Chapter 1

Functions and Graphs

1.5 Graphs of Functions

Chapter 1.5

Homework

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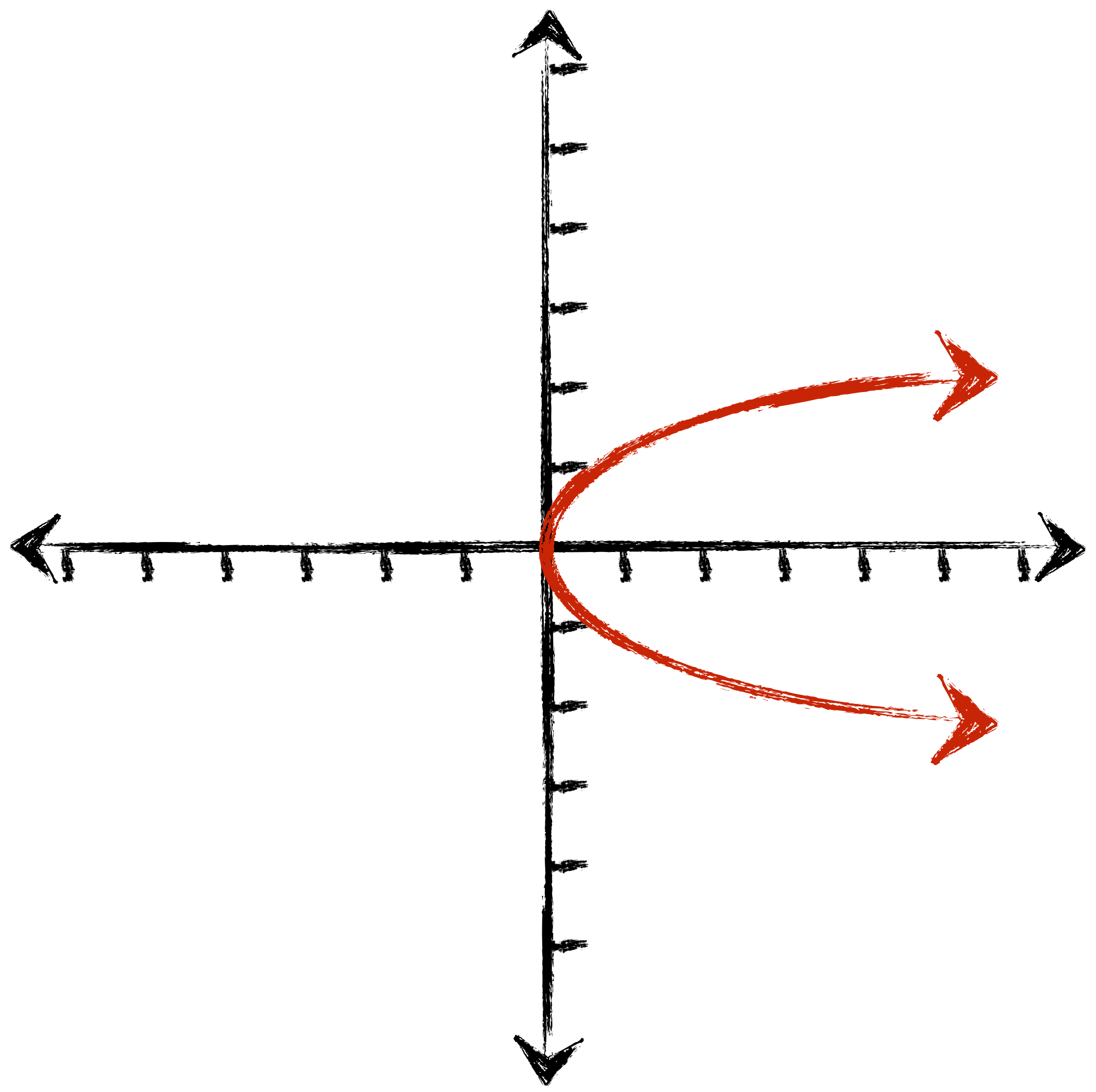
Chapter 1.5

Objectives

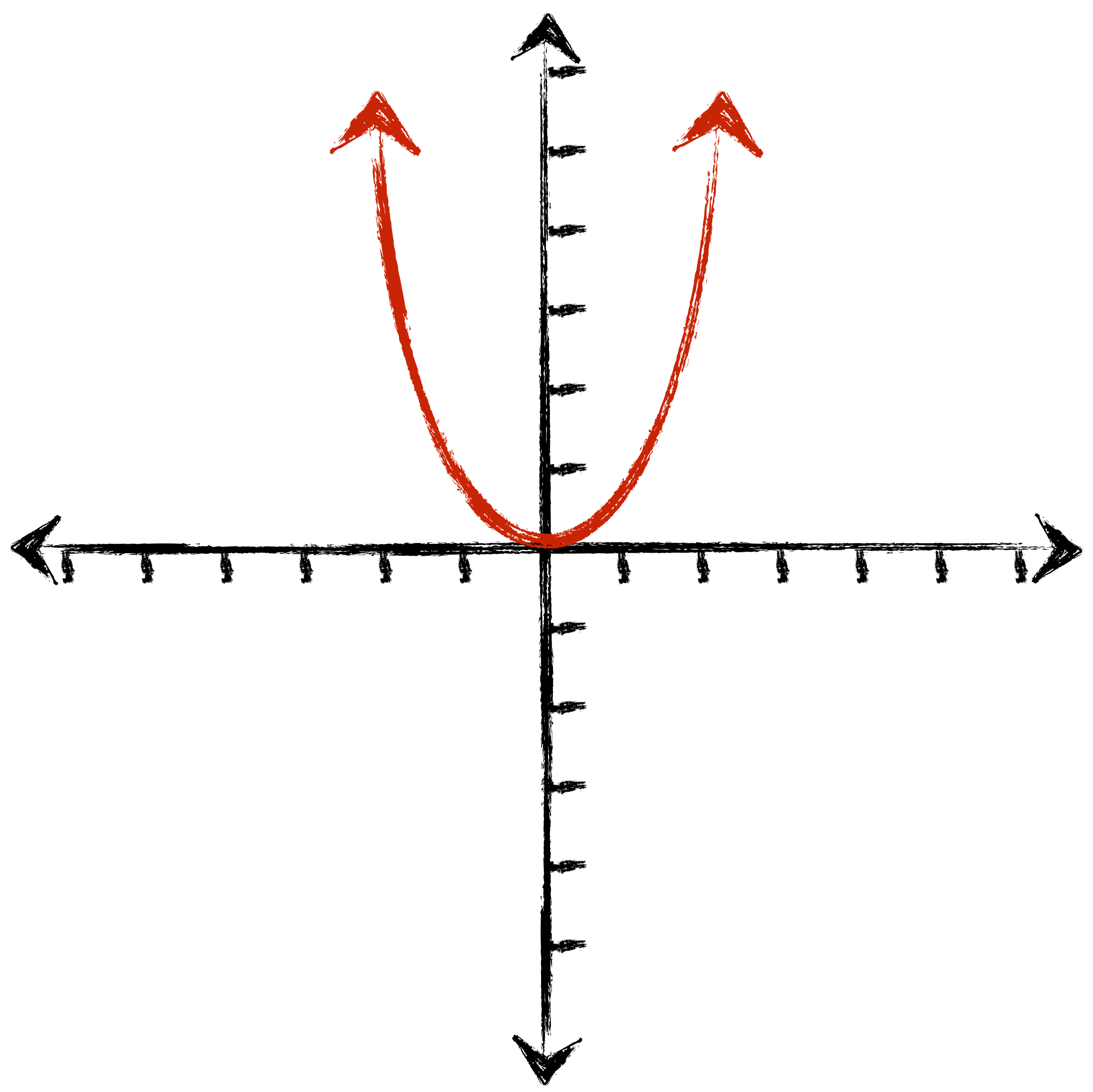
- Use Vertical Line Test
- Find the zeros of a functions
- Identify intervals on which a function increases, decreases, or is constant.
- Use graphs to locate relative maxima or minima.
- Identify even or odd functions and recognize their symmetries.
- Determine the average rate of change of a function.

Function

- The graph of a function is the graph of its ordered pairs (x, f(x)).
- Remember: To be a function there must be **exactly** one output value for each input value in the domain.
    - Is the relation $y^2 = x$ a function?



    - Is the relation $y = x^2$ a function?



Zeros of a Function

- If graph of a function of has an x-intercept at (a, 0) then a is a zero of the function.
- The zeros of a function are the values of x for which f(x) = 0. The zeros are input values that return an output value of zero.

> **Zeros of a Function**
>
> The **zeros of a function** f of x are the x-values for which $f(x) = 0$.

- To find the zeros of a function, set the function equal to zero and solve for the independent variable.

Zeros of a Function

- The zeros of a function are the roots of an equation and the x-intercepts of the graph of a function if there are x-intercepts.

- Those three values for a function all refer to the same quantities. We will be explicit in the use of those terms.

    - Zeros of a function.
    - Roots of an equation.
    - x-intercepts (if existing) of the graph of the function.

