Chapter 1

Functions and Graphs

1.4 Basics of Functions and Their Graphs

Chapter 1.4

Homework

1.4 Pg 48, 5-10, 13-37 odd, 45, 47, 49, 53-69 odd, 79-85 odd

Objectives:

- Find the domain and range of a relation.
- Determine whether a relation is a function.
- Determine whether an equation represents a function.
- Evaluate a function.
- Evaluate the difference quotient

Definition of a Relation

- A relation is any set of ordered pairs.
  - The set of all first components (X) of the ordered pairs is called the domain of the relation
    - The set of all second components (Y) is called the range of the relation.
- Find the domain and range of the relation:

{(0, 9.1), (10, 6.7), (20, 10.7), (30, 13.2), (40, 21.2)}.

domain: {0, 10, 20, 30, 40}

range: {9.1, 6.7, 10.7, 13.2, 21.2}

Definition of a Function

- A **function** is a correspondence from a first set, called the **domain**, to a second set, called the **range**, ...

  such that each element in the domain corresponds to exactly one element in the range.

- Consider a trip to the doughnut shoppe. If you are allowed to choose more than one doughnut that would be a relation, but not a function. One person from the domain can be matched with more than one doughnut from the range of doughnuts. If, howsomever, you are only allowed one doughnut, that would define a function. Even if your friends choose the same doughnut it is still a function. One person, one doughnut.

Definition of Function

A **function** f from a set A to a set B is a relation that assigns to each element x in the set A exactly one element y in the set B. The set A is the **domain** (or set of inputs) of the function f, and the set B contains the **range** (or set of outputs).

Characteristics of a Function from Set *A* to Set *B*

1. Each element in A must be matched with an element in B.
2. Some elements in B may not be matched with any element in A.
3. Two or more elements in A may be matched with the same element in B.
4. An element in A (the domain) cannot be matched with two different elements in B.

Example: Determining Whether a Relation is a Function

- Determine whether the relation is a function: {(1, 2), (3, 4), (6, 5), (8, 5)}.
  - Yeperdoo - No two ordered pairs in the given relation have the same first component and different second components. Thus, the relation is a function.
- If an equation is solved for y and **more than one value of y can be obtained for a given x**, then the equation does **not** define y as a function of x.
- Occasionally the variable representing the **input value**, usually represented by **x**, is called the **independent variable**. The variable representing the **output value**, usually represented by **y**, is called the **dependent variable**.

Example: Determining Whether an Equation Represents a Function

- Determine whether the equation defines y as a function of x.

$$x^2 + y^2 = 1 \qquad y^2 = 1 - x^2 \qquad y = \pm\sqrt{1 - x^2}$$

- The ± indicates that for certain values of x, there are two values of y. For this reason, the equation does not define y as a function of x.

- Please note: $\sqrt{1 - x^2} \neq 1 - x$

- Making this mistake will make my head explode.

Function Notation

- The special notation f(x), read "f of x" or "f at x", represents the value of the function (commonly known as "y") at the number x.

- Students are occasionally confused by function notation. Especially when the input value is an expression.

- If $f(x) = x^2 + 2$, then $f(x+1) = (x+1)^2 + 2$. Note that the input value is x+1 , and that is what is squared.

