Chpt 10

10.2 Parabolas

Chpt 10

10.2 Homework

p740 19, 21, 23, 45, 47, 49, 57

Objectives

Graph parabolas with vertices at the origin.

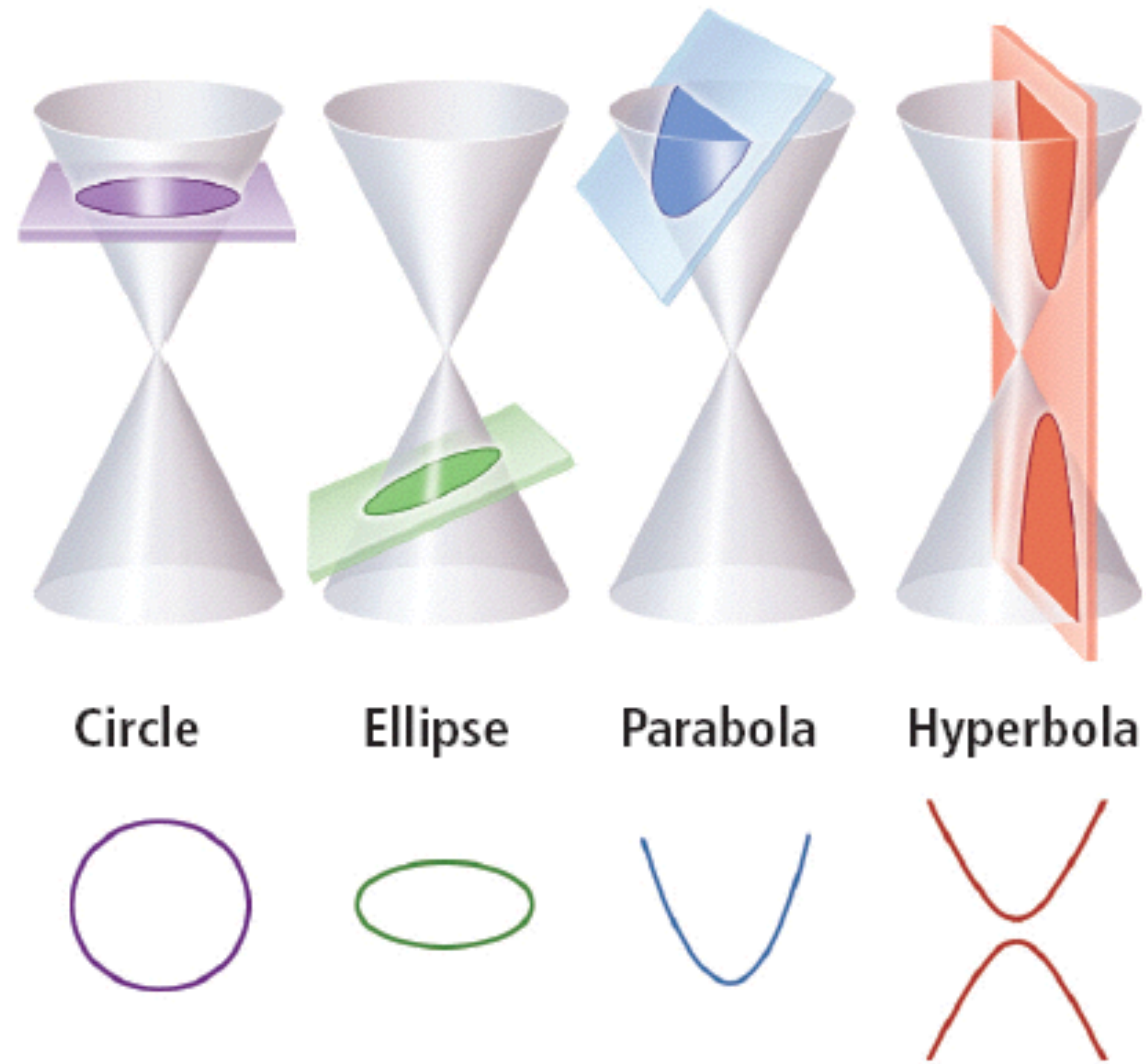
Write equations of parabolas in standard form.

Graph parabolas with vertices not at the origin.

Solve applied problems involving parabolas.

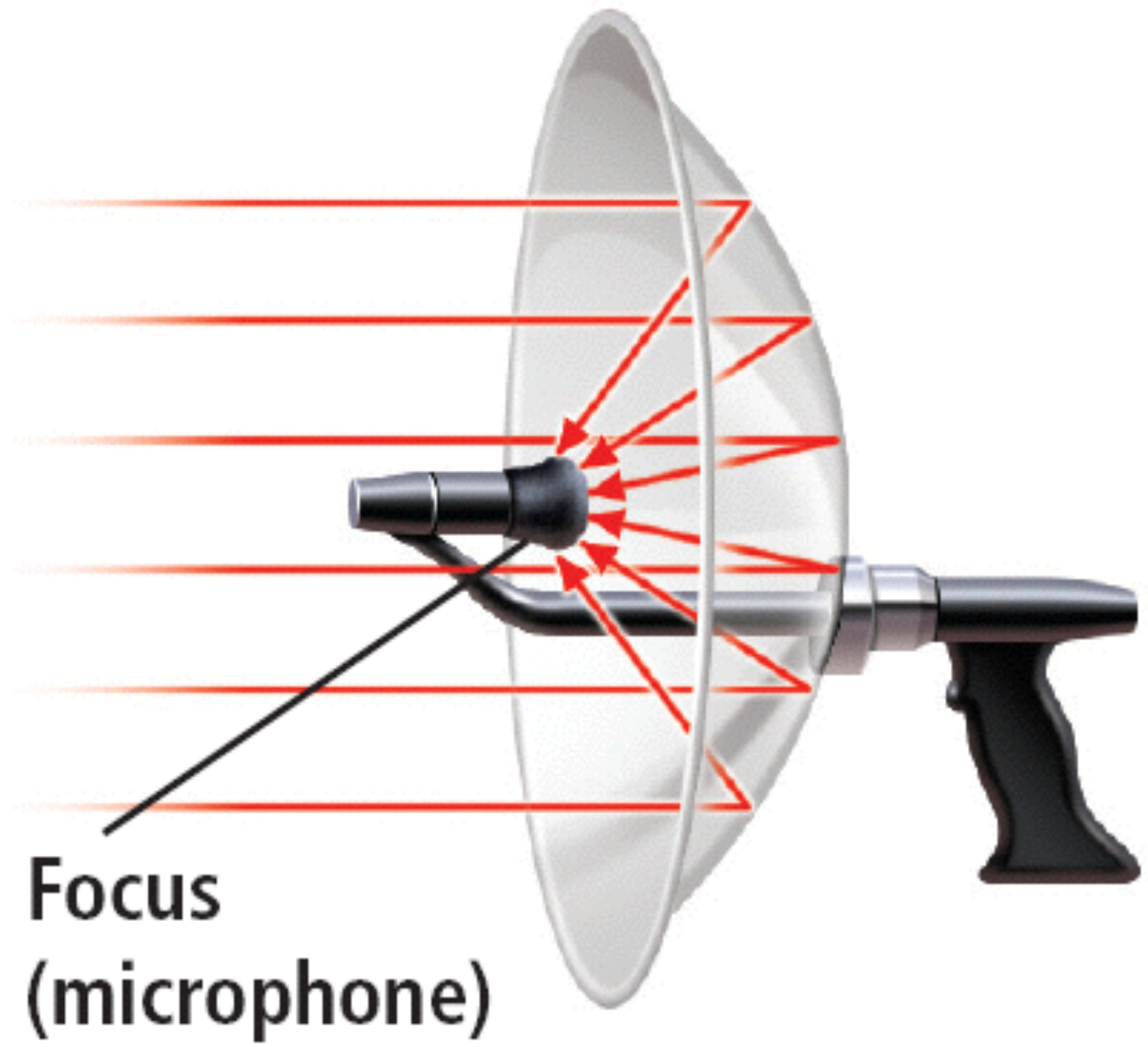
Conics

Conic sections are formed by the intersection of a double right cone and a plane. There are four types of conic sections: ***circles, ellipses, hyperbolas, and parabolas***.



Parabola

Light or sound waves captured by a parabola will be reflected by the curve of the parabolic shape through the **focus** of the parabola, (see figure). Waves emitted from the focus will be reflected out parallel to the axis of symmetry of a parabola. This is how you get your very important texts.