Zeros

- Find the zeros of
  - $2x^2 - x - 1$

$2x^2 - x - 1 = 0$

$(2x + 1)(x - 1) = 0$

$(2x + 1) = 0$ or $(x - 1) = 0$

$x = -1/2$ or $x = 1$

The zeros are -1/2 and 1

$$\sqrt[3]{x-1}$$

$$\sqrt[3]{x-1} = 0$$

$$x - 1 = 0$$

$$x = 1$$

The zero is 1

$$\frac{x-5}{2x-1}$$

$$\frac{x-5}{2x-1} = 0$$

$$x - 5 = 0$$

$$x = 5$$

The zero is 5

Increasing, Decreasing, and Constant Functions

1. A function is increasing on an open interval, I, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.

2. A function is decreasing on an open interval, I, if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.

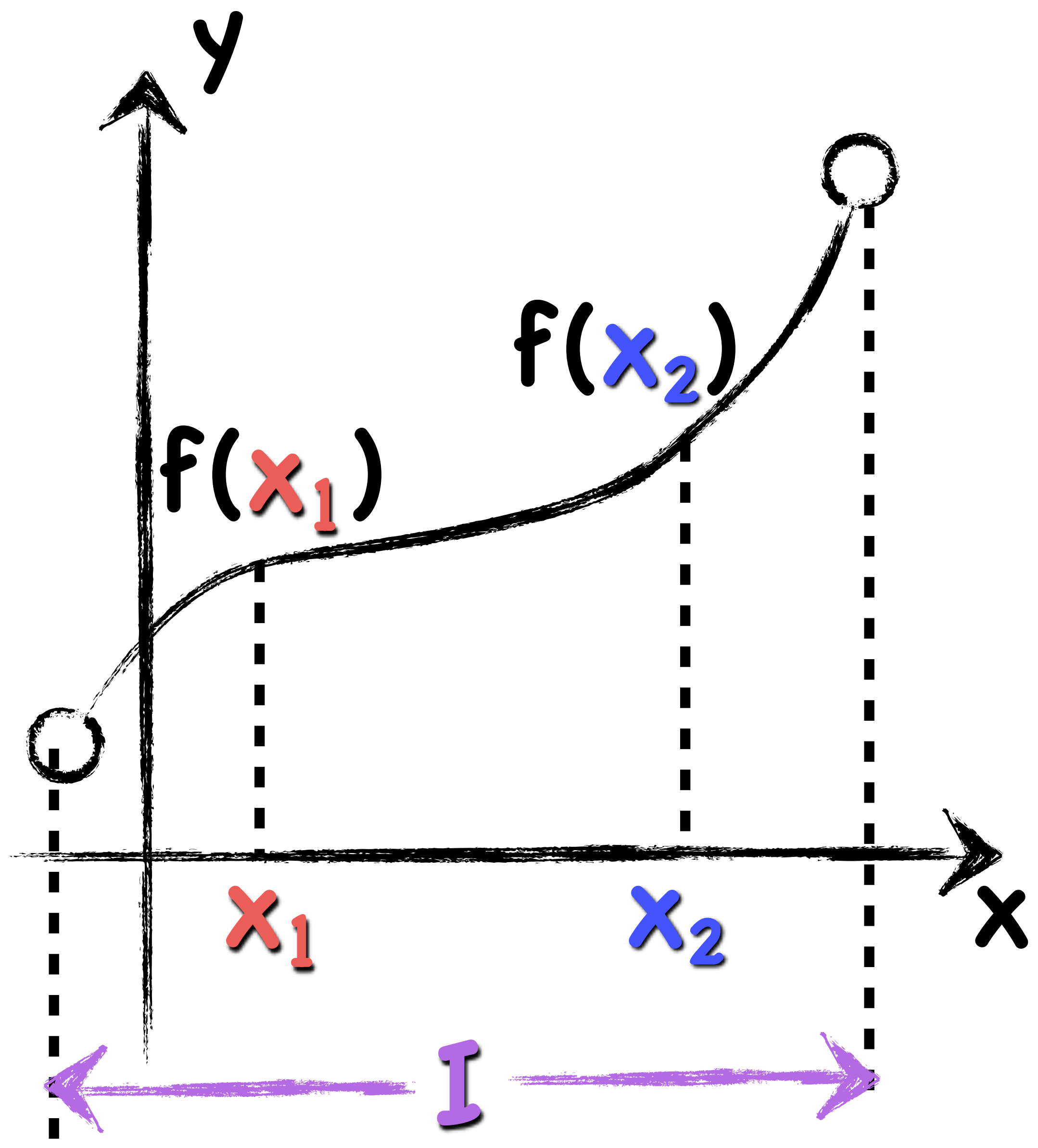
3. A function is constant on an open interval, I, if $f(x_1) = f(x_2)$ for any x_1 and x_2 in the interval.

Increasing, Decreasing, and Constant Functions

The open intervals, I, describing where functions increase, decrease, or are constant, use x-coordinates and not the y-coordinates.

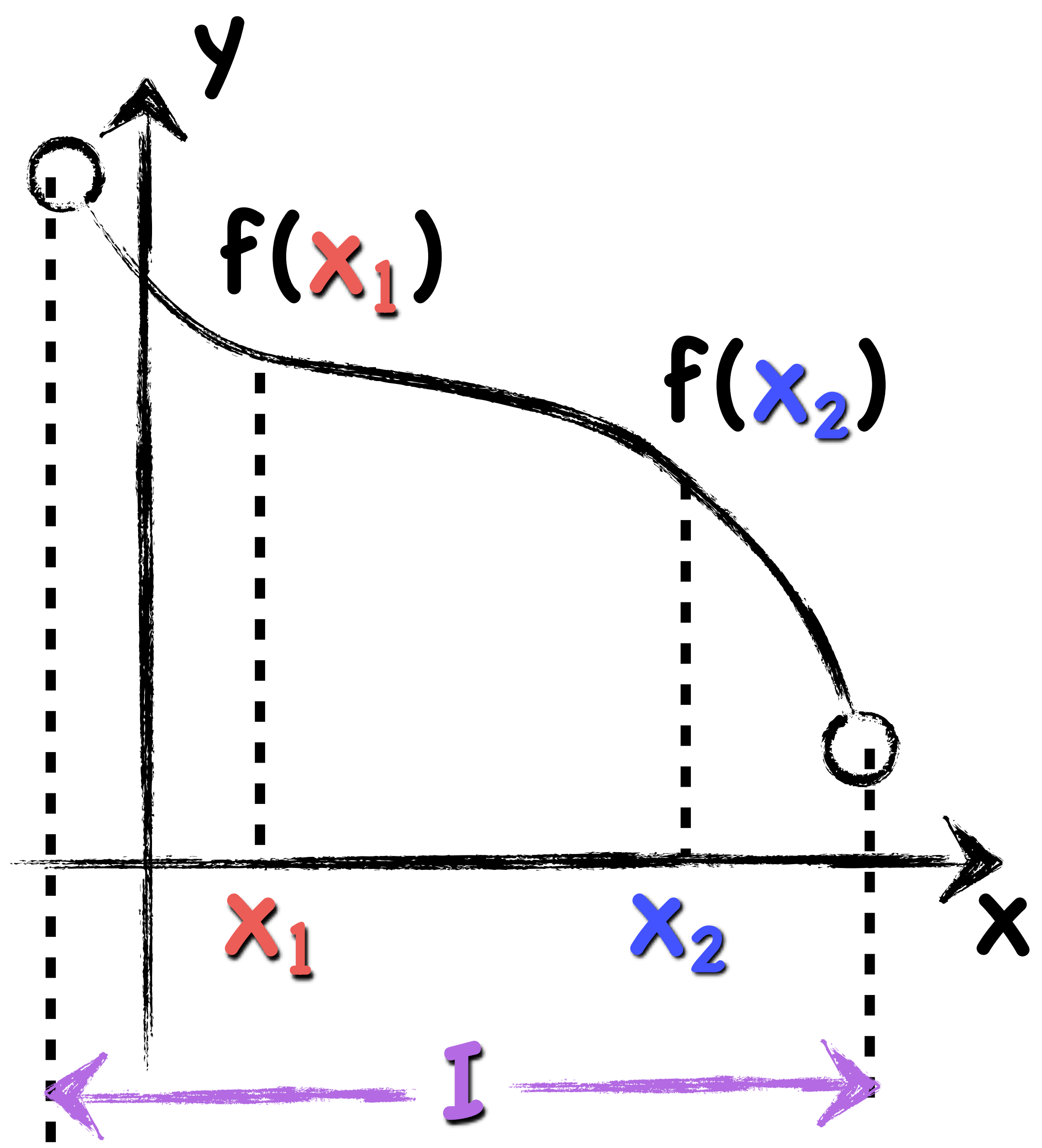
Increasing, Decreasing, and Constant Functions

