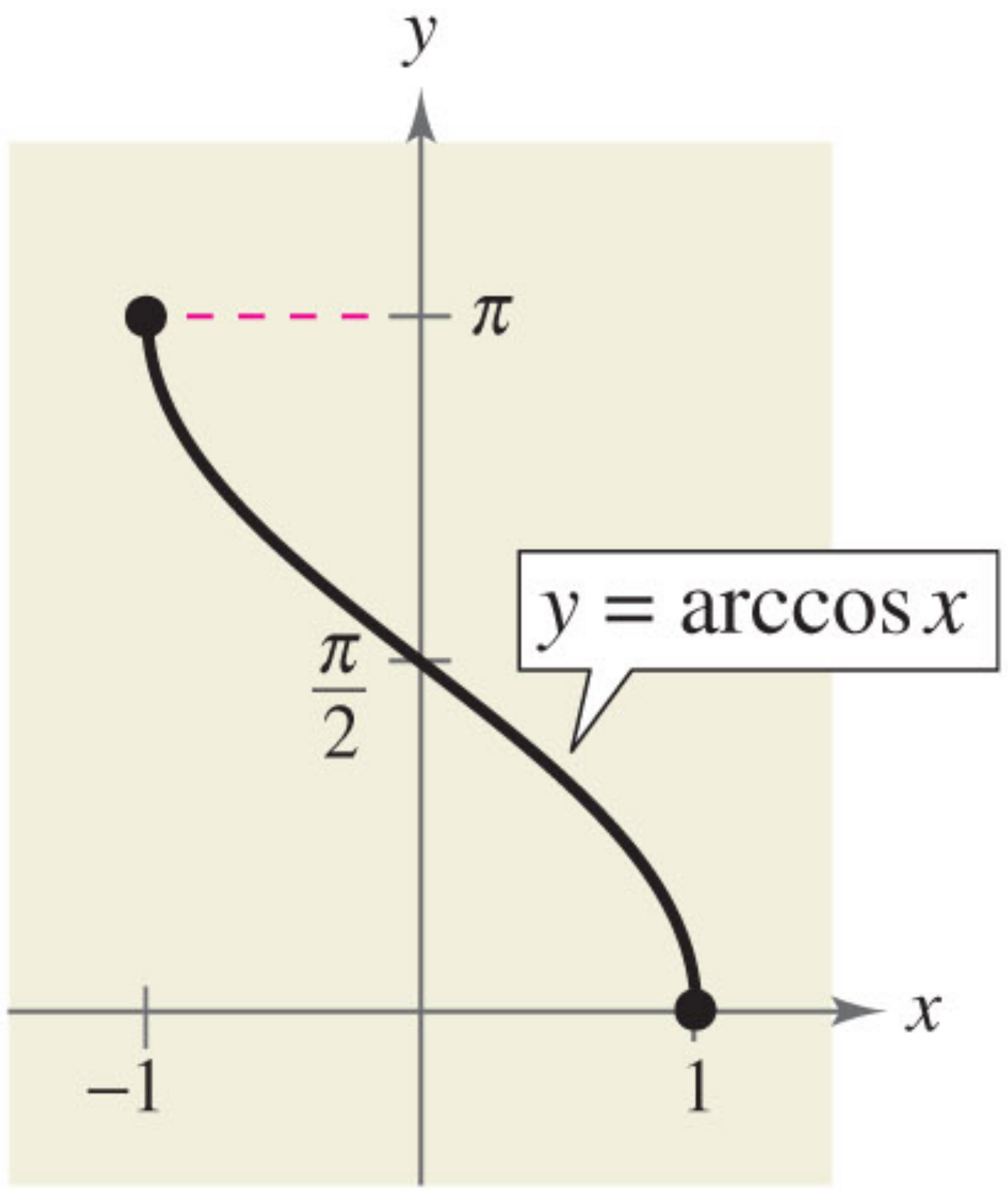
Step 3 The angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ whose tan is $\sqrt{3}$ = $\frac{\pi}{3}$