Parabola

Here is what I expect you to already know about parabolas and graphing.

$$y = a(x-h)^2 + k \qquad\qquad y = ax^2 + bx + c$$

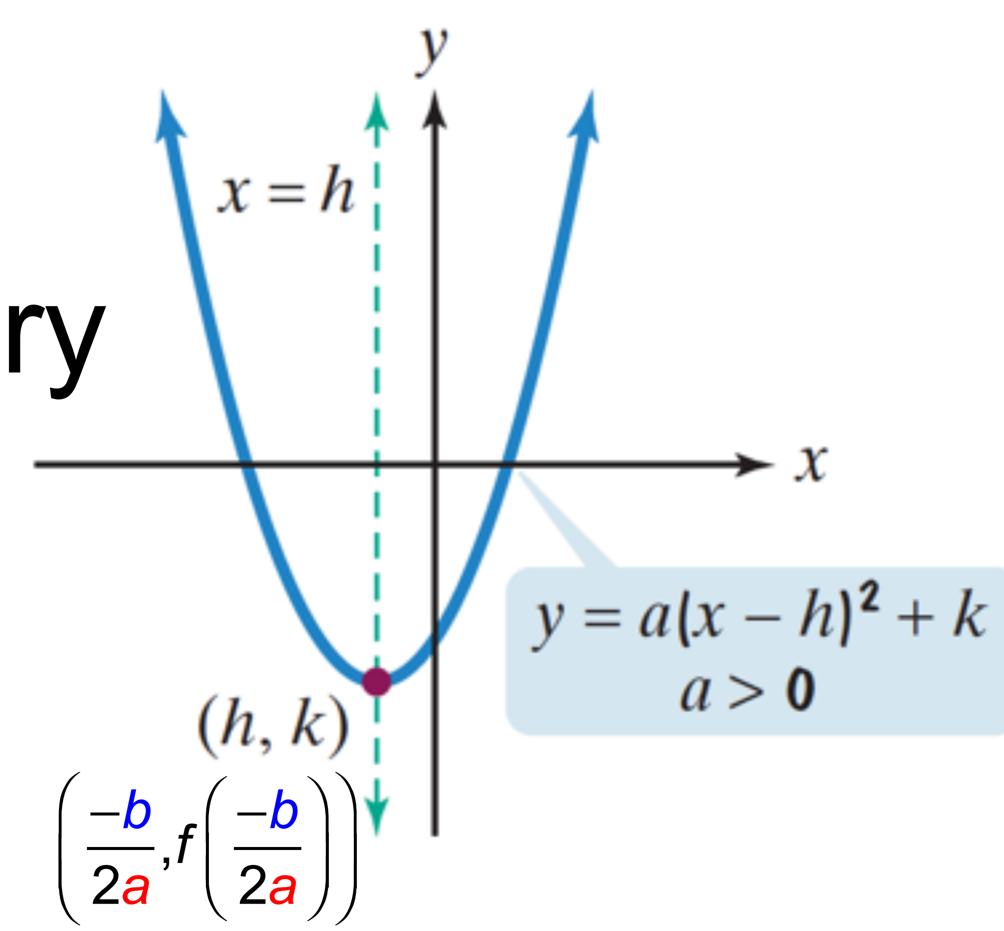
1. If $a > 0$, the graph opens upward. If $a < 0$, the graph opens downward.

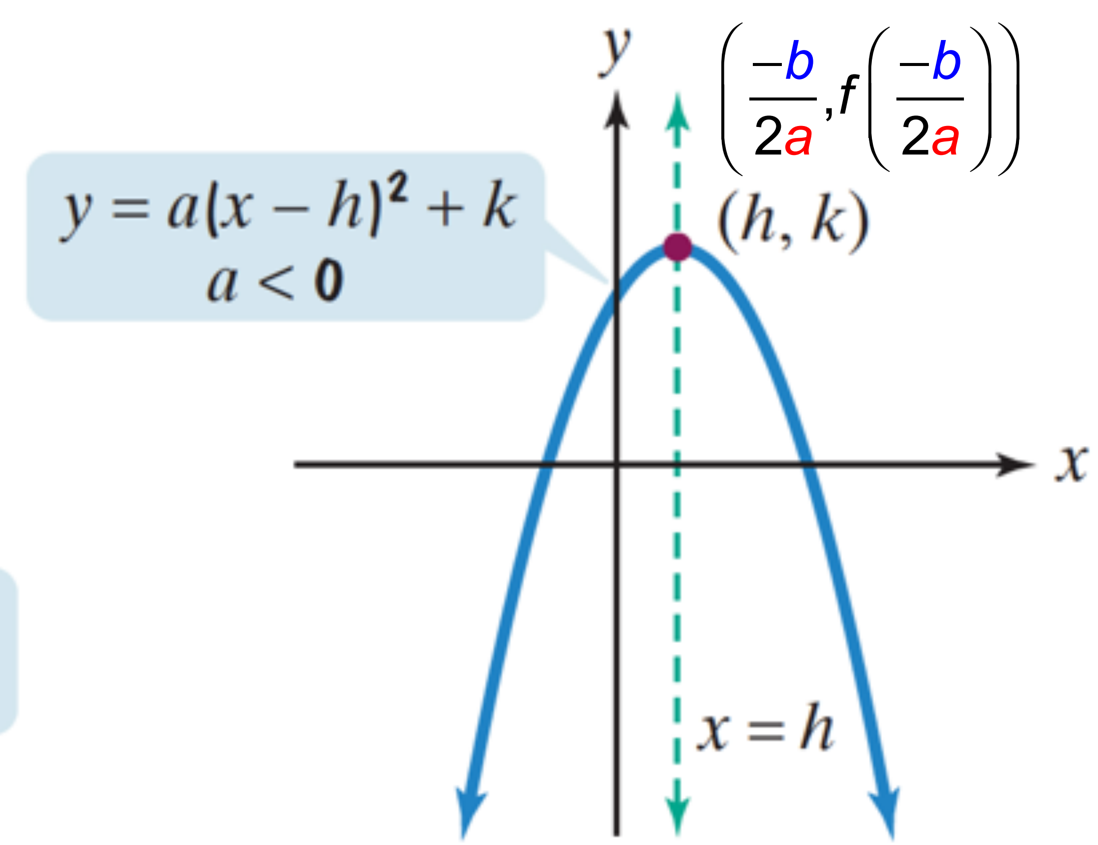
2. The vertex of $y = a(x - h)^2 + k$ is (h, k) and the axis of symmetry is x = h.

