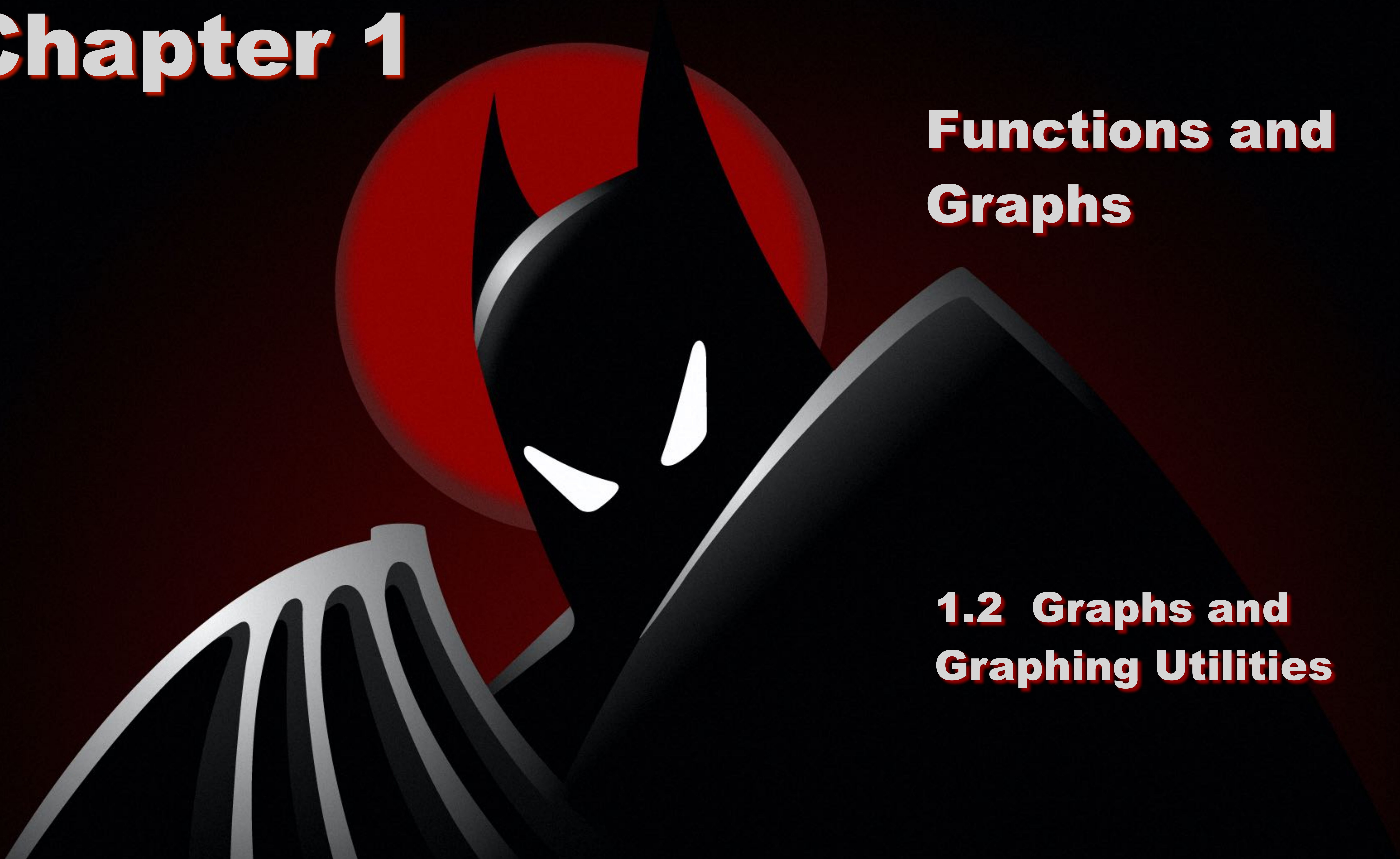
Chapter 1

Functions and Graphs



1.2 Graphs and Graphing Utilities

Chapter 1.2

Homework

1.2 p22 5-43 odd, 57-69 odd

Chapter 1.2

Learning Target

F–IF.4

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Chapter 1.2

Success Criteria

I can graph equations in two variables in the rectangular coordinate system.

I can identify intercepts of the graph of an equation.

I can use symmetry of an equation to draw the graph.

I can determine the equation and graph of a circle.

I can interpret information about mathematical models using technology and key features.

I can graph equations in two variables in the rectangular coordinate system.

Graphs of Equations

I can graph equations in two variables in the rectangular coordinate system.

- An equation in two variables is exactly that, an equation with two variables. Typically the variables are x and y, but any variable might be used. The formula $A = \pi r^2$ is an equation in two variables, **A** and **r**.

  - The solution set of an equation in two variables consists of ordered pair (x,y) **(POINTS)**. The solution set is the set of all **POINTS** that satisfy the equation (make the equation true).

  - The graph of an equation in two variables is simply the collection of all **POINTS** that are solutions to the equation, plotted on the Cartesian (coordinate) plane. You might think of the graph as a picture of the answer key to the equation.

So to repeat,

The solutions to an equation in two variables are...

ordered pair,

(x, y)

In other words, the solutions are …

POINTS

Graphing Equations

I can graph equations in two variables in the rectangular coordinate system.

- The **graph of an equation** is the graph of its ordered pairs.

  - Graph the functions $y = 2x$ and $y = 2x - 3$ in the same rectangular coordinate system. Select integers for x, starting with –2 and ending with 2.

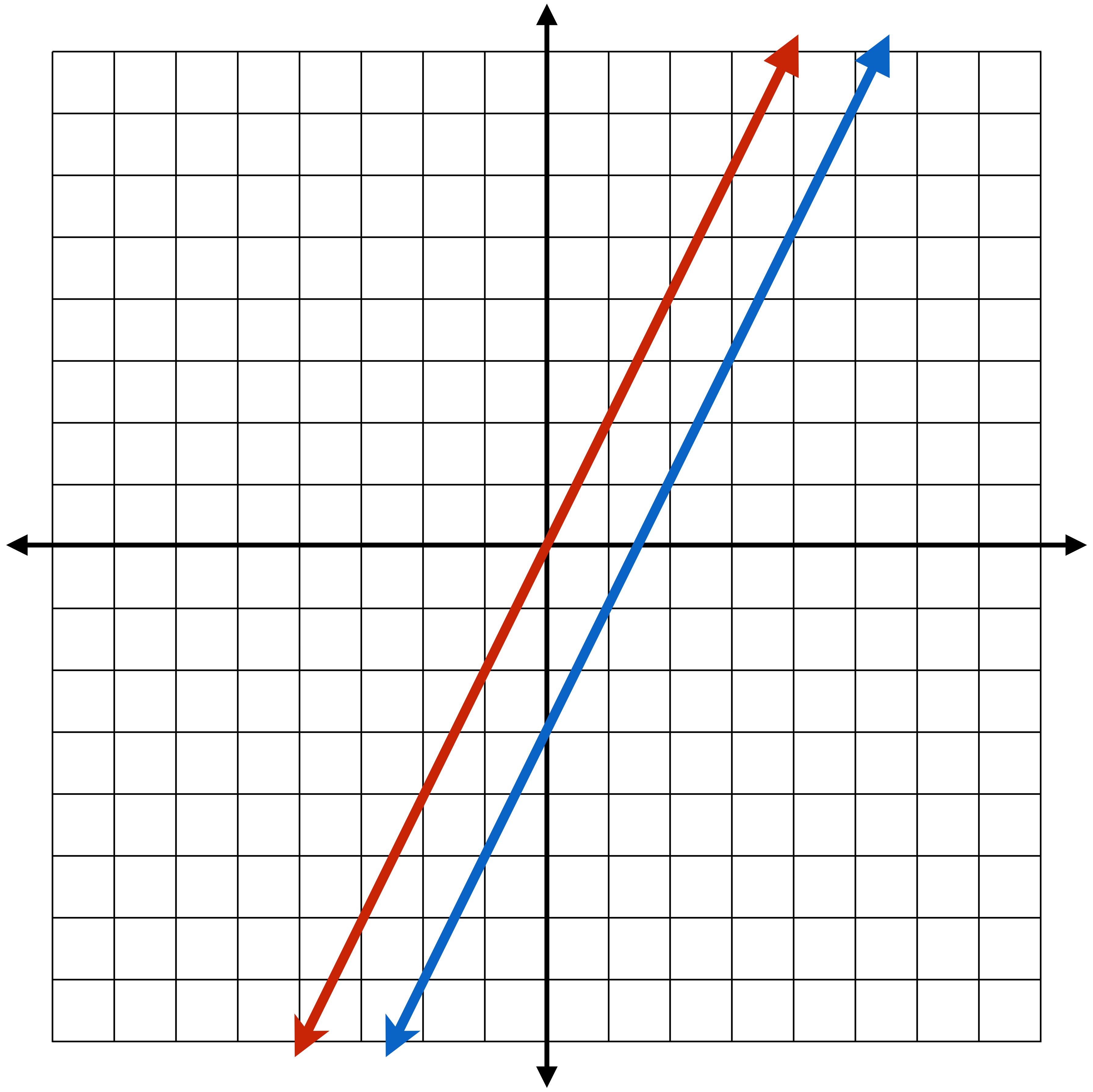
Why do we choose –2 to 2 for our x values?

- There are many ways to find the graph of an equation, and I am certain you have been shown many, but the only method that works **every single time** is by **using a table of values**. The only conditions necessary to graph using a table of values are that you are able to find points and you know the basic shape of the graph (parent function).

- We set up a partial table of values for each equation, plot the points, and connect the dots.

$y = 2x$

x	y = 2x
-2	-4
-1	-2
0	0
1	2
2	4



$y = 2x - 3$

x	y = 2x-3
-2	-7
-1	-5
0	-3
1	-1
2	1

- Your book lists the steps for graphing an equation by using a table of values.