Example: Evaluating a Function

- If $f(x) = x^2 - 2x + 7$ evaluate $f(-5)$

$$f(x) = x^2 - 2x + 7$$

$$f(-5) = (-5)^2 - 2(-5) + 7$$

$$f(-5) = 25 + 10 + 7 = 42$$

- Thus $f(-5) = 42$

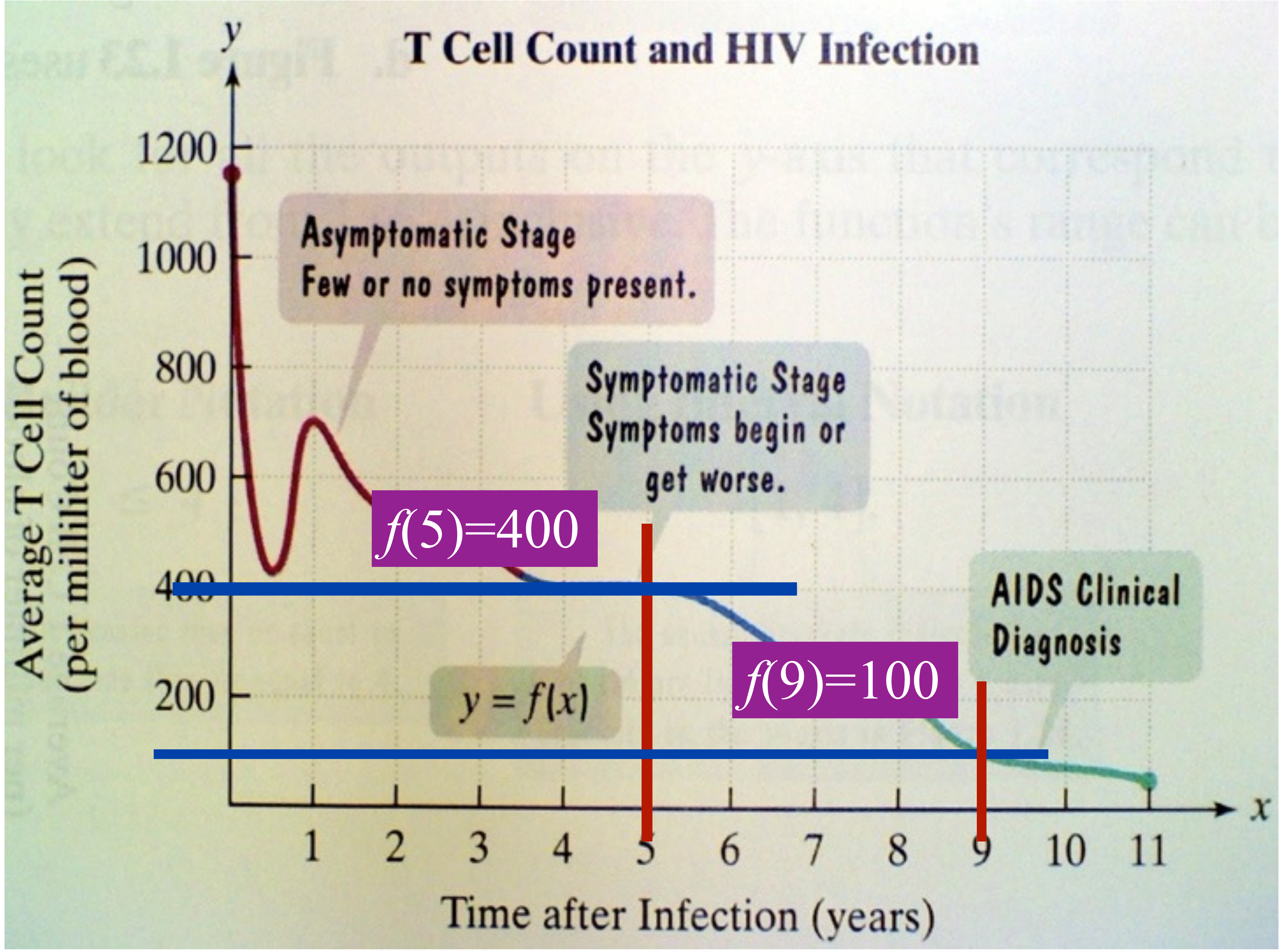
- If $f(x) = x^2 - 2x + 7$ evaluate $f(x+2)$

$$f(x+2) = (x+2)^2 - 2(x+2) + 7$$

$$f(x+2) = (x^2 + 4x + 4) - 2x - 4 + 7 = x^2 + 2x + 7$$

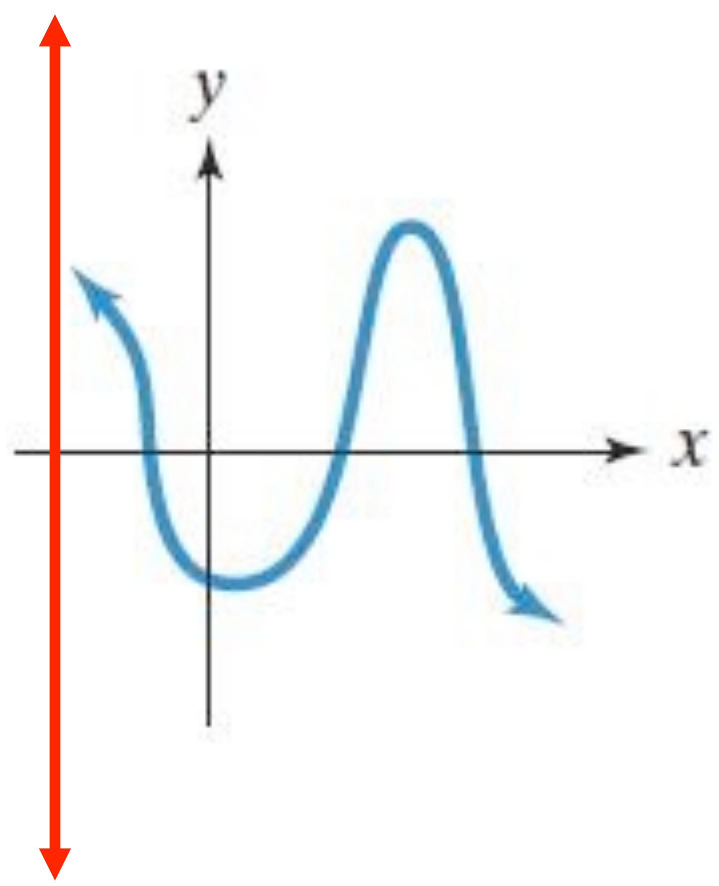
- Thus $f(x+2) = x^2+2x+7$

- Use the graph to find f(5)
  - f(5) = 400
- For what value of x is f(x) = 100?
  - f(9) ≈ 400, so x ≈ 9