Increasing



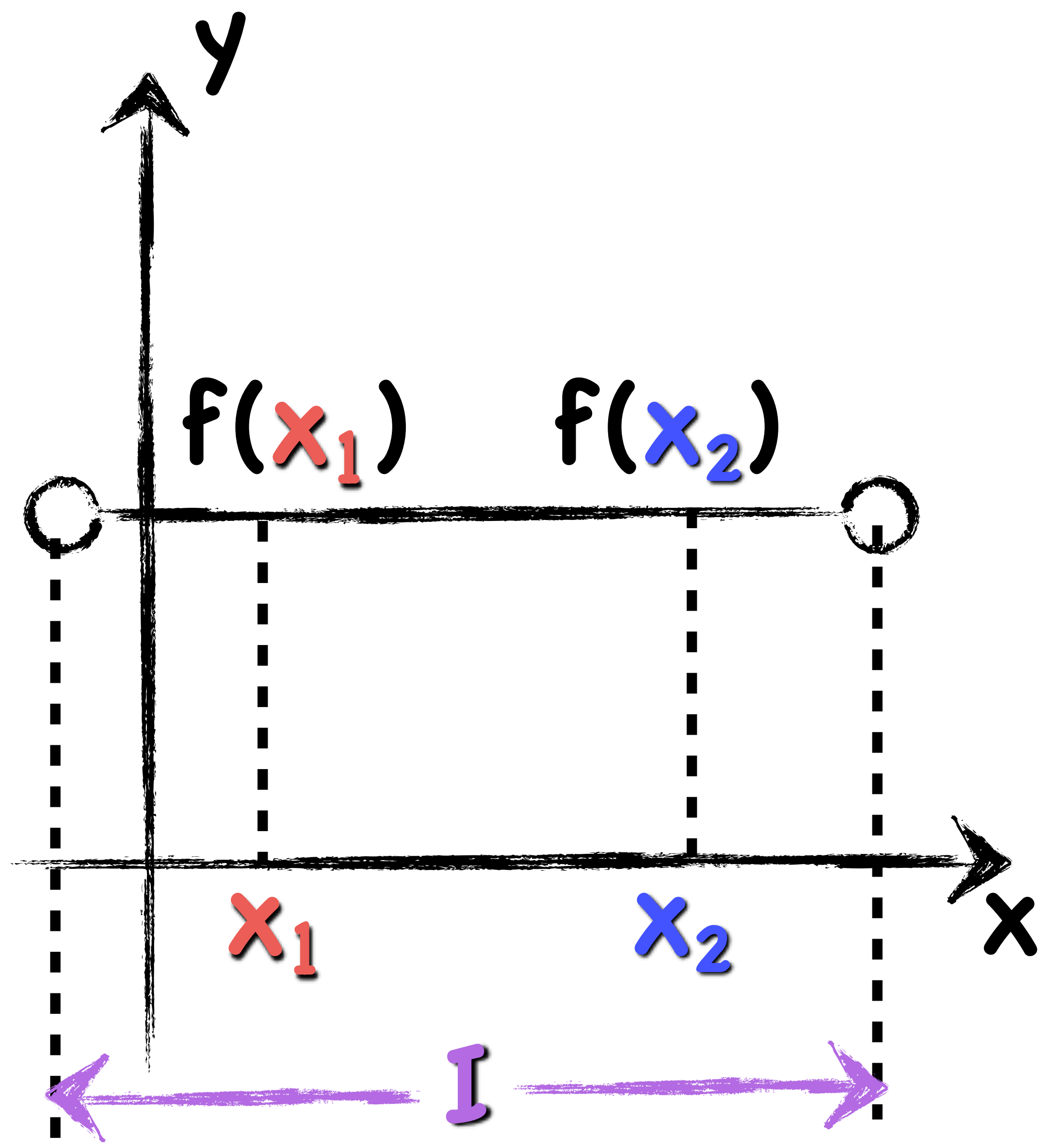
In I, $f(x_1) < f(x_2)$
whenever $x_1 < x_2$

Decreasing



In I, $f(x_1) > f(x_2)$
whenever $x_1 < x_2$

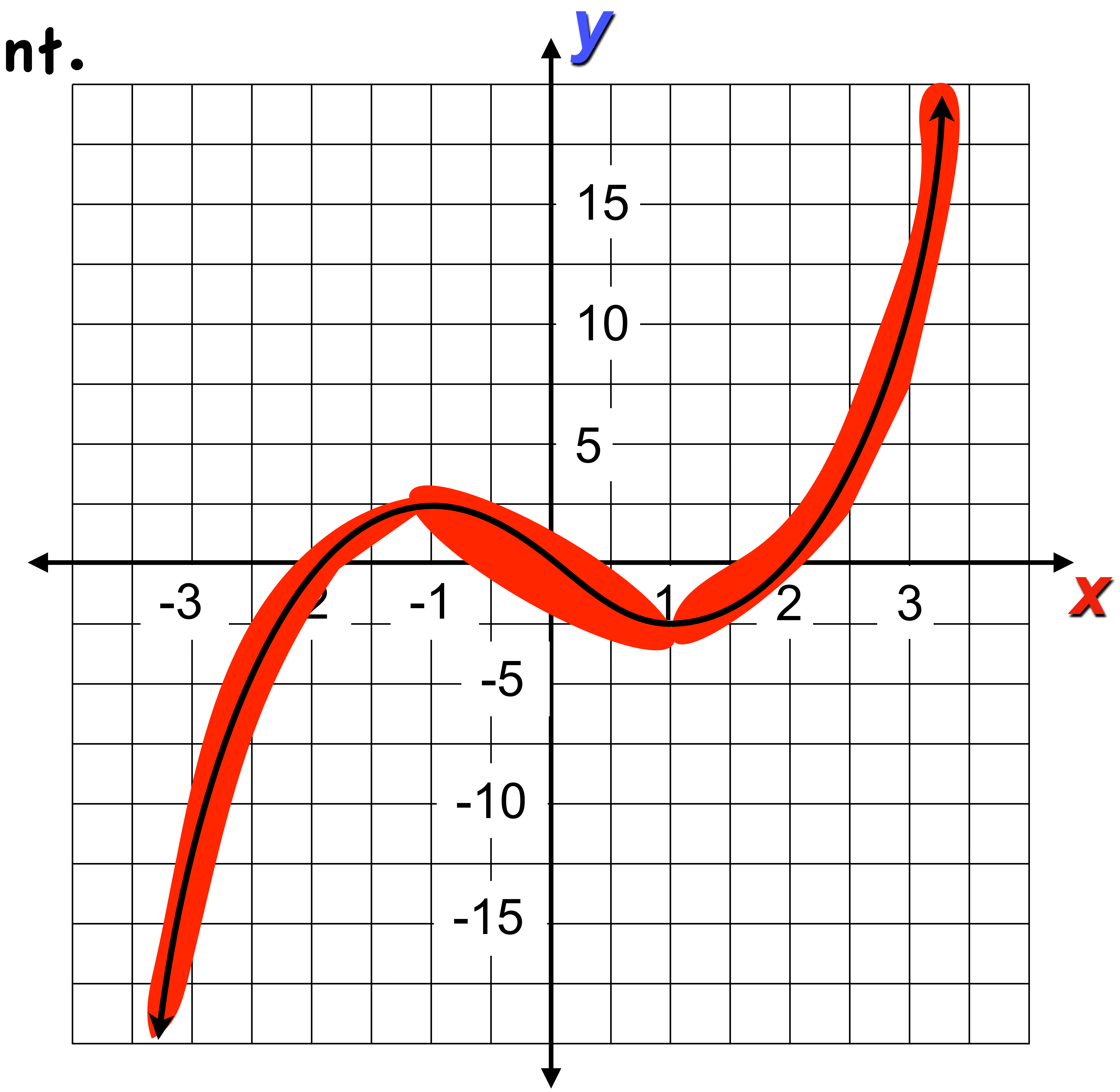
Constant



In I, $f(x_1) = f(x_2)$
whenever $x_1 < x_2$

Example: Intervals on Which a Function Increases, Decreases, or is Constant

State the intervals on which the given function is increasing, decreasing, or constant.