Sketching the Graph of an Equation by Point Plotting

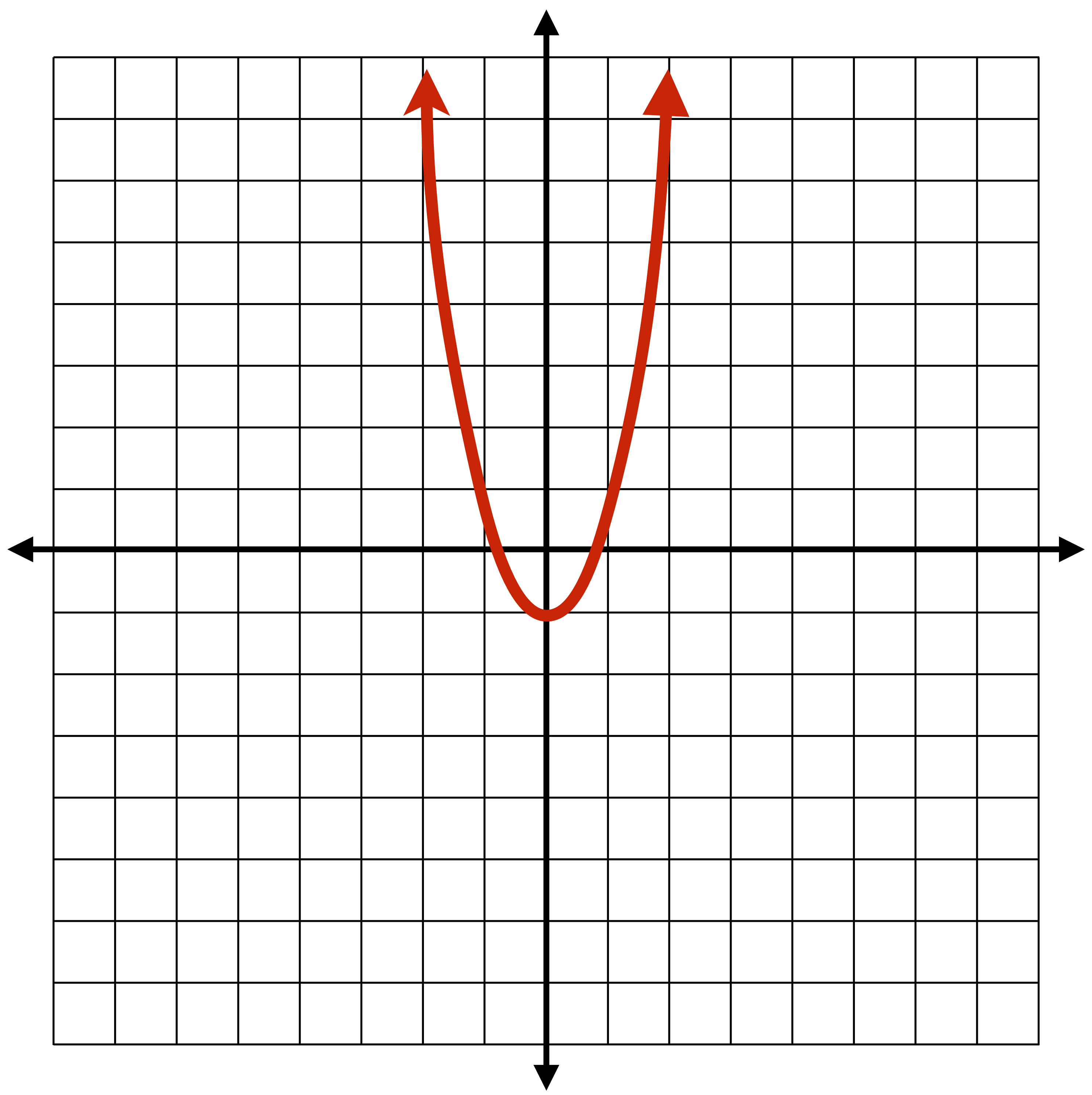
1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of values showing several solution points.
3. Plot these points on a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

I can graph equations in two variables in the rectangular coordinate system.

- Graph the function $y = 2x^2 - 1$. Select integers for x, starting with –2 and ending with 2.

Why do we choose –2 to 2 for our x values?

x	$y = 2x^2 - 1$
–2	7
–1	1
0	–1
1	1
2	7



STUDY TIP

One of your goals in this course is to learn to classify the basic shape of a graph from its equation. For instance, you will learn that the *linear equation* in Example 2 has the form

$$y = mx + b$$

and its graph is a line. Similarly, the *quadratic equation* in Example 3 has the form

$$y = ax^2 + bx + c$$

and its graph is a parabola.

Graphing an Equation by Plotting Points

I can graph equations in two variables in the rectangular coordinate system.

- Graph $y = |x + 1|$
  - To graph we need a few things.
    - We need some idea about the shape the graph will take.
    - And we need some points.
  - To find some points, select integers for *x*, (here we will start with –4 and end with 2)...

... and then find the appropriate **y-value.**

This gives us a **table of values**.

x	y=|x+1|	y
-4	y=|-4+1|	3
-3	y=|-3+1|	2
-2	y=|-2+1|	1
-1	y=|-1+1|	0
0	y=|0+1|	1
1	y=|1+1|	2
2	y=|2+1|	3

Solutions

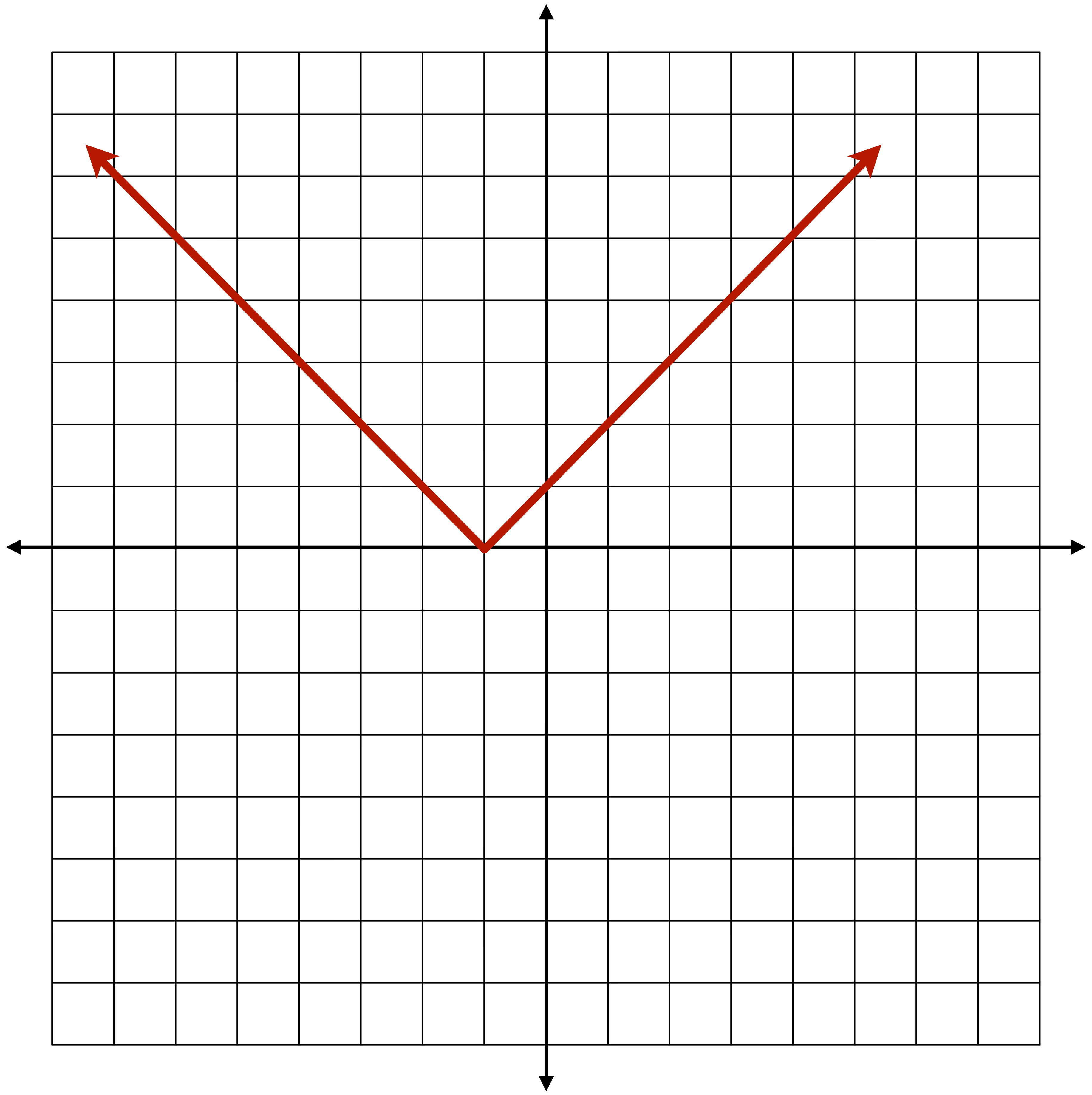
(x,y)
(-4,3)
(-3,2)
(-2,1)
(-1,0)
(0,1)
(1,2)
(2,3)

Graphing an Equation by Plotting Points

I can graph equations in two variables in the rectangular coordinate system.

- Graph $y = |x + 1|$
  - We plot the points from our table of values.
  - Then connect the dots to draw the graph
  - We will soon learn to add arrow notation for end behavior to the graph at the arrows.

x	y	(x,y)
-4	3	(-4,3)
-3	2	(-3,2)
-2	1	(-2,1)
-1	0	(-1,0)
0	1	(0,1)
1	2	(1,2)
2	3	(2,3)




I can identify intercepts of the graph of an equation.

Intercepts

I can identify intercepts of the graph of an equation.

- An ***x*-intercept** of a graph is the ***x*-coordinate** of a point where the ***y*-coordinate** is zero. It also happens to be where the graph intersects the ***x*-axis**. The ***y*-coordinate** corresponding to an ***x*-intercept** is **always zero**.

$$(x, 0)$$

  - To find the ***x*-intercept** of a graph; set the ***y*-coordinate** to 0 and solve for ***x*.**

- A ***y*-intercept** of a graph is the ***y*-coordinate** of a point where the ***x*-coordinate** is zero. It also happens to be where the graph intersects the ***y* axis**. **The *x*-coordinate** corresponding to a ***y*-intercept** **is always zero**.