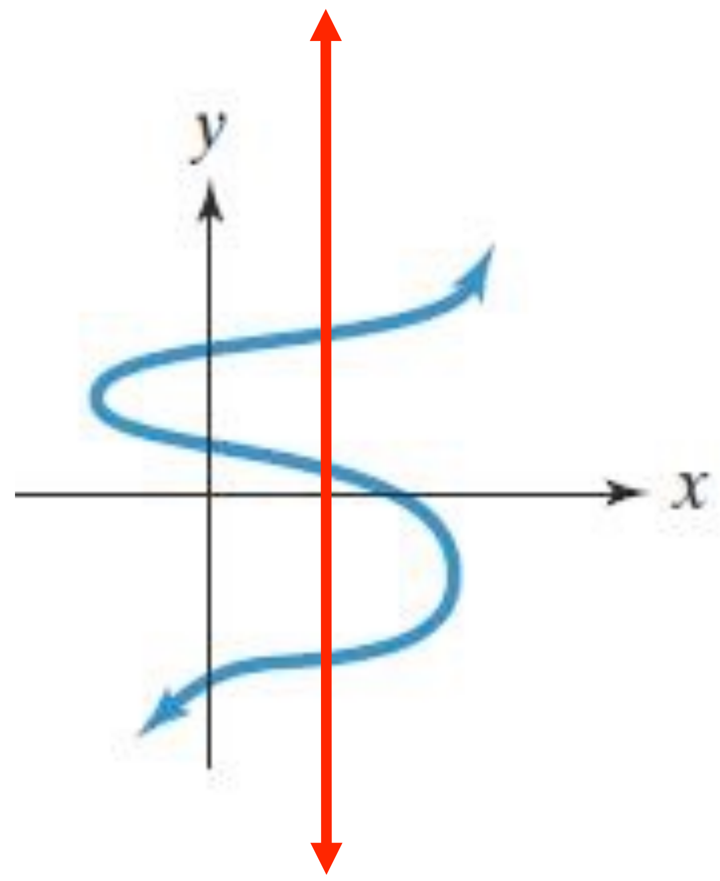


The Vertical Line Test for Functions

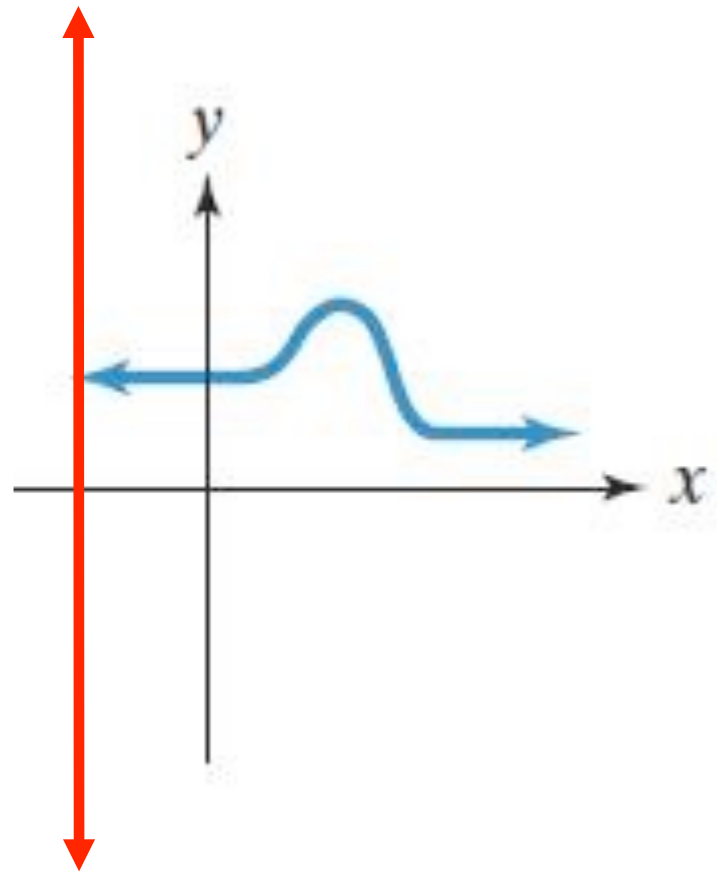
- If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x.
  - Use the vertical line test to identify graphs in which y is a function of x.