$$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

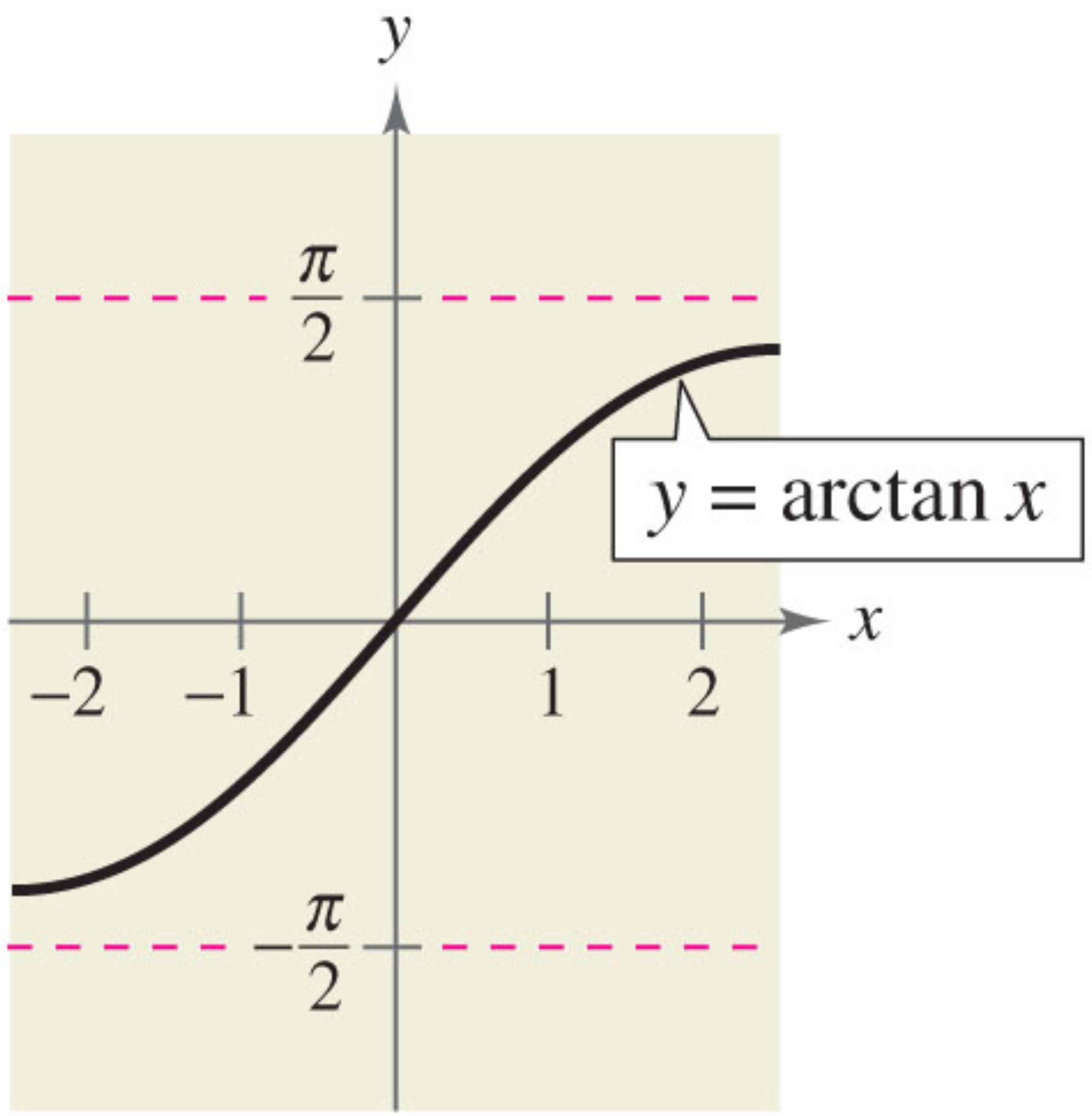
To summarize



DOMAIN: $[-1, 1]$
RANGE: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



DOMAIN: $[-1, 1]$
RANGE: $[0, \pi]$

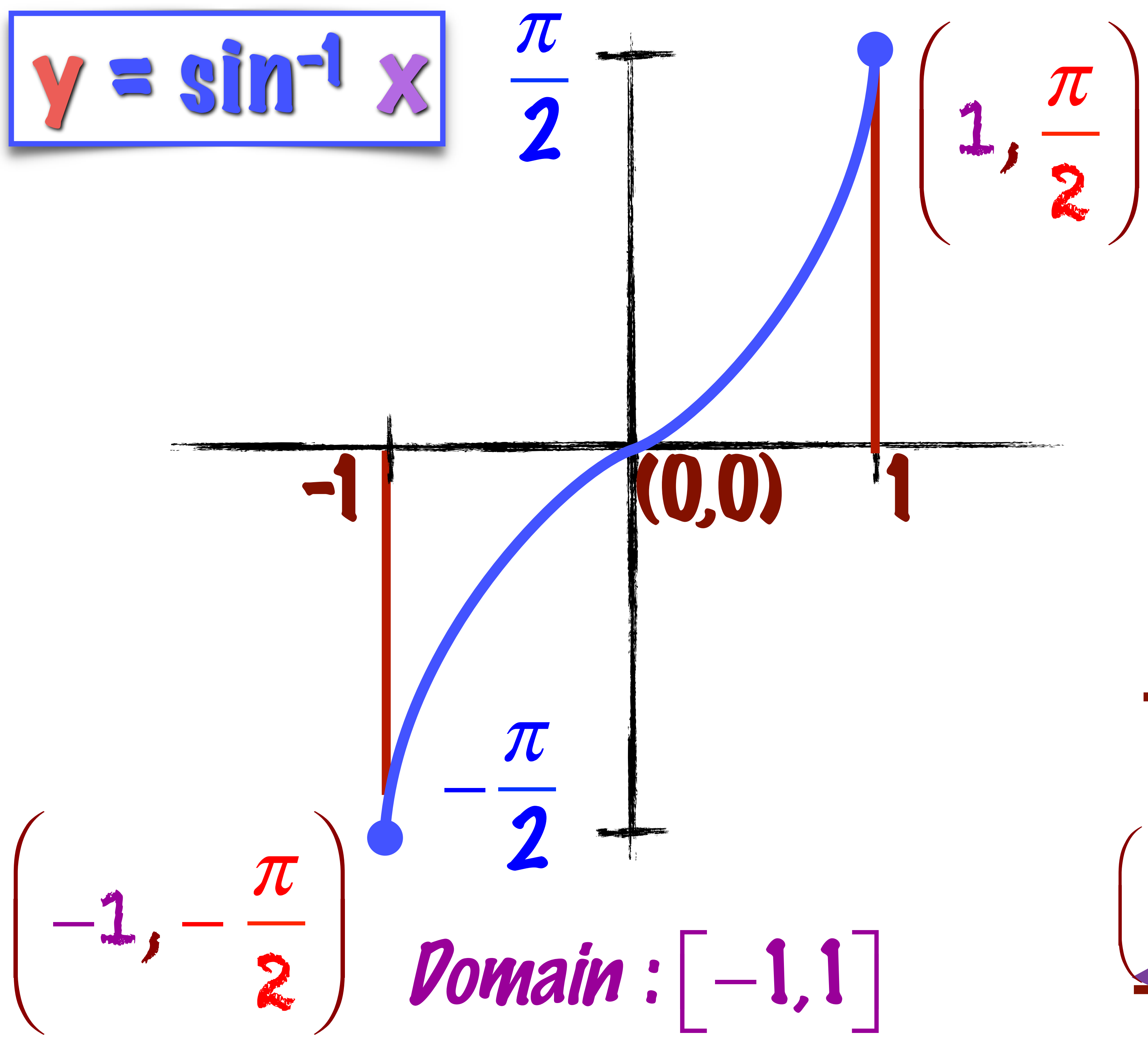


DOMAIN: $(-\infty, \infty)$
RANGE: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Inverse Trigonometric Functions