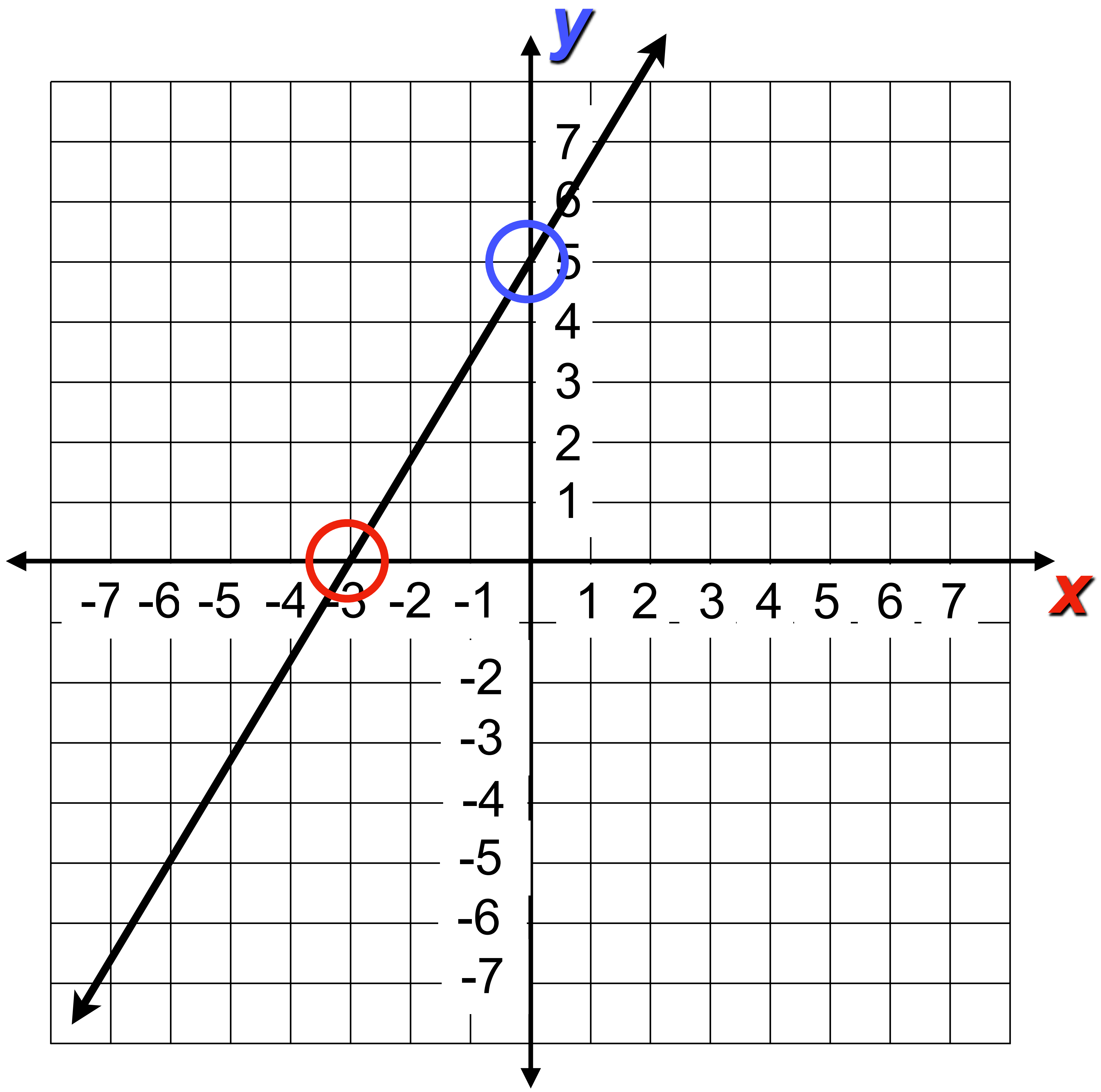
$$(0, y)$$

  - To find the ***y*-intercept** of a graph; set the ***x*-coordinate** to 0 and solve for ***y***.

Identifying Intercepts

I can identify intercepts of the graph of an equation.

- Identify the ***x***- and ***y***-intercepts.
  - The graph crosses the ***x***-axis at (–3, 0). Thus, the ***x*-intercept** is –3.
  - The graph crosses the ***y***-axis at (0, 5). Thus, the ***y*-intercept** is 5.



- Find the **x**- and **y**-intercepts for y = 2x – 6.

x-intercept

Set **y** = 0   **0** = 2**x** – 6

Solve for **x**   2**x** = 6   **x** = 3

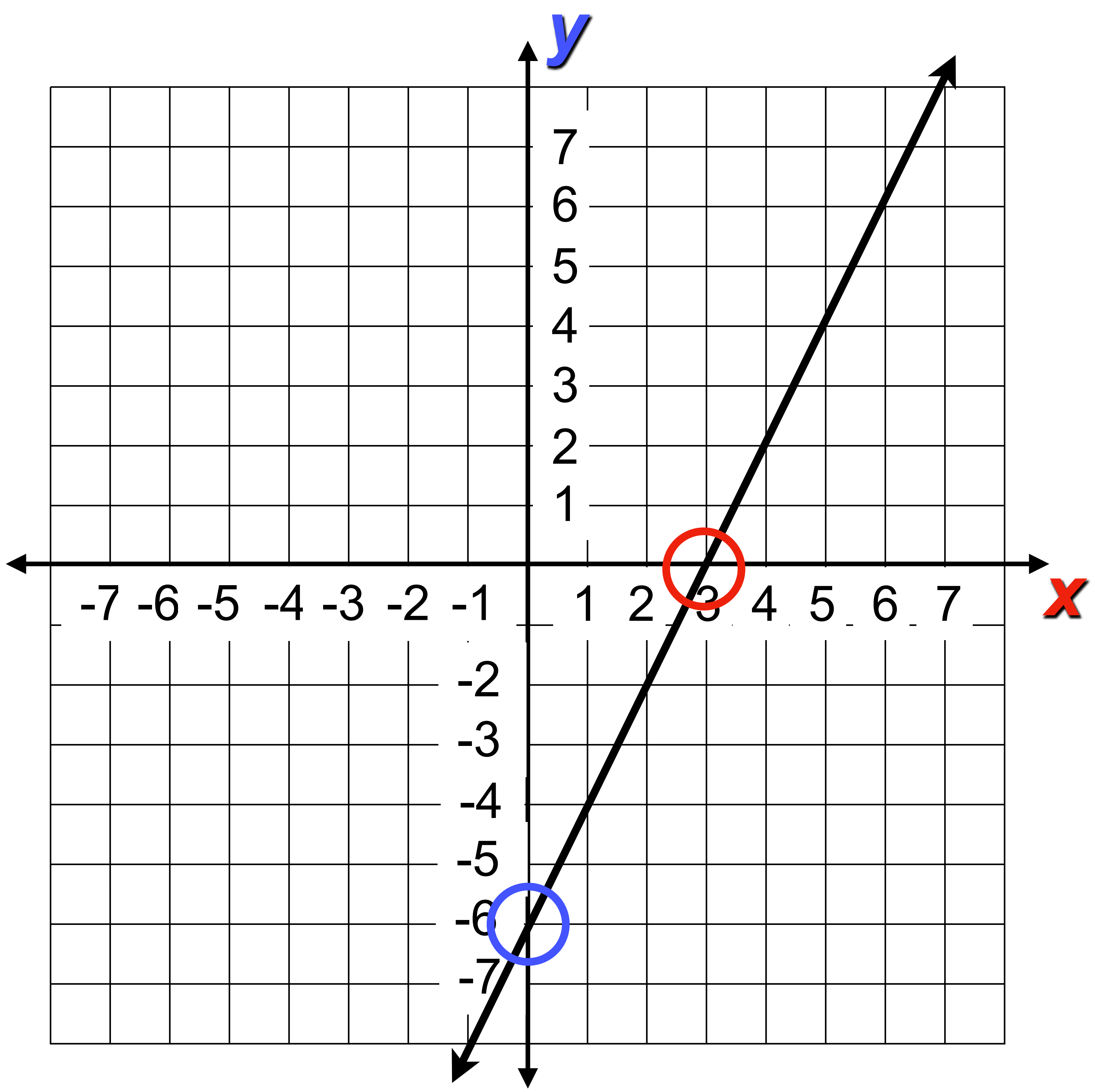
x-intercept = 3

y-intercept

Set **x** = 0   **y** = 2**(0)** – 6

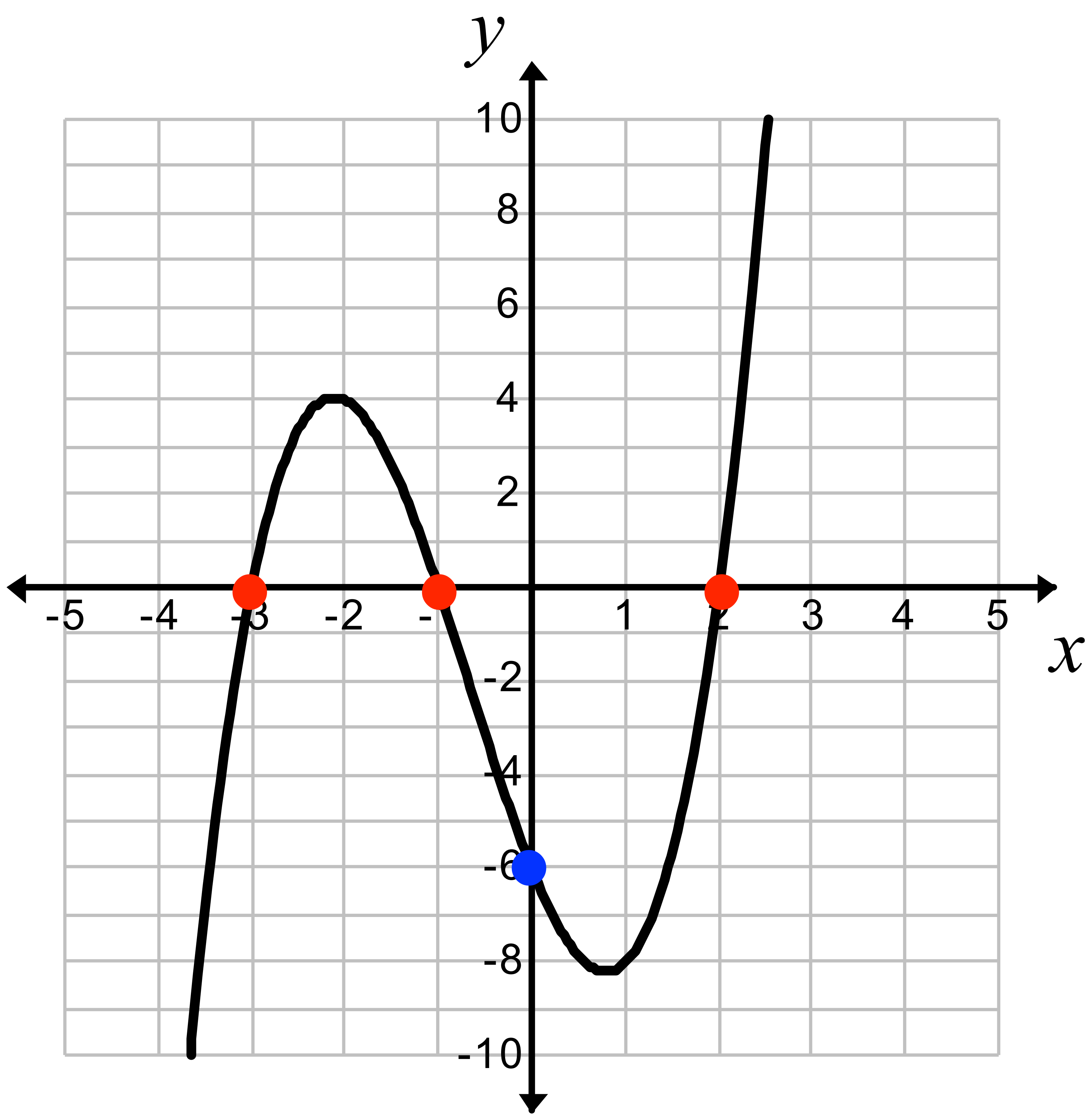
Solve for **y**   **y** = –6

y-intercept = –6



Identifying Intercepts

- Identify the *x*- and *y*-intercepts for the graph of *f(x)*.