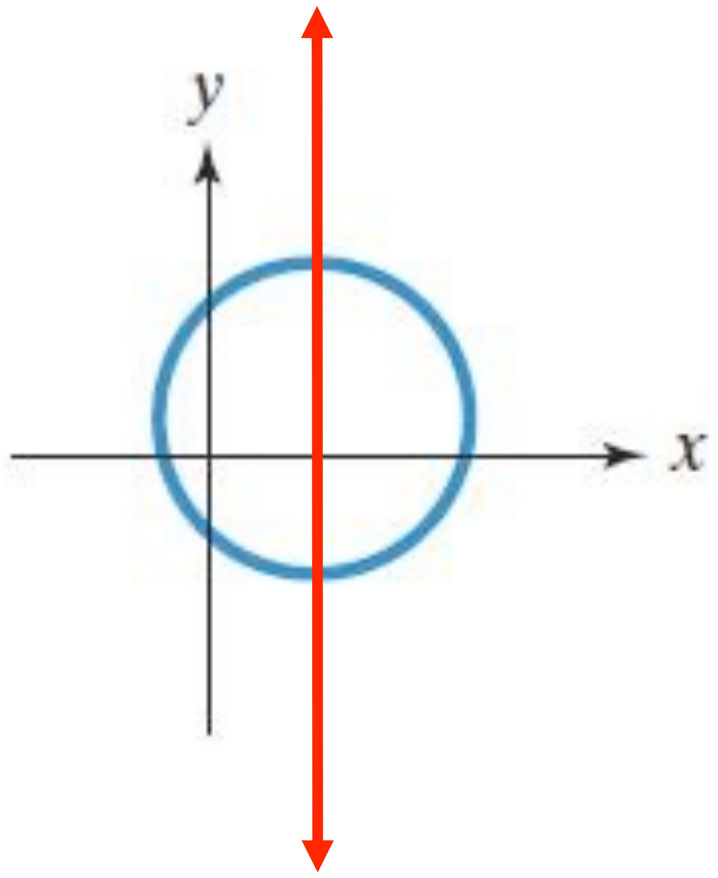
function



not a function



function



not a function

Domain

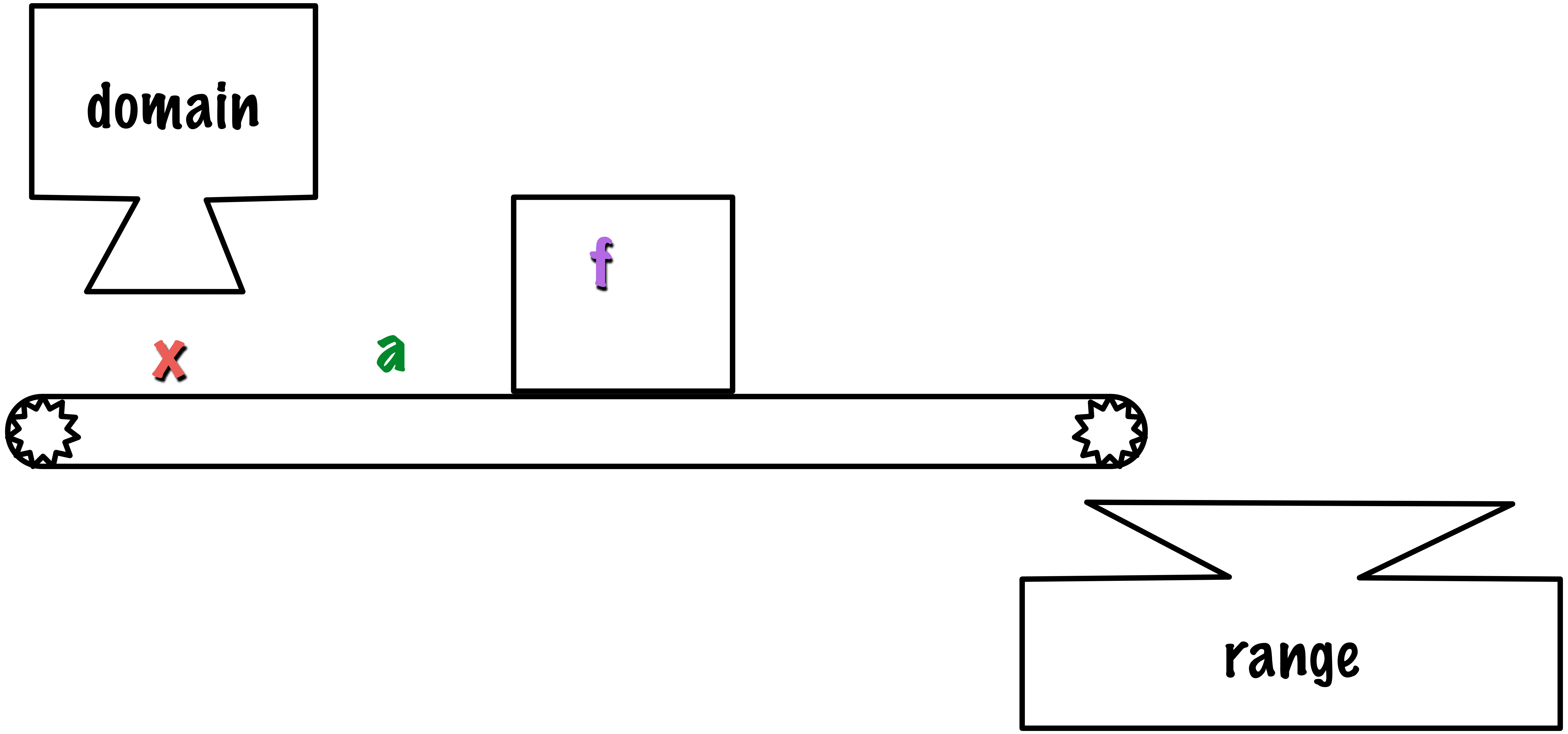
- The domain of a function is the set of all possible input values (all possible x values).

  - The domain can be explicit, meaning that it is decided apriori or defined for the function. Such as deciding ahead of time (apriori) that we will restrict the domain to positive integers.

  - The domain can be implicit, meaning that the function is not defined for some values. Taking the square root of negative numbers result in imaginary values, so if we are only interested in real numbers the domain of the square root function is implicitly defined as positive real numbers.

Range

- The range of a function is the set of all possible output values (all possible f(x) or y values).
  - The range is the resulting set of values from substituting all defined values (x) of the domain.



Domain

Technology

Use a graphing utility to graph the functions given by $y = \sqrt{4 - x^2}$ and $y = \sqrt{x^2 - 4}$. What is the domain of each function? Do the domains of these two functions overlap? If so, for what values do the domains overlap?

