Chapter 1

Functions and Graphs

1.4 Linear Functions and Slope

Chapter 14

Homework

14 p188 7, 9, 17, 23, 29, 37, 49, 51, 65, 67, 73, 75

Chapter 14

Learning Target

S-ID.7
Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Chapter 1

Success Criteria

- I can calculate a line's slope.
- I can graph horizontal or vertical lines.
- I can write the point-slope form of the equation of a line.
- I can write and graph the slope-intercept form of the equation of a line.
- I can recognize and use the general form of a linear equation.
- I can use intercepts to graph the general form of a line's equation.
- I can find the equation for parallel and perpendicular lines from data.

I can calculate a line's slope.

Rate of Change (Slope)

I can calculate a line's slope.

- A **rate of change** is a ratio that compares the amount of change in a **dependent variable** to the amount of change in an **independent variable**.

$$\text{Rate of change} = \frac{\text{Change in dependent variable (y)}}{\text{Change in independent variable (x)}}$$

- For any two points on a non-vertical **line**, this rate (ratio) is constant. The constant **rate of change** of a non-vertical **line** is most commonly called the **slope** of the line.

$$\text{Slope} = \frac{\text{Change in dependent variable (y)}}{\text{Change in independent variable (x)}}$$

Definition of Slope

I can calculate a line's slope.

The constant rate of change for a linear function is its slope. The slope of a linear function is the ratio $\frac{\text{change in } f(x)}{\text{change in } x}$, or $\frac{\Delta f(x)}{\Delta x} = \frac{\Delta y}{\Delta x}$, or, for the non-educated, $\frac{\text{rise}}{\text{run}}$.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

The rate of change of a linear function is constant. Thus the slope is the same between any two points on the line. You can graph lines by using two points, or the slope and a point.

Slope

I can calculate a line's slope.

- For a linear function, slope may be interpreted as the rate or ratio of change in the dependent variable (y) per unit change in the independent variable (x).

    If x and y are measured in the same units, slope is a ratio with no units.

    The ratio of Baptists to total population is about 1/10

    If x and y are measured in the different units, slope is a rate of change.

    The rate of growth of Baptists churches is about .006 per year

Using the Definition of Slope

I can calculate a line's slope.

- Find the slope of the line passing through the points (4, −2) and (−1, 5)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - -2}{-1 - 4} = \frac{7}{-5} = -\frac{7}{5}$$

- The slope of the line passing through the points (4, −2) and (−1, 5) is $-\frac{7}{5}$

- This tells us that for every 5 units x increases, f(x), or y, decreases 7 units.

Rate of Change and Slope

I can calculate a line's slope.

Find the rate of change for the data shown. Explain what the rate of change means. The table shows the amount of money the band boosters make washing cars.

No. Cars	Money ($)
5	40
10	80
15	120
20	160

+5 +5 +5

+40 +40 +40

$$\text{Rate of change} = \frac{\text{Change in dependent variable (y)}}{\text{Change in independent variable (x)}}$$

$$\text{Rate of change} = \frac{\Delta Y}{\Delta X} = \frac{40}{5} = \frac{8}{1}$$

The **unit rate of change** is $8 for each 1 car.

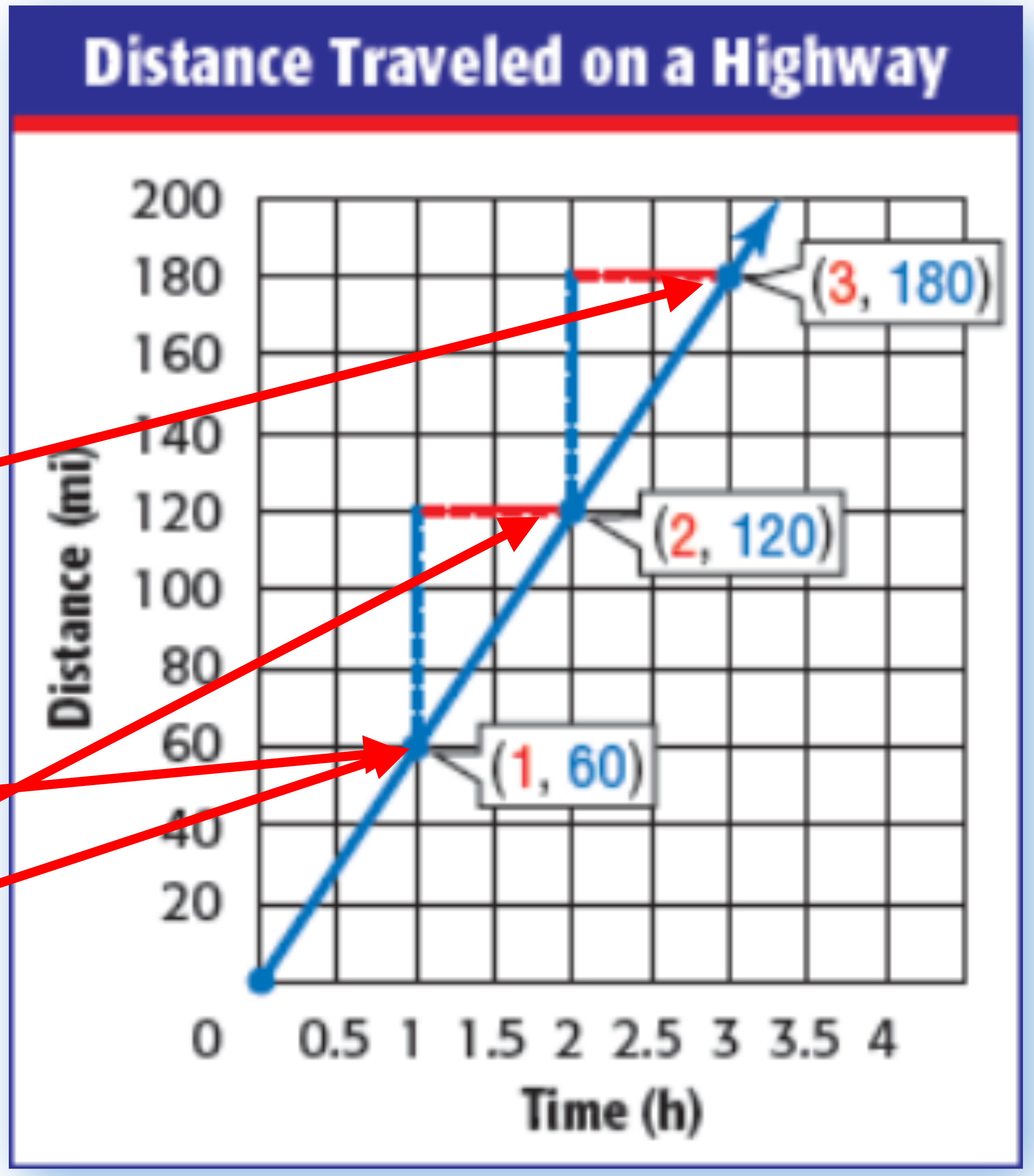
Rate of Change and Slope

I can calculate a line's slope.

- The graph represents the distance traveled while driving on a highway. Find the constant rate of change in miles per hour (mph).

  - Pick two points (any two points) to find the rate of change.

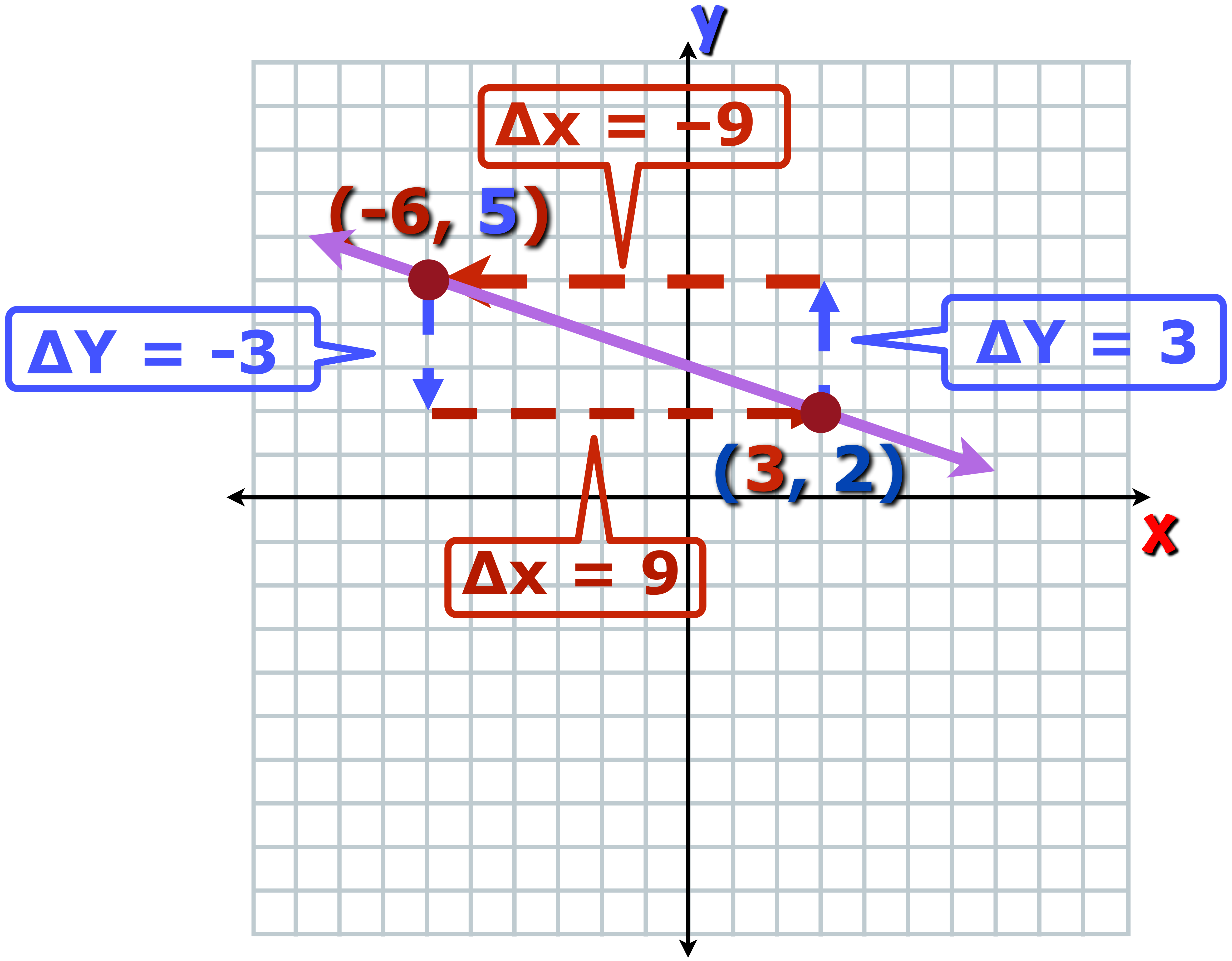
$$\text{Rate of change} = \frac{\text{Change in dependent variable (y)}}{\text{Change in independent variable (x)}}$$



$$\frac{\Delta Y}{\Delta X} = \frac{180-60}{3-1} = \frac{120}{2} = \frac{60}{1} \qquad \frac{\Delta Y}{\Delta X} = \frac{120-60}{2-1} = \frac{60}{1}$$

- The unit rate of change is 60 miles for each 1 hour. (60 mph)

Find the slope of the line through the two points.



$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{5 - 2}{-6 - 3} = \frac{3}{-9} = -\frac{1}{3}$$

$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{2 - 5}{3 - -6} = \frac{-3}{9} = -\frac{1}{3}$$

Rate of Change and Slope

I can calculate a line's slope.

- In 1990, 9 million adult men in the United States lived alone. In 2008, 14.7 million adult men in the United States lived alone. Use this information to find the slope of the linear function representing adult men living alone in the United States. Express the slope correct to two decimal places and describe what it represents.

We form the ordered pairs (year, number living alone). (1990, 9) and (2008, 14.7). Using these points, find the slope.

$$m = \frac{\Delta y}{\Delta x} = \frac{14.7 - 9}{2008 - 1990} = \frac{5.7}{18} = .31\overline{6}$$

The number of men living alone increased at a rate of about 0.32 million per year. (316667/year)

Rate of Change and Slope

I can calculate a line's slope.

- Use the data points (317, 57.04) and (354, 57.64) to obtain a linear function that models average global temperature, f(x), for an atmospheric carbon dioxide concentration of x parts per million.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{57.64 - 57.04}{354 - 317} = \frac{0.60}{37} \approx 0.016$$

What does this value $m \approx 0.016$ indicate?

It is the change in average global temperature (°F) for each change of one part per million in CO_2 concentration.

Rate of Change and Slope

I can calculate a line's slope.

- Now find the equation of the line:

$$m \approx 0.016 \quad (317, 57.04) \quad (354, 57.64)$$

- We have the slope and a point so use point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 57.64 = 0.016(x - 354)$$

$$y - 57.64 = 0.016x - 5.664$$

$$y = 0.016x - 51.976$$

$$f(x) = 0.016x - 52.0$$

What does the value 52.0 represent?

It is the average global temperature expected for a CO_2 concentration of 0.

f(x) models average global temperature for an atmospheric carbon dioxide concentration of x parts per million.

Rate of Change and Slope

I can calculate a line's slope.

- When the rate of change is constant no matter what points you choose, the relationship is **linear**. That is, **linear** relationships have a **constant rate of change**.