- The **x-intercepts** are -3, -1, and 2.

  or (–3, 0), (–1, 0), and (2, 0).

- The **y-intercept** is –6.

  or (0, –6)

Finding Intercepts

I can identify intercepts of the graph of an equation.

- Find the x and y intercepts for $y = x^2 - x - 6$

x-intercepts

$$0 = x^2 - x - 6$$

$$0 = (x+2)(x-3)$$

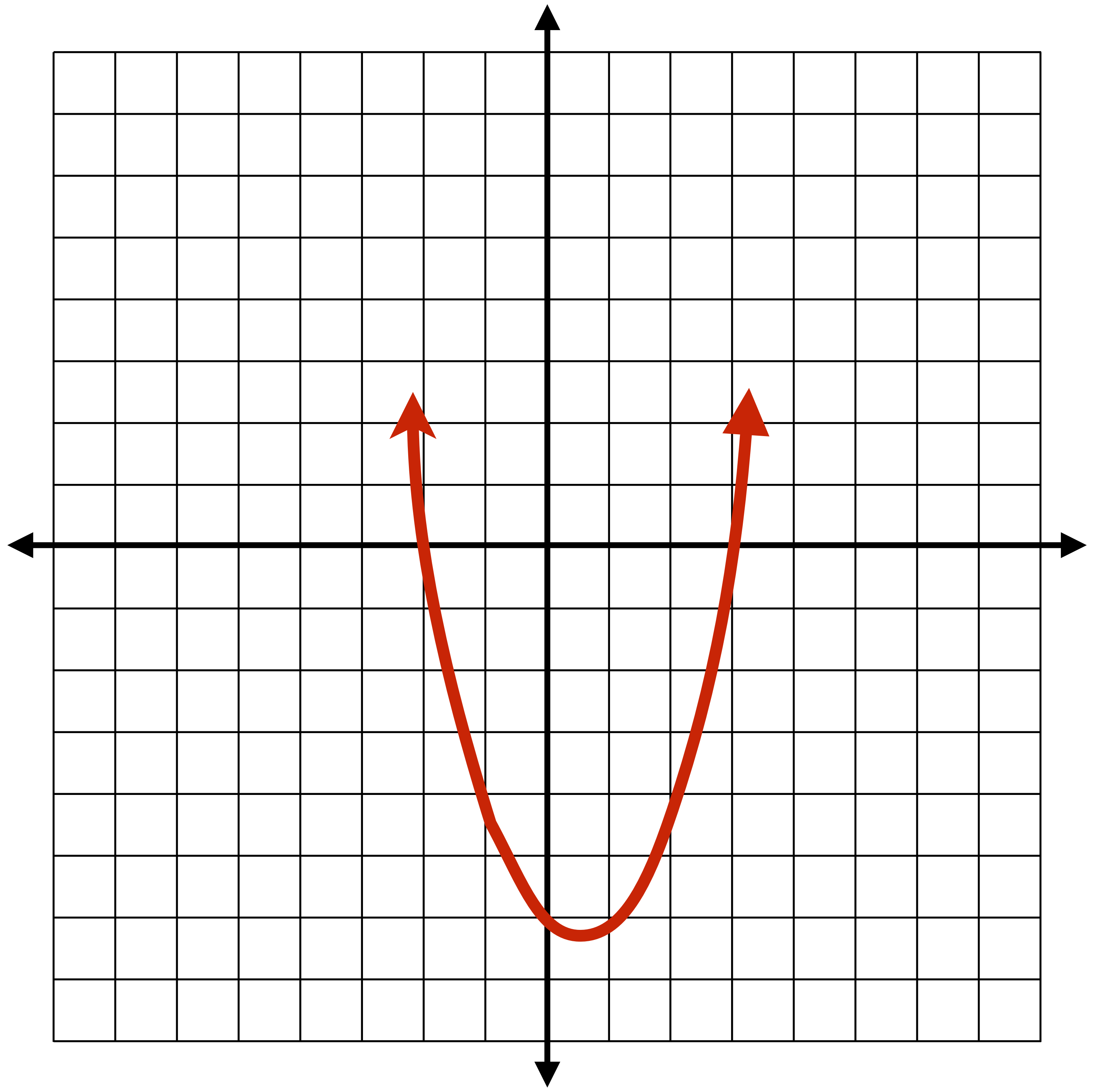
x + 2 = 0 or x – 3 = 0

x = –2 or x = 3

y-intercept

$$y = 0^2 - 0 - 6$$

y = –6



I can use symmetry of an equation to draw the graph.

- Graphs of equations are often symmetric with respect to lines or points.
  - For example, a parabola is symmetric with respect to the axis of symmetry, hence the name "**axis of symmetry**".
  - What we will focus on for the moment are three basic symmetries. Symmetry with respect to the x-axis, symmetry with respect to the y-axis, and symmetry with respect to the origin.

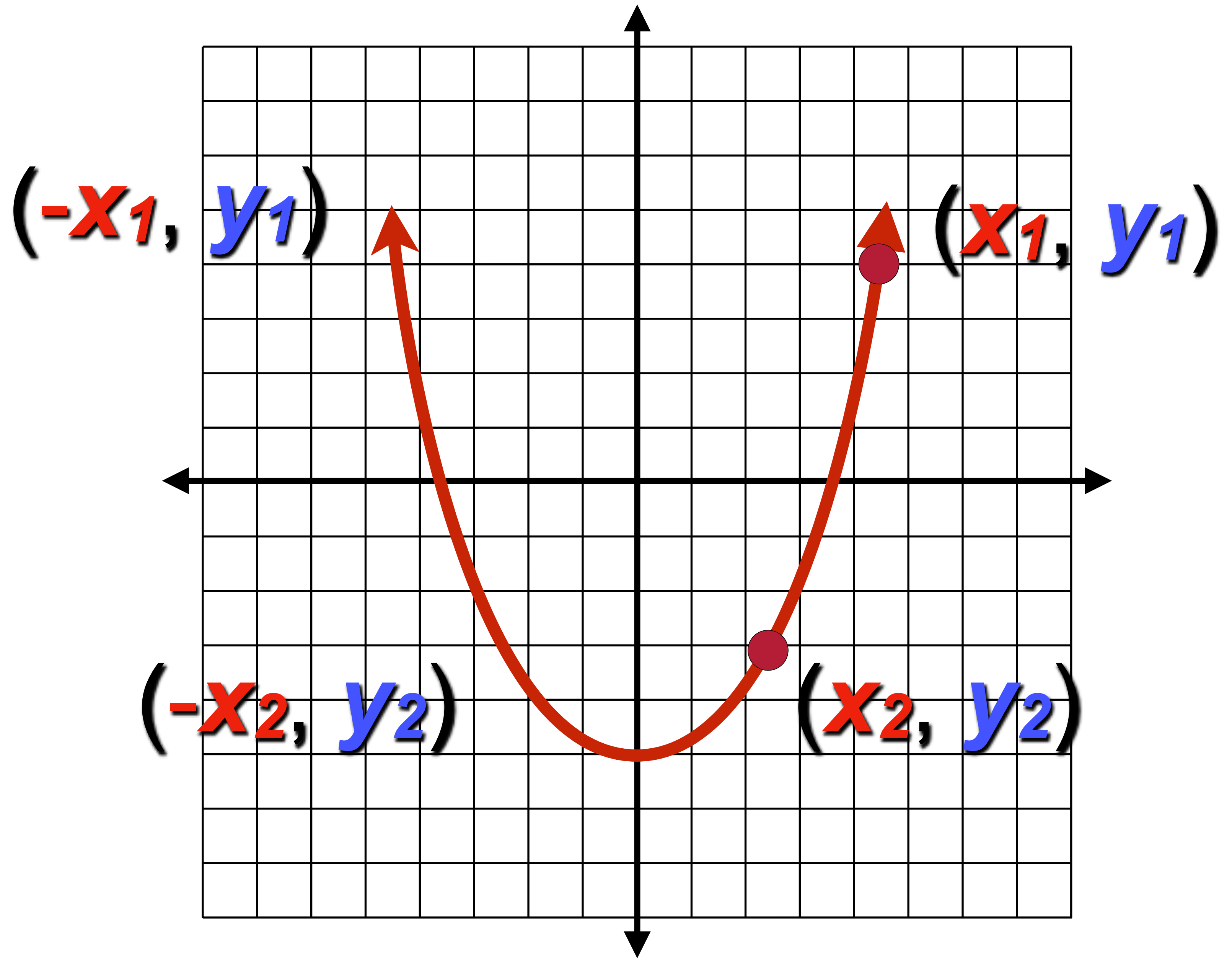
Graphical Tests for Symmetry

1. A graph is **symmetric with respect to the x-axis** if, whenever (x, y) is on the graph, $(x, -y)$ is also on the graph.
2. A graph is **symmetric with respect to the y-axis** if, whenever (x, y) is on the graph, $(-x, y)$ is also on the graph.
3. A graph is **symmetric with respect to the origin** if, whenever (x, y) is on the graph, $(-x, -y)$ is also on the graph.