- What is the domain of the first function?

$$f(x) = \sqrt{4 - x^2}$$

$$[-2, 2]$$

- What is the domain of the 2nd function?

$$f(x) = \sqrt{x^2 - 4}$$

$$(-\infty, -2] \cup [2, \infty)$$

Domain

- When determining the domain of a function ask yourself a couple of questions.
  - 1. What values make sense in the problem.
    - Do negative values make sense? Do fractional values make sense?
  - 2. What values are prohibited?
    - Even roots of negative values? Denominators of 0?

Domain

- Find the appropriate domains.

$$f(x)=\frac{1}{x^2-2x-8} \qquad =\frac{1}{(x+2)(x-4)}$$

D: All reals except -2 and 4,
$(-\infty, -2)\cup(-2, 4)\cup(4, \infty)$

$$f(x)=\sqrt{x^2-8} \qquad x^2\geq 8$$

$D: x\leq -\sqrt{8}, x\geq \sqrt{8}$

$\left(-\infty,-\sqrt{8}\right]\cup\left[\sqrt{8},\infty\right)$

Finding Domain and Range From a Function's Graph

- To find the domain of a function from it's graph, look for all the inputs on the x-axis that correspond to points on the graph.
- To find the range of a function from it's graph, look for all the outputs on the y-axis that correspond to points on the graph.
- A function may have more than one x-intercept, but a function can have only one y-intercept.

Finding Domain and Range From a Function's Graph

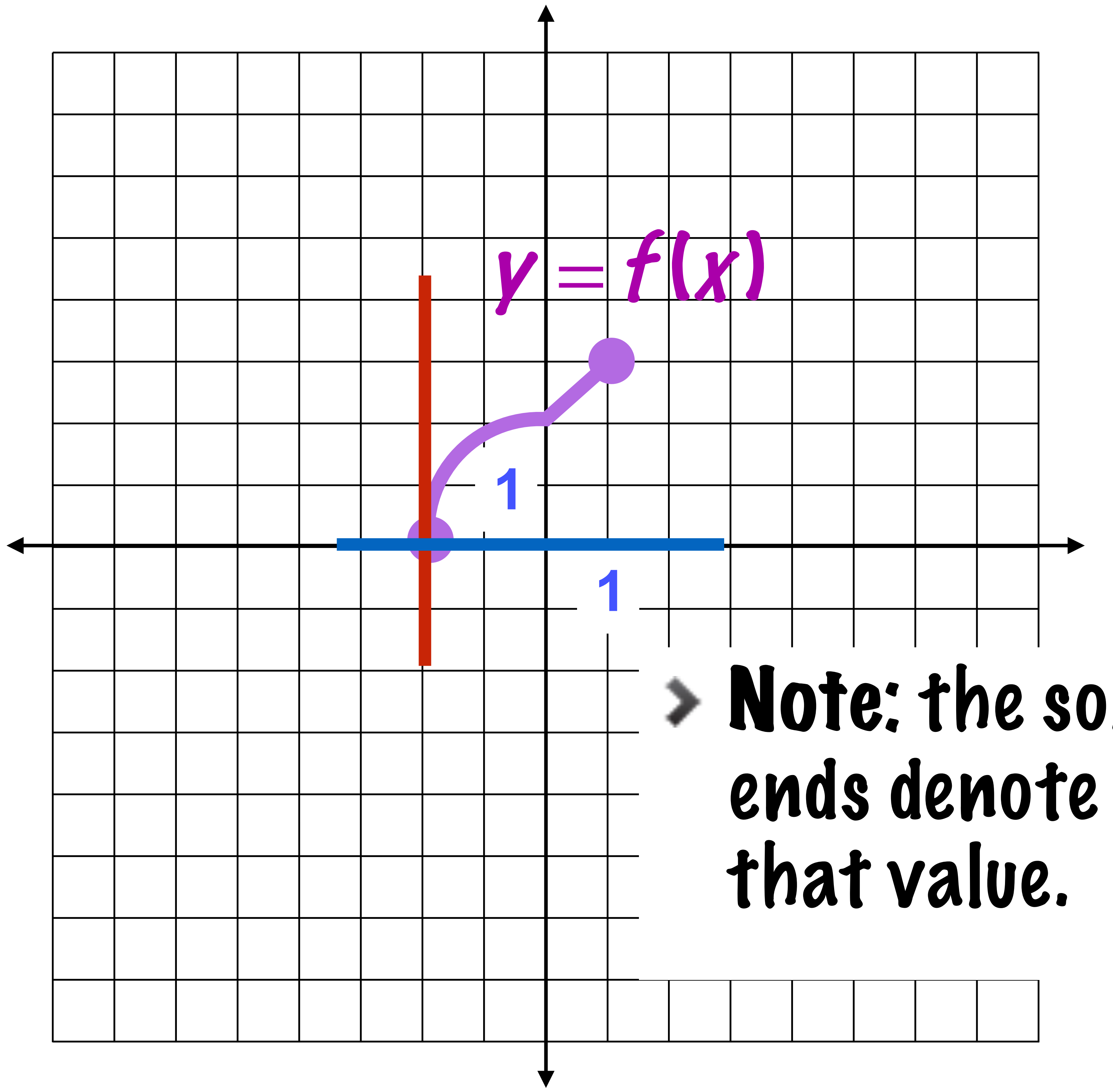
- Use the graph of the function to identify its domain and its range.