Increasing on $(-\infty, -1)$

Decreasing on $(-1, 1)$

Increasing on $(1, \infty)$

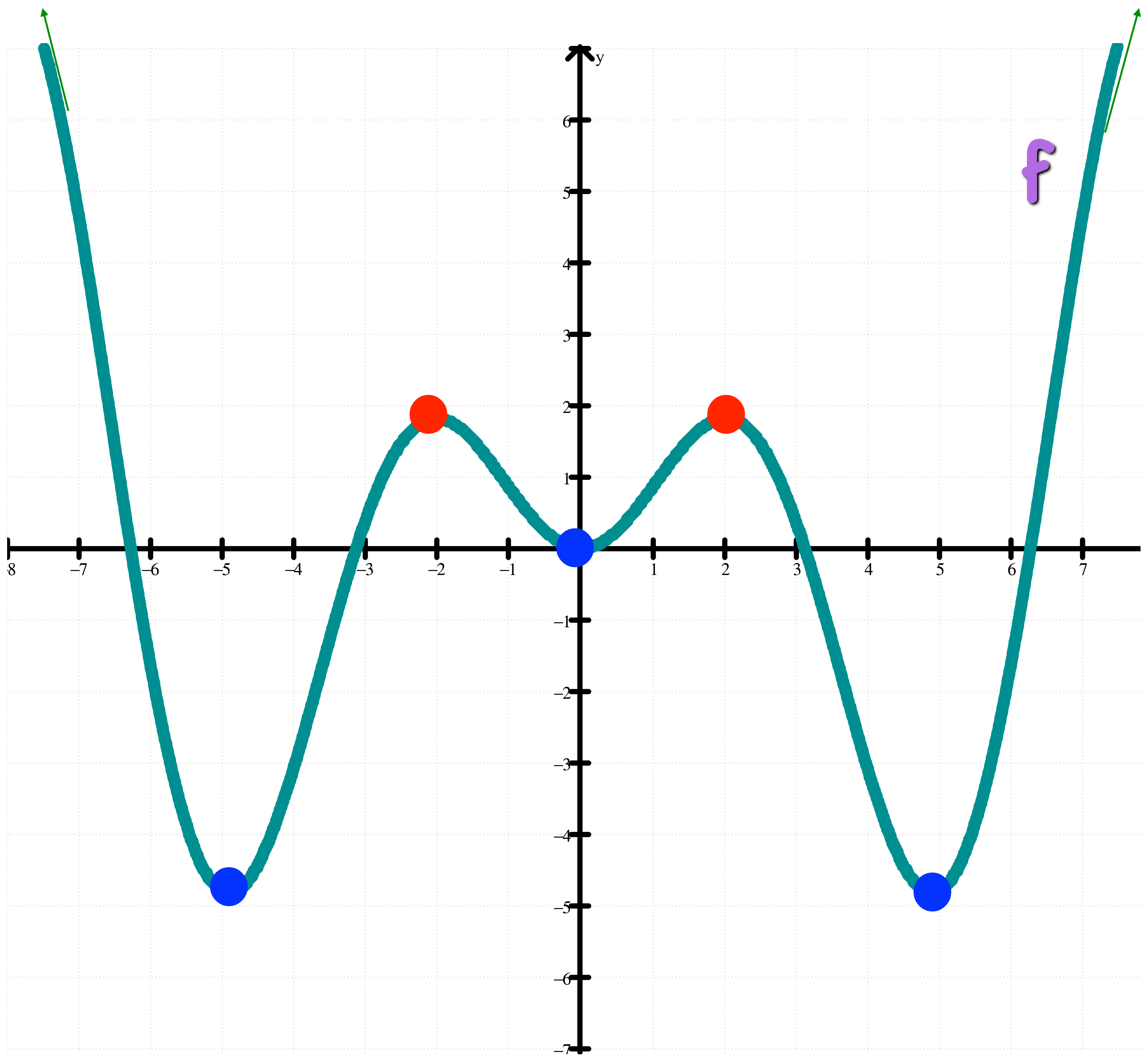
Definitions of Relative Extrema (Releative Maximum and Relative Minimum)

1. A function value $f(a)$ is a **relative maximum** of f if there exists an open interval containing a such that $f(a) > f(x)$ for all $x \neq a$ in the open interval.

2. A function value $f(b)$ is a **relative minimum** of f if there exists an open interval containing b such that $f(b) < f(x)$ for all $x \neq b$ in the open interval.

Use Graphs to Locate Relative Maxima or Minima

Identify the relative maxima and minima for the graph of f.



f has a relative maximum at
x = −2 and x = 2.

f has a relative minimum at
x = -5, x = 0, and x = 5

Relative extrema

Definitions of Relative Minimum and Relative Maximum

A function value $f(a)$ is called a **relative minimum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \leq f(x).$$

A function value $f(a)$ is called a **relative maximum** of f if there exists an interval (x_1, x_2) that contains a such that

$$x_1 < x < x_2 \quad \text{implies} \quad f(a) \geq f(x).$$

TI-84

- Using your calculator, graph $f(x) = -3x^2 - 2x + 1$ to estimate the relative extrema.

Y= (-) 3 X,T,θ,n x^2 − 2 X,T,θ,n + 1 ZOOM 6

2nd TRACE CALC ⋎ 4:Maximum Set your left boundary by moving the cursor. ENTER

Set your right boundary by moving the cursor. ENTER Guess? ENTER

$$x = -.3333314 \qquad y = 1.3333333$$

- The relative maximum is 1 1/3 at the point (-1/3, 1 1/3)

Definitions of Even and Odd Functions

The function f is an even function if $f(-x) = f(x)$ for all x in the domain of f. The right side of the equation of an even function does not change if x is replaced with −x.

$$\text{Even: } f(-x) = f(x)$$

The function f is an odd function if $f(-x) = -f(x)$ for all x in the domain of f. Every term on the right side of the equation of an odd function changes its sign (becomes the opposite) if x is replaced with −x.

$$\text{Odd: } f(-x) = -f(x)$$

Identifying Even or Odd Functions

Determine whether the function is even, odd, or neither.

$$h(x) = x^5 + 1$$

$$h(-x) = (-x)^5 + 1 = -x^5 + 1$$

$$h(-x) \neq h(x)$$

The function is not even.

$$-h(x) = -(x^5 + 1) = -x^5 - 1$$

$$h(-x) \neq -h(x)$$

The function is not odd.

The function is neither odd nor even.

Identifying Even or Odd Functions

- Determine whether the function is even, odd, or neither.

$$f(x) = x^4 - |x|$$

$$f(-x) = (-x)^4 - |-x| = x^4 - |x|$$

$$f(-x) = f(x)$$

The function is even.

$$g(x) = \frac{x}{x^2 + 1}$$

$$g(-x) = \frac{-x}{(-x)^2 + 1} = \frac{-x}{x^2 + 1} = -g(x)$$

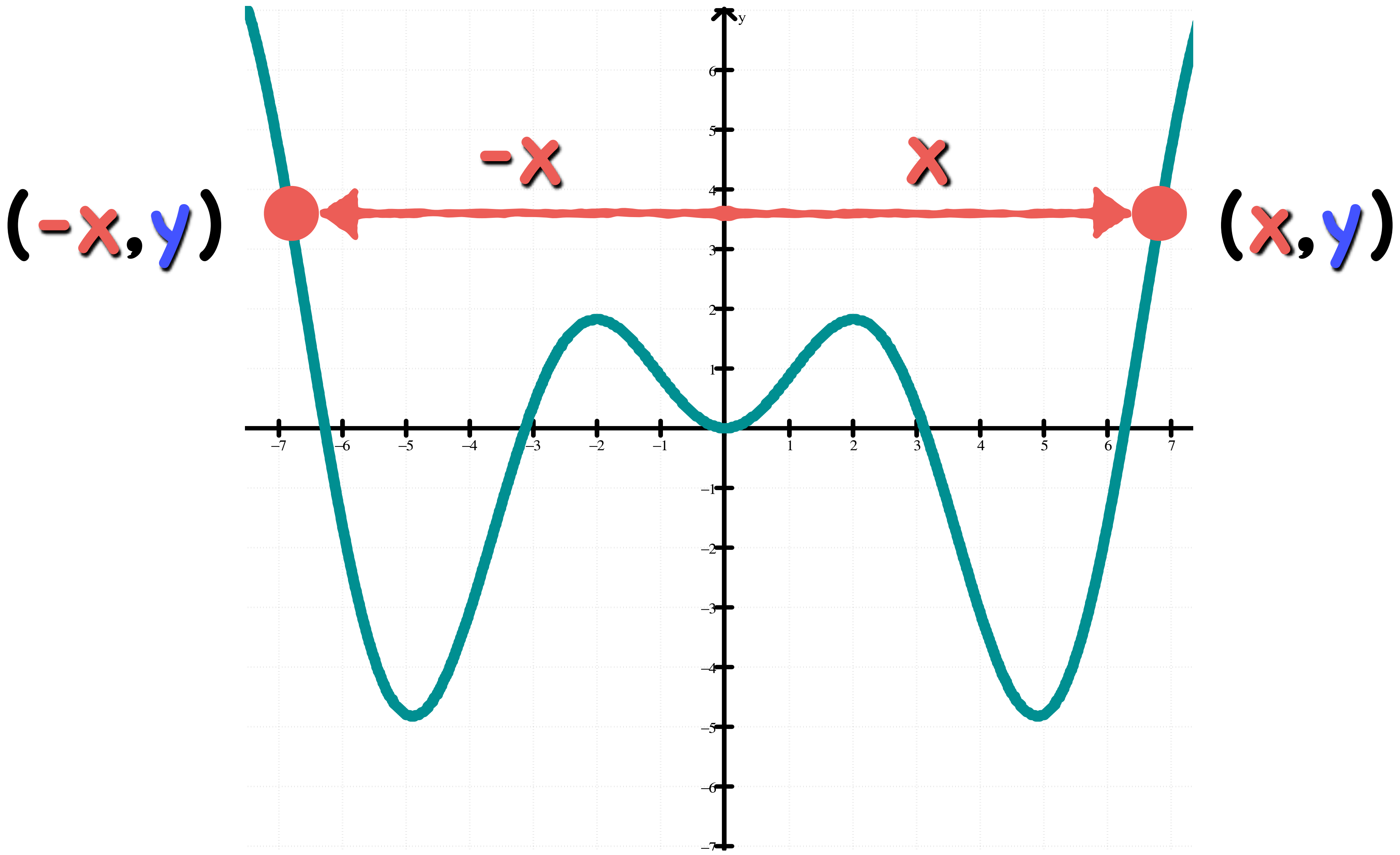
$$g(-x) = -g(x)$$

The function is odd.

Even Functions and *y*-Axis Symmetry

The graph of an even function in which f(−x) = f(x) is symmetric with respect to the y-axis.

A graph is symmetric with respect to the y-axis if, for every point (x,y) on the graph, the point (-x,y) is also on the graph. All even functions have graphs with this kind of symmetry.