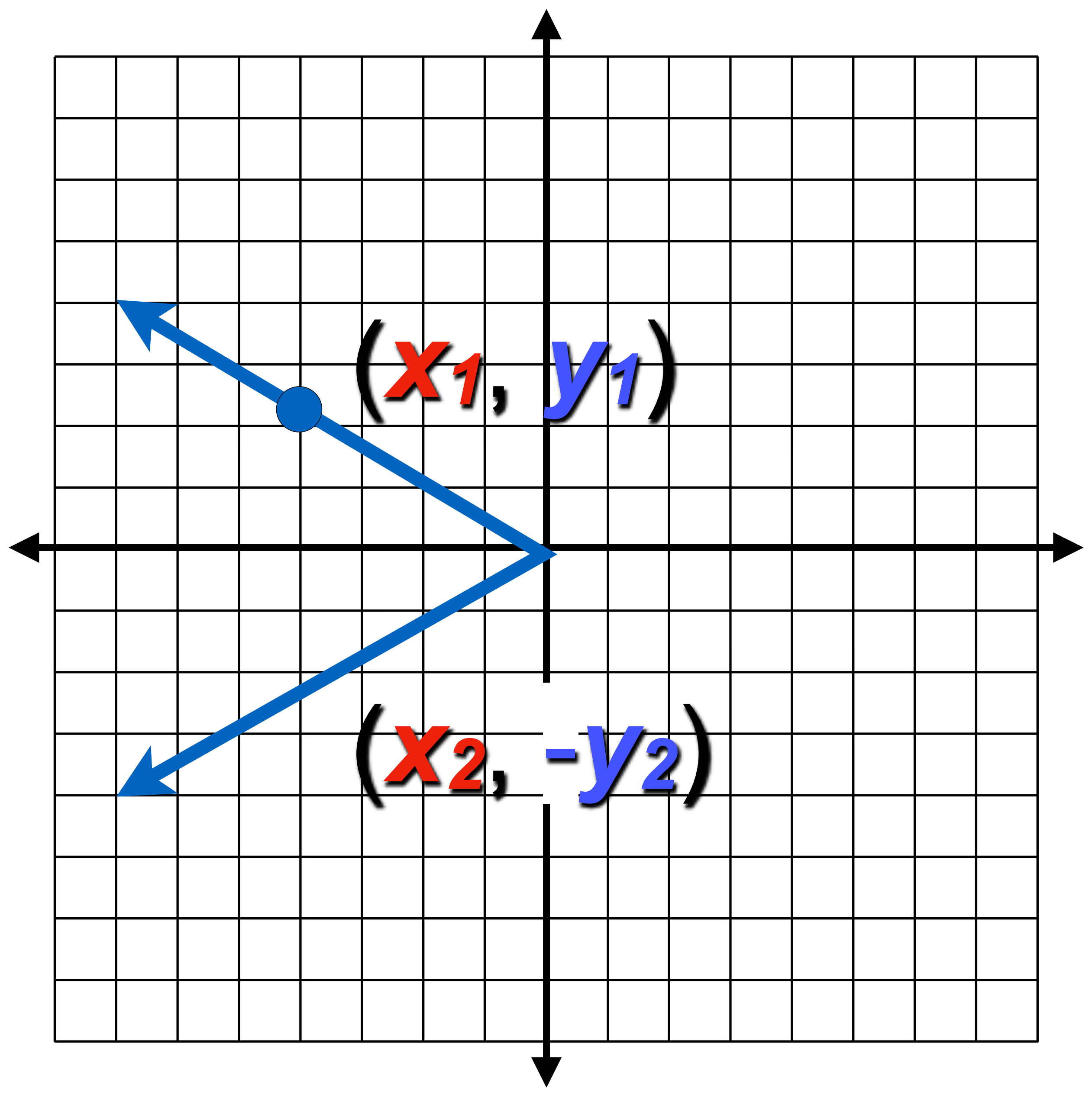
Symmetry

I can use symmetry of an equation to draw the graph.

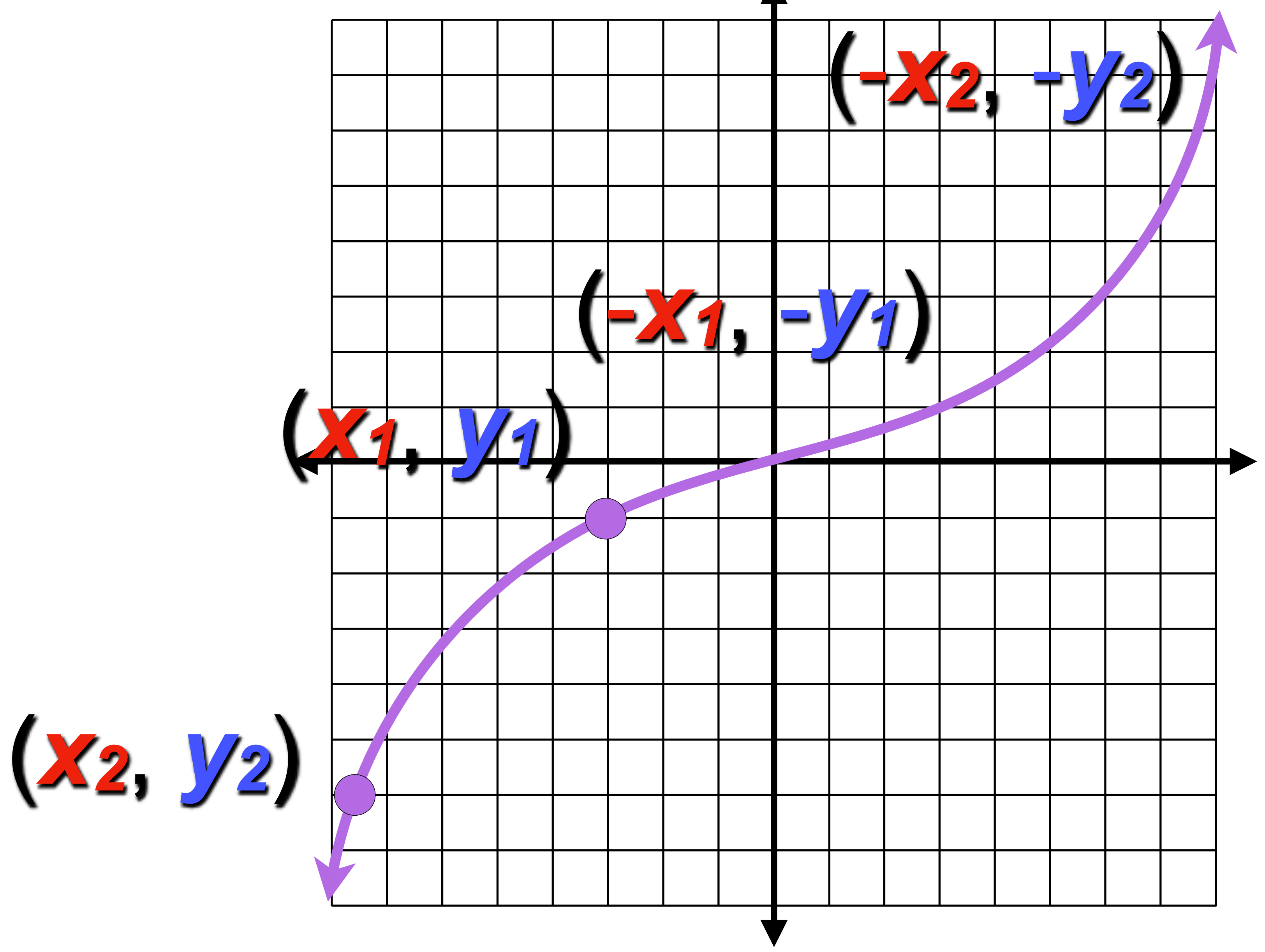
- Symmetric with respect to y-axis



- Symmetric with respect to x-axis



- Symmetric with respect to origin



- You can check symmetry without graphing by replacing x and/or y with the opposite value and test for changes.

Algebraic Tests for Symmetry

1. The graph of an equation is symmetric with respect to the x-axis if replacing y with $-y$ yields an equivalent equation.
2. The graph of an equation is symmetric with respect to the y-axis if replacing x with $-x$ yields an equivalent equation.
3. The graph of an equation is symmetric with respect to the origin if replacing x with $-x$ and y with $-y$ yields an equivalent equation.

- Determine, if any, the symmetry. $y = x^2 - x - 6$

$$-y = x^2 - x - 6$$
$$y = -x^2 + x + 6$$

$$y = (-x)^2 - (-x) - 6$$
$$y = x^2 + x - 6$$

$$-y = (-x)^2 - (-x) - 6$$
$$y = -x^2 - x + 6$$

- Not symmetric with the x-axis.
- Not symmetric with the y-axis.
- Not symmetric with the origin.

- Determine, if any, the symmetry. $y = |x| - 6$

$$-y = |x| - 6$$
$$y = -|x| + 6$$

- Not symmetric with the x-axis.

$$y = |-x| - 6$$
$$y = |x| - 6$$

- Symmetric with the y-axis.

$$-y = |-x| - 6$$
$$y = -|x| + 6$$

- Not symmetric with the origin.

- Determine, if any, the symmetry. $y^2 + x^2 = 4$

$(-y)^2 + x^2 = 4$

$y^2 + x^2 = 4$

- Symmetric with the x-axis.

$y^2 + (-x)^2 = 4$

$y^2 + x^2 = 4$

- Symmetric with the y-axis.

$(-y)^2 + (-x)^2 = 4$

$y^2 + x^2 = 4$

- Symmetric with the origin.

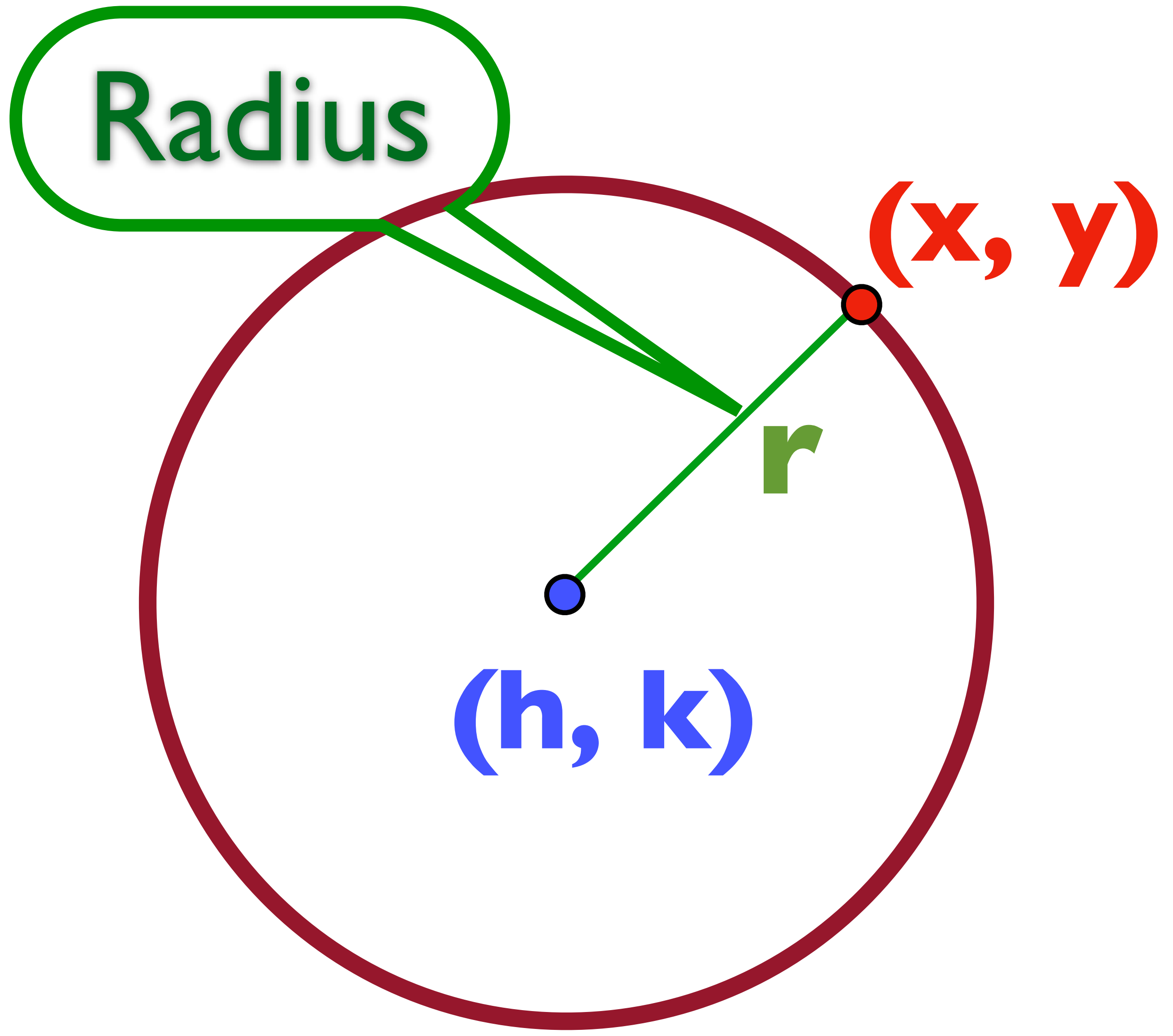
I can determine the equation and graph of a circle.