3. The vertex of $y = ax^2 + bx + c$ is

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$$

and the axis of symmetry is $x = \dfrac{-b}{2a}$





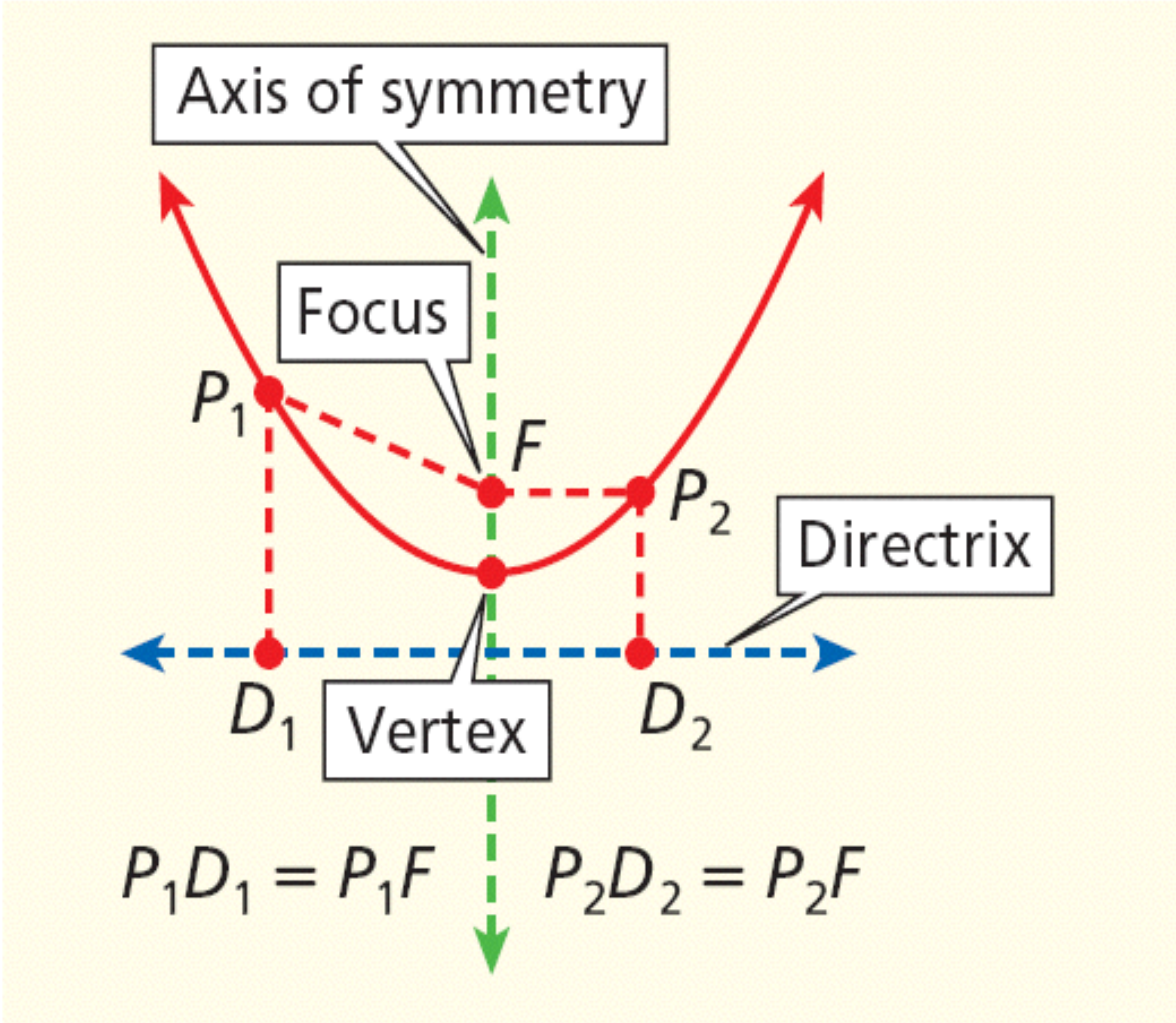
4. You should also be able to find the x and y intercepts of a parabola.

Definition of a Parabola

A **parabola** is the set of all points in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, that is not on the line.

A parabola has an **axis of symmetry** perpendicular to its **directrix** and passing through its vertex.

The vertex of a parabola is the **midpoint** of the perpendicular segment connecting the **focus** and the **directrix**.



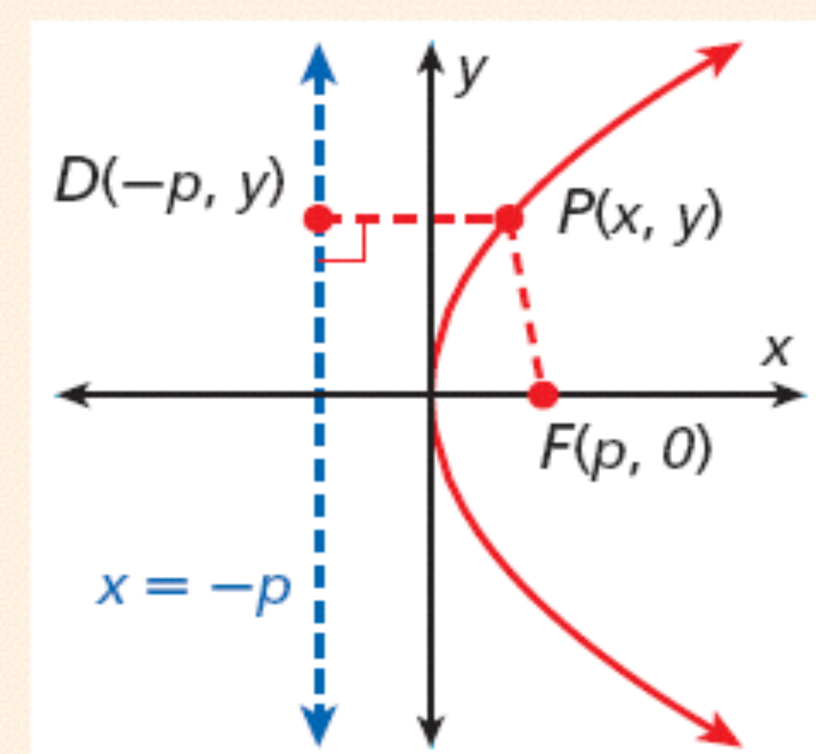
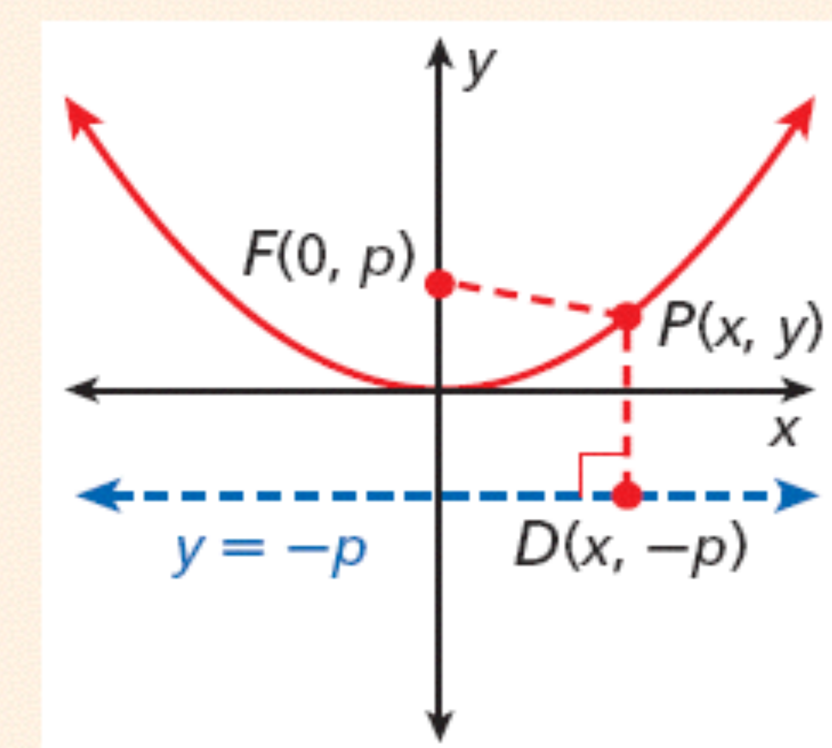
Standard Form of Equation of a Parabola

The **standard form** of the equation of a parabola with **vertex at the origin** is $x^2 = 4py$ or $y^2 = 4px$

Thus p is the **directed** distance from the vertex to the focus. **Directed** means the sign of p hold significance.

If p > 0, the focus is above or right of the vertex. If p < 0 the focus is below or left of the vertex.

Standard Form for the Equation of a Parabola — **Vertex at (0, 0)**

<table>
<tr><th>AXIS OF SYMMETRY</th><th>HORIZONTAL
$y = 0$</th><th>VERTICAL
$x = 0$</th></tr>
<tr><td>Equation</td><td>$x = \frac{1}{4p}y^2$</td><td>$y = \frac{1}{4p}x^2$</td></tr>
<tr><td>Direction</td><td>Opens right if $p > 0$
Opens left if $p < 0$</td><td>Opens upward if $p > 0$
Opens downward if $p < 0$</td></tr>
<tr><td>Focus</td><td>$(p, 0)$</td><td>$(0, p)$</td></tr>
<tr><td>Directrix</td><td>$x = -p$</td><td>$y = -p$</td></tr>
<tr><td>Graph</td><td>
</td><td>
</td></tr>
</table>