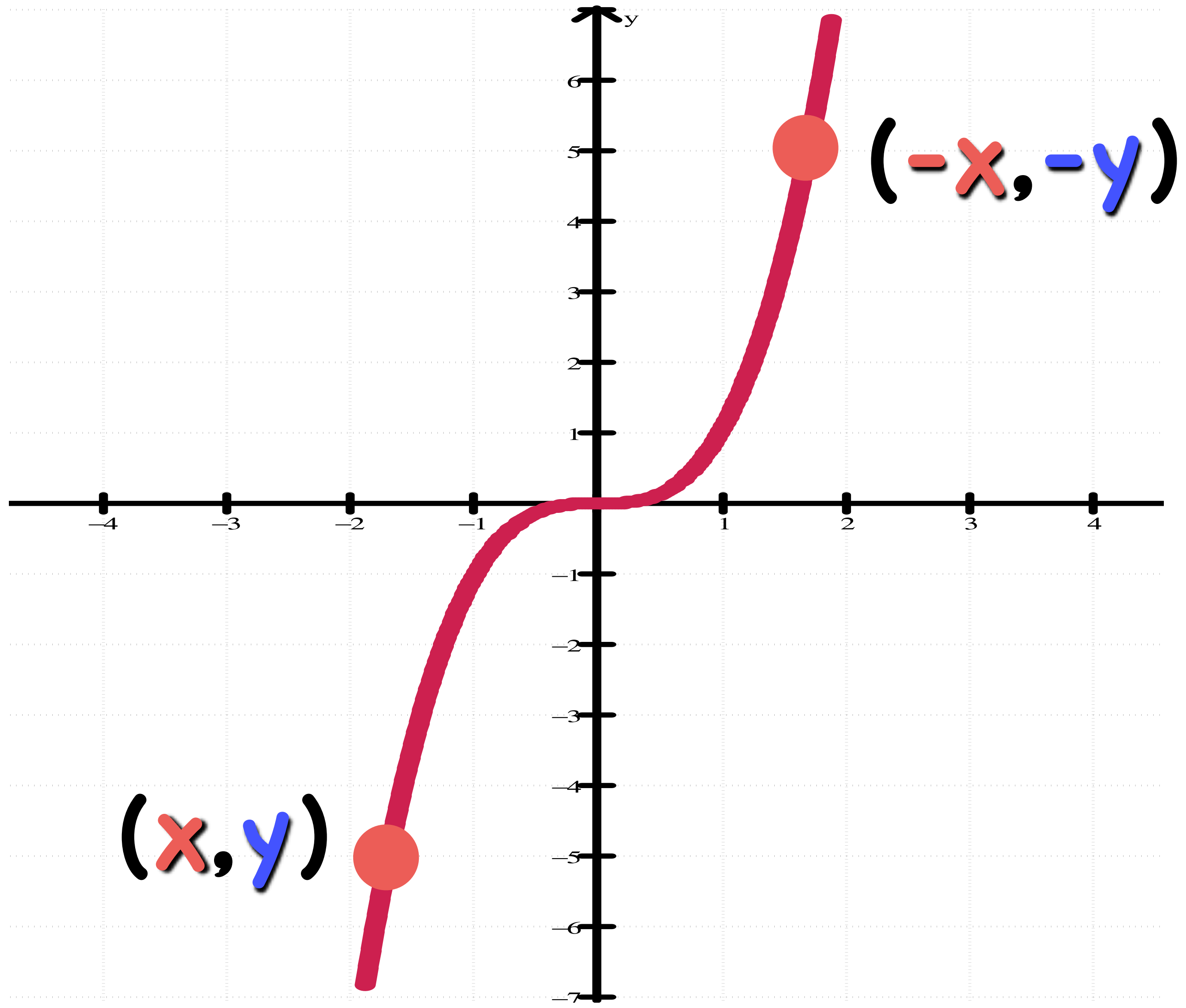


Odd Functions and Origin Symmetry

The graph of an odd function in which $f(-x) = -f(x)$ is symmetric with respect to the **origin**.

A graph is symmetric with respect to the origin if, for every point (x,y) on the graph, the point $(-x,-y)$ is also on the graph. All odd functions have graphs with origin symmetry.

Note that the 1st and 3rd quadrants of odd functions are reflections of each other with respect to the origin. The same is true for 2nd and 4th quads.

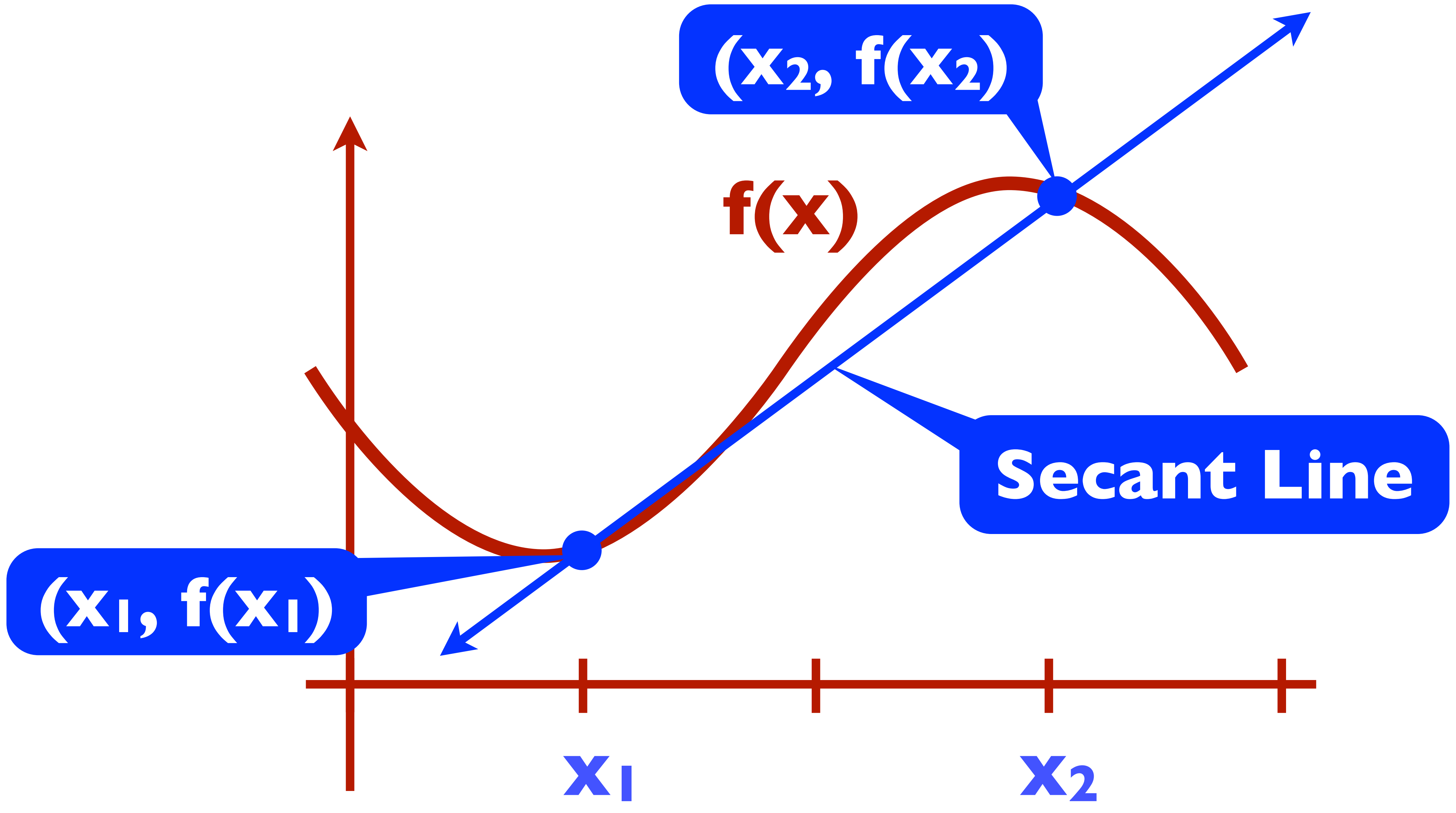


Also note that f(x) and f(-x) have opposite signs, so that $f(-x) = -f(x)$.