  - Decide which of the following tables define a linear function.

x	y
1	0
2	2
3	4
4	6

+1 +2

- Yep

x	y
1	1
2	4
3	5
4	7

+1 +1 +3 +1

- Nope

x	y
1	12
2	7
3	4
4	3

+1 +1 -5 -3

- Nope

x	y
2	24
4	20
8	12
14	0

+2 +4 +6 -4 -8 -12

- Yep

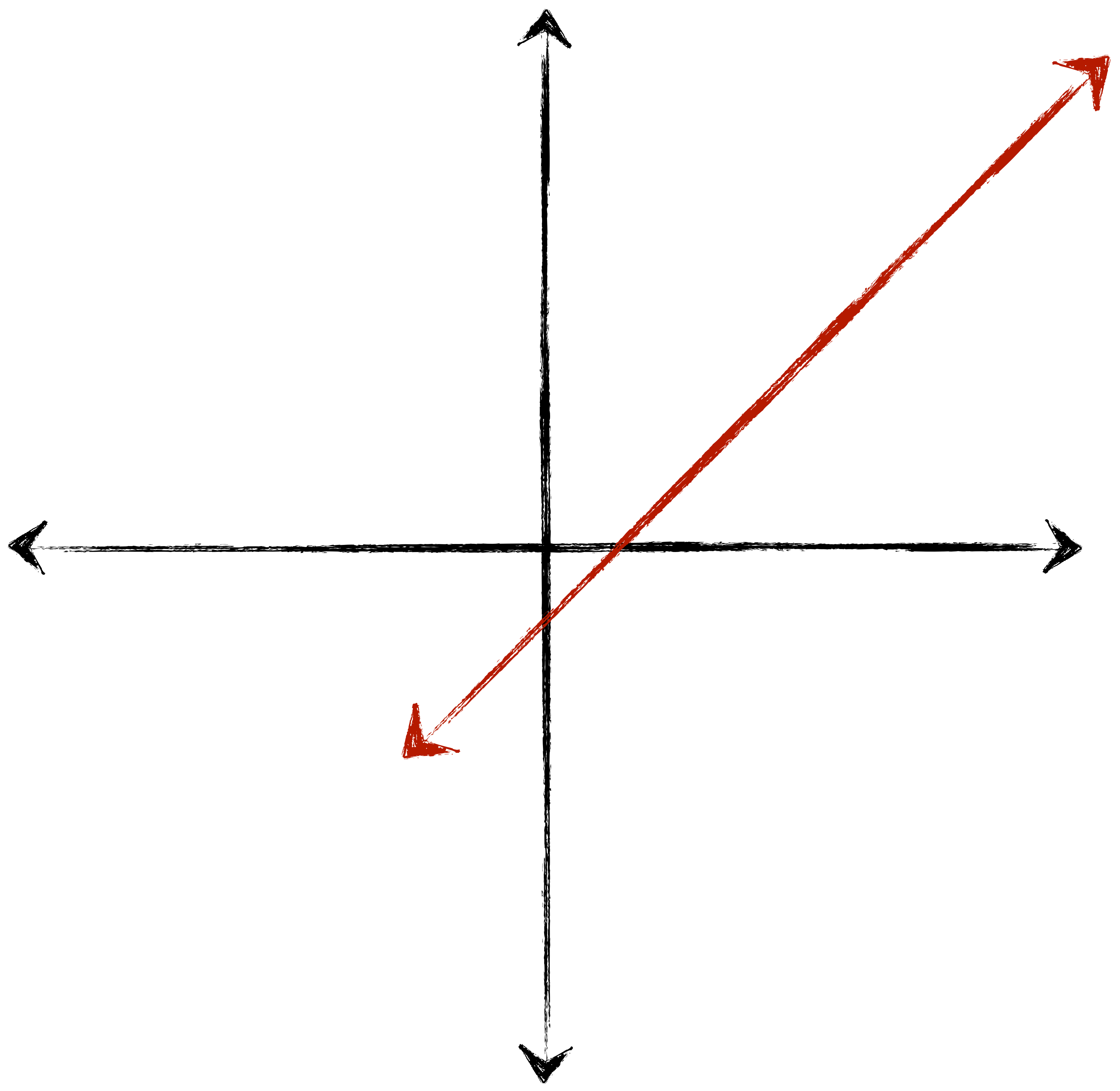
x	y
24	24
16	32
8	40
0	48

-8 +8

- Yep

Possibilities for Slope of a Line

- The slope of a linear relation can be positive, negative, zero, or undefined.
  - Positive Slope $m > 0$
    - As x increases, f(x) increases.
  - Negative Slope $m < 0$
    - As x increases, f(x) decreases.
  - Zero Slope $m = 0$
    - As x increases, f(x) remains constant.
  - Undefined Slope
    - x remains constant



Finding Coordinates Given Slope

I can calculate a line's slope.

- Find the value of r so that the line through (1, 7) and (3, r) has slope $m = -\frac{1}{2}$

Use the formula for slope.

$$\text{Slope} = m = \frac{\Delta Y}{\Delta X} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\Delta Y}{\Delta X} = \frac{r - 7}{3 - 1} = -\frac{1}{2} \qquad \frac{r-7}{2} = -\frac{1}{2} \qquad r - 7 = -1 \qquad r = 6$$

- Check: $m = \frac{\Delta Y}{\Delta X} = \frac{6 - 7}{3 - 1} = -\frac{1}{2}$

Finding Coordinates Given Slope

- Find the value of r so that the line through (-4, r) and (-8, 3) has slope $m = -5$

Use the formula for slope.

$$\text{Slope} = m = \frac{\Delta Y}{\Delta X} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\Delta Y}{\Delta X} = \frac{r - 3}{-4 - -8} = -5 \qquad \frac{r - 3}{4} = -5 \qquad r - 3 = -20 \qquad r = -17$$

- Check: $m = \frac{\Delta Y}{\Delta X} = \frac{-17 - 3}{-4 - -8} = -\frac{20}{4} = -5$

I can graph horizontal or vertical lines.

Vertical and Horizontal Lines

I can graph horizontal or vertical lines.

- An equation with only one variable can be represented by either a vertical line or a horizontal line.

- A **horizontal** line is given by an equation of the form $y = b$, where b is the y-intercept of the line.
  - The slope of a horizontal line is **zero**.

$$y = b$$

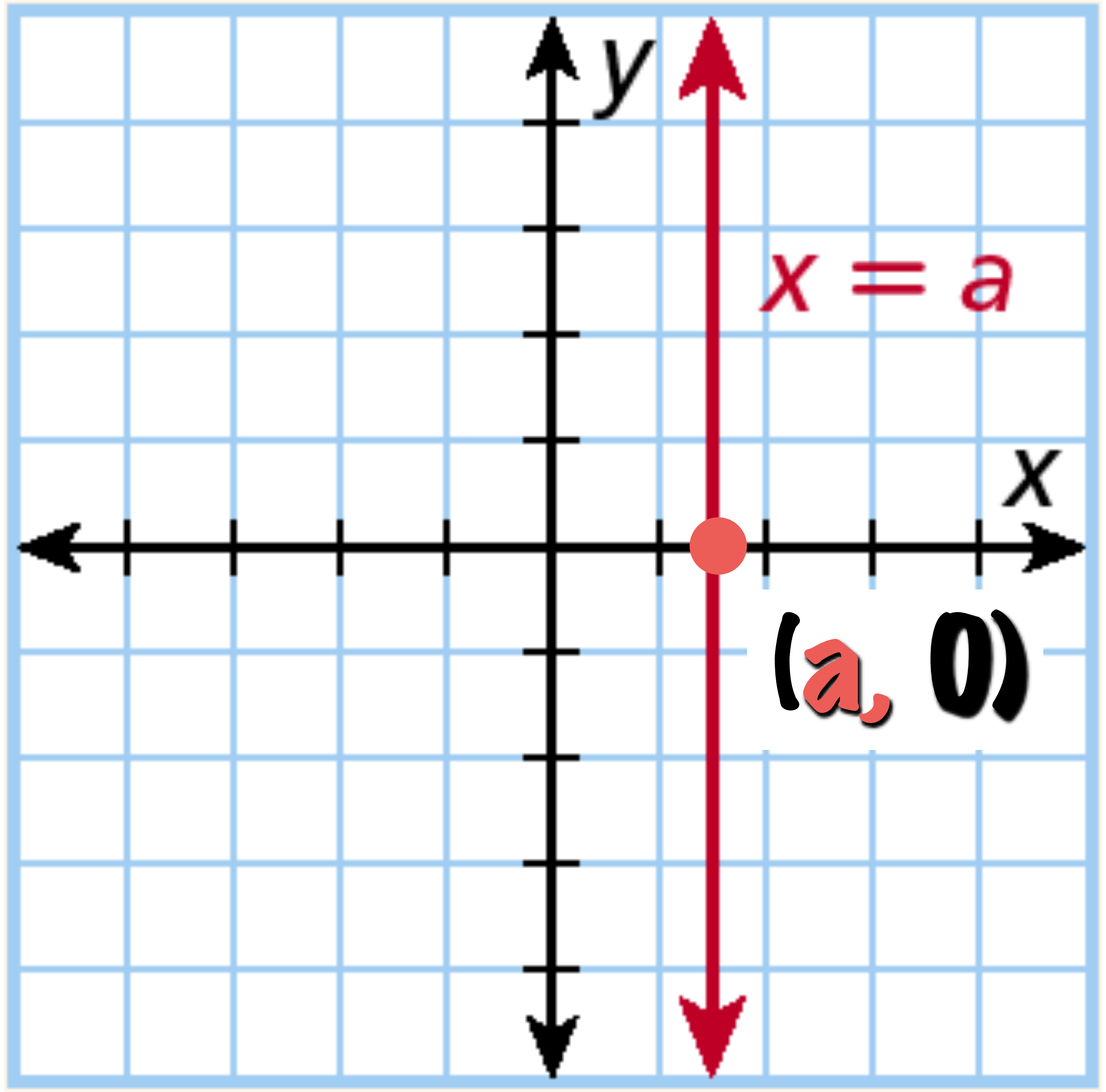
- A **vertical** line is given by an equation of the form $x = a$, where a is the x-intercept of the line.
  - The slope of a vertical line is **undefined**.

$$x = a$$

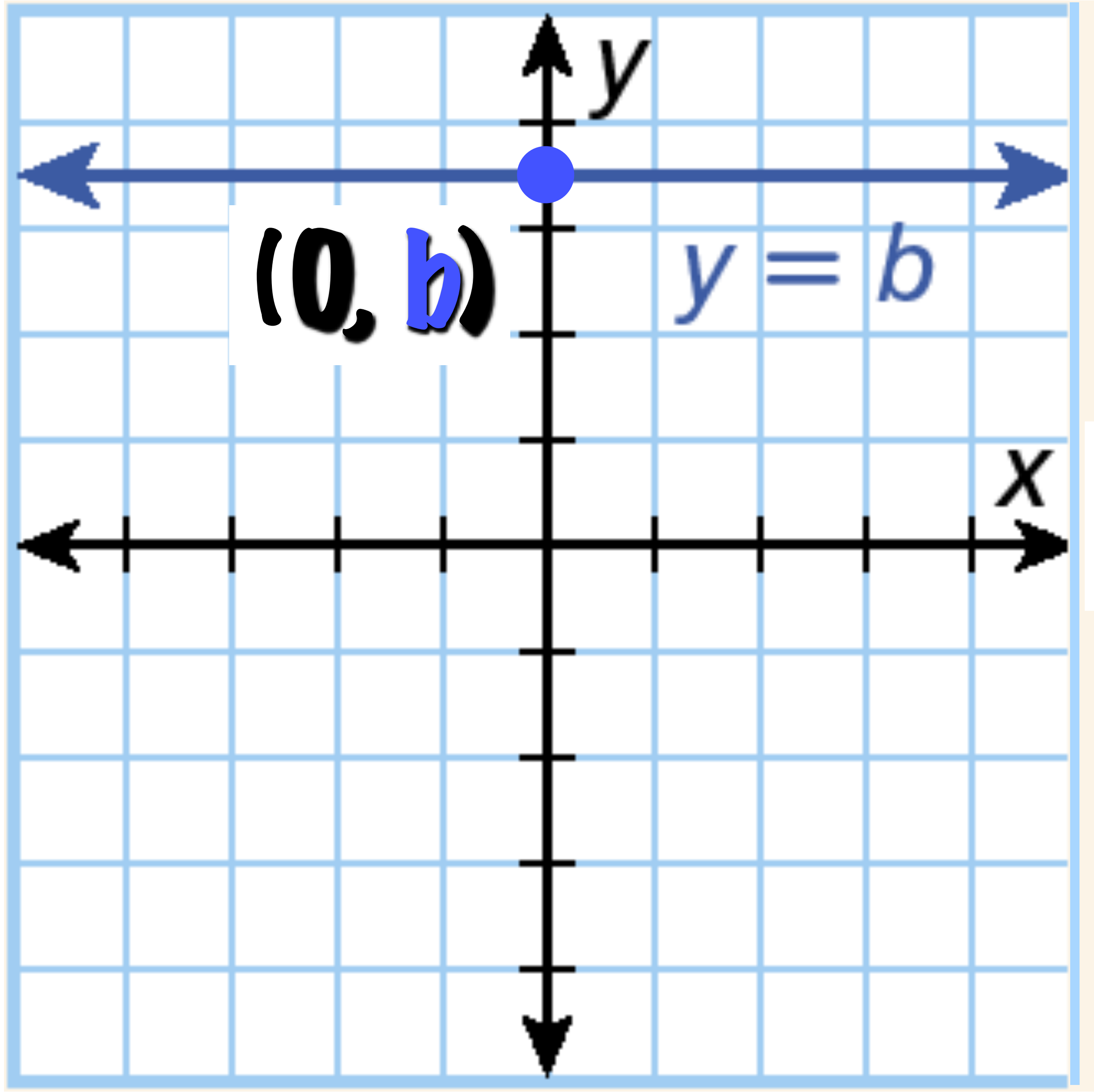
Vertical and Horizontal Lines

I can graph horizontal or vertical lines.

Vertical Lines
The line **x = a** is a vertical line at a.