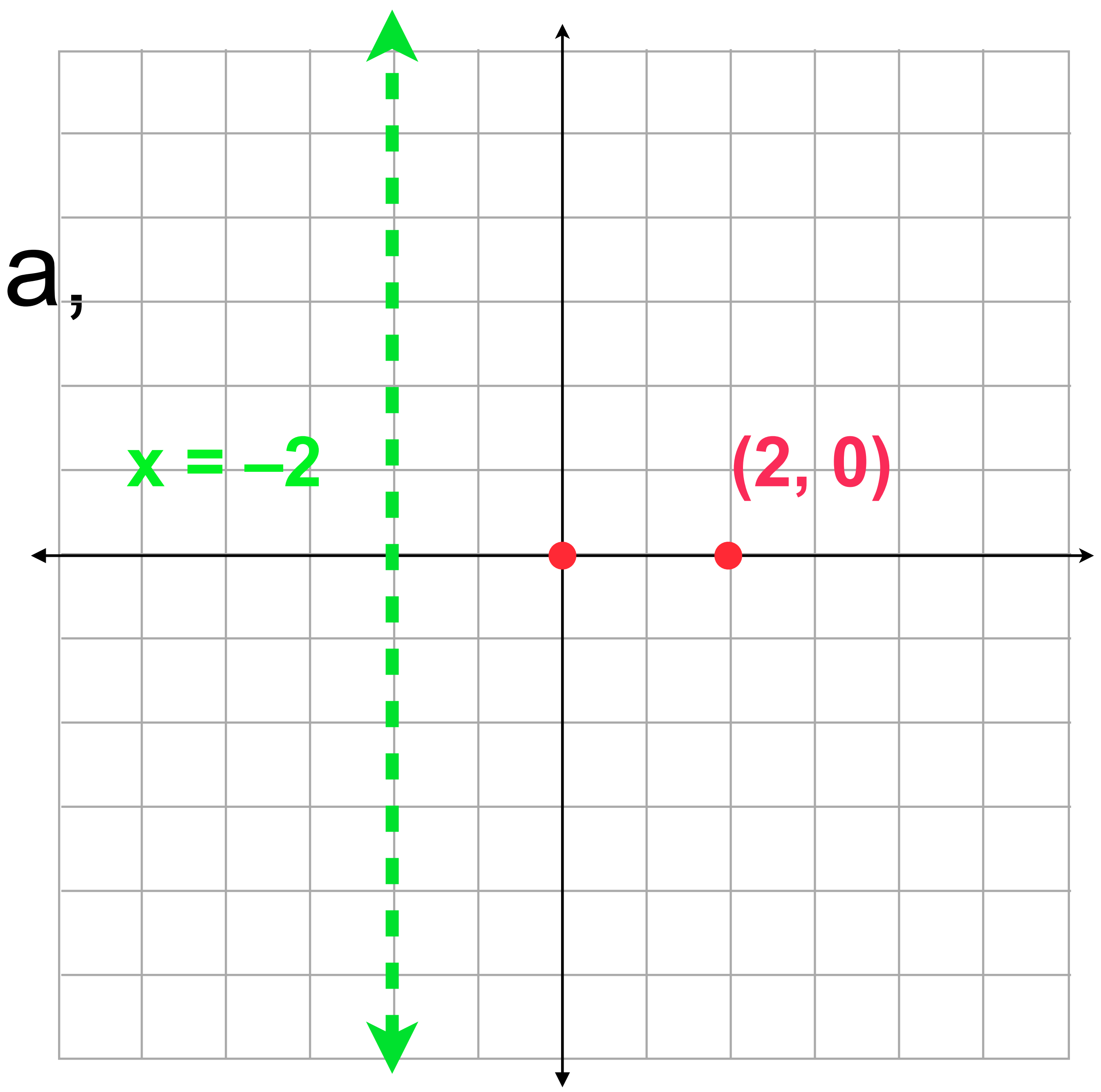
Finding the Focus and Directrix of a Parabola

Find the **focus** and directrix of the parabola given by $y^2 = 8x$. Then graph the parabola.

The given equation, $y^2 = 8x$, is in the standard form $y^2 = 4px$, so $4p = 8$.

The vertex is (0, 0) and $4p = 8$, $p = 2$

Because p is positive, and y is squared, the parabola, with its x-axis symmetry, opens right.



The **focus** is 2 units to the right of the vertex, (0, 0).

The **focus** is (2, 0).

The **directrix** is 2 units to the left of the vertex.

The **directrix** is $x = -2$.

Graphing the Parabola

Find the **focus** and **directrix** of the parabola given by $y^2 = 8x$. Then graph the parabola.

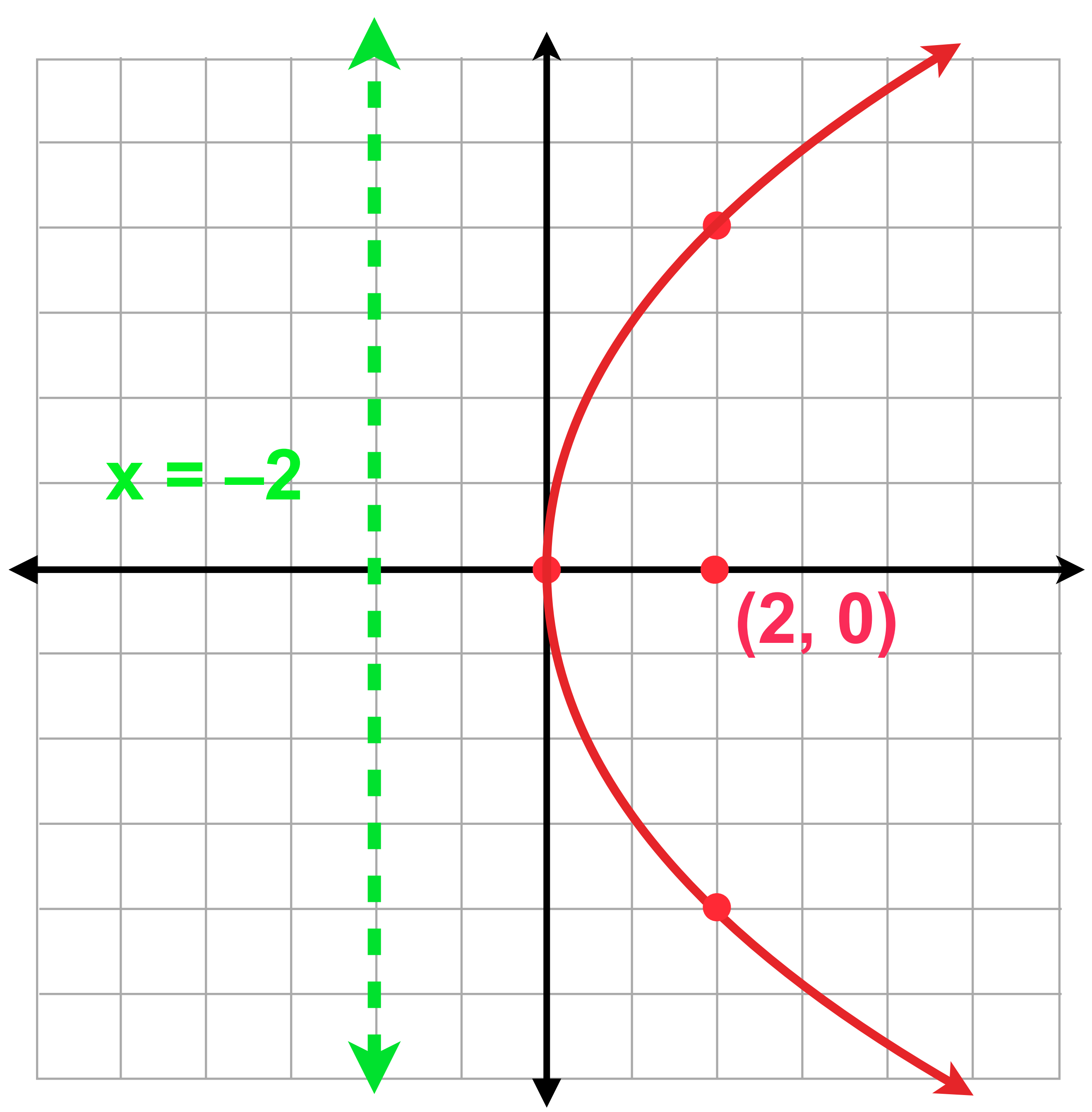
The **focus** is (2, 0). The **directrix** is $x = -2$.

To graph the parabola, we will do what always works. We plot points. To make things convenient, we use two points on the graph that are directly above and below the focus ($x = 2$).

Because the **focus** is at (2, 0), we simply substitute 2 for x in the parabola's equation, $y^2 = 8x$.

The points on the parabola above and below the **focus** are (2, 4) and (2, –4).

Ta Da! A parabola.