Average Rate of Change

- Let $(x_1, f(x_1))$ and $(x_2, f(x_2))$ be distinct points on the graph of a function f. The **average rate of change of f** from x_1 to x_2 is:

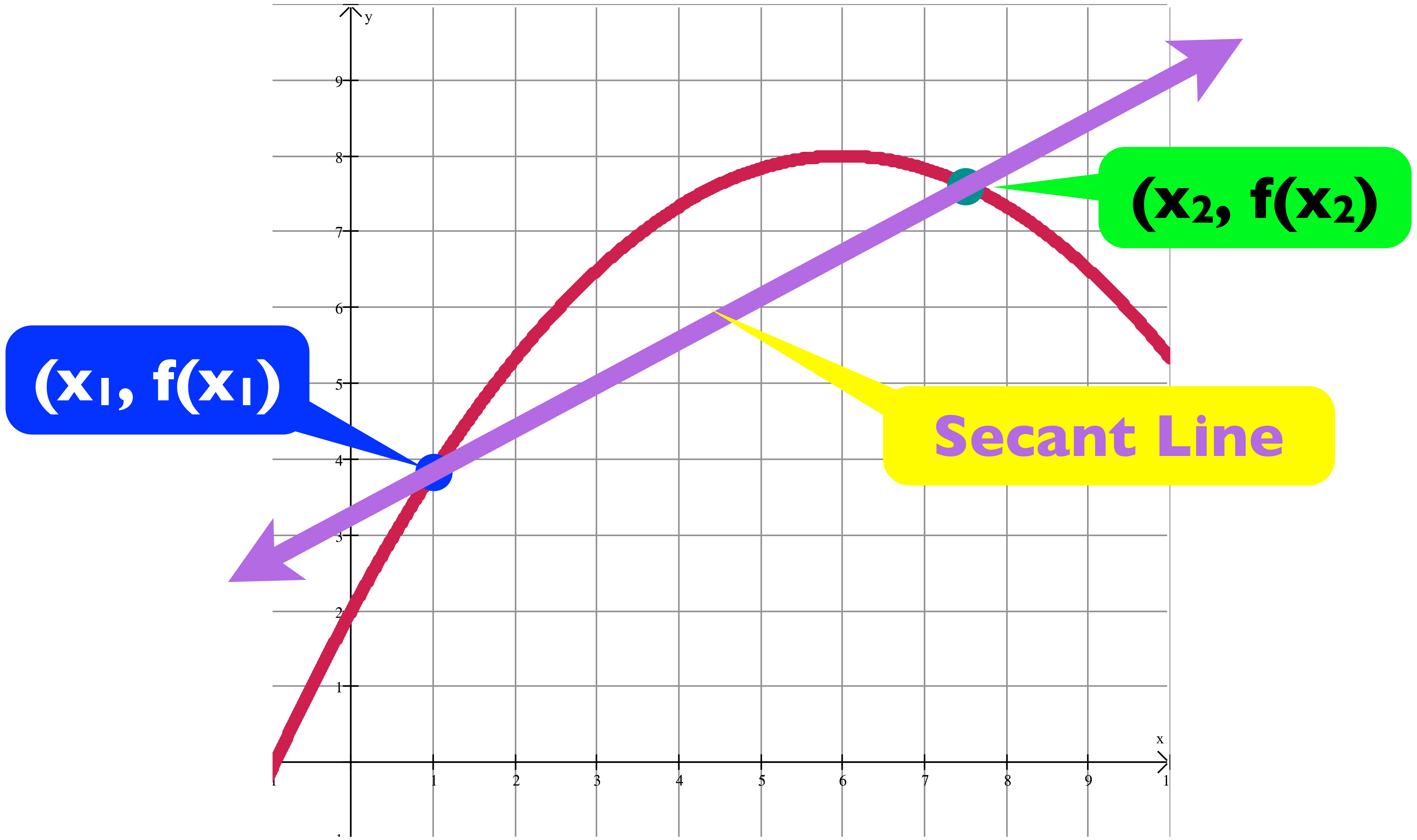
$$\frac{\Delta y}{\Delta x} = \frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



- The **average rate of change** is the slope of the line (called the secant line) containing two points on the graph of the function.

Average Rate of Change

- If the graph of a function is not a straight line, the **average rate of change of f** between any two points is the slope of the line containing the two points. This line is called a secant line.

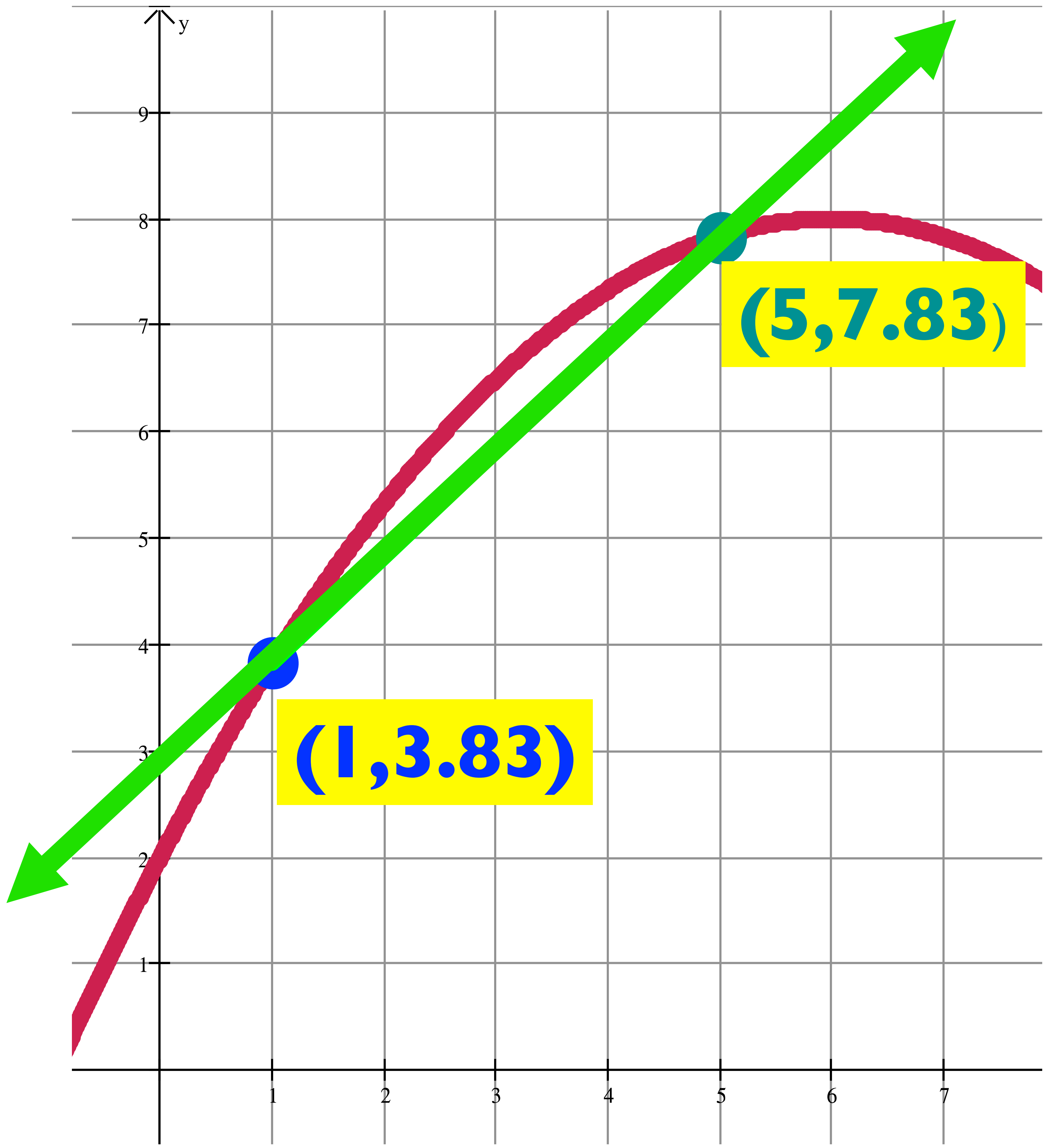


average rate of change

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Average Rate of Change

- Find the average rate of change of f between the points (1,3.83) and (5,7.83).