- Domain $\{x \mid -2 \le x \le 1\}$

  $[-2, 1]$

- Range $\{y \mid 0 \le y \le 3\}$

  $[0, 3]$



- Note: the solid dots on the ends denote inclusion of that value.

Example: Identifying the Domain and Range of a Function from Its Graph

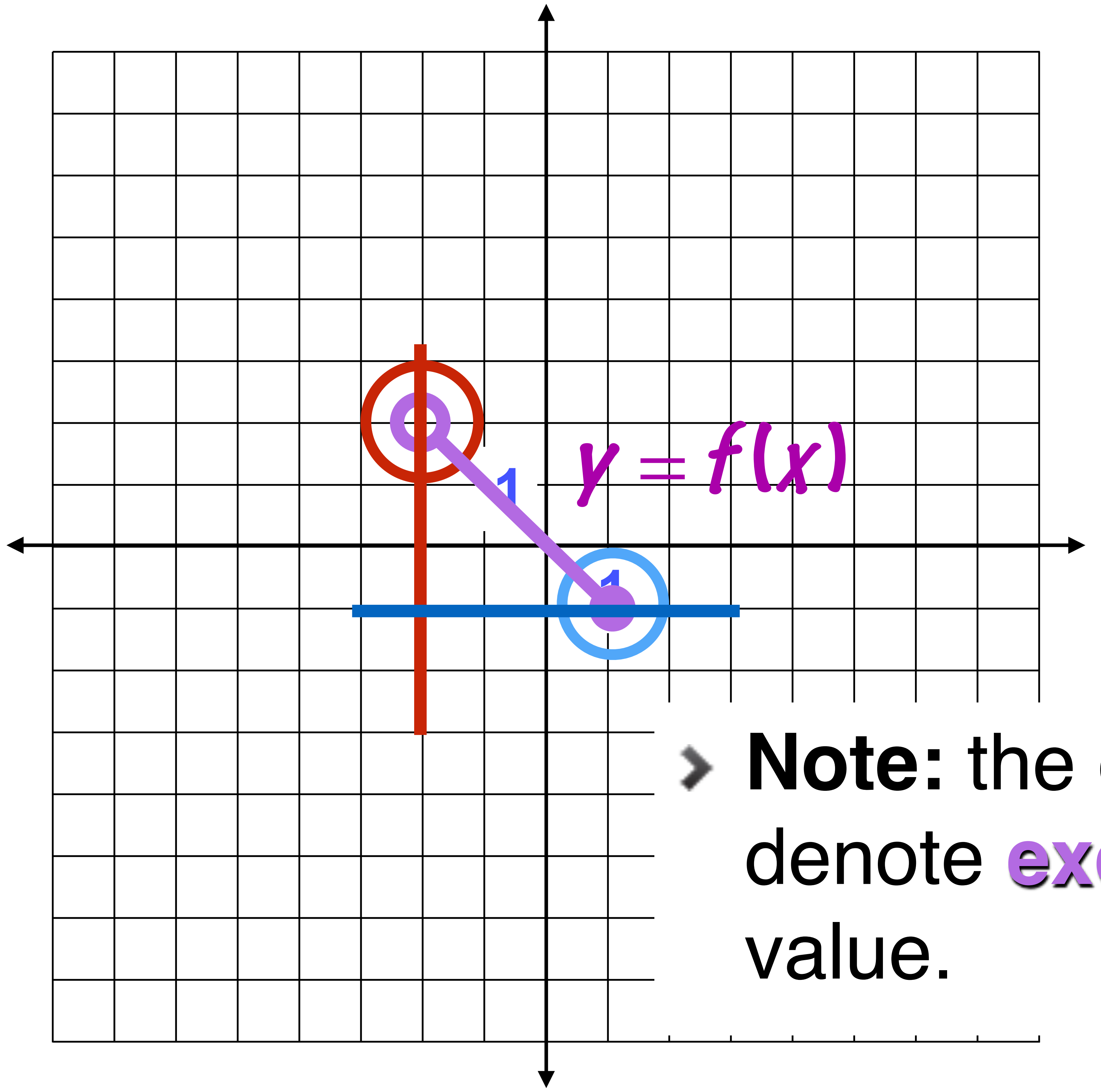
- Use the graph of the function to identify its domain and its range.

- Domain $\{x \mid -2 < x \leq 1\}$

  $(-2, 1]$

- Range $\{y \mid -1 \leq y < 2\}$

  $[-1, 2)$



- **Note:** the open ends denote **exclusion** of that value.