Inverse Trig Functions

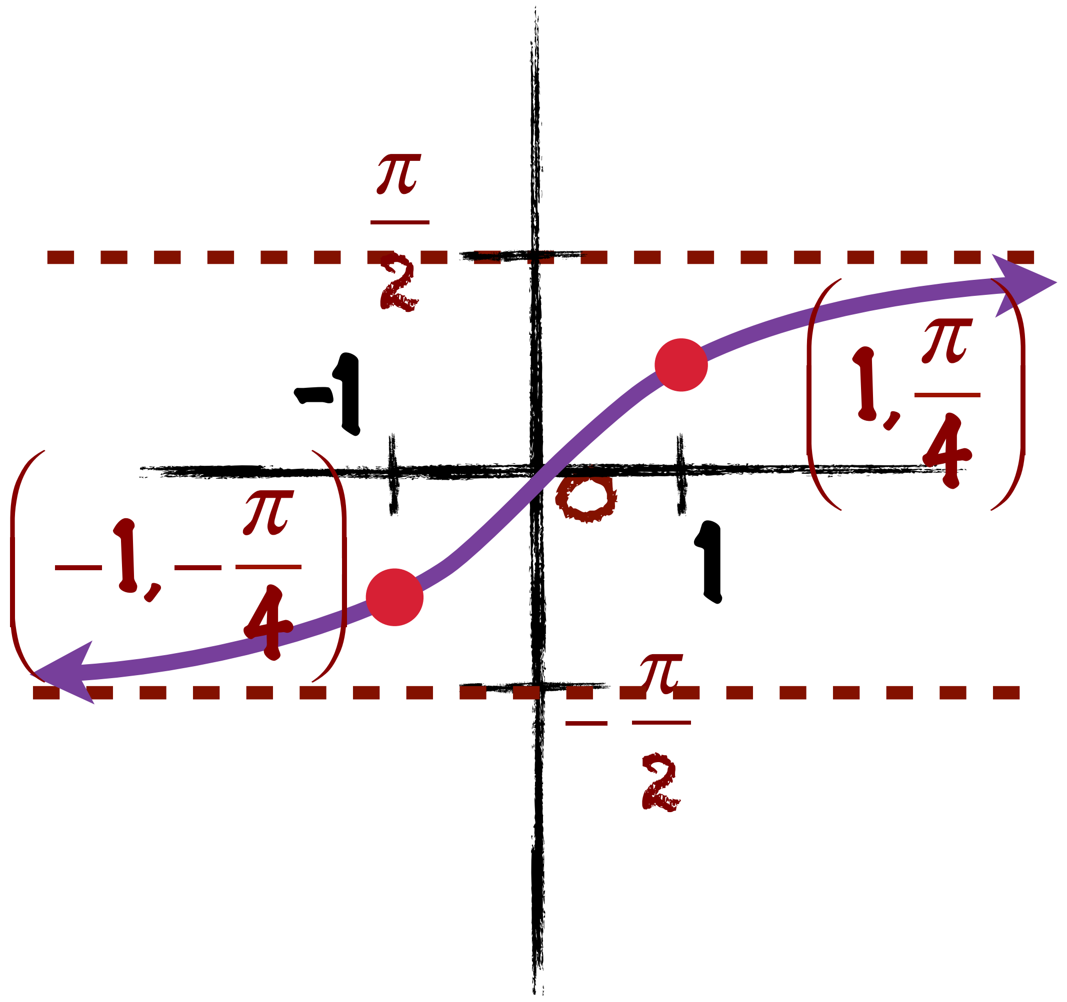
$y = \sin^{-1} x$



Domain : $[-1,1]$

Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

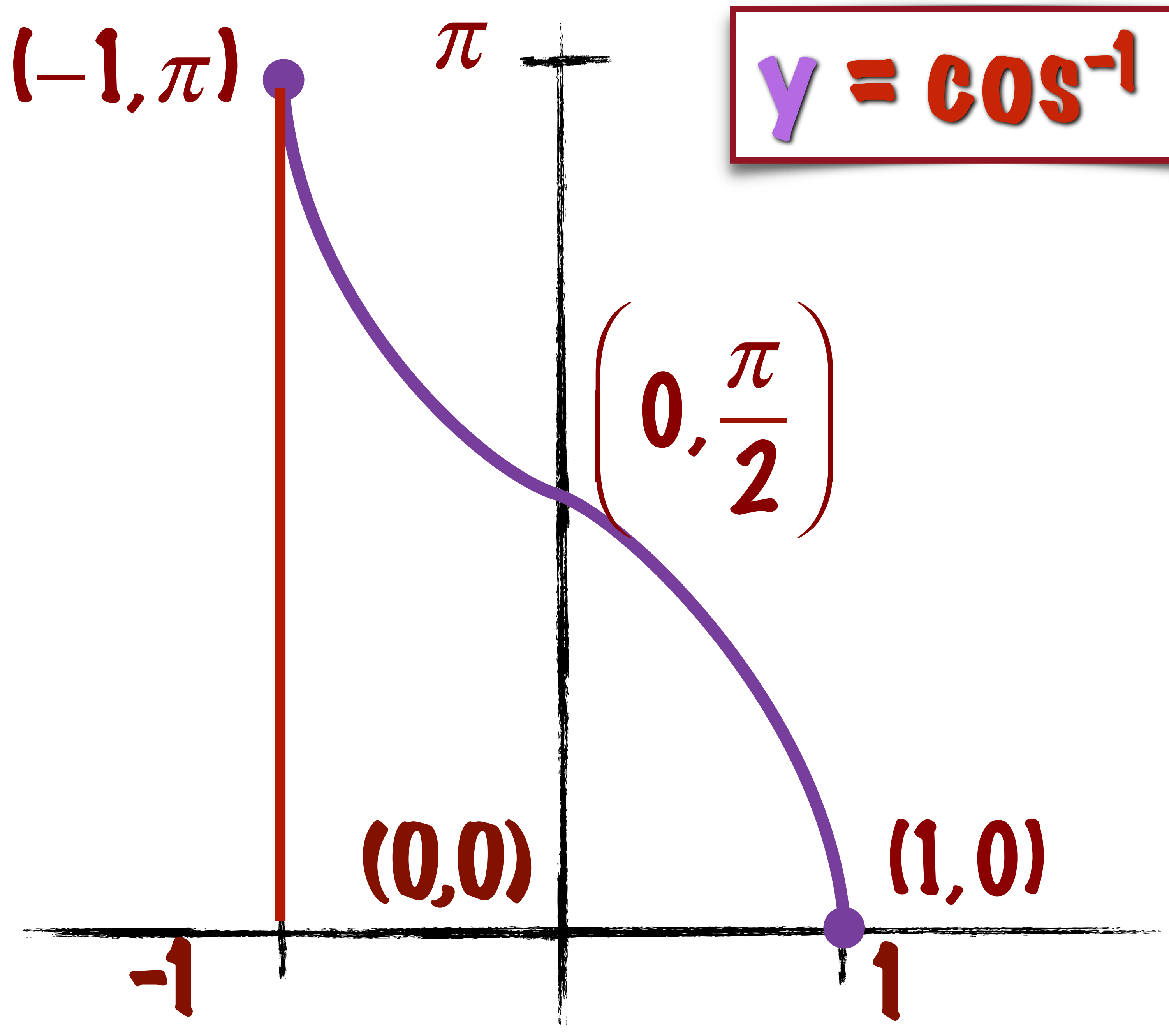
$y = \tan^{-1} x$



Domain : $[-\infty, \infty]$

Range : $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \cos^{-1} x$



Domain : $[-1,1]$

Range : $[0, \pi]$

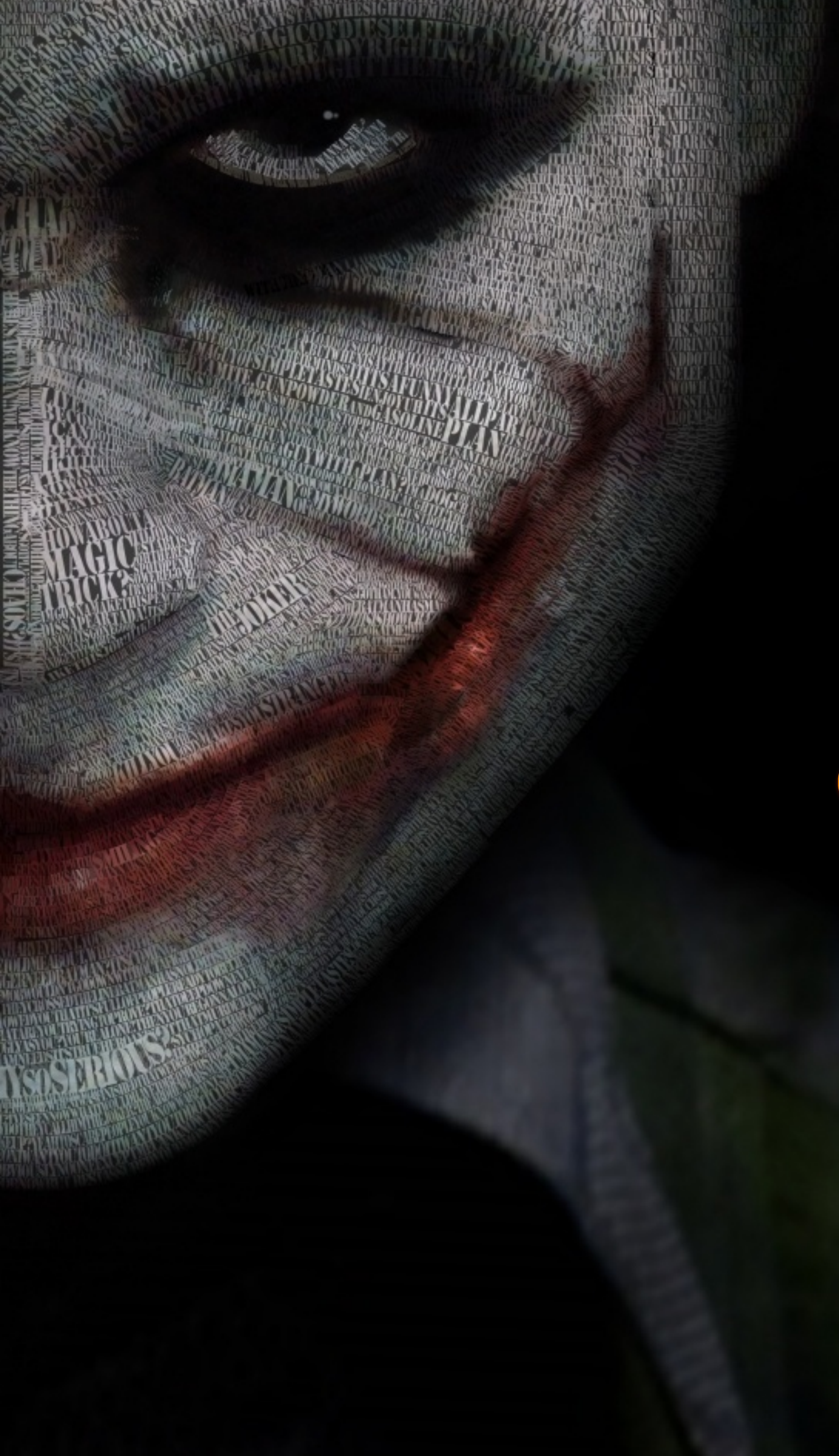
Now let us see how we can use the calculator.

a. $\cos^{-1}\frac{1}{3}$ — 2nd [COS⁻¹] COS, 1, ÷, 3,), ENTER — $\cos^{-1}\frac{1}{3} = 1.2310$

b. $\tan^{-1}(-35.85)$ — 2nd [TAN⁻¹] TAN, (–), 3, 5, ., 8, 5, ENTER — $\tan^{-1}(-35.85) = -1.5429$

c. $\sin^{-1}\frac{\sqrt{3}}{2}$ — 2nd [SIN⁻¹] SIN, 2nd [√] X², 3, ➢, ÷, 2,), ENTER — $\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

Shame on you!



Composition of Trigonometric Functions

Composition of Trigonometric Functions

- Keep in mind that we take the inverse function of a value that returns the measure of an angle.

$$\cos^{-1} x = \text{measure of an angle}$$

- We can then take a trigonometric function of that angle

$$\sin(\cos^{-1} x) = \text{a ratio}$$

- The converse is also possible

$$\sin^{-1}(\cos x) = \text{measure of an angle}$$

Composition of trigonometric functions

$$\sin\left(\sin^{-1} x\right) = x \quad \text{for } -1 \le x \le 1$$

$$\sin^{-1}\left(\sin x\right) = x \quad \text{for } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$

$$\cos\left(\cos^{-1} x\right) = x \quad \text{for } -1 \le x \le 1$$

$$\cos^{-1}\left(\cos x\right) = x \quad \text{for } 0 \le x \le \pi$$

$$\tan\left(\tan^{-1} x\right) = x \quad \text{for all } x$$

$$\tan^{-1}\left(\tan x\right) = x \quad \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

Try these (using appropriate values) on your calculator.