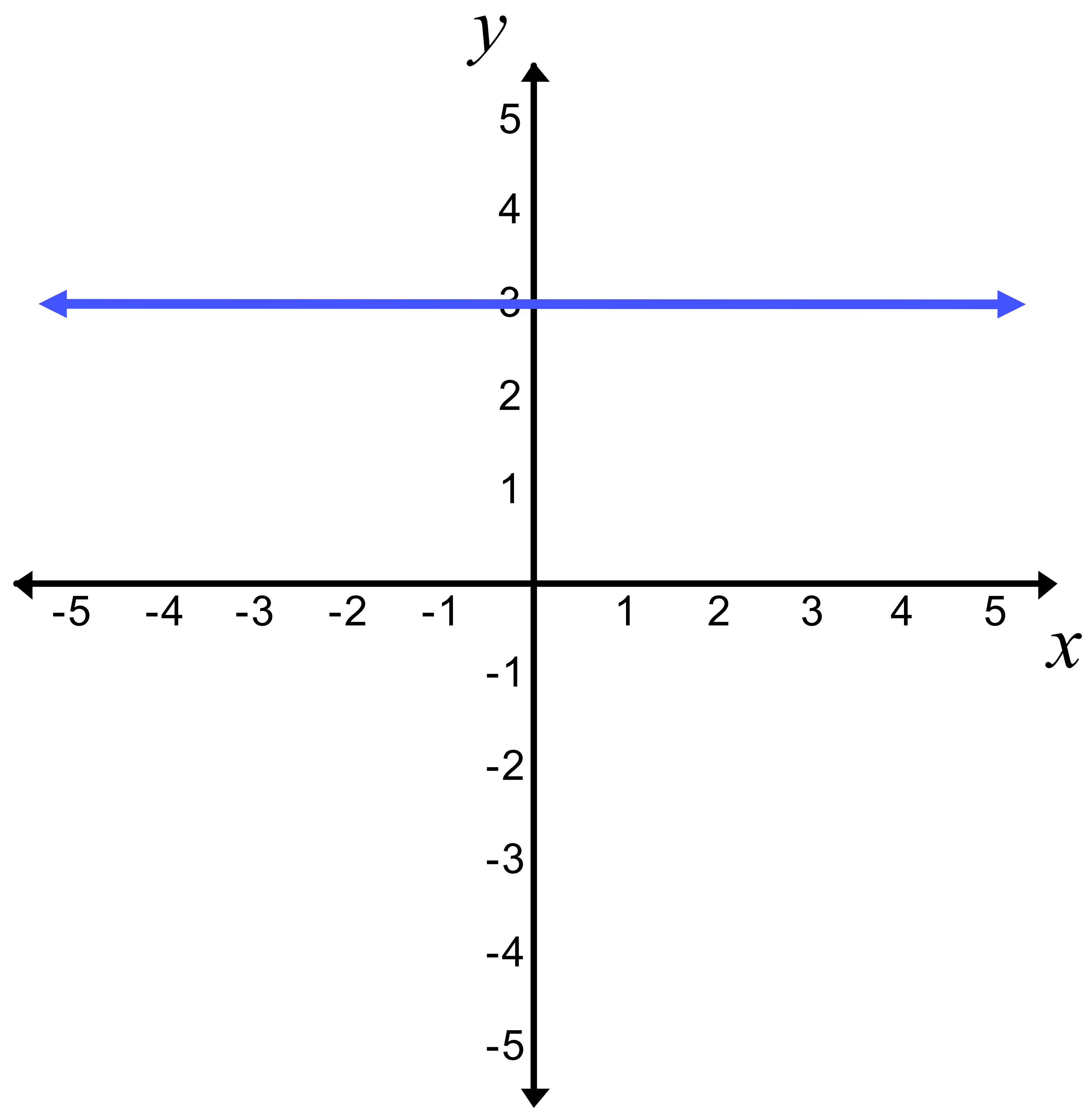
Horizontal Lines
The line **y = b** is a horizontal line at b.



Vertical and Horizontal Lines

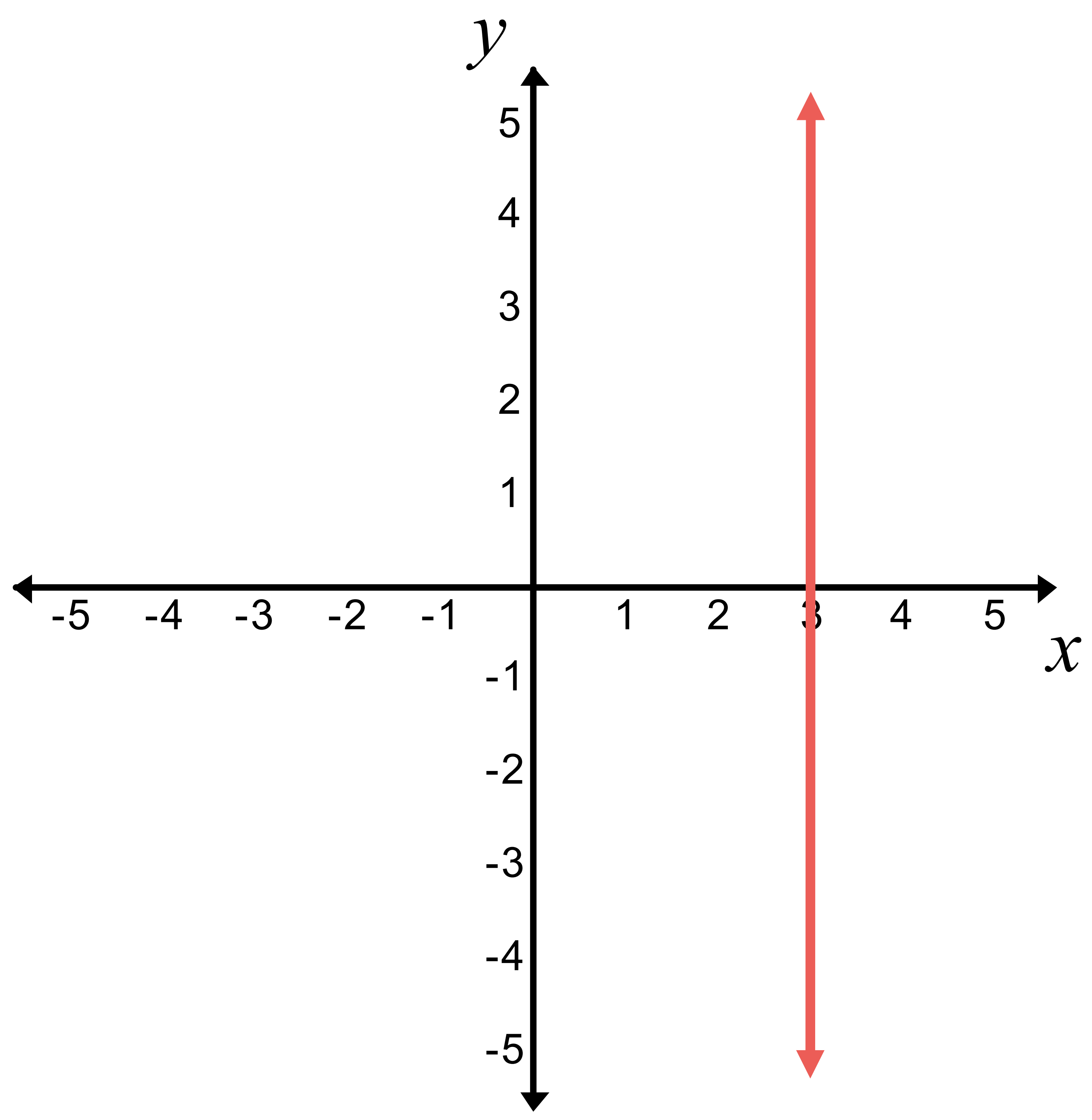
I can graph horizontal or vertical lines.

- Graph y = 3 in the rectangular coordinate system.



- The slope of a horizontal line is **zero**.

- Graph x = 3 in the rectangular coordinate system.

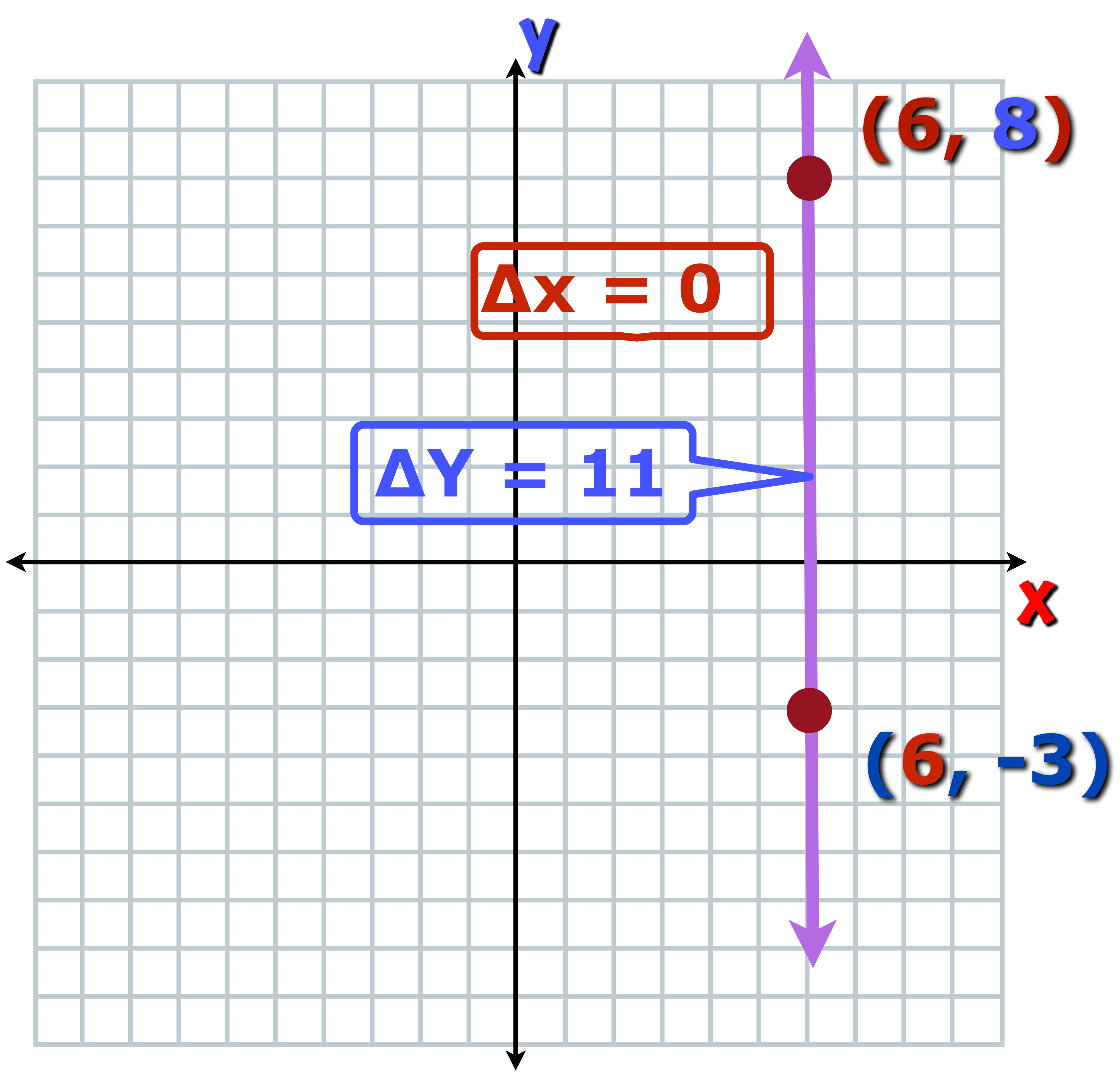


- The slope of a vertical line is **undefined**.

Rate of Change and Slope

I can graph horizontal or vertical lines.

Find the slope of the line through the two points.



$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{8 - -3}{6 - 6} = \frac{11}{0}$$

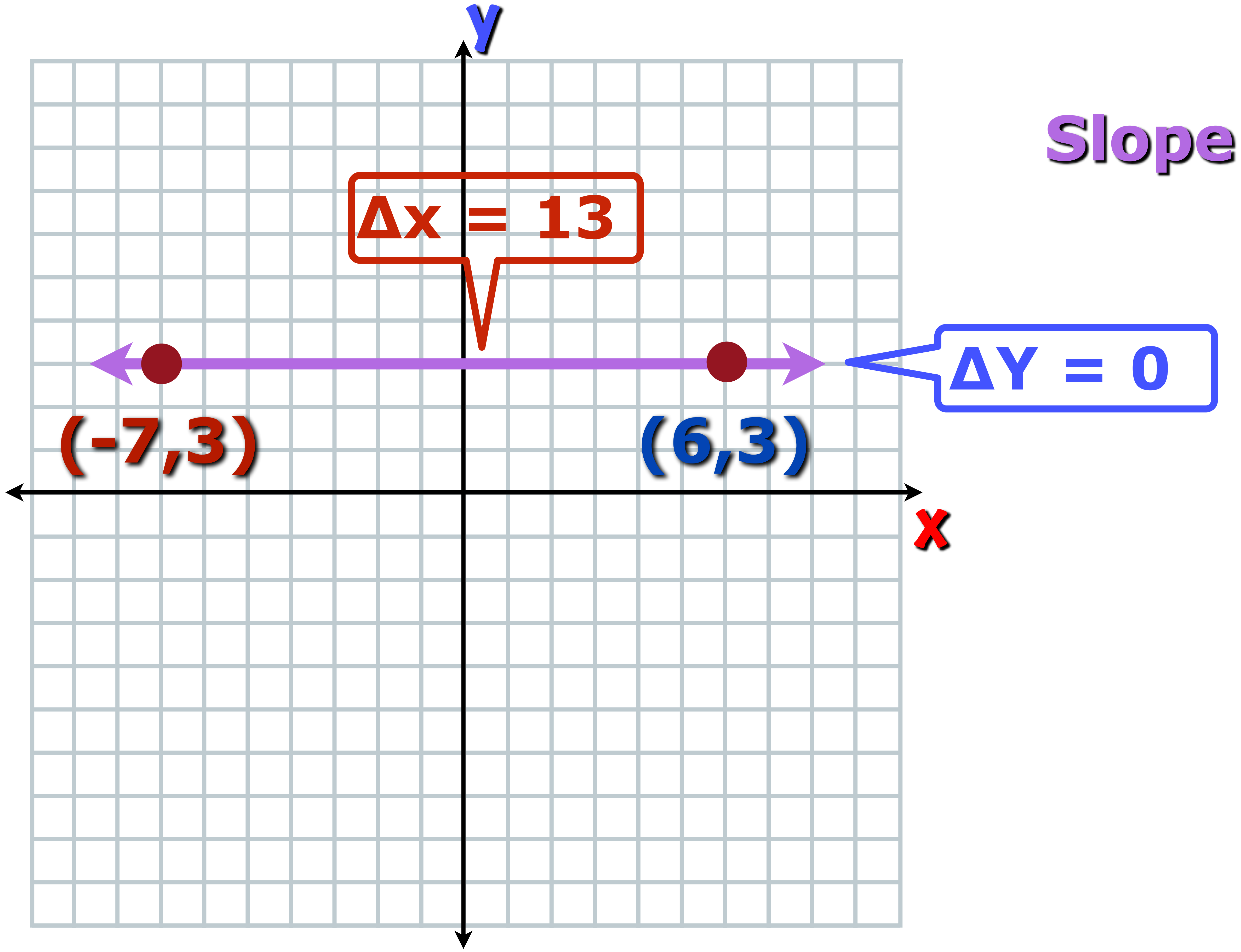
Slope = undefined

A vertical line has a slope that is **undefined.**

Rate of Change and Slope

I can graph horizontal or vertical lines.

- Find the slope of the line through the two points.



$$\text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{3 - 3}{-7 - 6} = \frac{0}{-13}$$

Slope = 0

A horizontal line
has a slope that is **0.**

Write the point-slope form of the equation of a line.

Point-Slope Form

I can write the point-slope form of the equation of a line.

- You probably remember that if you know the **slope** and the **y-intercept** of a line, you can graph the line. You can also graph a line if you know its **slope** and **any point** on the line.