Circles

I can determine the equation and graph of a circle.

- A **circle** is the set of points in a plane that are a constant distance, called the **radius**, from a fixed point, called the **center**.

- As all of the points on a circle are the same distance from the center of the circle, you can use the Distance Formula to find the equation of a circle.



$$r = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

or

$$(x - h)^2 + (y - k)^2 = r^2$$

Circles

I can determine the equation and graph of a circle.

- Write the standard form of the equation of the circle with center (0, –6) and radius 10.

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - 0)^2 + (y - -6)^2 = 10^2$$

$$x^2 + (y + 6)^2 = 100$$

Find the center and radius, then graph the circle whose equation is

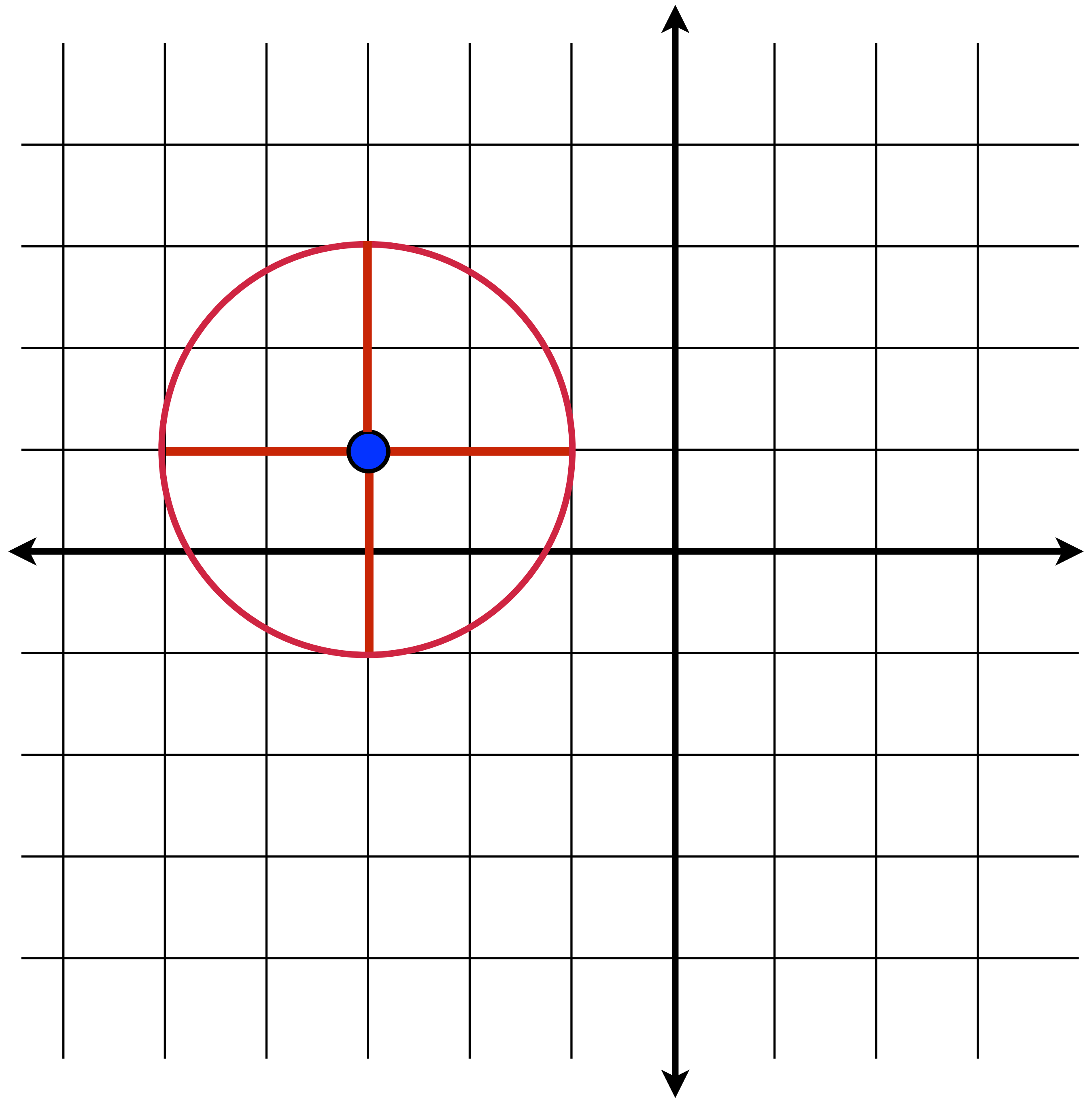
$$(x + 3)^2 + (y - 1)^2 = 4$$

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(x - -3)^2 + (y - 1)^2 = 4$$

The center is (–3, 1).

The radius is $\sqrt{4} = 2$



- Write the standard form of the equation of the circle with center (-3, -6) and containing the point (2, 1).

$$(x - h)^2 + (y - k)^2 = r^2$$

$$(2 - -3)^2 + (1 - -6)^2 = r^2$$

$$5^2 + 7^2 = r^2$$

$$74 = r^2$$

$$r = \sqrt{74}$$

I can interpret information about mathematical models using technology and key features.

Mathematical Modeling

I can interpret information about mathematical models using technology and key features.

The table shows the life expectancies of a child (at birth) in the United States for selected years from 1920 to 2000. A model for the life expectancy during this period is:

$$y = -0.0014t^2 + 0.4129t + 46.6573, \quad 20 \le t \le 120$$

Where y = life expectancy, and t = time (years from 1900)

a. Plot the points from the table (scatterplot)

b. Graph the model and compare with scatterplot.

c. Determine the life expectancy in 1949 using the model and graph.

d. Determine the life expectancy in 2007.

e. Do you think the model is still valid today? Should we use the model to predict life expectancy for a child born this year?

Year	Life Expectancy, y
1900	46.41
1910	50.08
1920	54.50
1930	57.96
1940	61.43
1950	65.63
1960	66.66
1970	67.15
1980	69.94
1990	71.82
2000	74.03
2010	75.40
2020	76.50

I can interpret information about mathematical models using technology and key features.

a. Plot the points from the table (scatterplot)

Enter the data from the table into two lists. For the first list enter the value (year - 1900), so 1920 = 20.

Year	Life Expectancy, y
1900	46.41
1910	50.08
1920	54.50
1930	57.96
1940	61.43
1950	65.63
1960	66.66
1970	67.15
1980	69.94
1990	71.82
2000	74.03
2010	75.40
2020	76.50

Stat | 1:Edit | Select List "L1" | Enter first value "20" | Enter

Enter 2nd value "30" | Enter | Repeat to end of list

Move to List "L2" | Enter first value "54.1" | Enter | Repeat to end of list | 2nd | Quit