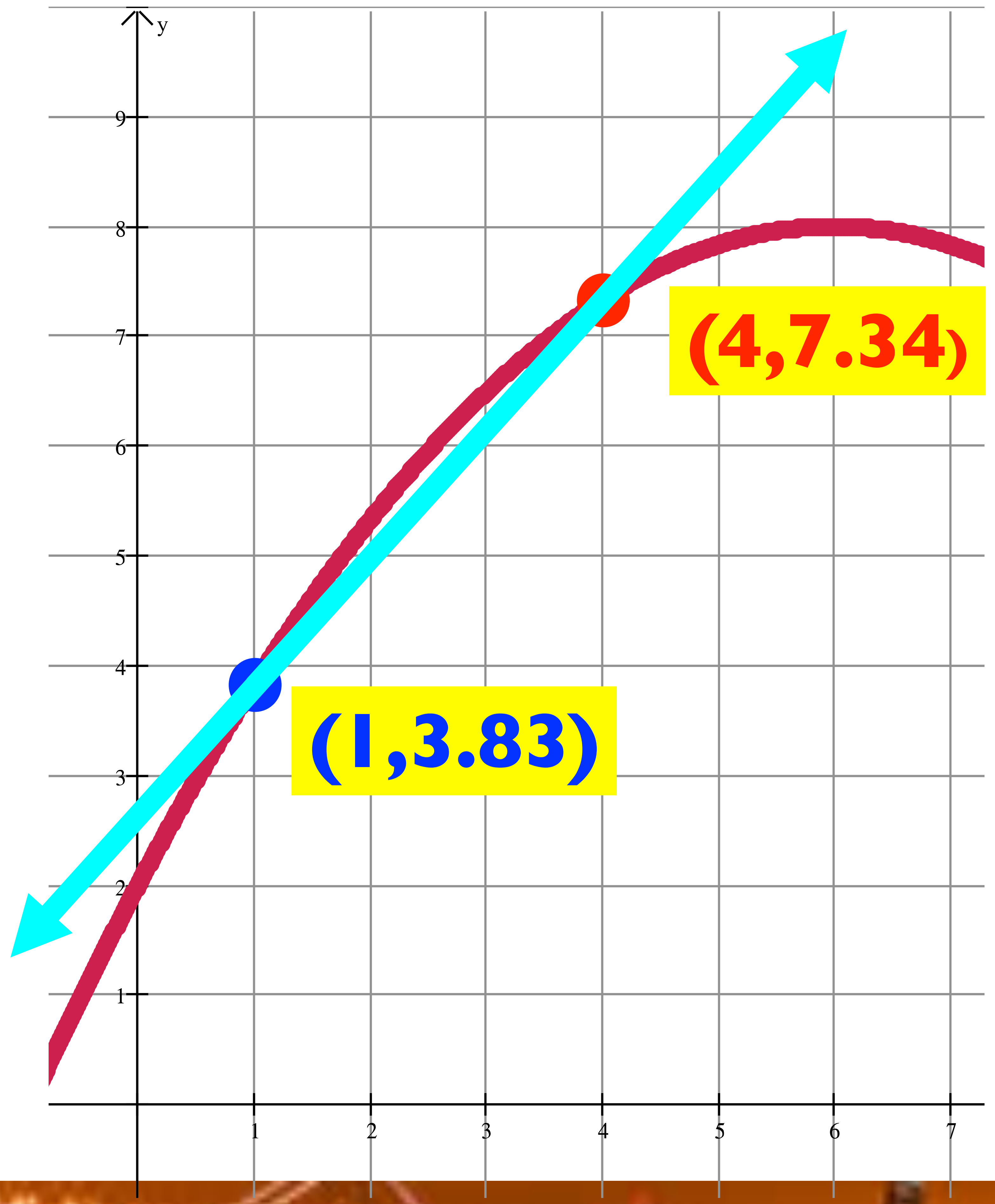


$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{7.83 - 3.83}{5 - 1} = \frac{4}{4} = 1$$

Average Rate of Change

- Find the average rate of change of f between the points (1,3.83) and (4,7.34).

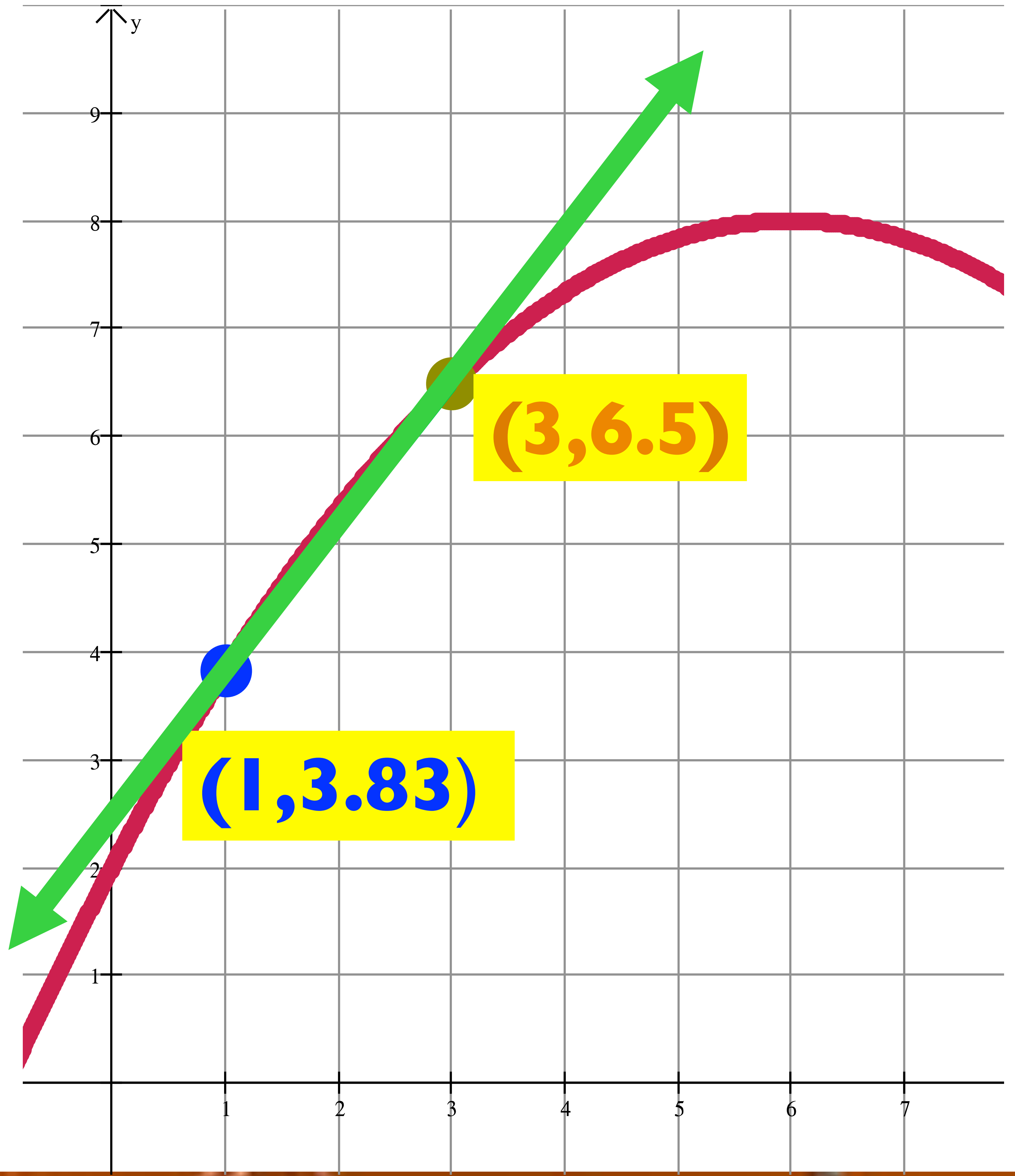


$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{7.34 - 3.83}{4 - 1} = \frac{3.51}{3} = 1.17$$

Average Rate of Change

- Find the average rate of change of f between the points (1,3.83) and (3,6.5).

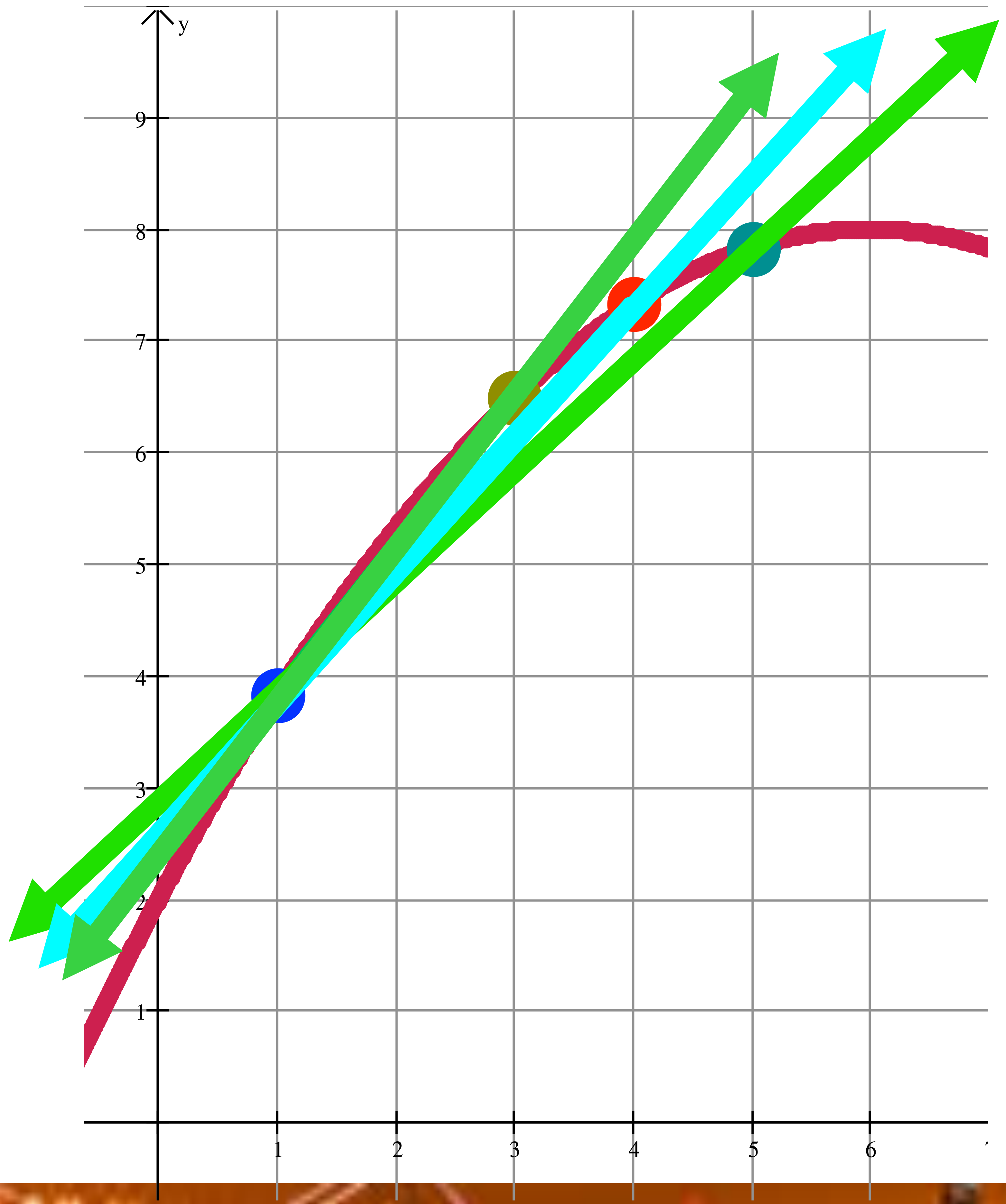


$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$= \frac{6.5 - 3.83}{3 - 1} = \frac{2.67}{2} = 1.34$$

Average Rate of Change

- Let us look at the 3 cases together



x	y	Slope of the secant line
3	6.5	1.34
4	7.34	1.17
5	7.83	1

•Notice how the slope changes depending upon the point that you choose because this function is a curve, not a line. So the average rate of change varies depending upon which points you may choose.