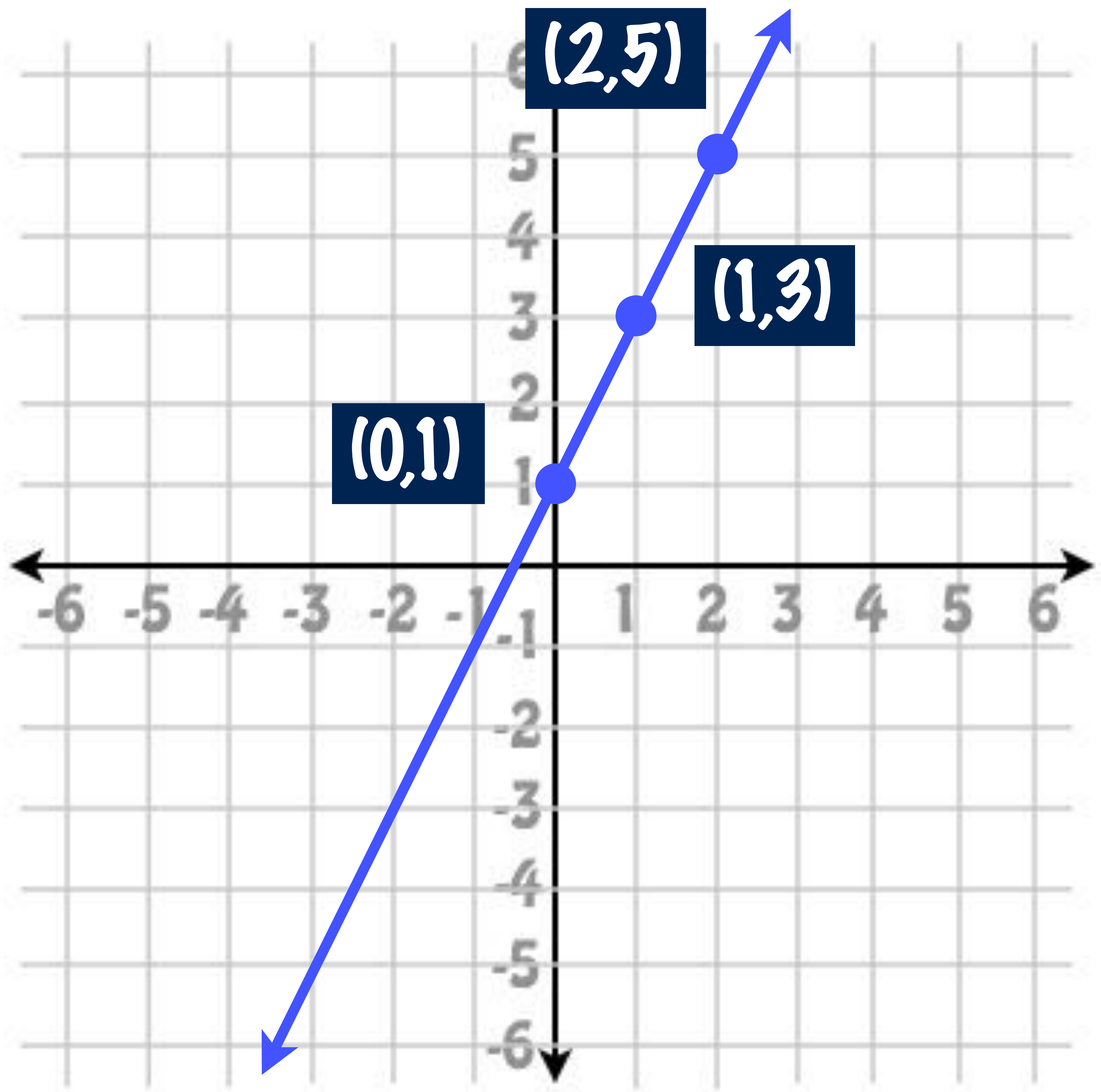
- Thus, to graph any linear function, we need the **slope** and **any point on the line**. It is also true that if we have enough information to graph a line, then we have enough information to find the **function** defined by that line.

- Thus, to find the function representing any line we need the **slope** and a **point**.

Point-Slope Form

I can write the point-slope form of the equation of a line.

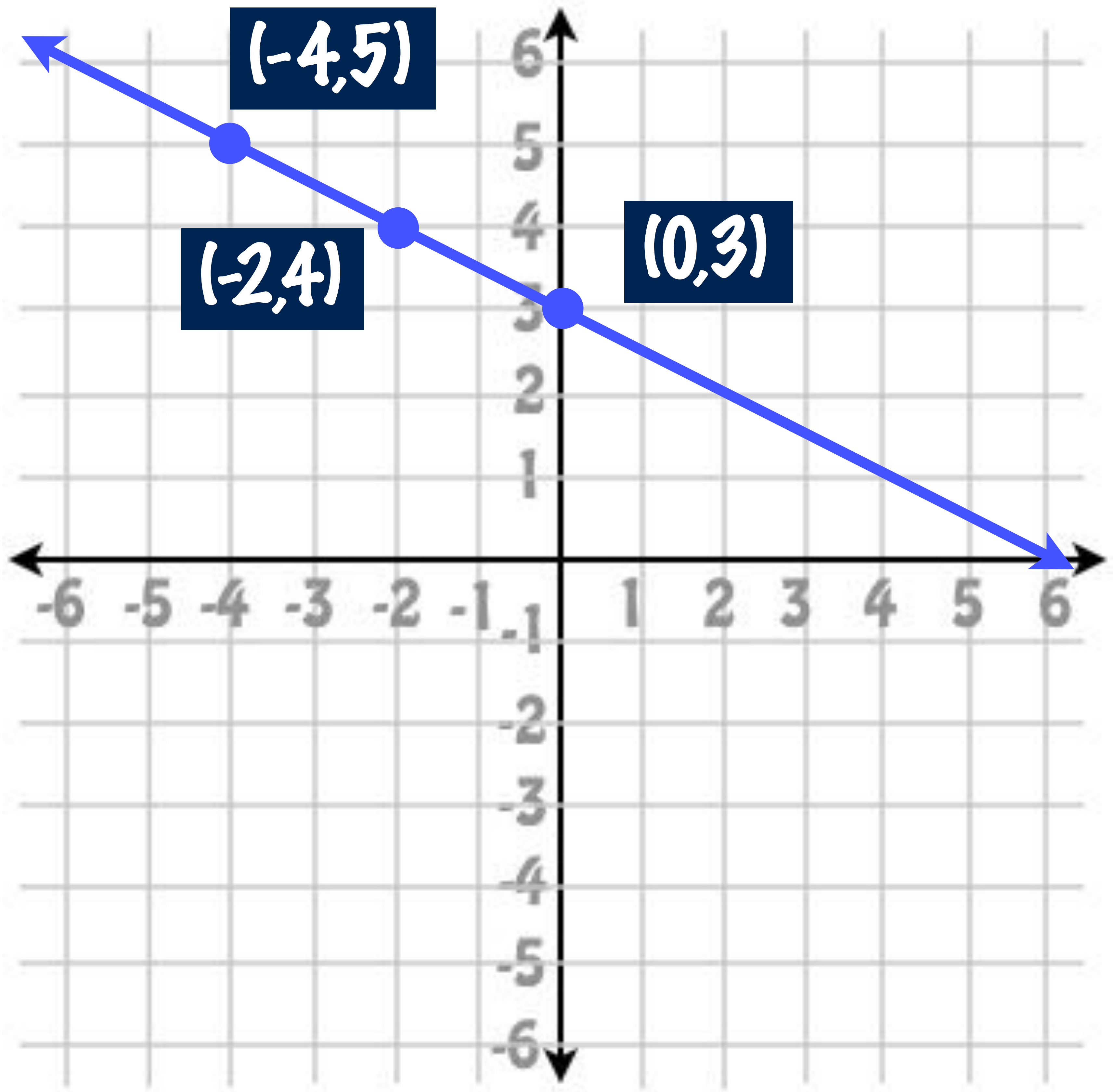
- Graph the line with the slope = 2 and containing the point (1, 3).
  - Plot the point (1, 3)
  - Slope = $m = \frac{\Delta y}{\Delta x} = \frac{-2}{-1} = \frac{2}{1}$
  - Count **2 units down** and **1 unit left** from (3, 1) and plot another point.
    - and/or
  - Count **2 units up** and **1 unit right** from (3, 1) and plot another point.
  - Draw the line



Point-Slope Form

I can write the point-slope form of the equation of a line.

- Graph the line with the slope = $-\frac{1}{2}$ and containing the point (-2, 4).
  - Plot the point (-2, 4)
  - Slope = $m = \frac{\Delta y}{\Delta x} = \frac{-1}{2} = \frac{1}{-2}$
  - Count **1 unit down** and **2 units right** from (-2, 4) and plot another point.
    - and/or
  - Count **1 unit up** and **2 units left** from (-2, 4) and plot another point.
  - Draw the line

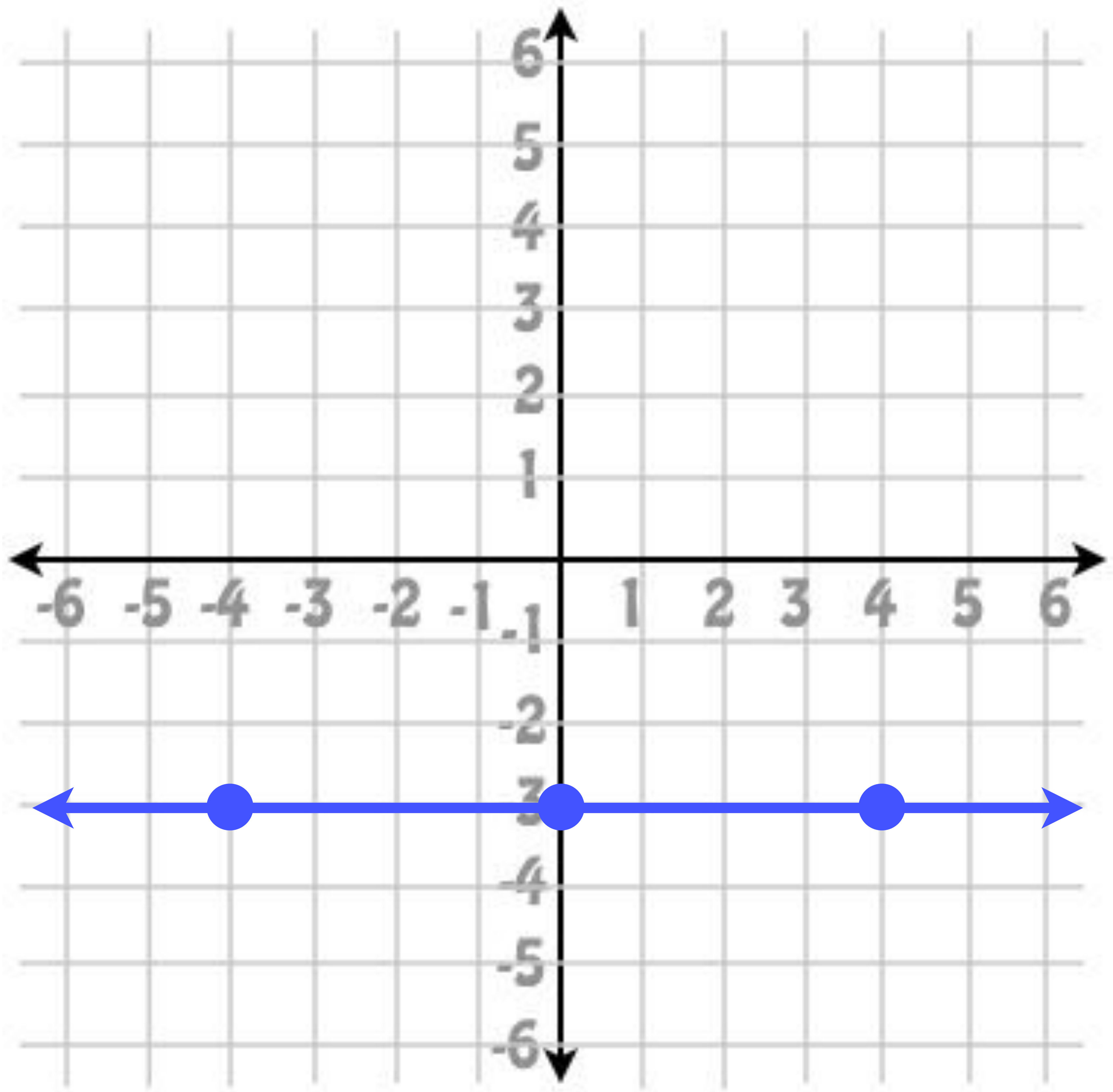


Point-Slope Form

I can write the point-slope form of the equation of a line.

- Graph the line with the slope = 0 and containing the point (4, -3).

  - A line with slope of 0 is a horizontal line through the point (4, -3).



Point-Slope Form

I can write the point-slope form of the equation of a line.

- If you know the slope and any point on a line, you can write an equation of the line derived by using the slope formula.
- Slope Formula $m = \frac{\Delta f(x)}{\Delta x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- Multiply both sides by denominator $m(x_2 - x_1) = y_2 - y_1$
- Point-Slope Form of Linear Equation $y_2 - y_1 = m(x_2 - x_1)$ ⇨ $y - y_1 = m(x - x_1)$
- Note that the equation includes x and y representing all the points (x, y) on the line.

Point-Slope Form of the Equation of a Line

I can write the point-slope form of the equation of a line.