To plot the points you just entered

stat plot

2nd | y= | Enter | ON | TYPE" ⁘ | XList | 2nd | 1 | YList | 2nd | 2 | Mark

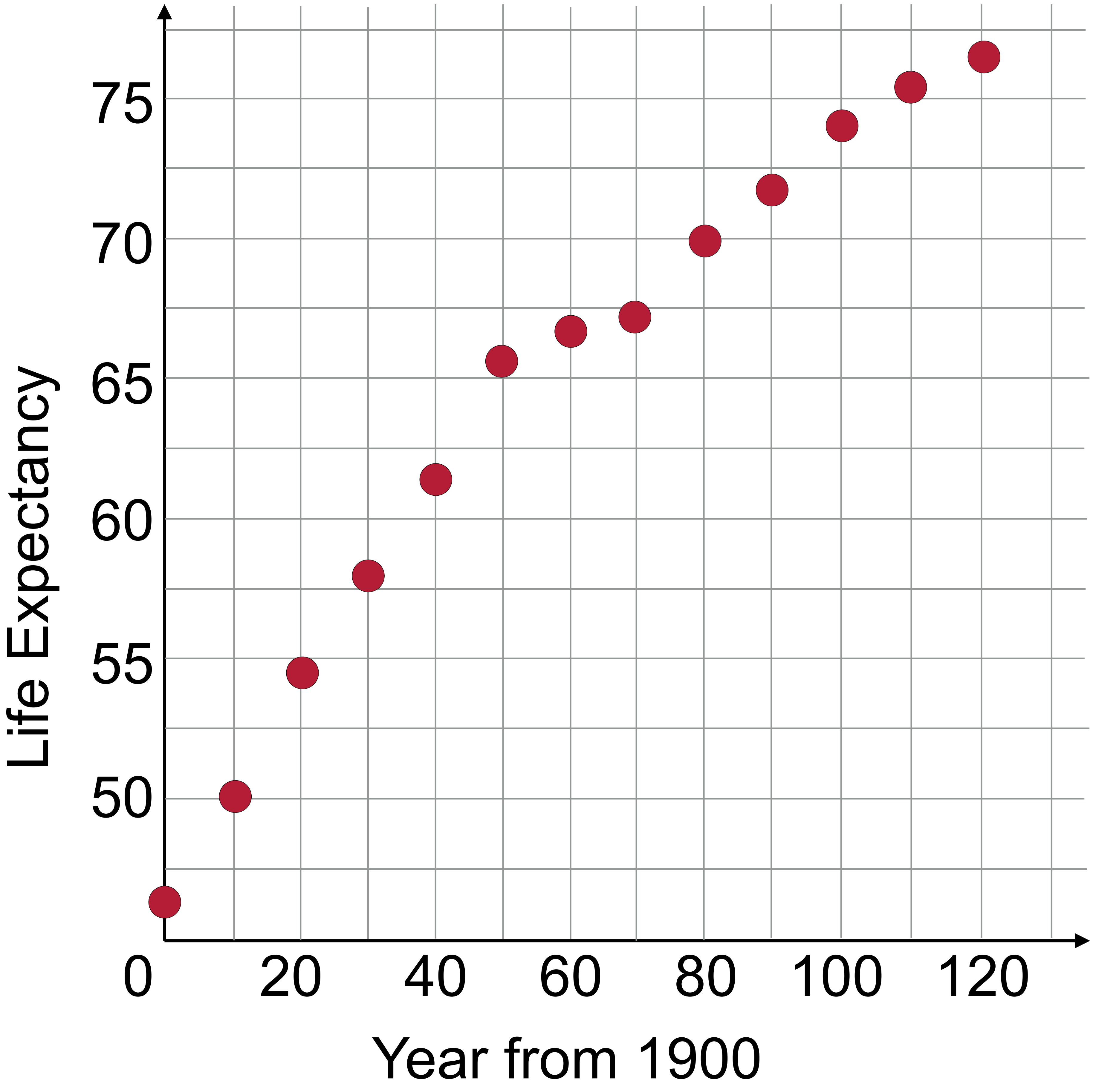
Zoom | 9

Modeling

I can interpret information about mathematical models using technology and key features.

- Your graph should look something like this

Year	Life Expectancy, y
1900	46.41
1910	50.08
1920	54.50
1930	57.96
1940	61.43
1950	65.63
1960	66.66
1970	67.15
1980	69.94
1990	71.82
2000	74.03
2010	75.40
2020	76.50



Modeling

I can interpret information about mathematical models using technology and key features.

b. Graph the model $y = -0.0014t^2 + 0.4129t + 46.6573,\ 20 \le t \le 120$

Y= (-) 0 . 0 0 1 4 X,T,θ,n X^2 + 0 . 4 1 2 9 X,T,θ,n

\+ 4 6 . 6 5 7 3

Once again, to see your graph Zoom 9

We can also rearrange the window to account for the x and y values.

Window Xmin= 0 Xmax= 1 0 0 Xscl= 1 Ymin= 0 Ymax= 1 2 0

Now we can graph both the points and the model Trace

Looks pretty good to me.

Year	Life Expectancy, y
1900	46.41
1910	50.08
1920	54.50
1930	57.96
1940	61.43
1950	65.63
1960	66.66
1970	67.15
1980	69.94
1990	71.82
2000	74.03
2010	75.40
2020	76.50

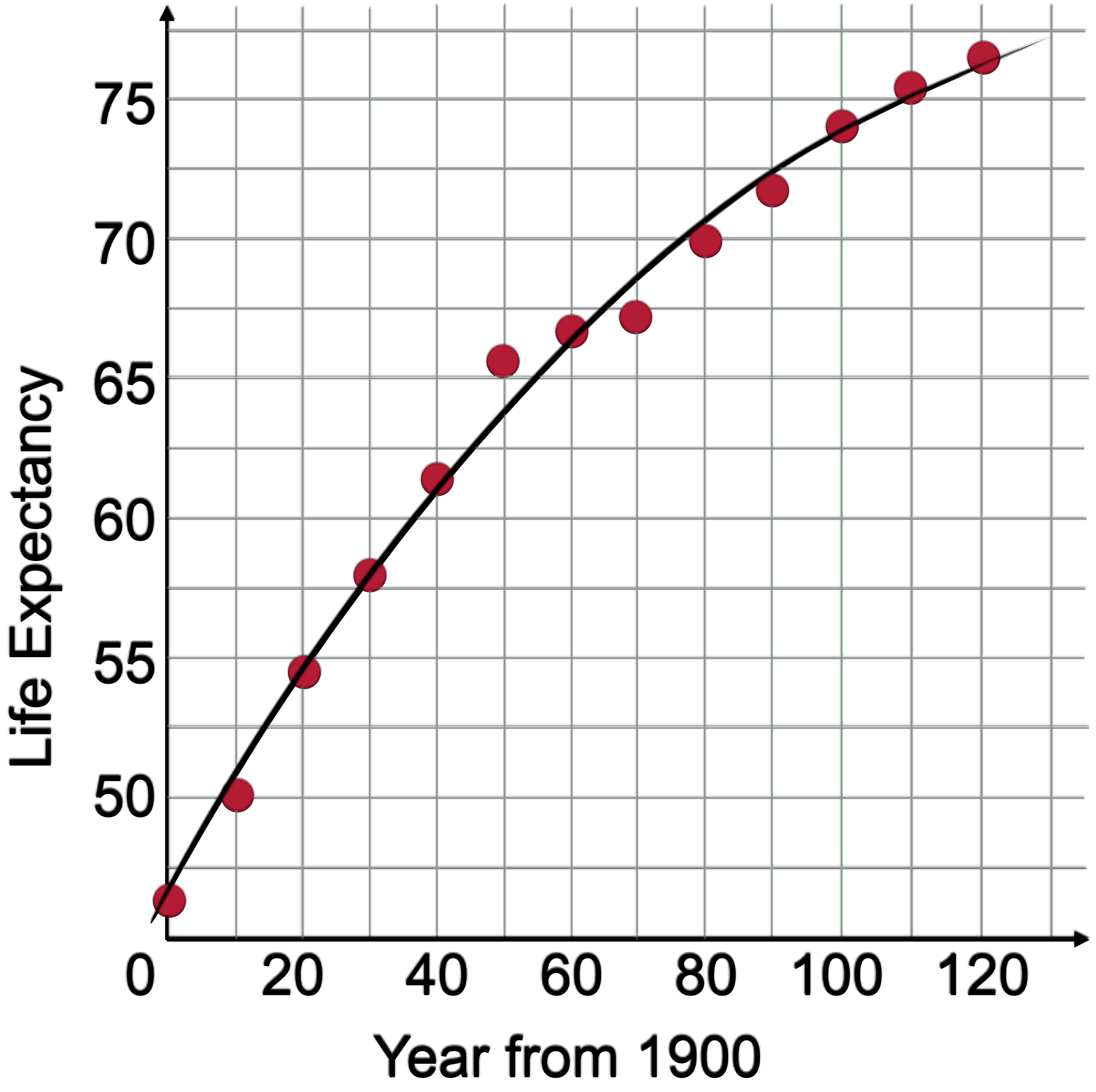
Modeling

I can interpret information about mathematical models using technology and key features.

- Here are the points and the graph of the model.

$$y = -0.0014t^2 + 0.4129t + 46.6573$$

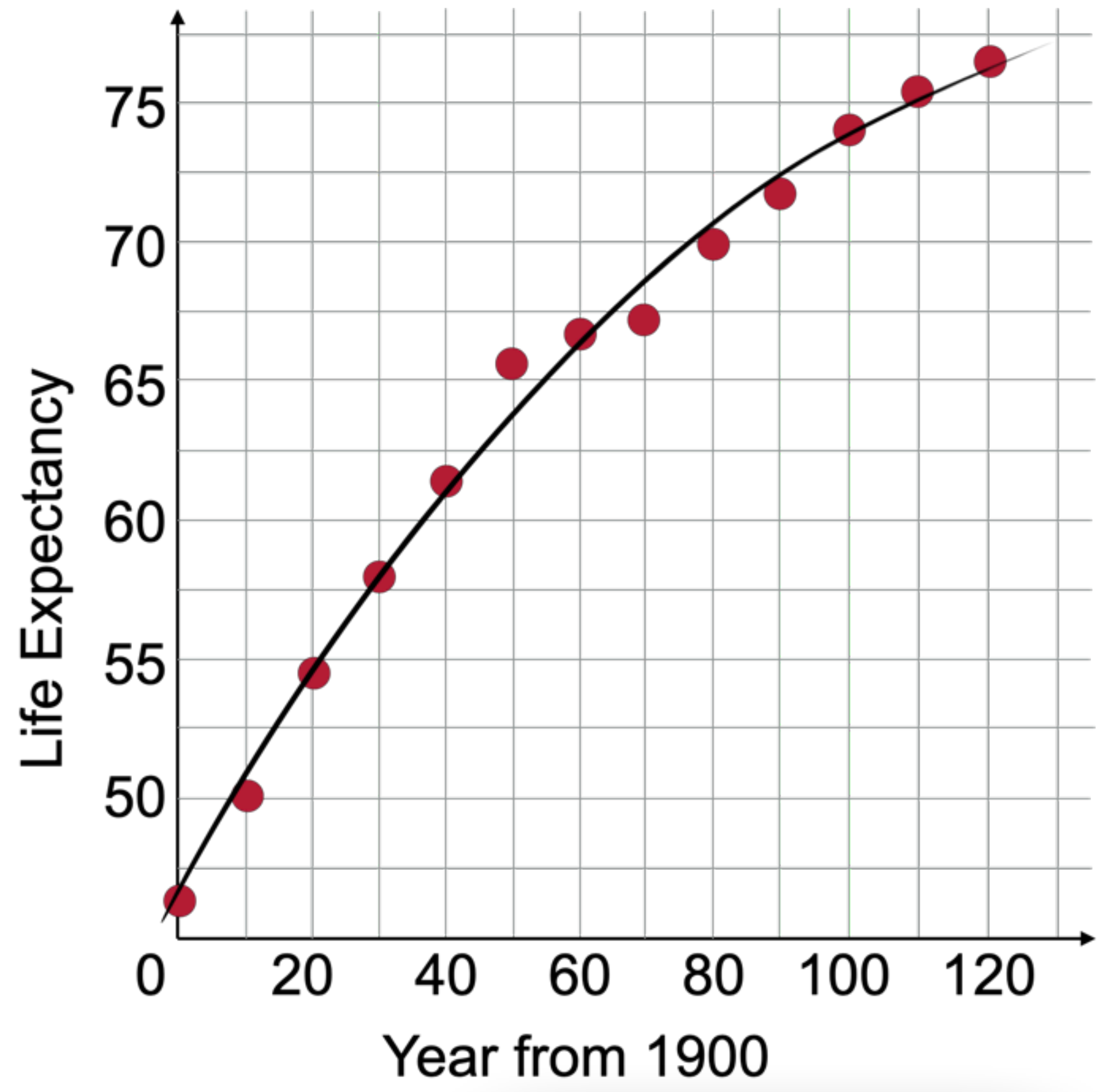
Year	Life Expectancy, y
1900	46.41
1910	50.08
1920	54.50
1930	57.96
1940	61.43
1950	65.63
1960	66.66
1970	67.15
1980	69.94
1990	71.82
2000	74.03
2010	75.40
2020	76.50



Modeling

I can interpret information about mathematical models using technology and key features.