Average Rate of Change

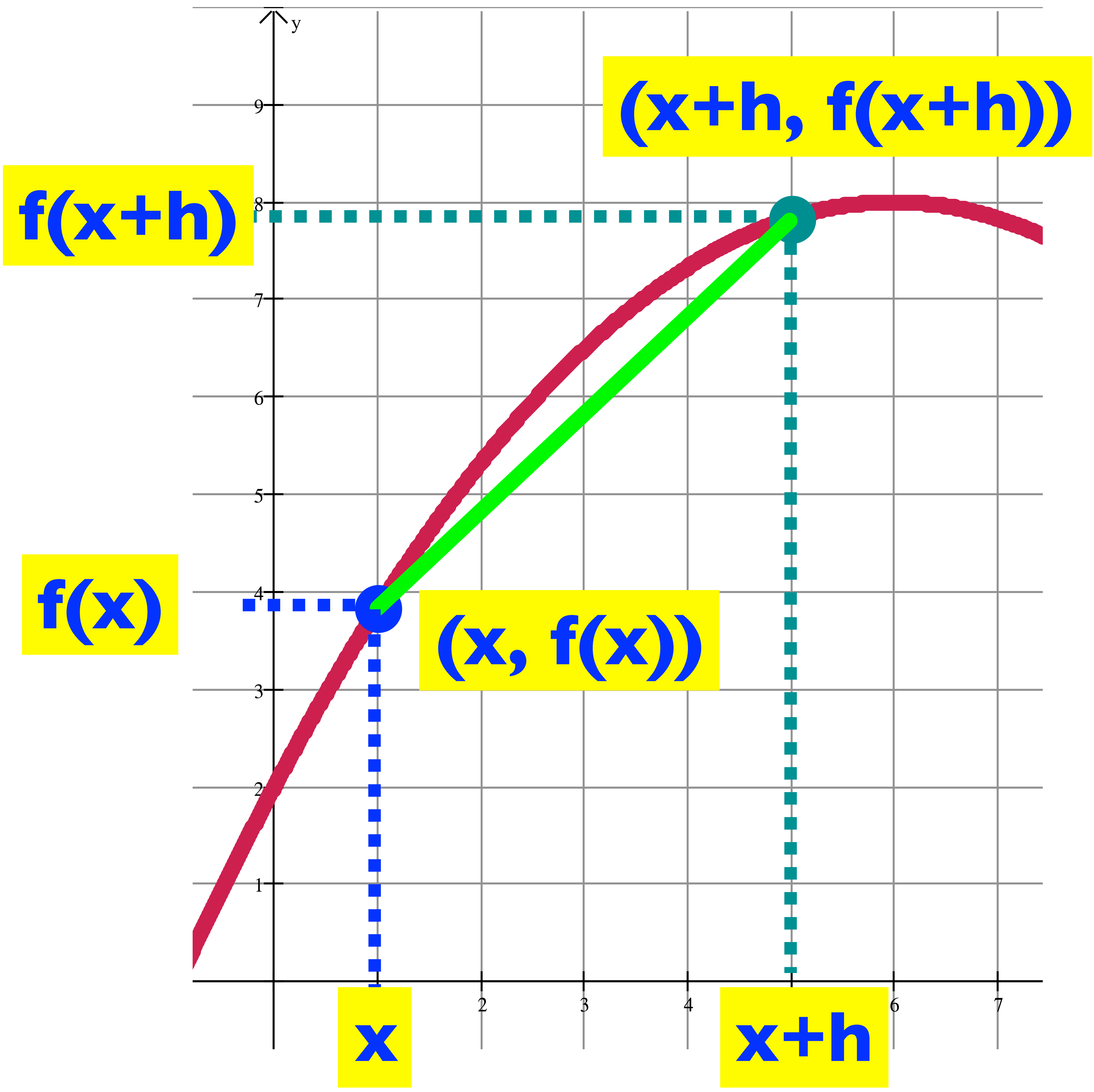
- The average rate of change from $x_1 = x$ to $x_2 = x + h$ is:

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x+h) - f(x)}{x+h-x} = \frac{f(x+h) - f(x)}{h}$$

  - The last expression is the difference quotient.

- The difference quotient gives the average rate of change of a function from x to x + h. In the difference quotient, h is thought of as a number very close to 0. In this way the average rate of change can be found for a very short interval.

Average Rate of Change



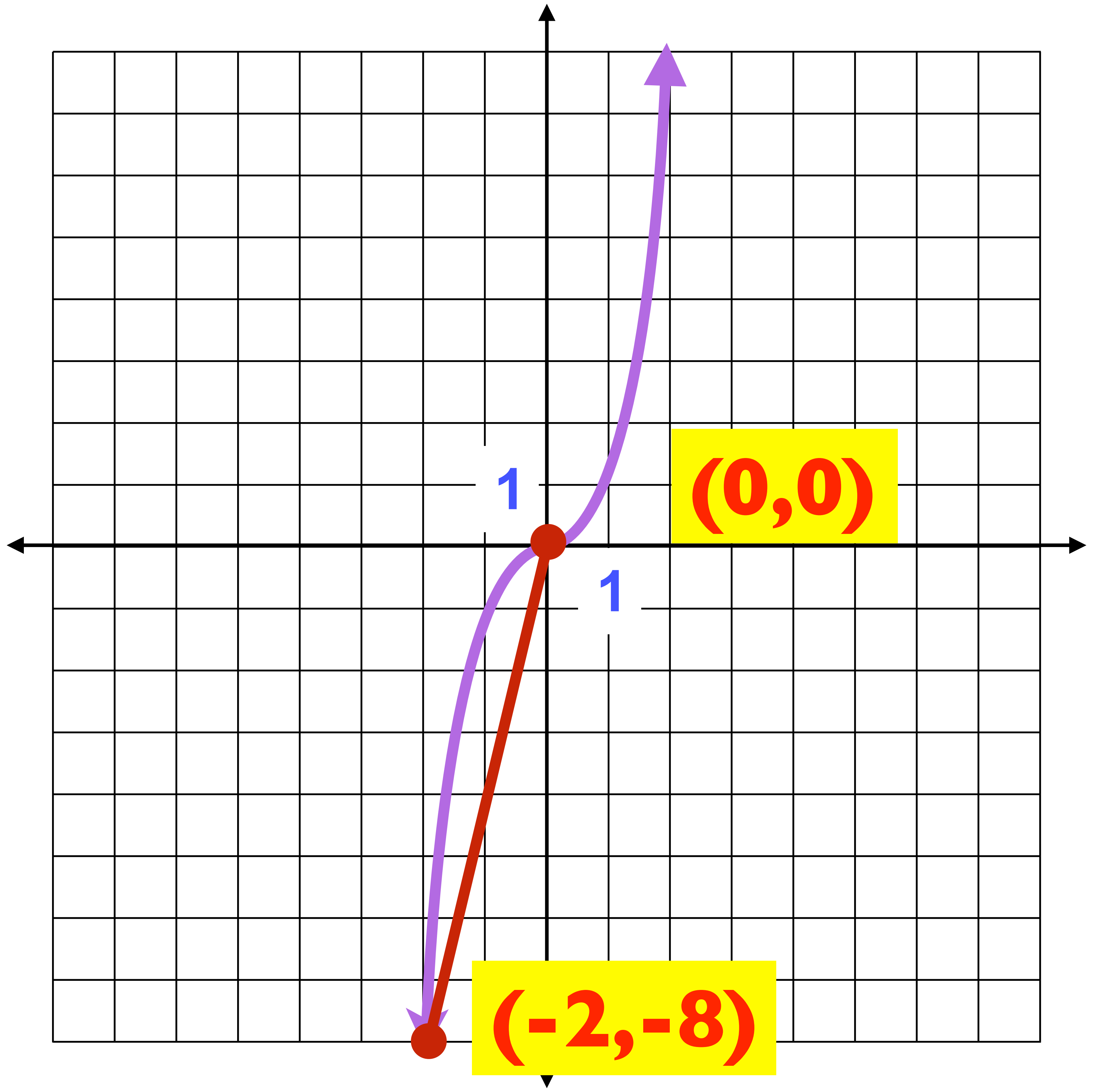
$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(x+h) - f(x)}{h}$$

The difference quotient becomes very important in higher level math.

Average Rate of Change

- Find the average rate of change for $f(x) = x^3$ from $x_1 = -2$ to $x_2 = 0$.

$$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(0) - f(-2)}{0-(-2)} = \frac{0^3 - (-2)^3}{0-(-2)} = \frac{8}{2} = 4$$



The average rate of change for $f(x) = x^3$ from $x_1 = -2$ to $x_2 = 0$ is 4 units of change in y for every unit change in x.