- Given the slope m and any point (x_1, y_1) on the line, the point-slope form is:

$$y - y_1 = m(x - x_1)$$

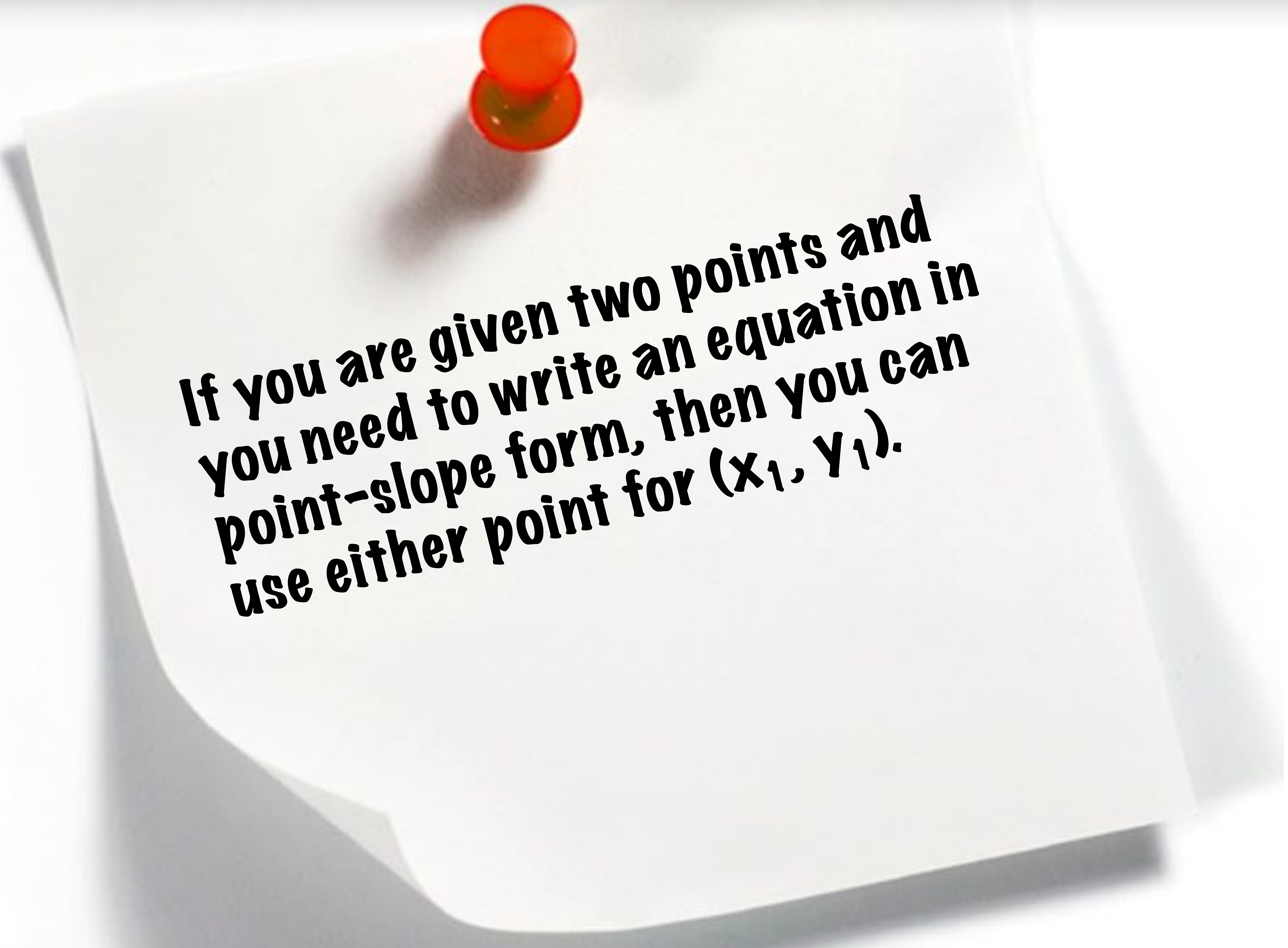
y-coordinate of given point

slope

x-coordinate of given point

Point-Slope Form

I can write the point-slope form of the equation of a line.



Point-Slope Form

I can write the point-slope form of the equation of a line.

Write an equation in point-slope form for the line with slope 2 containing the point (3, 4).

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 3)$$

y-coordinate

slope

x-coordinate

Equation of a Line in Point-Slope Form

I can write the point-slope form of the equation of a line.

- Write an equation in point-slope form for the line with slope 6 that passes through the point (2, –5). Then solve the equation for y.

$$y - y_1 = m(x - x_1)$$

$$y - (-5) = 6(x - 2)$$

$$y + 5 = 6(x - 2)$$

The equation in point-slope form.

Solving for y...

$$y + 5 = 6x - 12$$

$$y = 6x - 17$$

The equation in slope-intercept form.

Equation of a Line in Point-Slope Form

I can write the point-slope form of the equation of a line.

- Write an equation in point-slope form for the line with slope -1 containing the point (5, 1).

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -1(x - 5)$$

- Write an equation in point-slope form for the line with slope -1 containing the point (-2, 4)

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -1(x - -2)$$

$$y - 4 = -1(x + 2)$$

- Write an equation in point-slope form for the line with slope $\frac{3}{4}$ containing the point (-3, -6).

$$y - y_1 = m(x - x_1) \qquad y - -6 = \frac{3}{4}(x - -3) \qquad y + 6 = \frac{3}{4}(x + 3)$$

Equation of a Line in Point-Slope Form

I can write the point-slope form of the equation of a line.

- Write an equation in point-slope form for the line with the given slope that contains the given point.

A. slope = $\frac{1}{6}; (5,1)$

$$y-1=\frac{1}{6}(x-5)$$

B. slope = $-4; (0,3)$

$$y-3=-4(x-0)$$

$$y-3=-4x$$

C. slope = $1; (-1,-4)$

$$y--4=1(x--1)$$

$$y+4=x+1$$

D. slope = $0; (3,-4)$

$$y--4=0(x-3)$$

$$y+4=0$$

- I can write and graph the slope-intercept form of the equation of a line.

Slope-intercept Form

- I can write and graph the slope-intercept form of the equation of a line.

- You have learned one way you can graph the line defined by a linear function is to determine two (three) points from the function, plot those points and draw the line that goes through those two points (and the third point). To check our accuracy we plot that third point to ensure that point also falls on the line.

- You also now know that given a point and slope you can graph the point, use the slope to find another point and draw the line. You can also find the equation in point-slope form.

- Another method of graphing a linear equation is to use the specific point containing the **y-intercept** and using the **slope** of the line to determine additional points.

Slope-Intercept Form

I can write and graph the slope-intercept form of the equation of a line.

- Linear functions can also be expressed as linear equations of the form $y = mx + b$. When a linear function is written in the form $y = mx + b$, the function is said to be in slope-intercept form because m is the slope of the graph and b is the y-intercept.

$$y = mx + b$$

Remember that slope-intercept form is the equation solved for y in terms of x.

Slope-Intercept Form

I can write and graph the slope-intercept form of the equation of a line.

- The **slope-intercept** form of the linear **function** is:

$$f(x) = mx + b$$

y

slope

y-intercept

- The **point** $(x, f(x))$ is on the line defined by $f(x) = mx + b$.

Caution

The slope of a line does not contain x in the value!

$$f(x) = mx + b$$

slope

Caution

The slope of the equation is m, NOT mx

Writing Function in Slope-intercept Form

I can write and graph the slope-intercept form of the equation of a line.

Write the equation of the line in slope-intercept form.

$\text{slope} = 3 \quad \text{y-intercept} = \frac{1}{2}$

$f(x) = mx + b$

$f(x) = 3x + \frac{1}{2}$

$\text{slope} = -\frac{1}{4} \quad \text{y-intercept} = -6$

$f(x) = mx + b$

$f(x) = -\frac{1}{4}x + -6$

Graphing y = mx + b

1. Begin by taking note of the sign of the slope. A **positive** slope indicates the line will **rise to the right**. A **negative** slope indicates the line will **fall to the right**.

2. Plot the point containing the **y-intercept** on the y-axis. This is the point (0, **b**).

3. Obtain a second point using the slope, **m**. Write **m** as a fraction, and use $\frac{\Delta y}{\Delta x}$, starting at the point containing the **y-intercept**, to plot this 2nd point.

   - Because we are using slope to find a second point, you can use the slope to find a third point but it is not necessary as it provides no additional benefit.

4. Use a straightedge to draw a line through the two points. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

Graphing y = mx + b Using the Slope (m) and the y-Intercept (b)

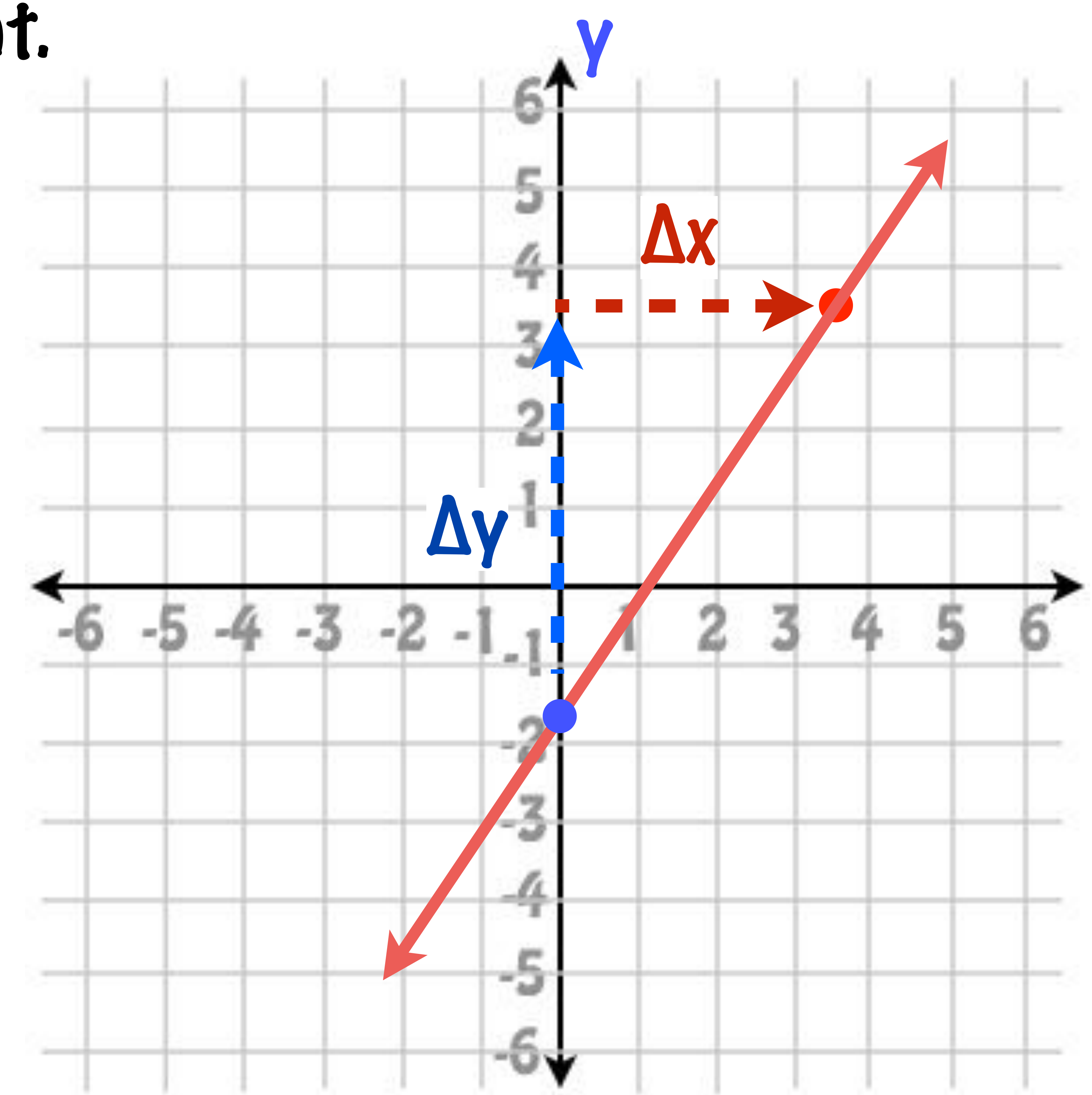
I can write and graph the slope-intercept form of the equation of a line.

1. Begin by taking note of the sign of the slope. A **positive** slope indicates the line will **rise** to the right. A **negative** slope indicates the line will **fall** to the right.

2. Plot the point containing the **y-intercept** on the y-axis. This is the point (0, b).

3. Obtain a second point using the slope, m. Write m as a fraction, and use $\frac{\Delta y}{\Delta x}$, starting at the point containing the **y-intercept**, to plot this 2nd point.

4. Draw the line, indicating that the line continues forever.



Writing Math

A negative fraction can be written with the negative in the numerator, denominator, or as a negative fraction.

$$-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}$$

Writing Math

Any integer can be written as a fraction with 1 in the denominator.

$$-2 = \frac{-2}{1} = \frac{2}{-1}$$

Graphing y = mx + b Using the Slope (m) and the y-Intercept (b)

I can write and graph the slope-intercept form of the equation of a line.

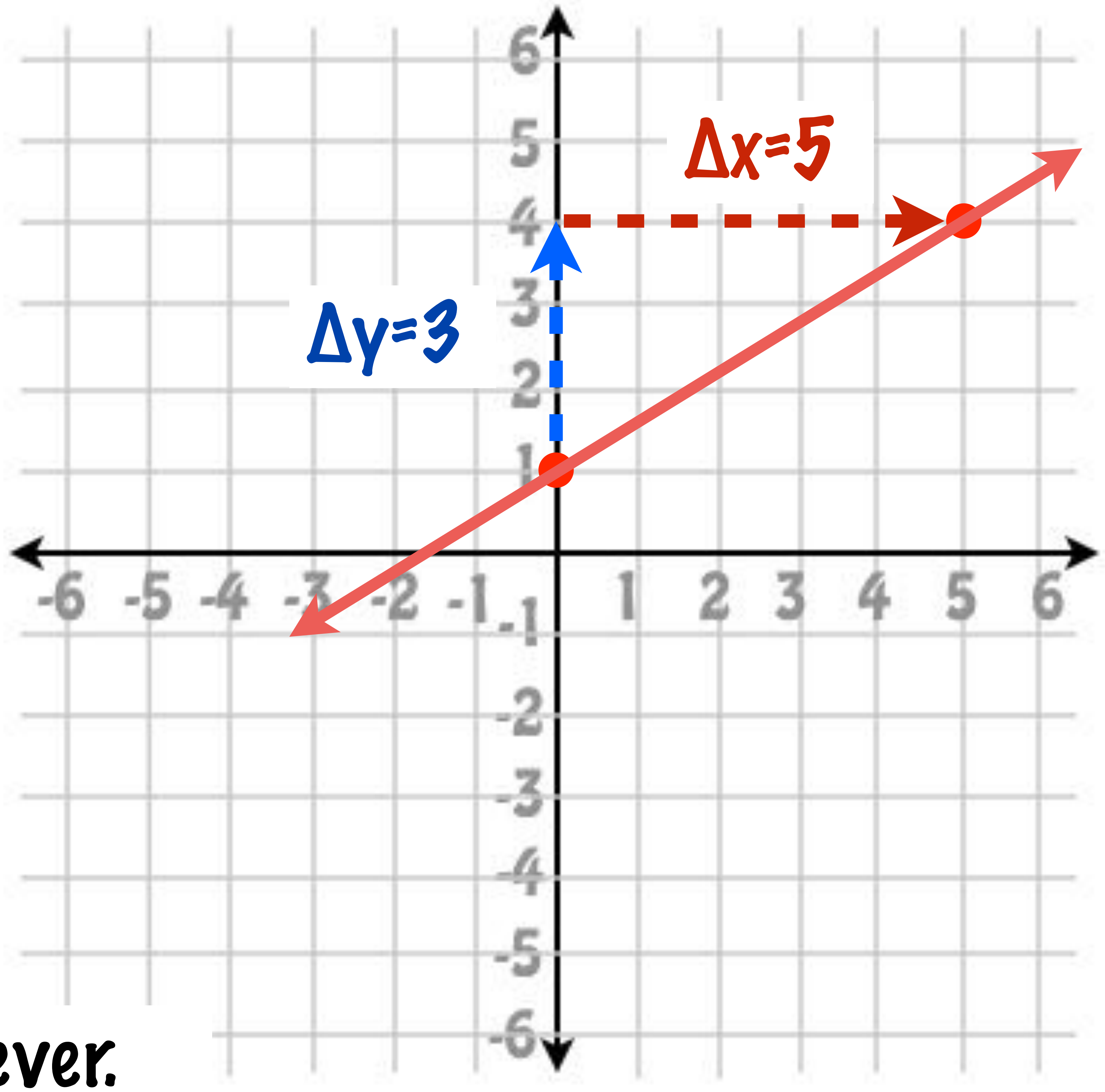
Graph the linear function: $f(x) = \frac{3}{5}x + 1$

Step 1 Plot the point containing the y-intercept on the y-axis.