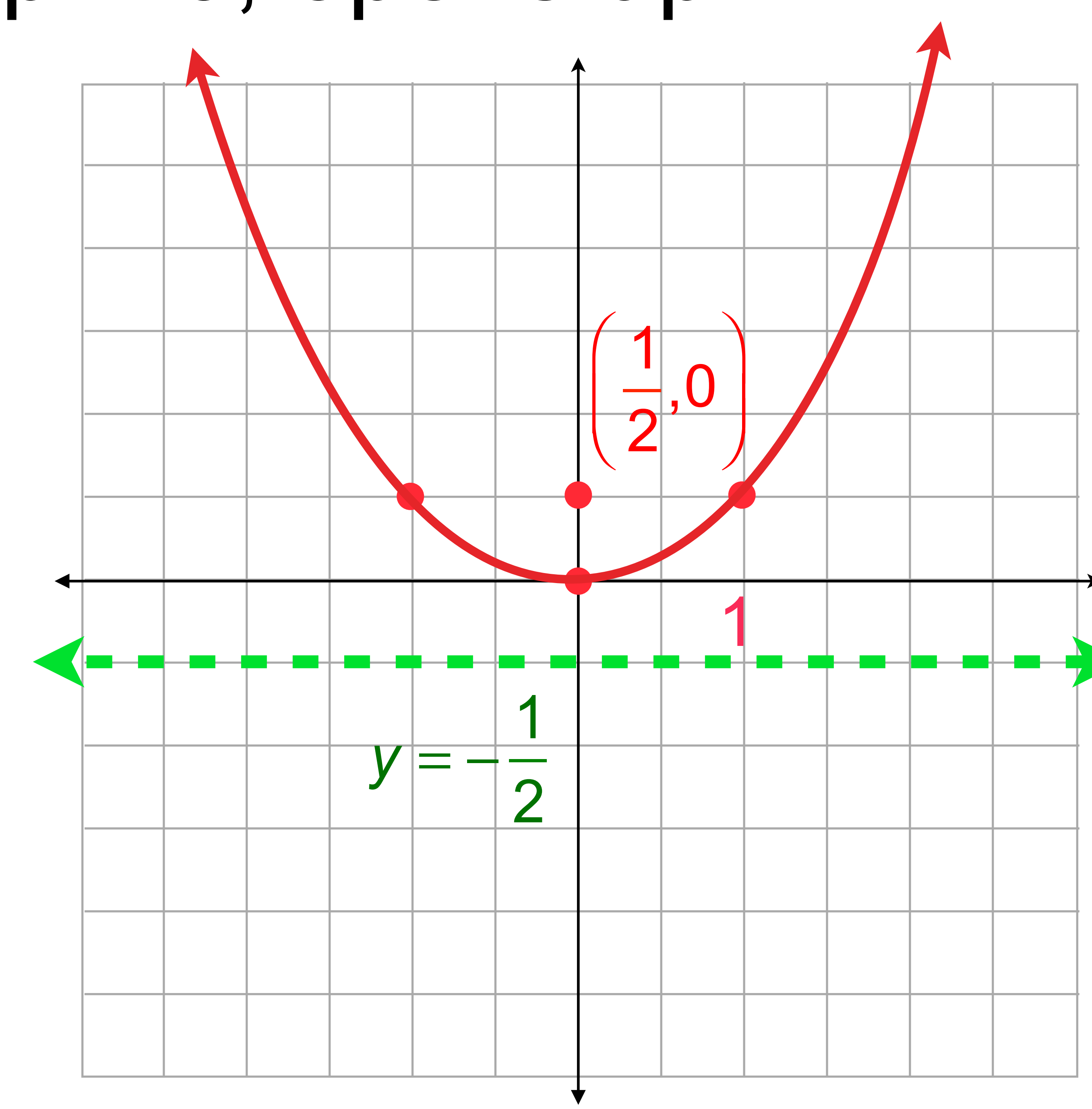


Finding the Focus and Directrix of a Parabola

Find the **focus** and directrix of the parabola given by $y = \frac{1}{2}x^2$. Then graph the parabola.

$$y = \frac{1}{2}x^2 \rightarrow x^2 = 2y \qquad x^2 = 4py \qquad 4p = 2 \quad p = \frac{1}{2}$$

p > 0, opens up



The **focus** is 1/2 unit above the vertex, (0, 0).

The **directrix** is 1/2 unit below the vertex.

The points on the parabola right and left of the **focus** are (1, 1/2) and (-1, 1/2).

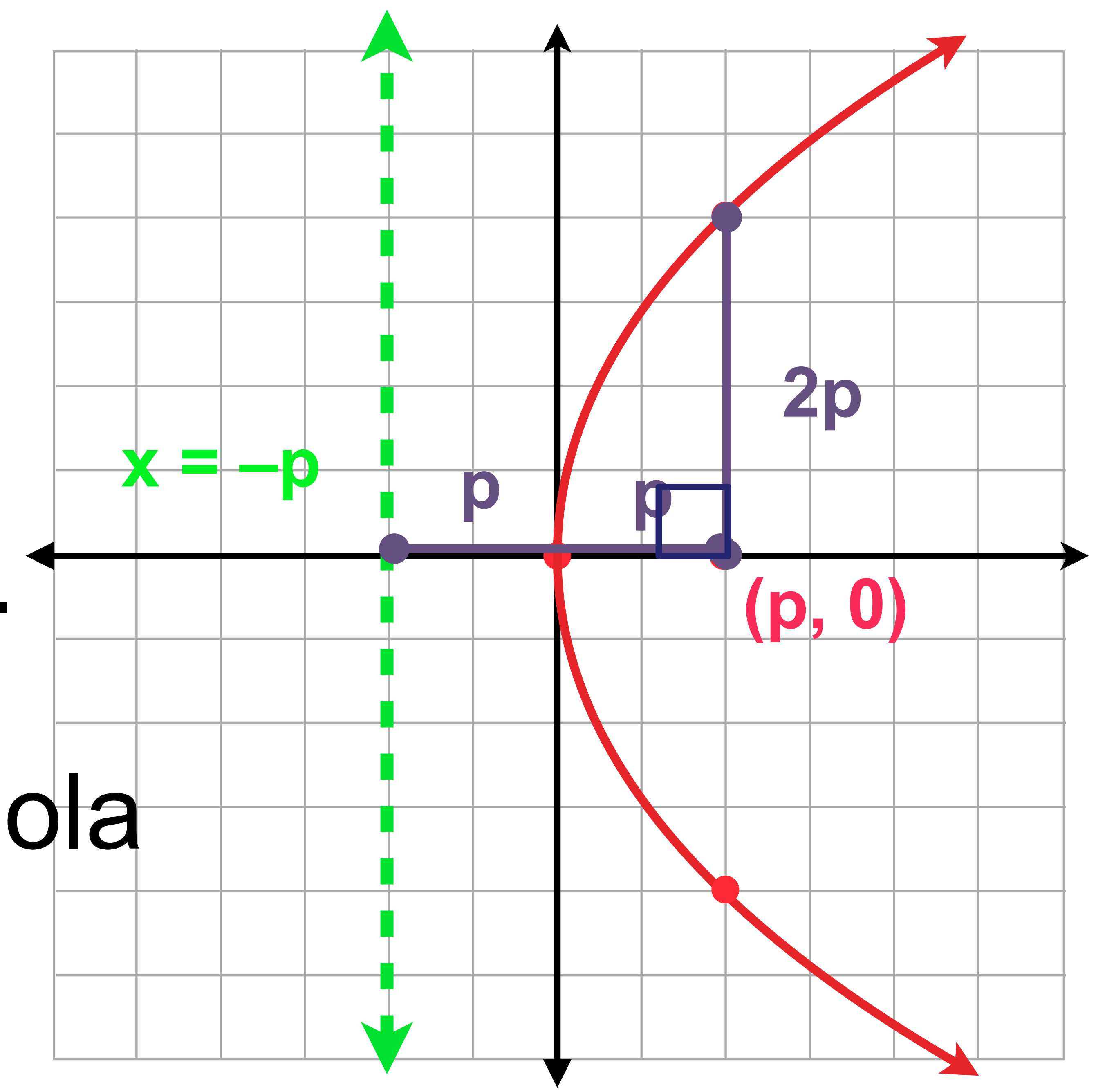
Latus Rectum

Find the **focus** and **directrix** of the parabola with vertex at (0,0) is given by $y^2 = 4px$. The **focus** is (p, 0). The **directrix** is $x = -p$.