$y = -0.0014t^2 + 0.4129t + 46.6573$



c. Determine the life expectancy in 1949 using the model and the graph.

To use the graph to calculate 1949, while in the “trace” function and the graph is showing:

calc

2nd Trace 1:value ENTER X= 4 9 63.5247

To use the model to calculate 1949

quit

2nd Mode back to home screen

These are important!

VARS ➢ YVARS ∀ 1:Function ENTER 1:Y_1 ENTER (4 9) ENTER

63.5247 same result.

Year	Life Expectancy, y
1900	46.41
1910	50.08
1920	54.50
1930	57.96
1940	61.43
1950	65.63
1960	66.66
1970	67.15
1980	69.94
1990	71.82
2000	74.03
2010	75.40
2020	76.50

I can interpret information about mathematical models using technology and key features.

d. Determine the life expectancy in 2004.

calc

2nd Trace 1:value ENTER X= 1 0 4 74.44

Use the model to estimate life expectancy.

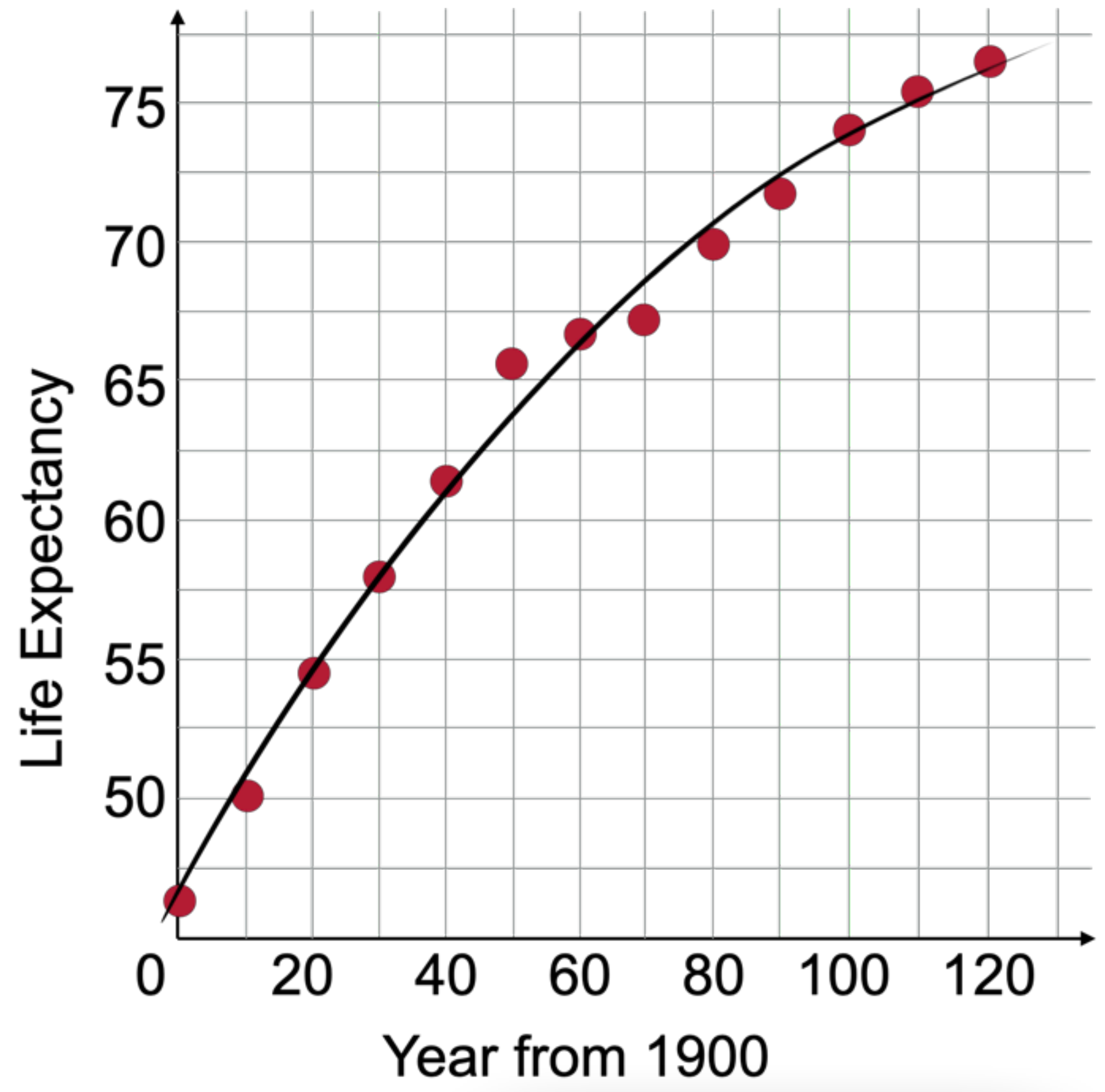
quit

2nd Mode back to home screen

VARS ➢ YVARS ∨ 1:Function ENTER 1:Y_1 ENTER (1 0 4) ENTER

74.44, the model seems to work pretty well.

This points out a caution when modeling data. Do not extrapolate far beyond the data. There is no guarantee the model will work beyond the known data.



Year	Life Expectancy, y
1900	46.41
1910	50.08
1920	54.50
1930	57.96
1940	61.43
1950	65.63
1960	66.66
1970	67.15
1980	69.94
1990	71.82
2000	74.03
2010	75.40
2020	76.50

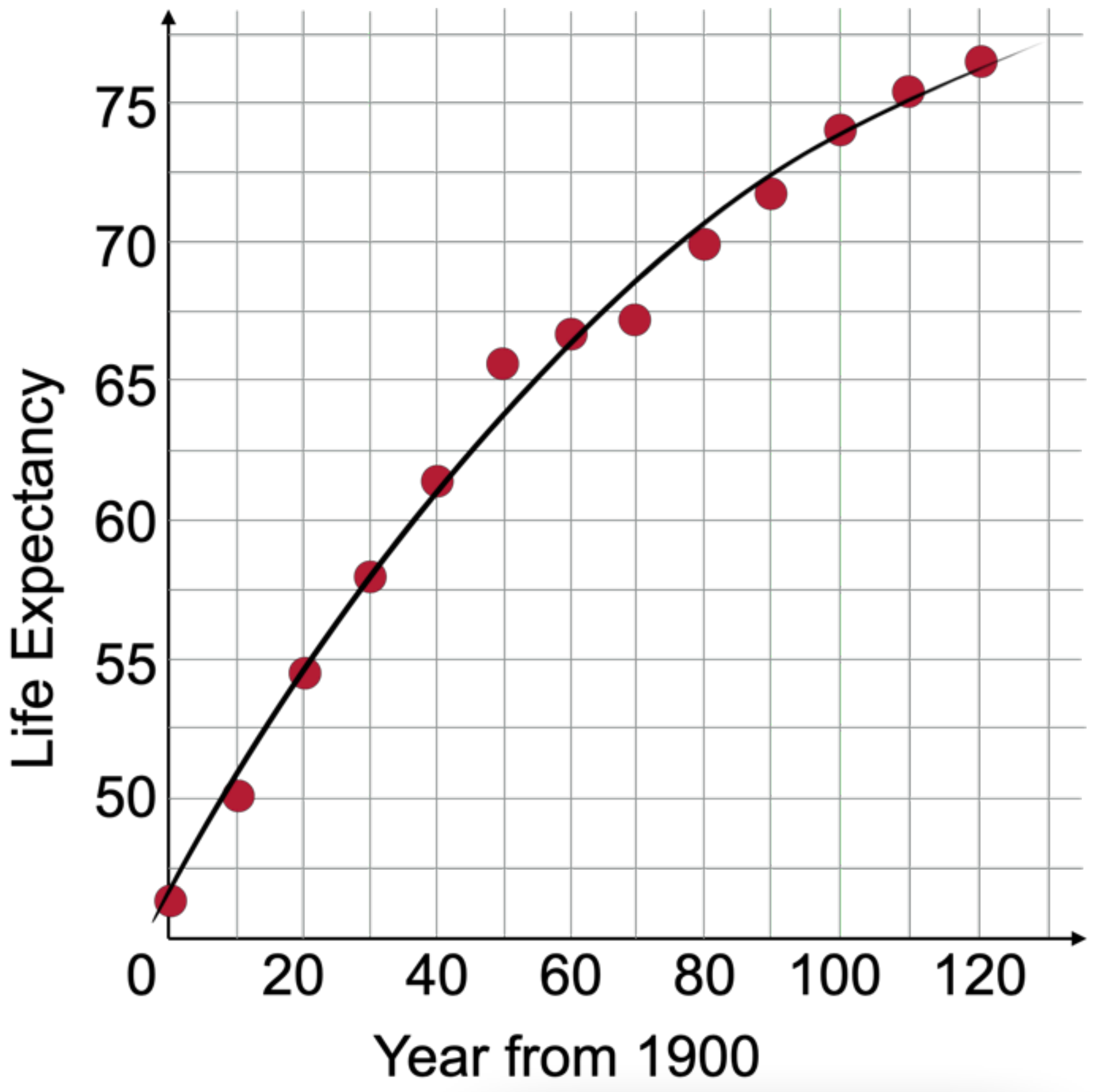
I can interpret information about mathematical models using technology and key features.

e. Do you think the model is still valid today? Should we use the model to predict life expectancy for a child born this year?

We can, but we should not be too confident in the accuracy of the prediction.

VARS ➢ YVARS ∨ 1:Function ENTER 1:Y_1 ENTER (1 2 4) ENTER

76.3



Year	Life Expectancy, y
1900	46.41
1910	50.08
1920	54.50
1930	57.96
1940	61.43
1950	65.63
1960	66.66
1970	67.15
1980	69.94
1990	71.82
2000	74.03
2010	75.40
2020	76.50

First: much has changed, health care has improved, we know much more about dietary risks, and people are more conscious of the life choices they make.

More importantly, try not to extrapolate a model far beyond your data. Any conclusions from extrapolating beyond your data cannot be trusted.

Math Drawings from Text

I can interpret information about mathematical models using technology and key features.

- On occasion your test makes reference to enlarged copies of graphs available at mathgraphs.com. Should you go to that site, search through the PreCalculus & College Algebra list until you find the cover of our text. It takes awhile (about 23 clicks).

The actual URL for the book we are using is below.

http://www.mathgraphs.com/mg_pl1e.html