The y-intercept is 1. We plot the point (0, 1).

Step 2 Using the slope, plot another point.

$$m = \frac{3}{5} = \frac{\Delta y}{\Delta x}$$



Step 3 Draw the line, indicating that the line continues forever.

Graphing Using the Slope and y-intercept

I can write and graph the slope-intercept form of the equation of a line.

Graph the linear function: $f(x) = -\frac{2}{3}x - 5$

Step 1 We note the negative slope. The line will fall to the right.

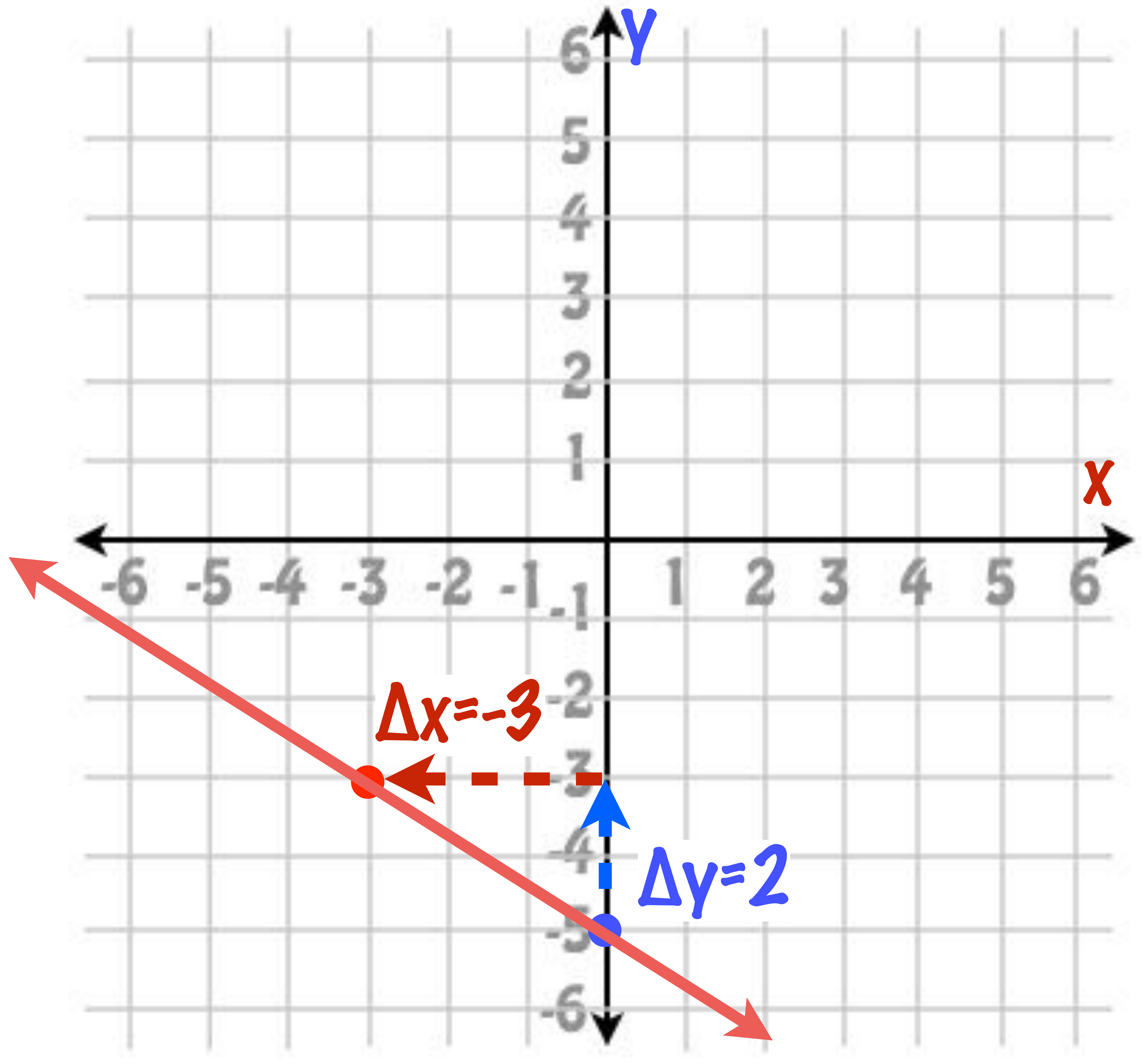
Step 2 Plot the point containing the y-intercept on the y-axis.

The y-intercept is -5. We plot the point (0, -5).

Step 3 Using the slope, plot another point.

$$m = -\frac{2}{3} = \frac{\Delta y}{\Delta x}$$

Step 4 Draw the line, indicating that the line continues forever.



- I can recognize and use the general (standard) form of a linear equation.

General (Standard) Form of the Equation of a Line

I can recognize and use the general form of a linear equation.

- Every line has an equation that can be written in the **general (or standard) form** $ax + by + c = 0$ where a, b, and c are **integers**, a and b are not both zero.

$$ax + by + c = 0$$

Standard Form from Point-Slope

I can recognize and use the general form of a linear equation.

Write an equation in **standard form** for the line with slope $-\frac{2}{5}$ that contains (-4, -5).

Step 1 Write the equation in point-slope form: $y - -5 = -\frac{2}{5}(x - -4)$ $y + 5 = -\frac{2}{5}(x + 4)$

Step 2 Write the equation in **standard form**.

$y + 5 = -\frac{2}{5}(x + 4)$ Distributive Property.

$y + 5 = -\frac{2}{5}x - \frac{8}{5}$

$5y + 25 = -2x - 8$ Multiply **both sides** by 5

$2x + 5y + 33 = 0$ Make one side 0

Standard Form from Point-Slope

I can recognize and use the general form of a linear equation.

Write an equation in **standard form** for the line containing the points (1, -3) and (6, -7)

Step 1 Find the slope

$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - -7}{1 - 6} = -\frac{4}{5}$$

Step 2 Write the equation in point-slope form:

$$y - -3 = -\frac{4}{5}(x - 1)$$

$$y + 3 = -\frac{4}{5}(x - 1)$$

Step 3 Write the equation in standard form:

$$y + 3 = -\frac{4}{5}(x - 1)$$ Distributive Property.

$$y + 3 = -\frac{4}{5}x + \frac{4}{5}$$

$$5y + 15 = -4x + 4$$ Multiply **both sides** by 5

$$4x + 5y + 11 = 0$$ Make one side 0

Standard Form from Point-Slope

- Write an equation in **standard form** for the line containing the points (6, 3) and (0, -1)

Step 1 Find the slope

$$\text{slope} = m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - -1}{6 - 0} = \frac{4}{6} = \frac{2}{3}$$

Step 2 Write the equation in point-slope form:

$$y - -1 = \frac{2}{3}(x - 0) \qquad y + 1 = \frac{2}{3}x$$

Step 3 Write the equation in standard form:

$$y + 1 = \frac{2}{3}x$$

$$3y + 3 = 2x$$ Multiply **both sides** by 3

$$2x - 3y - 3 = 0$$ Make one side zero

Finding the Slope and the y-Intercept from Standard Form

I can recognize and use the general form of a linear equation.

- Find the slope and the y-intercept of the line whose equation is: 3x + 6y – 12 = 0

Rewrite the equation in slope intercept form by **solving for y in terms of x**.

$$3x + 6y - 12 = 0$$

$$6y = -3x + 12$$

$$y = -\frac{1}{2}x + 2$$

The slope is $-\frac{1}{2}$

NOT $-\frac{1}{2}x$

The y-intercept is 2.

- I can use intercepts to graph the general form of a line's equation.

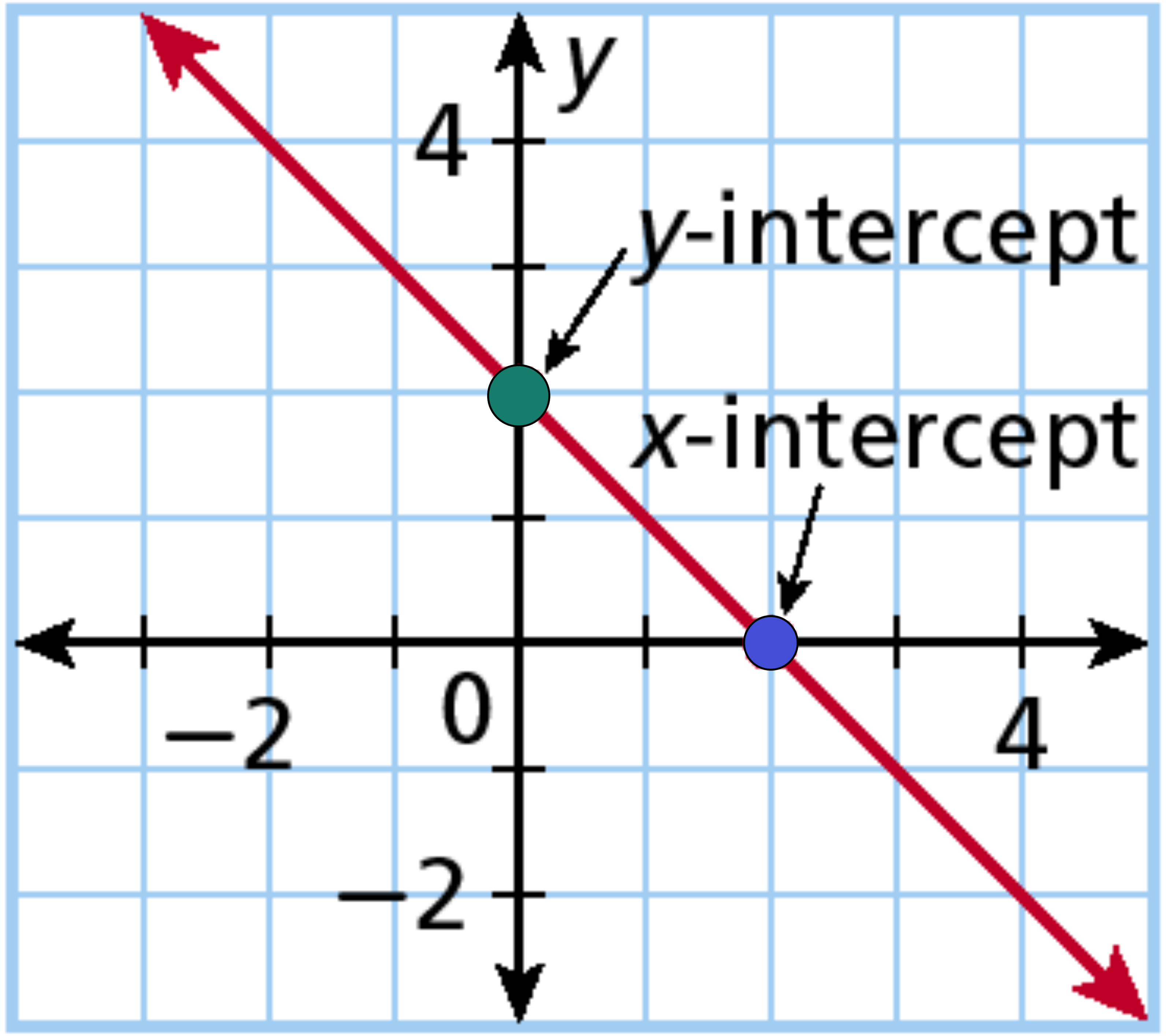
Intercepts

I can use intercepts to graph the general form of a line's equation.

- Recall from geometry that two points determine a line. Often the easiest points to find are the points where a line crosses the axes. **The x- and y-intercepts**.

The **x-intercept** is the **x**-coordinate of the point with **y-**coordinate 0, also where the line crosses the y-axis. (**Note that y = 0**.)

The **y-intercept** is the **y**-coordinate of the point with **x-**coordinate 0, also where the line crosses the x-axis. (**Note that x = 0**.)



Graphing With Intercepts

1. To find the x-intercept: Let $y = 0$ and solve for x. Plot the point containing the x-intercept on the x-axis.

2. Find the y-intercept. Let $x = 0$ and solve for y. Plot the point containing the y-intercept on the y-axis.

3. Draw a line through the points containing the intercepts. Draw arrowheads at the ends of the line to show that the line continues indefinitely in both directions.

Using Intercepts to Graph a Linear Equation

I can use intercepts to graph the general form of a line's equation.

Graph using intercepts: $3x - 2y - 6 = 0$

Step 1: Find the y-intercept.

Let $x = 0$ and solve for y.

$$3x - 2y - 6 = 0$$
$$3(0) - 2y - 6 = 0$$
$$-2y = 6$$
$$y = -3$$

The y-intercept is -3, the line passes through (0, -3).

Step 2: Find the x-intercept.

Let $y = 0$ and solve for x.

$$3x - 2y - 6 = 0$$
$$3x - 2(0) - 6 = 0$$
$$3x = 6$$
$$x = 2$$

The x-intercept is 2, the line passes through (2, 0).