Recall that the definition of a parabola is the set of points that are equidistant from the **focus** and the **directrix.**

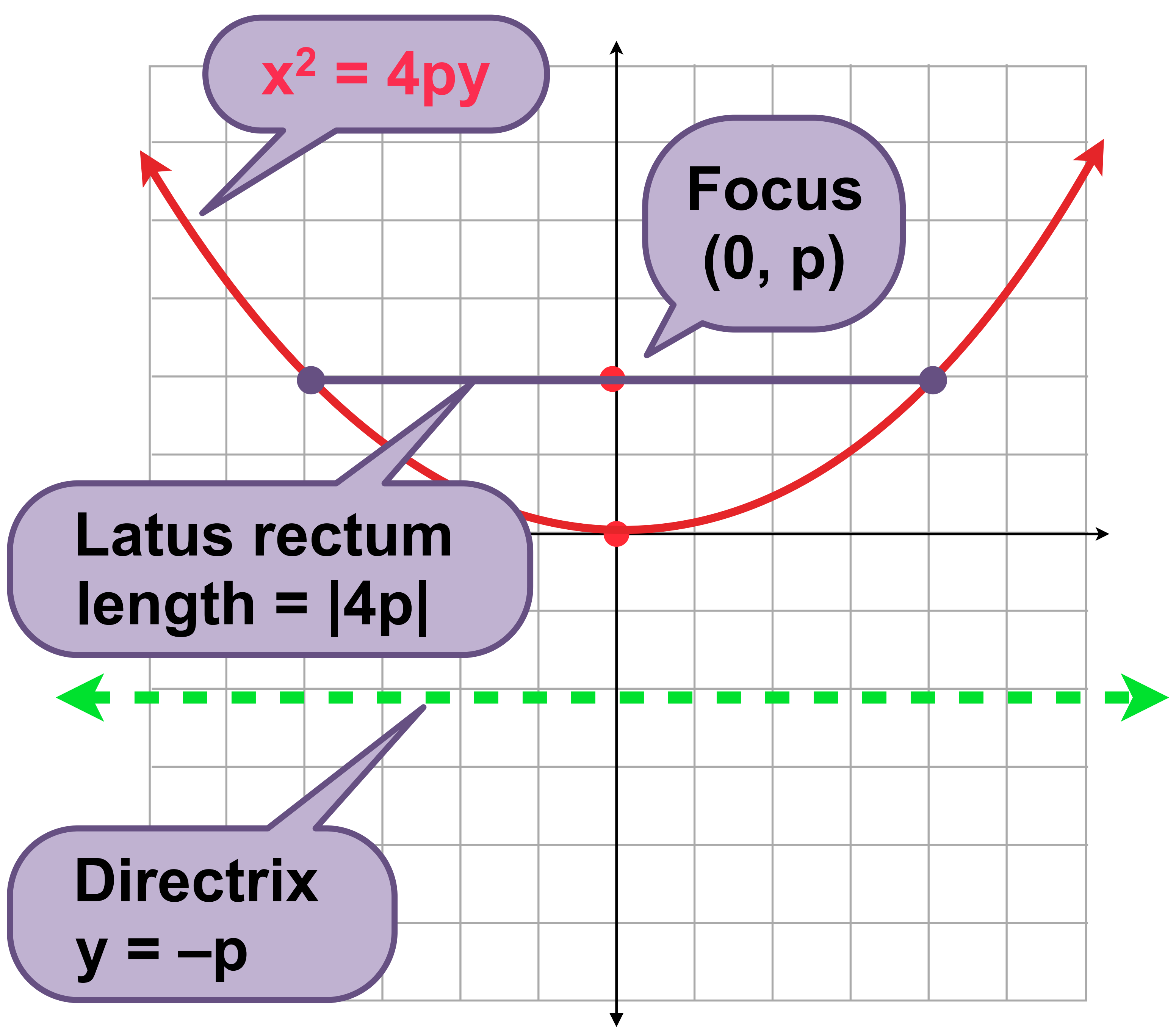
The distance from the **focus** to the **vertex**, and the distance from the **directrix** to the **vertex** are both **p**.

The distance from the **focus** to a point on the parabola and a segment perpendicular to the axis is **2p**.

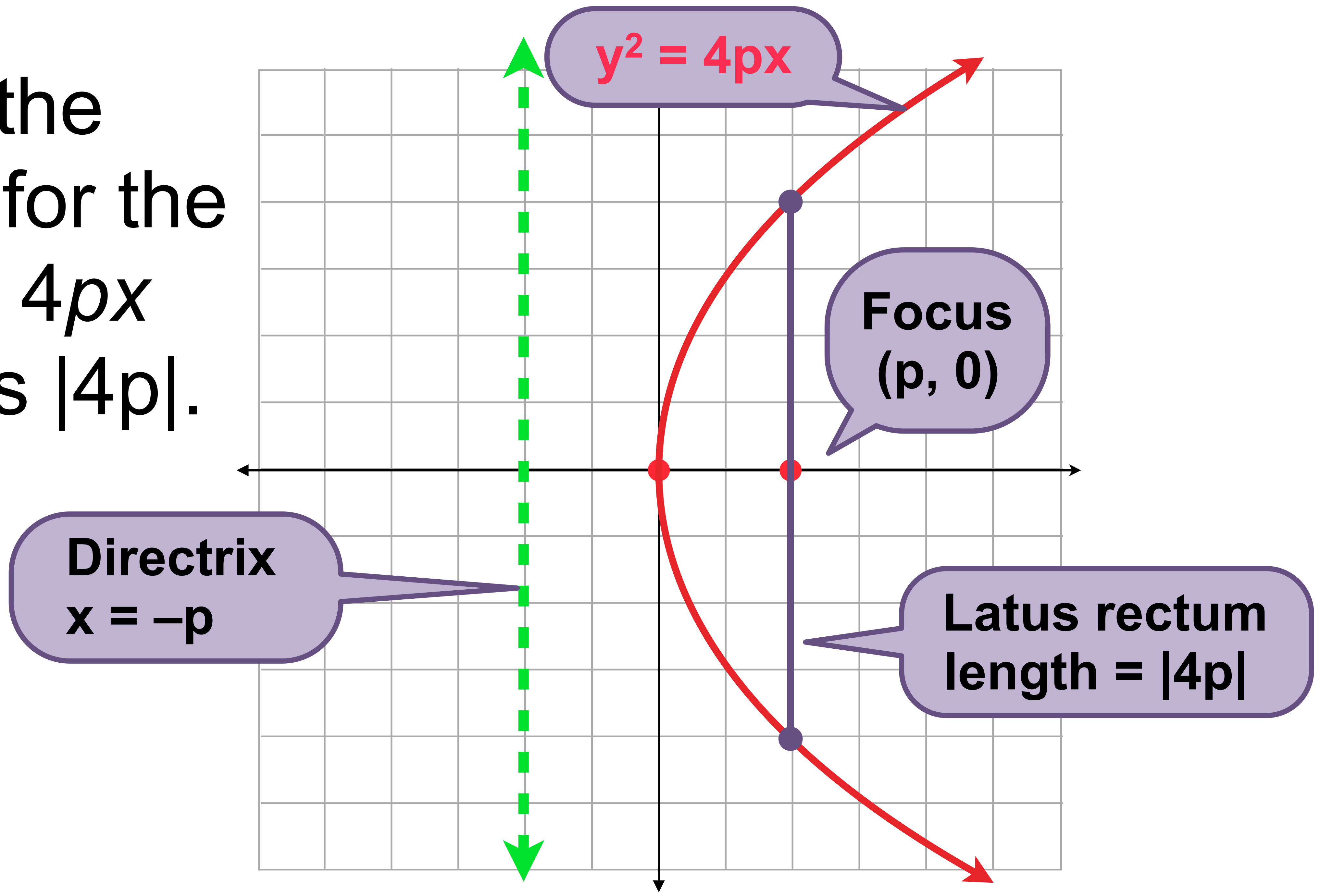


Latus Rectum (Straight Side)

A line segment with endpoints on a parabola and containing the **focus** of the parabola is called a **focal chord**. A specific **focal chord**, the **latus rectum** (pause for the 6th graders) of a parabola is a line segment that passes through its **focus**, is parallel to its **directrix**, and has its endpoints on the parabola.