Inverse Properties

Find the exact value, if possible:

a. $\cos\left(\cos^{-1} 0.7\right)$ $\quad$ $\cos\left(\cos^{-1} 0.7\right) = 0.7$

b. $\sin\left(\sin^{-1} \pi\right)$ $\quad$ Careful! Cannot find $\sin^{-1} \pi$

c. $\sin^{-1}\left(\sin \pi\right)$ $\quad$ $\sin \pi = 0$ $\quad$ $\sin^{-1}(0) = 0$

d. $\cos\left(\cos^{-1}(-1.2)\right)$ $\quad$ Careful! Cannot find $\cos^{-1}(-1.2)$

e. $\sin^{-1}\left(\sin \frac{3\pi}{2}\right)$ $\quad$ $\sin \frac{3\pi}{2} = -1$ $\quad$ $\sin^{-1}(-1) = -\frac{\pi}{2}$

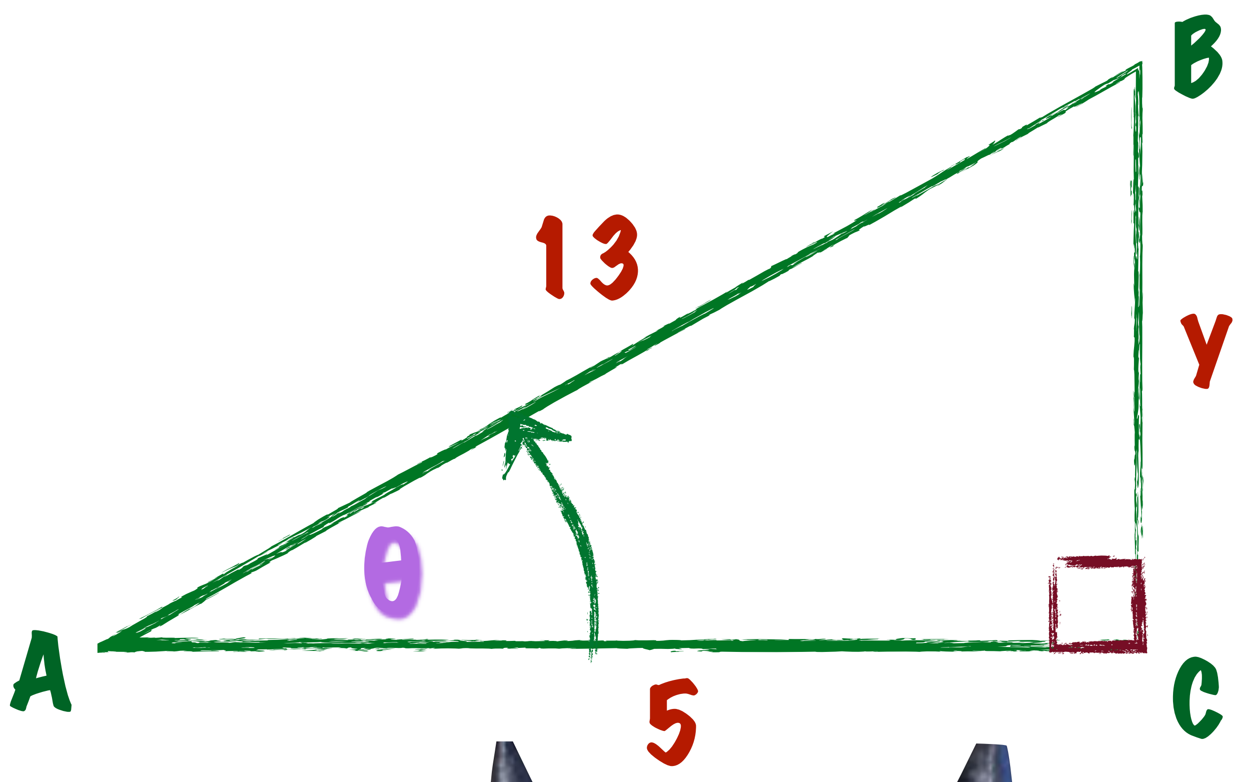
Let us make life a little more interesting.

STUDY TIP

As with the trigonometric functions, much of the work with the inverse trigonometric functions can be done by *exact* calculations rather than by calculator approximations. Exact calculations help to increase your understanding of the inverse functions by relating them to the right triangle definitions of the trigonometric functions.

Evaluating Composite Trigonometric Functions

Find the exact value, if possible: $\sin\left(\cos^{-1}\frac{5}{13}\right)$



First: Let $\theta = \cos^{-1}\frac{5}{13}$ $\quad \cos\theta = \frac{5}{13} = \frac{adj}{hyp}$

θ is an angle in the first quadrant (0 ≤ θ ≤ π).

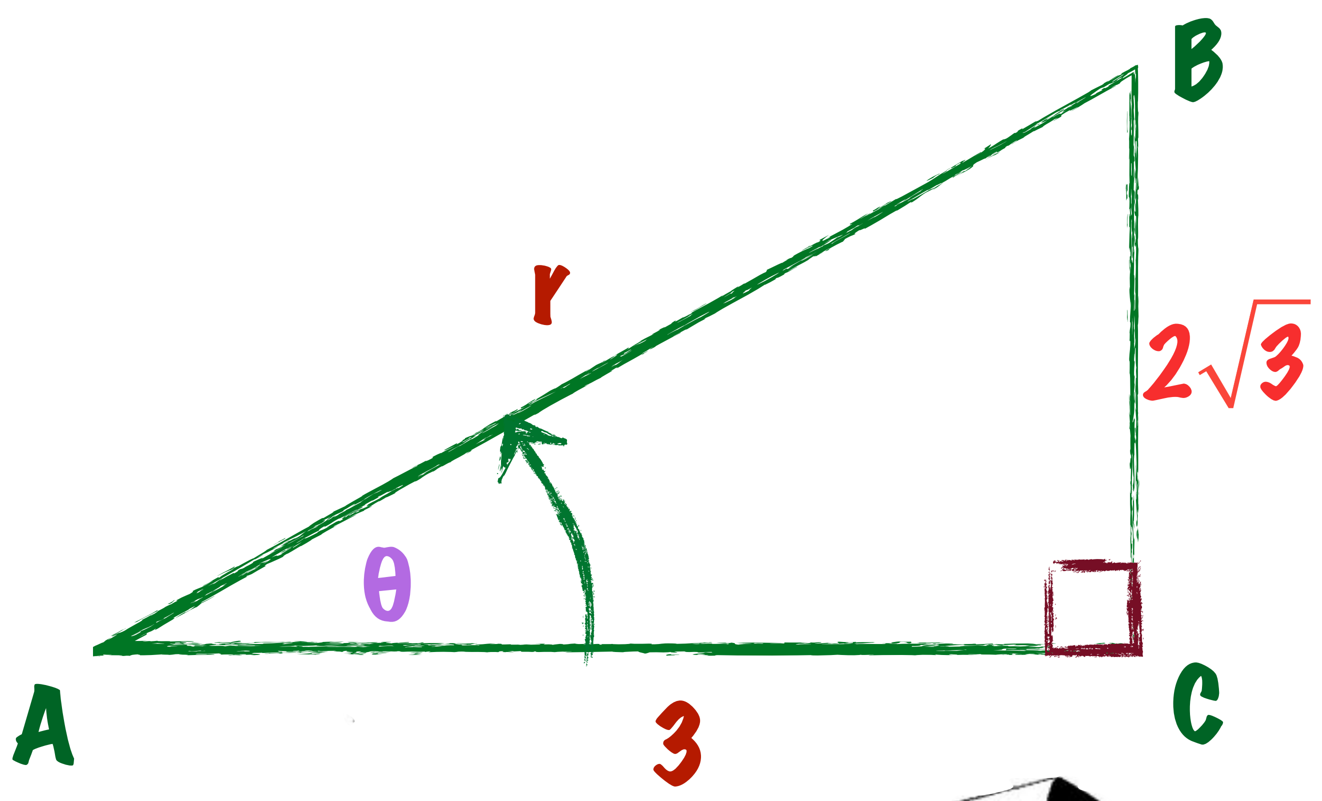
$\sin\theta = \frac{opp}{hyp}$ We need to find opp