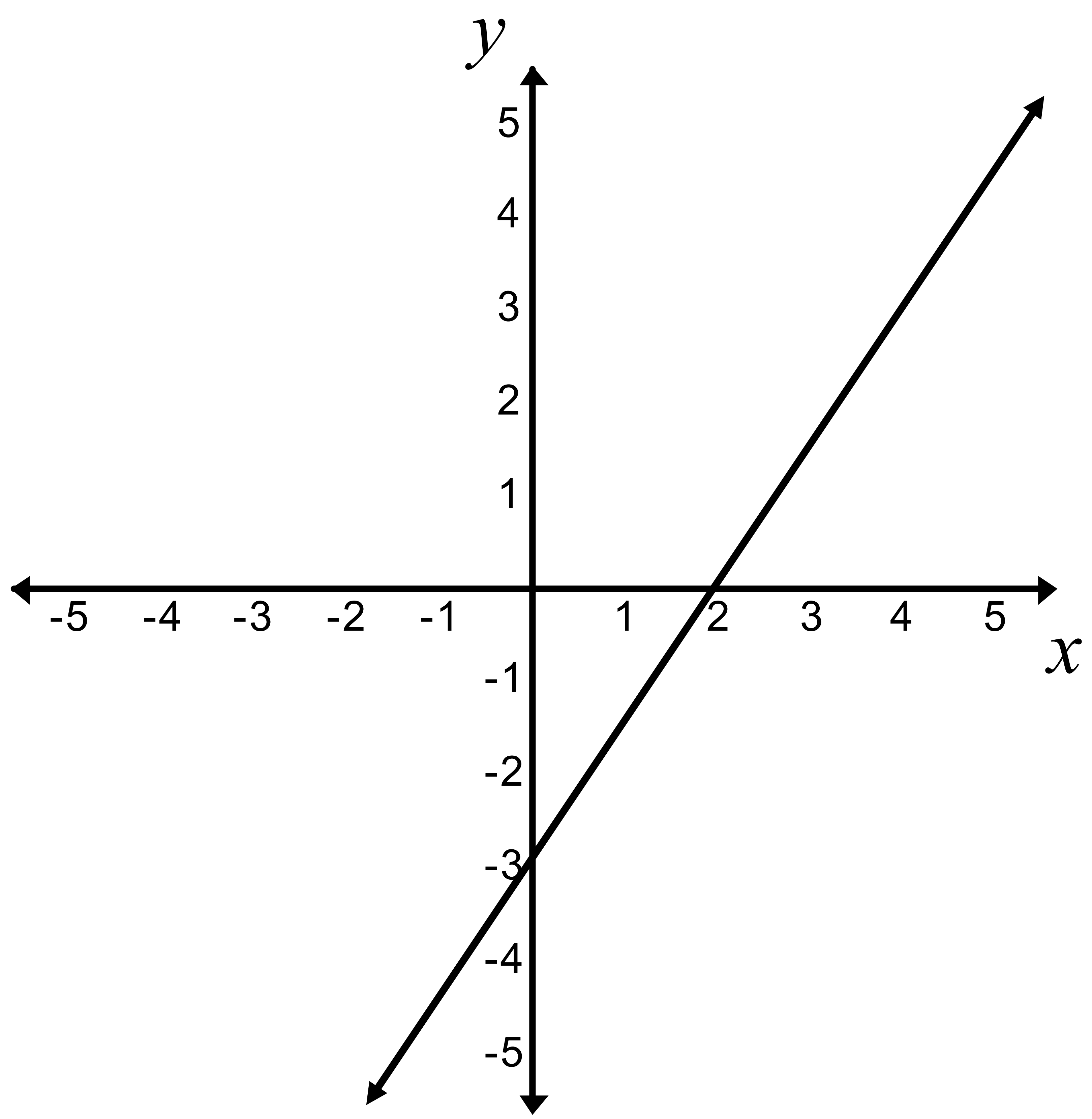
Using Intercepts to Graph a Linear Equation

I can use intercepts to graph the general form of a line's equation.

Step 3: Graph the equation by drawing a line through the two points containing the intercepts.

The x-intercept is 2, the line passes through (2, 0).

The y-intercept is -3, the line passes through (0, -3).



Another Example

I can use intercepts to graph the general form of a line's equation.

Graph the linear function: $2y + 4x = 7$

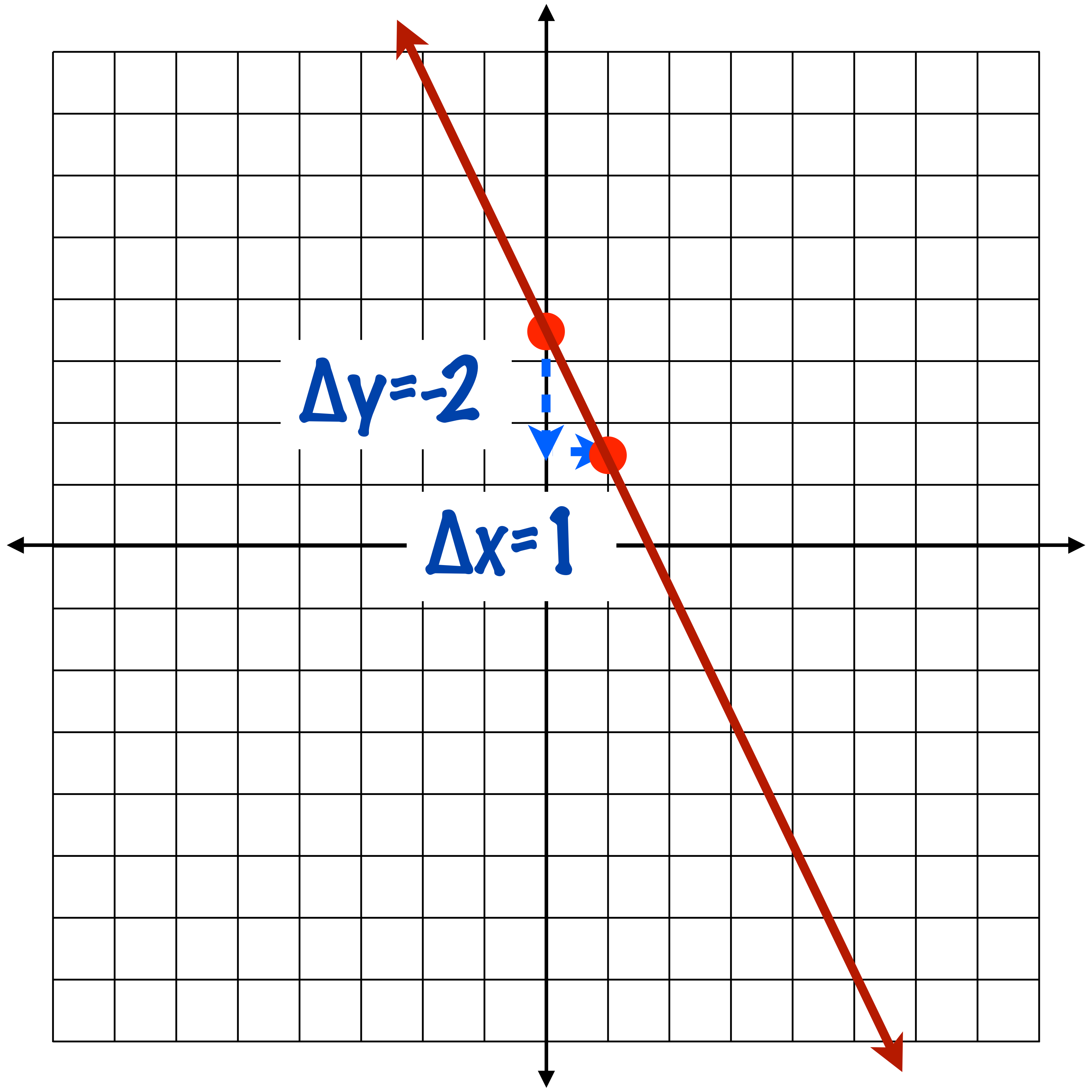
Step 0 Write the equation in slope intercept form. $y = -2x + \frac{7}{2}$

Step 1 Plot the y-intercept on the y-axis.

We plot the point $\left(0, \frac{7}{2}\right)$.

Step 2 Using the slope, plot a second point. $m = \frac{-2}{1} = \frac{\Delta y}{\Delta x}$

Step 3 Draw the line.




TI-84

TI-84

- Be careful when graphing linear equations (or any equations) on the TI. The window is not square. To ensure a square graph use ZOOM 5:Zsquare.

This will ensure the slope of your line looks as you would expect.

You can also set the window with y = 2/3x

Technology

On a graphing utility, lines will not appear to have the correct slope unless you use a viewing window that has a square setting. For instance, try graphing the lines in Example 4 using the standard setting $-10 \le x \le 10$ and $-10 \le y \le 10$. Then reset the viewing window with the square setting $-9 \le x \le 9$ and $-6 \le y \le 6$. On which setting do the lines $y = \frac{2}{3}x - \frac{5}{3}$ and $y = -\frac{3}{2}x + 2$ appear to be perpendicular?

TI-84

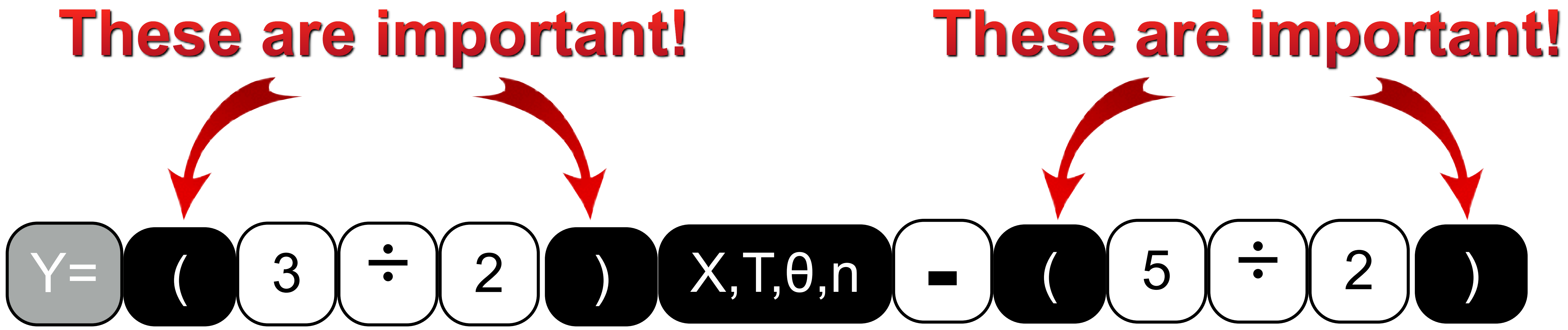
- Graph the line 3x - 2y - 5 = 0
  - To use the TI-84 to graph a linear function, you need the equation in slope-intercept form.

$$3x - 2y - 5 = 0$$

$$-2y = -3x + 5$$

$$y = \frac{-3}{-2}x + \frac{5}{-2}$$

$$y = \frac{3}{2}x - \frac{5}{2}$$



- I can find the equation for parallel and perpendicular lines from data.

Parallel Lines

I can find the equation for parallel and perpendicular lines from data.

- By **comparing slopes**, you can determine if lines are parallel or perpendicular. You can also write equations of lines that meet certain criteria.

Parallel Lines

1. If two non-vertical lines are **parallel**, then they have the **same slope**.
2. If two distinct non-vertical lines have the **same slope**, then they are **parallel**.
3. Two distinct **vertical** lines, both with undefined slopes, are **parallel**.

Parallel Lines

I can find the equation for parallel and perpendicular lines from data.

- Write an equation of the line passing through (–2, 5) and parallel to the line whose equation is $y = 3x + 1$.

The slope of the line $y = 3x + 1$ is 3. A parallel line will also have slope of 3.

In point-slope form, the equation of the new line is

$y - 5 = 3(x - -2)$ or $y - 5 = 3(x + 2)$

In slope-intercept form, the equation of the new line is

$y = 3x + 11$

Perpendicular Lines

1. If two non-vertical lines are **perpendicular**, then the **product of their slopes is -1** (opposite reciprocal).

2. If the **product of their slopes is -1**, then the lines are **perpendicular**.

3. A horizontal line having zero slope is **perpendicular** to a vertical line having undefined slope.

Perpendicular Lines

I can find the equation for parallel and perpendicular lines from data.

- Find the slope of any line that is perpendicular to the line whose equation is $x + 3y - 12 = 0$.

First we must determine the slope of the line.

$$x + 3y - 12 = 0$$

$$3y = -x + 12$$

$$y = \frac{-1}{3}x + 4$$

The slope of the original line is $-\frac{1}{3}$

The slope of any line perpendicular to the original line is 3.

$$-\frac{1}{3} \cdot 3 = -1$$

Parallel and Perpendicular

I can find the equation for parallel and perpendicular lines from data.