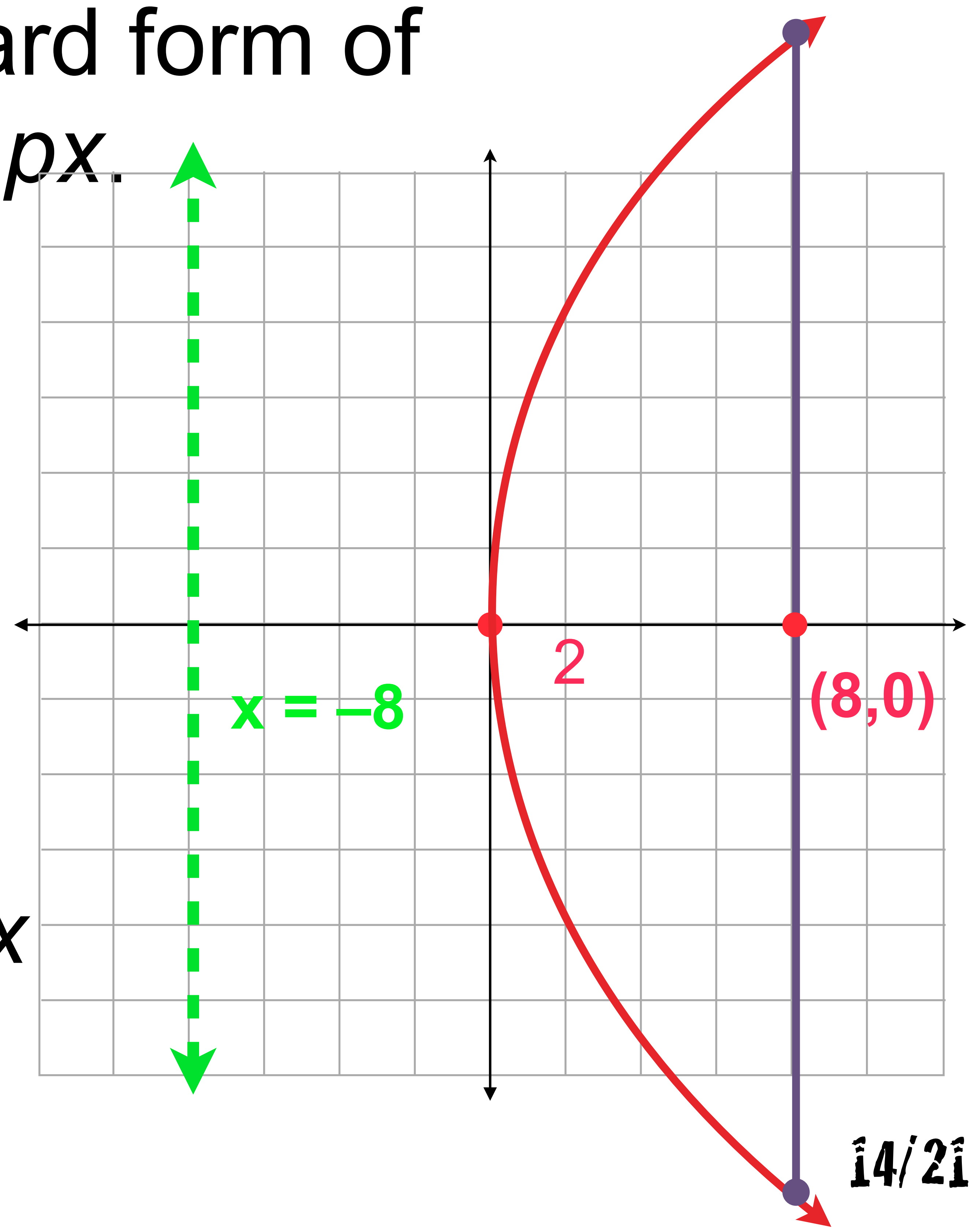
The length of the **latus rectum** for the graphs of $y^2 = 4px$ and $x^2 = 4py$ is $|4p|$.



Finding Equation from Focus and Directrix

Find the standard form of the equation of a parabola with focus (8, 0) and directrix $x = -8$.

The focus, (8, 0), is on the x-axis. We use the standard form of the equation in which there is x-axis symmetry, $y^2 = 4px$.



The focus is 8 units to the right of the vertex, (0, 0). Thus, p is positive and $p = 8$.

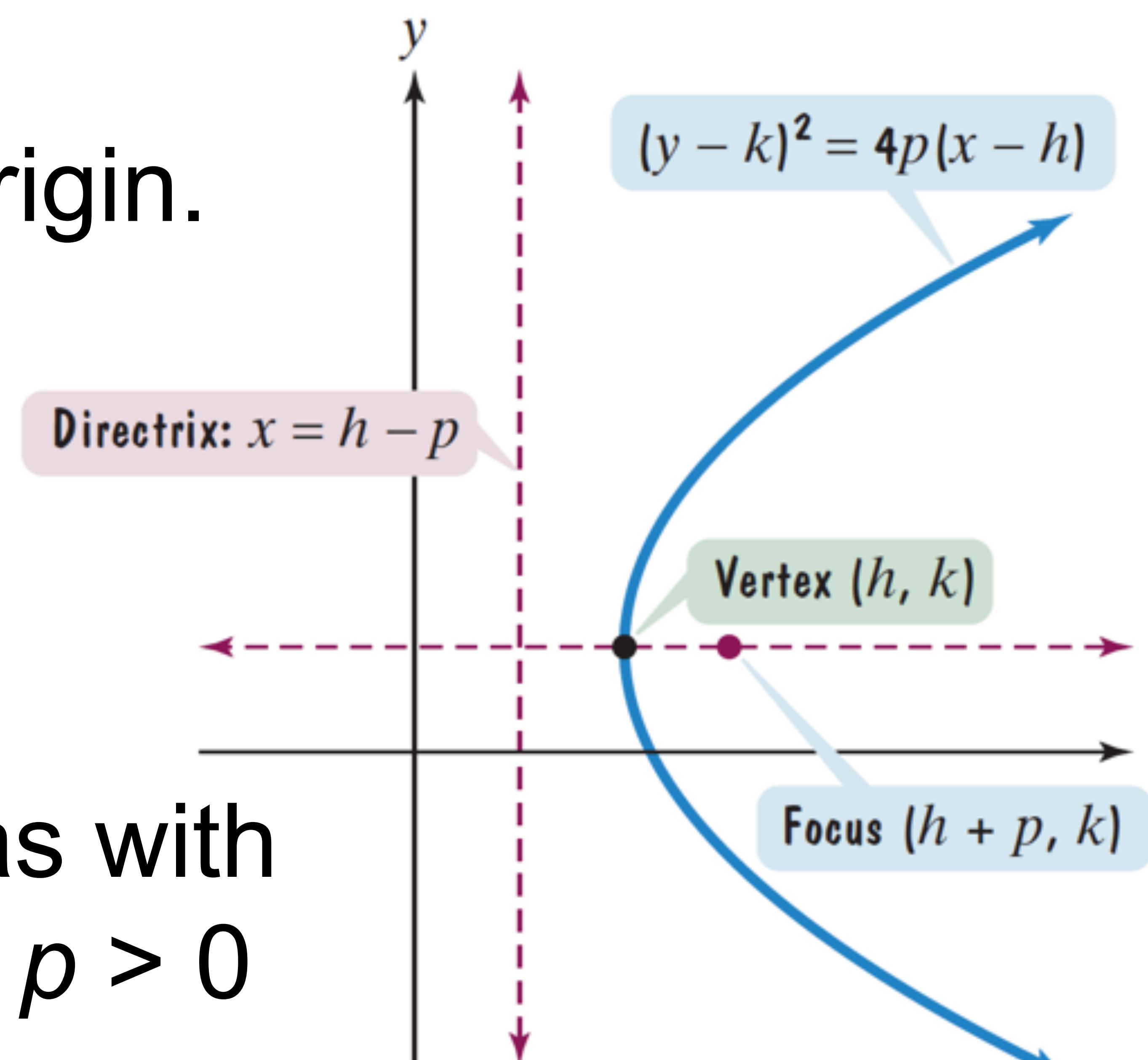
$$y^2 = 4px \qquad y^2 = 32x \quad \text{or} \quad x = \frac{y^2}{32}$$