$5^2 + y^2 = 13^2 \quad y^2 = 13^2 - 5^2 = 144$ $\quad$ y = 12

$\sin\theta = \frac{12}{13} \quad \sin\left(\cos^{-1}\frac{5}{13}\right) = \frac{12}{13}$

Evaluating Compositions of Functions and Their Inverses

Find the exact value, if possible: $\cos\left(\tan^{-1}\dfrac{2\sqrt{3}}{3}\right)$



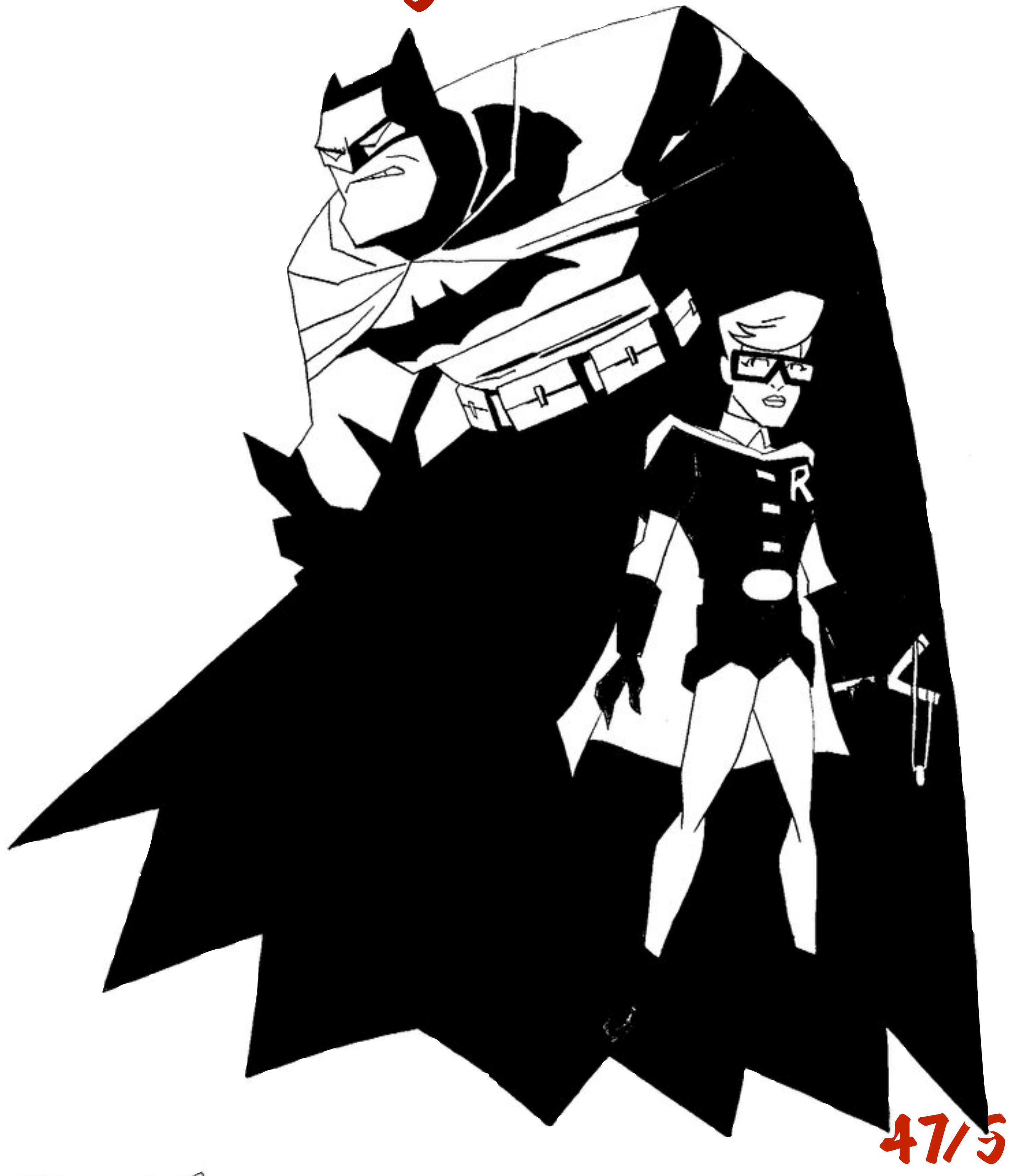
First: Let $\theta = \tan^{-1}\dfrac{2\sqrt{3}}{3}$ $\qquad \tan\theta = \dfrac{2\sqrt{3}}{3} = \dfrac{y}{x}$

θ is an angle in the first quadrant. $\left(-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}\right)$

$\cos\theta = \dfrac{x}{r}$ $\qquad$ We need to find r.