Piecewise Functions

- A function that is defined by two (or more) equations over a specified domain is called a
  - **piecewise function.**

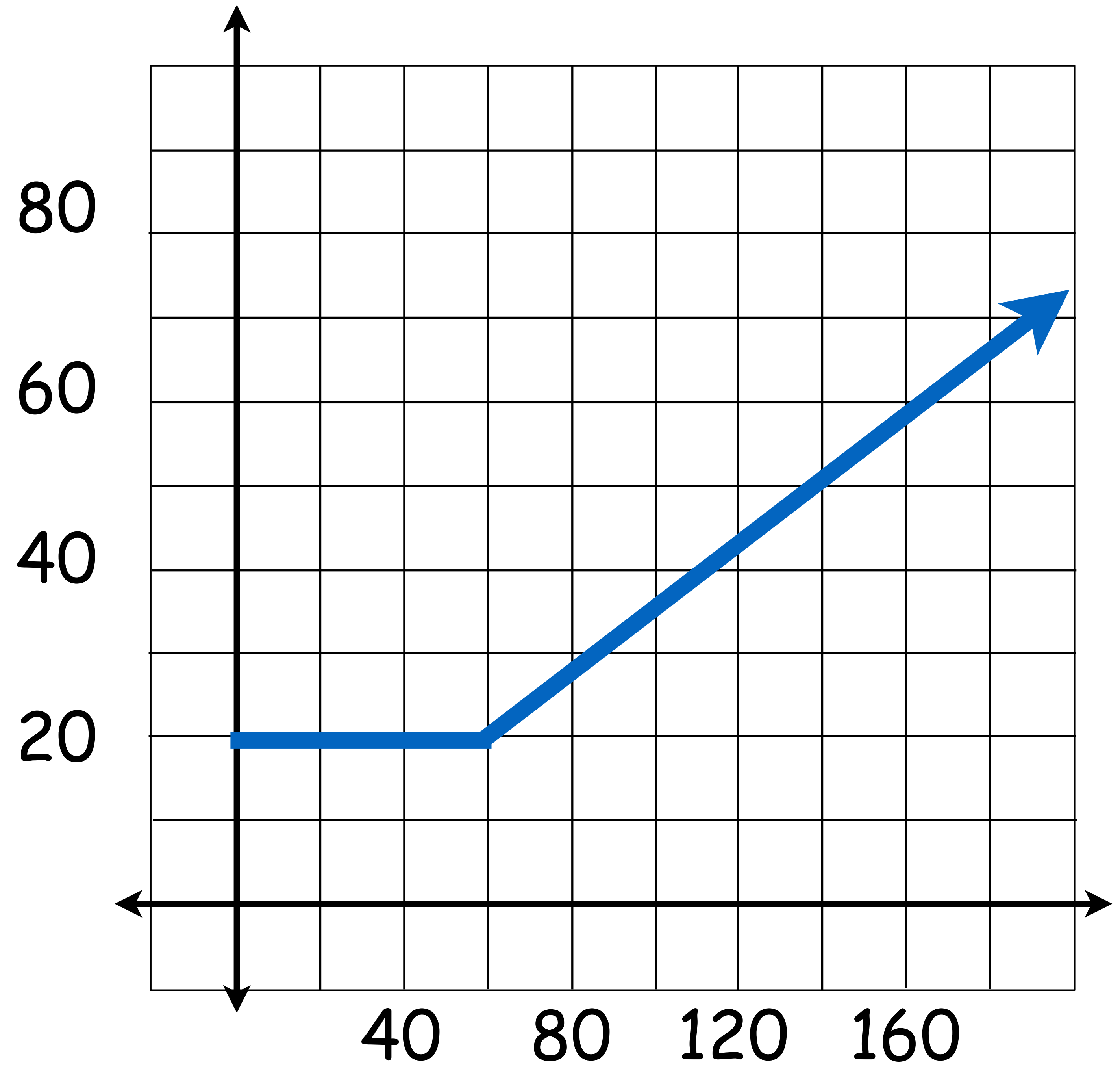
- Given the function $C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t-60) & \text{if } t > 60 \end{cases}$

Find C(40)

$0 \le 40 \le 60 \text{ so } C(40) = 20$

Find C(80)

$$80 > 60 \text{ so } C(80) = 20 + 0.40(80-60)$$
$$= 20 + 0.4(20)$$
$$= 20 + 8 = 28$$



Piecewise Functions

- Graph the piecewise function defined by $C(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x-2 & \text{if } x > -1 \end{cases}$

  - We will graph f in two parts, using a partial table of coordinates for each piece of the graph.

x	f(x)=3	(x,f(x))
-1	3	(-1,3)
-2	3	(-2,3)
-3	3	(-3,3)