- Parallel lines have equal slopes.

$$// \quad m_1 = m_2$$

- Perpendicular lines have opposite reciprocal slopes.

$$\perp \quad m_1 \cdot m_2 = -1$$

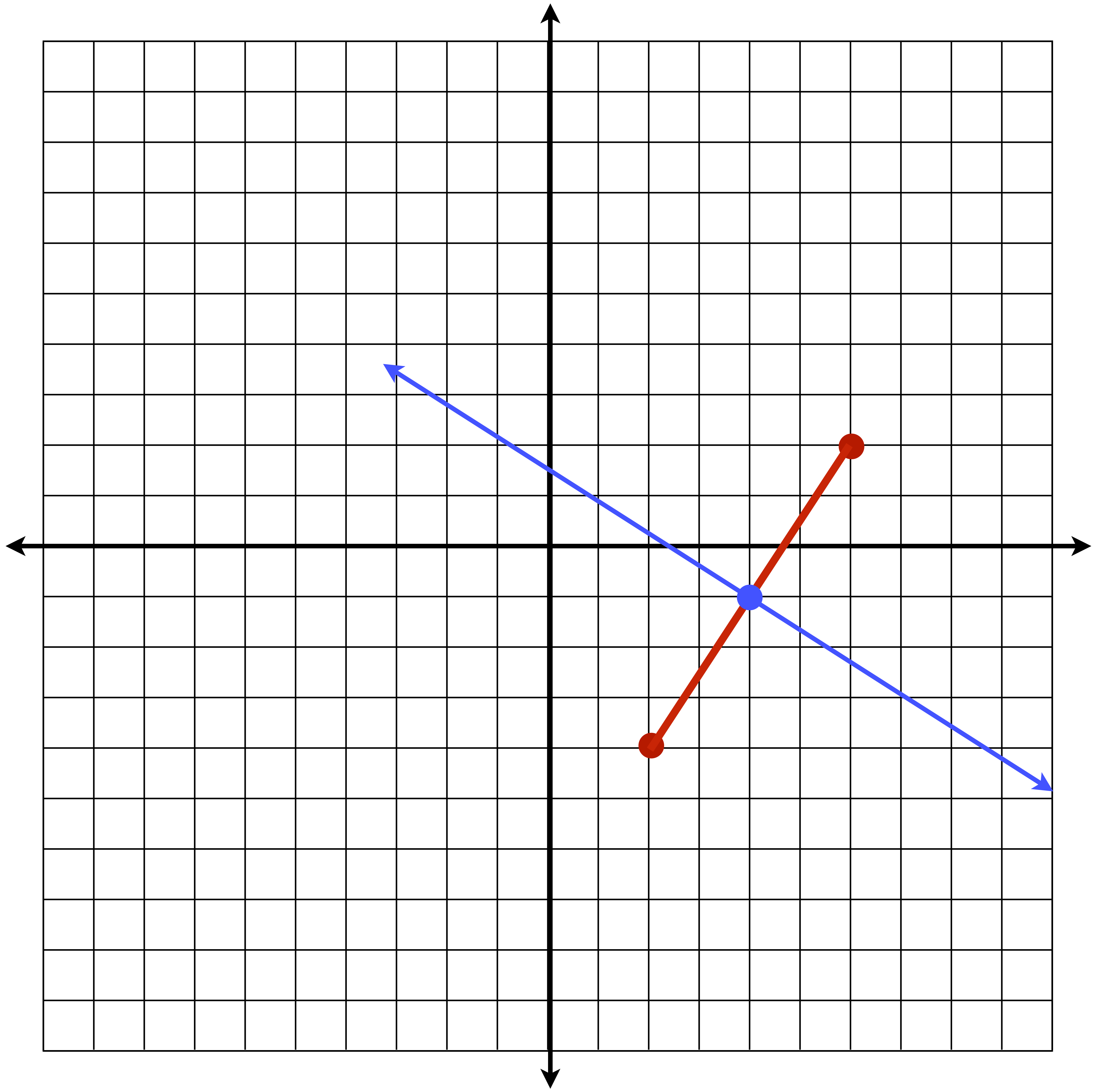
Interpolation, Extrapolation

- Interpolation is finding unknown data values **within** known values.
  - Linear interpolation is finding a point **between** given points on a line.
- Extrapolation is finding unknown data values **beyond** known values.
  - Linear extrapolation is finding a point **outside** given points on a line.

Equidistant

I can find the equation for parallel and perpendicular lines from data.

- Find a relationship between x and y such that (x, y) is equidistant (the same distance) from the two points (2, -4) and (6, 2).
  - If we were to plot those two points, what points would be equidistant?
  - The midpoint (4, -1) would satisfy the conditions, but is it the only point?
  - Every point on a line through the midpoint and perpendicular to the segment joining (2, -4) and (6, 2) would be equidistant from the endpoints.



Equidistant

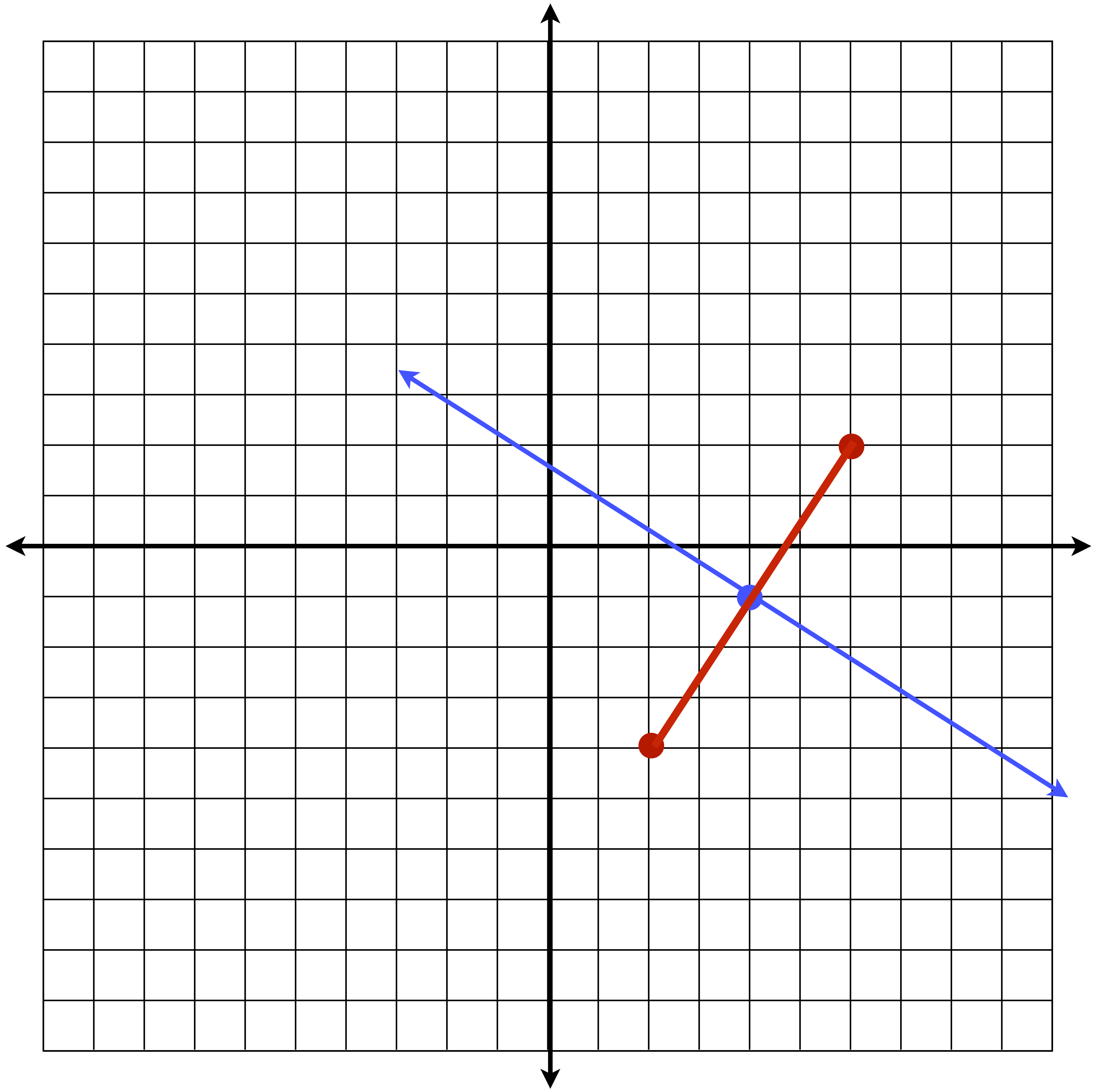
I can find the equation for parallel and perpendicular lines from data.

- Find a relationship between x and y such that (x, y) is equidistant (the same distance) from the two points (2, -4) and (6, 2).
  - The slope of the segment joining our original points is

$$m = \frac{\Delta y}{\Delta x} = \frac{2 - -4}{6 - 2} = \frac{6}{4} = \frac{3}{2}$$

  - The slope of the new relationship must then be $m_2 = -\frac{2}{3}$
  - The new line must go through our midpoint (4, -1)
  - So our line has slope $-\frac{2}{3}$ and contains (4, -1)

$$y - y_1 = m(x - x_1) \qquad y - -1 = -\frac{2}{3}(x - 4) \qquad y = -\frac{2}{3}x + \frac{5}{3}$$



Round Up

Interchangeable Forms

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation

I can graph horizontal or vertical lines.

I can use intercepts to graph a linear equation.

I can model data with linear functions.

- We have learned three ways to write a linear equation.
  - 1. Standard Form $ax + by + c = 0$
  - 2. Slope-Intercept Form $y = mx + b$
  - 3. Point-Slope Form $y - y_1 = m(x - x_1)$

- We can change any of these forms of the linear equations to any other form of the linear equation.

A Summary of the Various Forms of Linear Equations

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation

I can graph horizontal or vertical lines.

I can use intercepts to graph a linear equation.

I can model data with linear functions.

1. Point-Slope Form	$y - y_1 = m(x - x_1)$
2. Slope-Intercept Form	$y = mx + b$
3. Horizontal Line	$y = b$
4. Vertical Line	$x = a$
5. General (Standard) Form	$ax + by + c = 0$

Point-Slope to Slope-Intercept

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation
I can graph horizontal or vertical lines.
I can use intercepts to graph a linear equation.
I can model data with linear functions.

- Write an equation in slope-intercept form for the line with slope 3 that contains (-1, 4).
  - Step 1 Write the equation in point-slope form:

$$y - 4 = 3[x - (-1)]$$
$$y - 4 = 3(x + 1)$$

  - Step 2 Write the equation in slope-intercept form by solving for y.

$$y - 4 = 3(x + 1)$$ Distributive Property.
$$y - 4 = 3x + 3$$
$$y - 4 + 4 = 3x + 3 + 4$$ Add 4 to both sides.
$$y = 3x + 7$$

Point-Slope to Slope-Intercept

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation

I can graph horizontal or vertical lines.

I can use intercepts to graph a linear equation.

I can model data with linear functions.

- Write an equation in slope-intercept form for the line with slope $\frac{1}{3}$ that contains (3, -1).
  - Step 1 Write the equation in point-slope form:

    $$y - -1 = \frac{1}{3}(x - 3) \qquad y + 1 = \frac{1}{3}(x - 3)$$

  - Step 2 Write the equation in slope-intercept form by solving for y.

    $$y + 1 = \frac{1}{3}(x - 3)$$ Distributive Property.

    $$y + 1 = \frac{1}{3}x - 1$$

    $$y = \frac{1}{3}x - 2$$ Add -1 to both sides.

Point-Slope to Slope-Intercept

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation

I can graph horizontal or vertical lines.

I can use intercepts to graph a linear equation.

I can model data with linear functions.

- Write an equation in slope-intercept form for the line with slope $-\frac{2}{5}$ that contains (-4, -5).
  - Step 1 Write the equation in point-slope form:

    $$y - -5 = -\frac{2}{5}(x - -4) \qquad y + 5 = -\frac{2}{5}(x + 4)$$

  - Step 2 Write the equation in slope-intercept form by solving for y.

    $$y + 5 = -\frac{2}{5}(x + 4)$$ Distributive Property.

    $$y + 5 = -\frac{2}{5}x - \frac{8}{5}$$

    $$y = -\frac{2}{5}x - \frac{33}{5}$$ Add -5 to both sides.

Finding Slope-Intercept Form from 2 points.

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation
I can graph horizontal or vertical lines.
I can use intercepts to graph a linear equation.
I can model data with linear functions.

- There is another method for finding the slope-intercept form given two points.
  - What do you think we do first?
    - 1. Find the slope determined by the two given points.

$$m = \frac{\Delta f(x)}{\Delta x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

    - 2. Use the slope and **either** of the two given points to find the equation.

Finding Slope-Intercept Form from 2 points.

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation
I can graph horizontal or vertical lines.
I can use intercepts to graph a linear equation.
I can model data with linear functions.

- Write the equation of the line through the points (2, -3) and (4, 1).
  - Step 1 is to find the slope through those two points. $m = \frac{\Delta Y}{\Delta X} = \frac{1 - -3}{4 - 2} = \frac{4}{2} = 2$
  - Step 2: Choose one point to use. $y = mx + b$
  - (4, 1) We know three values in the equation. We have an x value (4), a y value (1), and slope (2). $1 = 2(4) + b \quad 1 = 8 + b \quad -7 = b$
  - (2, -3) Or we know three values in the equation. We have an x value (2), a y value (-3), and slope (2). $-3 = 2(2) + b \quad -3 = 4 + b \quad -7 = b$
  - Our equation is $f(x) = 2x + -7$ or $f(x) = 2x - 7$

Finding Slope-Intercept Form from 2 points.

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation
I can graph horizontal or vertical lines.
I can use intercepts to graph a linear equation.
I can model data with linear functions.

- Write the equation of the line through the points (0, 1) and (-2, 9).
  - Step 1 is to find the slope through those two points.

$$m = \frac{\Delta Y}{\Delta X} = \frac{9 - 1}{-2 - 0} = \frac{8}{-2} = -4$$

  - Step 2: Choose one point to use. $y = mx + b$
    - Come on now! The y-intercept is 1.
  - Our equation is $f(x) = -4x + 1$

Finding Slope-Intercept Form from 2 points.

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation
I can graph horizontal or vertical lines.
I can use intercepts to graph a linear equation.
I can model data with linear functions.

- Write the equation of the line through the points (1, -3) and (6, -7).
  - Step 1 is to find the slope through those two points.

$$m = \frac{\Delta Y}{\Delta X} = \frac{-3 - -7}{1 - 6} = \frac{4}{-5} = -\frac{4}{5}$$

  - Step 2: Choose one point to use. $y = mx + b$
  - Choose (1, -3) $\quad -3 = -\frac{4}{5}(1) + b \quad -3 = -\frac{4}{5} + b \quad -\frac{11}{5} = b$
  - Or choose (6, -7) $\quad -7 = -\frac{4}{5}(6) + b \quad -7 = -\frac{24}{5} + b \quad -\frac{11}{5} = b$
  - Our equation is $y = -\frac{4}{5}x + -\frac{11}{5}$ or $y = -\frac{4}{5}x - \frac{11}{5}$

Standard Form to Slope-Intercept Form

I can calculate a line's slope, write and graph the equation of a line in general, slope-intercept, and point-slope form of the equation
I can graph horizontal or vertical lines.
I can use intercepts to graph a linear equation.
I can model data with linear functions.

- Find the slope and the y-intercept of the line whose equation is: $3x + 6y - 12 = 0$
  - Rewrite the equation in slope intercept form by **solving for y in terms of x**.

$$3x + 6y - 12 = 0$$

$$6y = -3x + 12$$

$$\frac{1}{6}(6y) = \frac{1}{6}(-3x + 12)$$

$$y = -\frac{1}{2}x + 2$$

The slope is $-\frac{1}{2}$

NOT $-\frac{1}{2}x$

The y-intercept is 2.

Standard Form to Slope-Intercept Form

Graph the linear function $2y + 4x = 7$ using slope-intercept form.

Step 1 Write the equation in slope intercept form.

$$2y + 4x = 7$$

$$2y = -4x + 7$$

$$\frac{1}{2}(2y) = \frac{1}{2}(-4x + 7)$$

$$y = -2x + \frac{7}{2}$$

Step 2 Plot the y-intercept on the y-axis. $\left(0, \frac{7}{2}\right)$

Step 3 Using the slope, plot a second point. $m = \frac{-2}{1} = \frac{\Delta y}{\Delta x}$

Step 4 Draw the line.