$3^2 + \left(2\sqrt{3}\right)^2 = r^2 \qquad r^2 = 21 \qquad r = \sqrt{21}$

$\cos\theta = \dfrac{3}{\sqrt{21}} \qquad \cos\left(\tan^{-1}\dfrac{2\sqrt{3}}{3}\right) = \dfrac{3}{\sqrt{21}}$



Evaluating Compositions of Functions and Their Inverses

Find the exact value, if possible: $\sec\left(\tan^{-1}\left(-\frac{4}{7}\right)\right)$

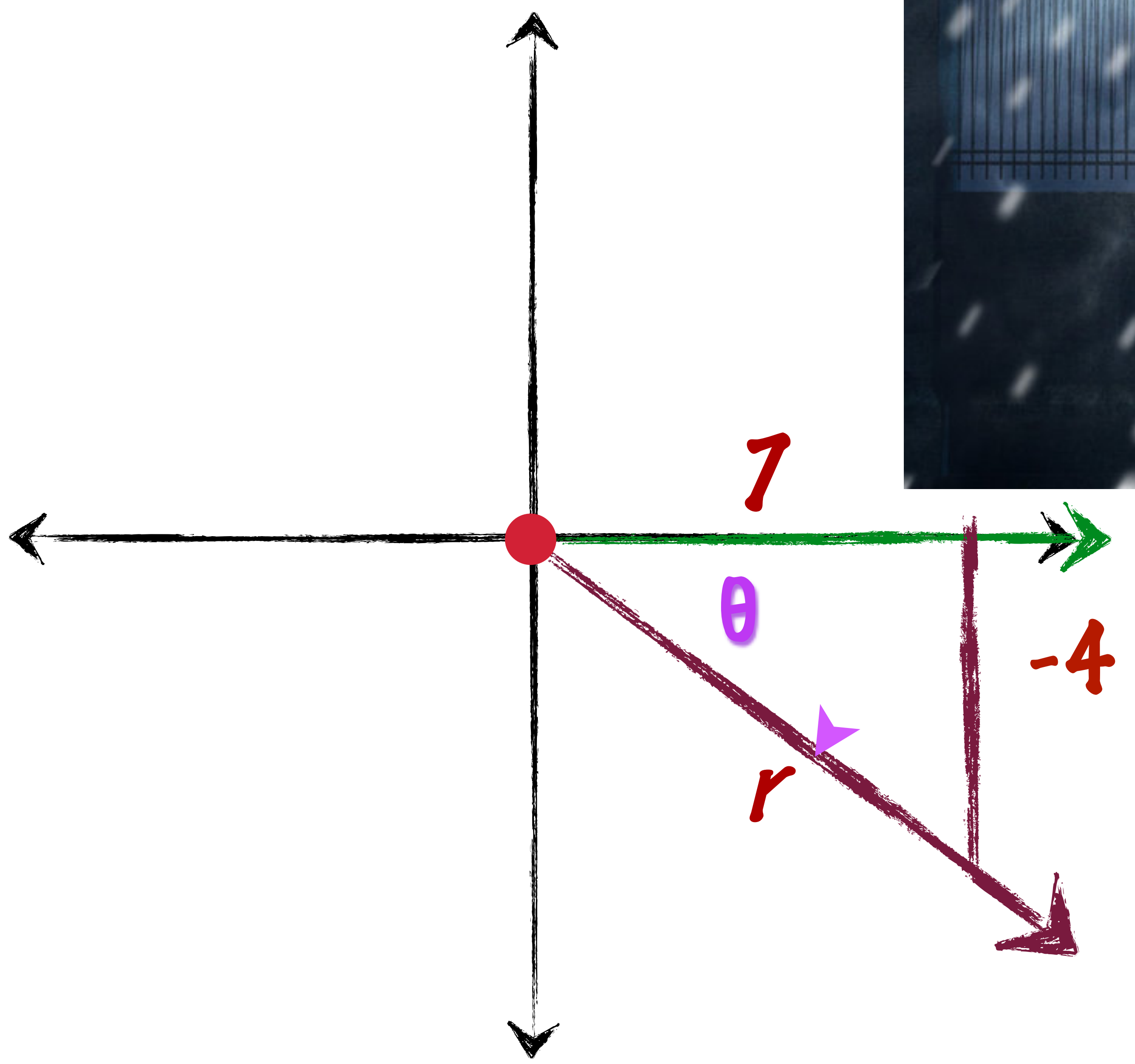
First: Let $\theta = \tan^{-1}\left(-\frac{4}{7}\right)$ $\quad \tan\theta = -\frac{4}{7} = \frac{y}{x}$

θ is an angle in the 4th quadrant. $\left(-\frac{\pi}{2} < \theta < \frac{\pi}{2}\right)$

$\sec\theta = \frac{r}{x}$ We need to find r.

$(-4)^2 + 7^2 = r^2 \quad r^2 = 65 \quad r = \sqrt{65}$

$\sec\theta = \frac{\sqrt{65}}{7} \quad \sec\left(\tan^{-1}\left(-\frac{4}{7}\right)\right) = \frac{\sqrt{65}}{7}$



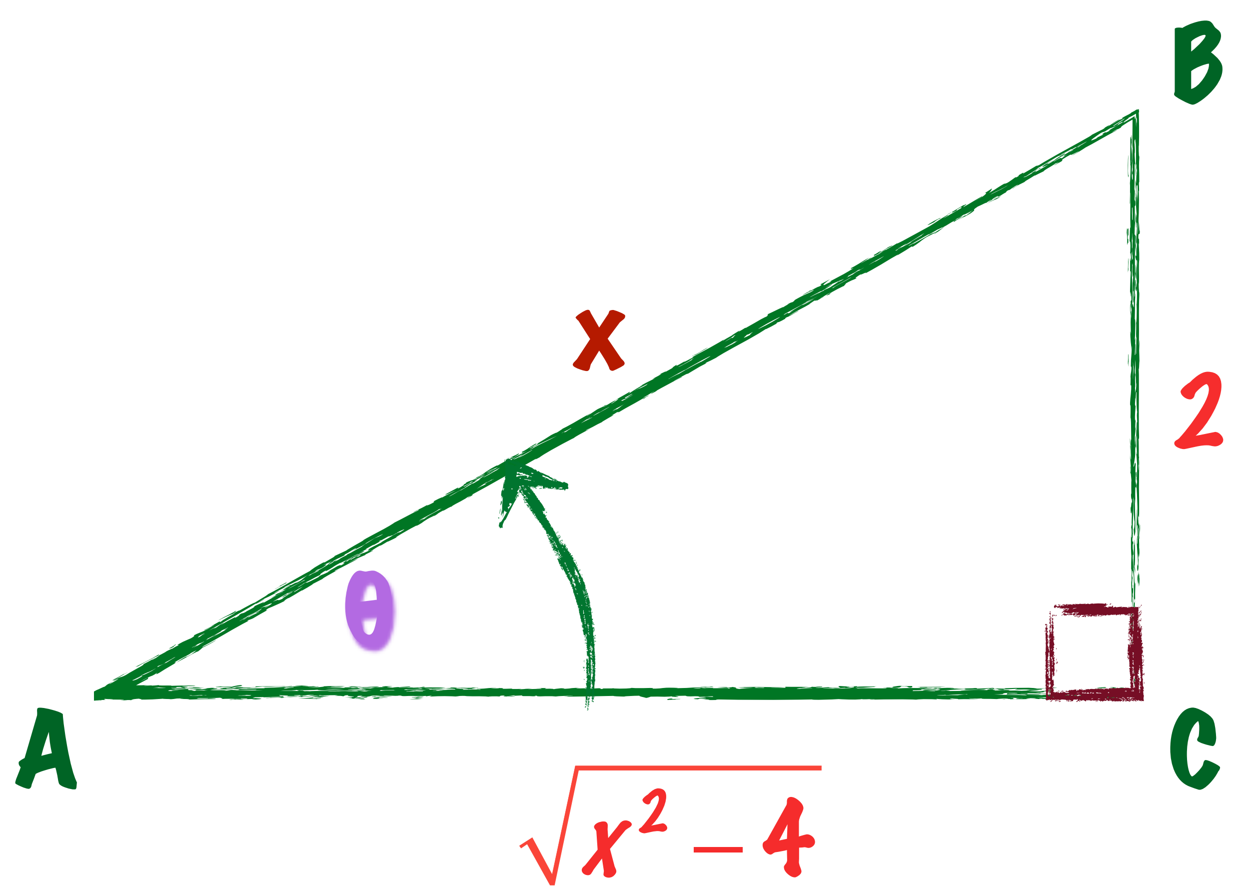
Evaluating Compositions of Functions and Their Inverses