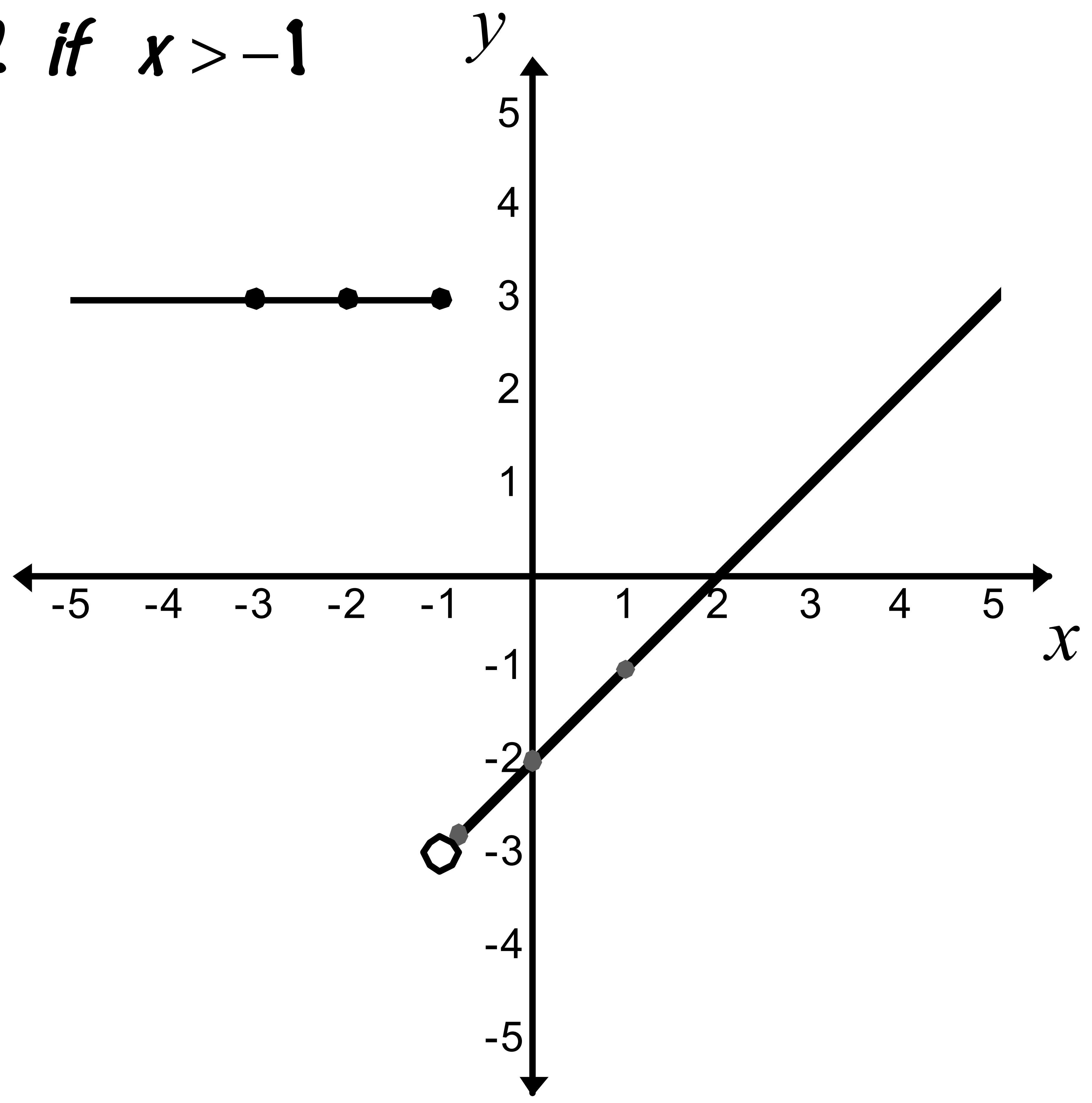
x	f(x)=x-2	(x,f(x))
-1	-3	(-1,-3)
0	-2	(0,-2)
1	-1	(1,-1)

Piecewise Functions

- Graph the piecewise function defined by $C(x) = \begin{cases} 3 & \text{if } x \le -1 \\ x-2 & \text{if } x > -1 \end{cases}$

x	f(x)=3	(x,f(x))
-1	3	(-1,3)
-2	3	(-2,3)
-3	3	(-3,3)

x	f(x)=x-2	(x,f(x))
-1	-3	(-1,-3)
0	-2	(0,-2)
1	-1	(1,-1)



Difference Quotient

- The expression $\dfrac{f(x+h)-f(x)}{h}$ for $h \neq 0$ is called the **difference quotient** of the function f.

If $f(x) = -2x^2 + x + 5$, find and simplify the **difference quotient** $\dfrac{f(x+h)-f(x)}{h}$

$$f(x+h) = -2(x+h)^2 + (x+h) + 5$$

$$= -2(x^2 + 2hx + h^2) + (x+h) + 5$$

$$f(x+h) = -2x^2 - 4hx - 2h^2 + x + h + 5$$

$$f(x) = -2x^2 + x + 5$$

Difference Quotient

If $f(x) = -2x^2 + x + 5$, find and simplify the **difference quotient** $\dfrac{f(x+h) - f(x)}{h}$

$f(x+h) = -2x^2 - 4hx - 2h^2 + x + h + 5 \qquad f(x) = -2x^2 + x + 5$

$$\frac{f(x+h) - f(x)}{h} = \frac{\left(\cancel{-2x^2} - 4hx - 2h^2 + \cancel{x} + h + \cancel{5}\right) - \left(\cancel{-2x^2} + \cancel{x} + \cancel{5}\right)}{h}$$

$$= \frac{-4hx - 2h^2 + h}{h} \qquad = \frac{h(-4x - 2h + 1)}{h}$$

$$= -4x - 2h + 1, h \neq 0$$

Functions

- Summary
  - Function: A function is a relationship between two variables such that to each value of the independent variable there corresponds exactly one value of the dependent variable.
  - $f(x) = y$
    - f is the name of the function.
    - x is the input value (of independent variable) from the domain.
    - $f(x)$ is the output value (of dependent variable) from the range when x is the input value.

Wrap Up

- Summary

  - Domain: The domain is the set of all possible values of the inputs x (independent variable) for which the function is defined.

    - The domain can be implicit, determined by the values for which the function is defined, or explicit, specifically defined.

  - Range: The range is the set of all possible values of the outputs f(x) (dependent variable).