Use a right triangle to write $\cos\left(\sin^{-1}\dfrac{2}{x}\right)$ as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x.

First: Let $\theta = \sin^{-1}\left(\dfrac{2}{x}\right)$ $\quad \sin\theta = \dfrac{2}{x} = \dfrac{opp}{hyp}$ $\quad \left(-\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}\right)$

We need to find adj.

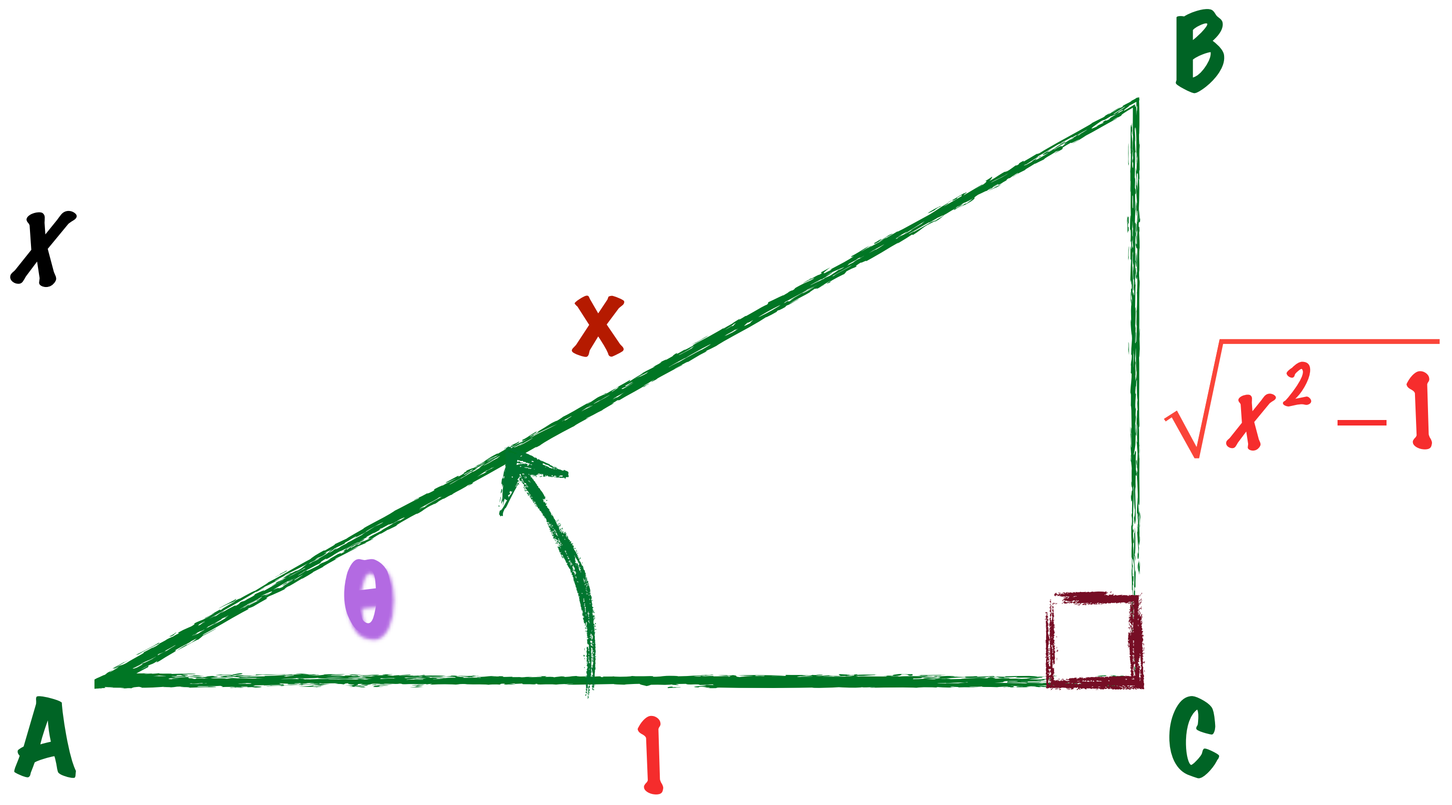
$$\cos\left(\sin^{-1}\left(\frac{2}{x}\right)\right) = \cos\theta = \frac{\sqrt{x^2-4}}{x}$$



Use a right triangle to write $\sec\left(\cos^{-1}\frac{1}{x}\right)$ as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x.

First: Let $\theta = \cos^{-1}\left(\frac{1}{x}\right)$ $\quad \cos\theta = \frac{1}{x}$ $\quad (0 < \theta < \pi)$

$$\sec\theta = \frac{r}{x} = \frac{1}{\frac{1}{x}} = x$$



Find the exact value, if possible: $\cos^{-1}\left(\cos\frac{9\pi}{4}\right)$

$$\cos\left(t+2\pi n\right)=\cos t$$

$$\cos\frac{9\pi}{4}=\cos\left(2\pi+\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

$$\cos^{-1}\left(\cos\frac{\pi}{4}\right)=\frac{\pi}{4}$$