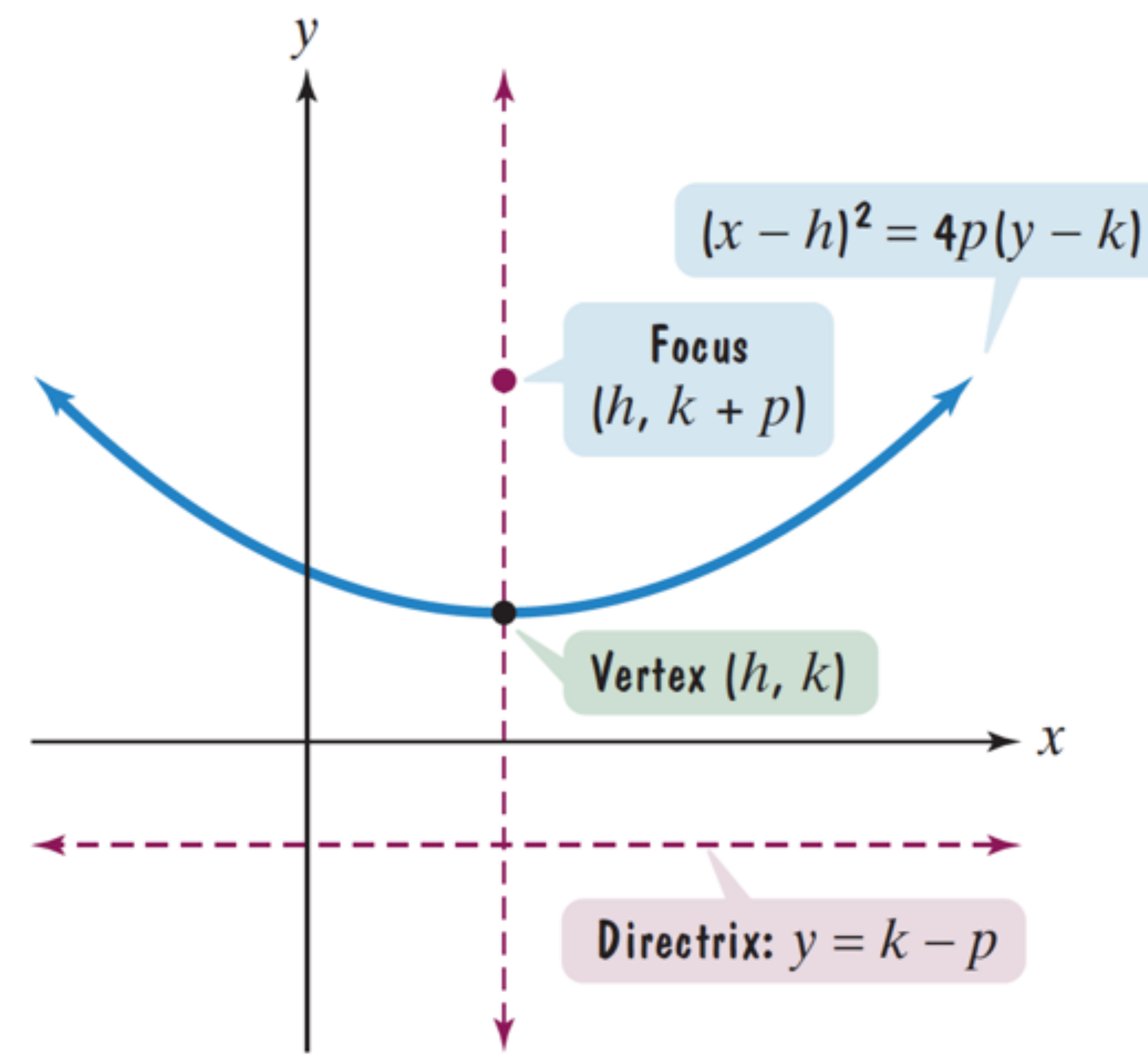
The length of the **latus rectum** for the graph of $y^2 = 32x$ is $|4x8| = 32$.

Translations for parabolas in Standard Form $(x-h)^2 = 4p(y-k)$ or $(y-k)^2 = 4p(x-h)$

Equation	Vertex	Axis of Sym	Focus	Directrix	Direction
$(y-k)^2=4p(x-h)$	(h, k)	y = k	(h + p, k)	x = h – p	p > 0 right, p < 0 left
$(x-h)^2=4p(y-k)$	(h, k)	x = h	(h, k+ p)	y = k – p	p > 0 up, p < 0 down

Let us get off the origin.





Graphs of parabolas with vertex at (h, k) and $p > 0$

Graphing a Parabola with Vertex (h, k)

Find the vertex, focus, and directrix of the parabola given by $(x-2)^2 = 4(y+1)$

This may look more familiar as $y = \frac{1}{4}(x-2)^2 - 1$

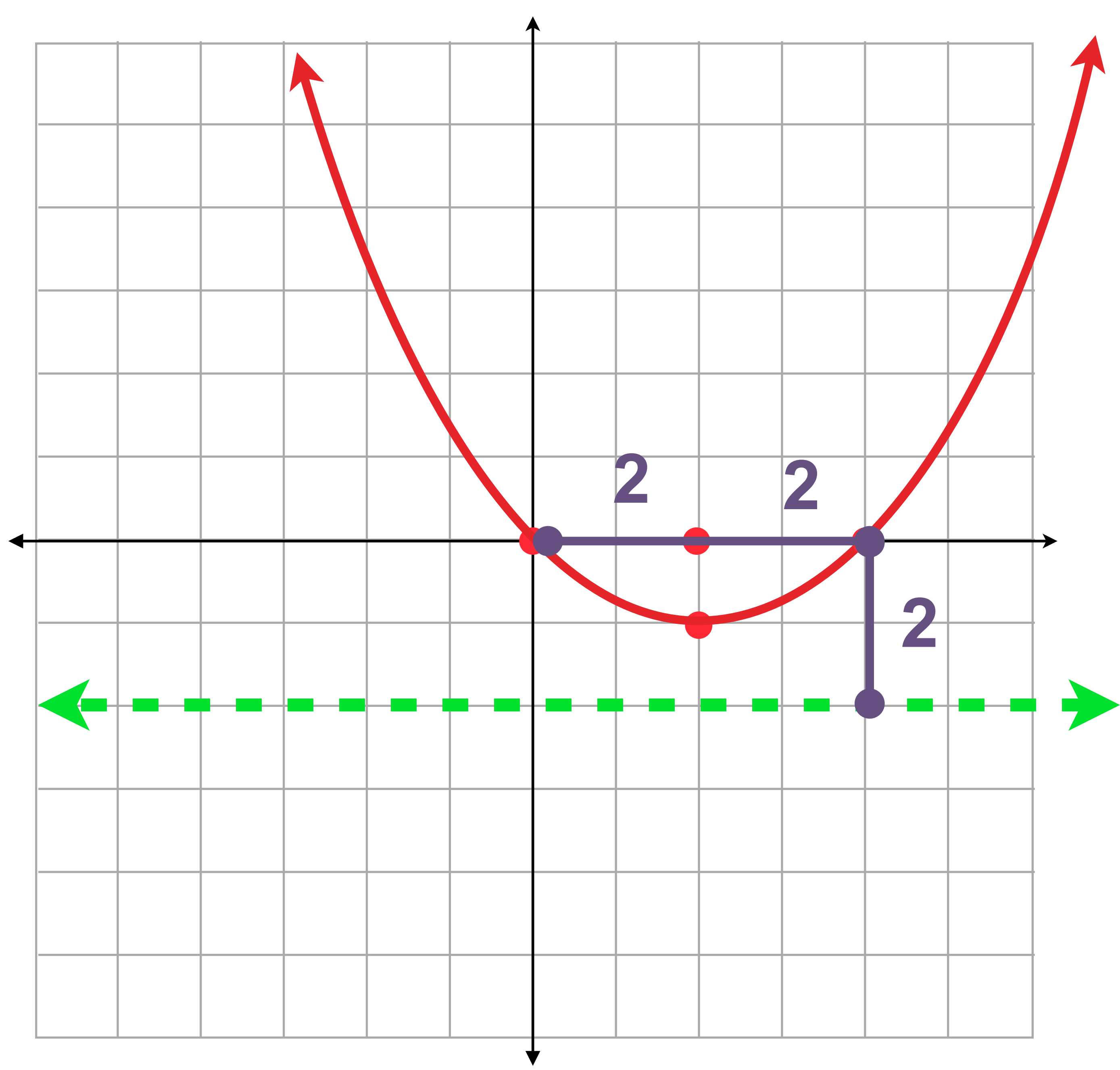
The equation is in the form $(x-h)^2 = 4p(y-k)$

So 4p = 4 and p = 1. The vertex is (2, –1).

Thus the focus is 1 unit above the vertex at (2, 0).

The directrix is 1 unit below the vertex at y = –2.

Keep in mind, when all else fails, a table of values will help you out.



Finding vertex, focus, directrix and graph of ax^2+bx+c

Find the vertex, focus, and directrix of the parabola given by

$\boldsymbol{x^2 - 2x - 16y - 31 = 0}$ To get the form we would like, complete the square.

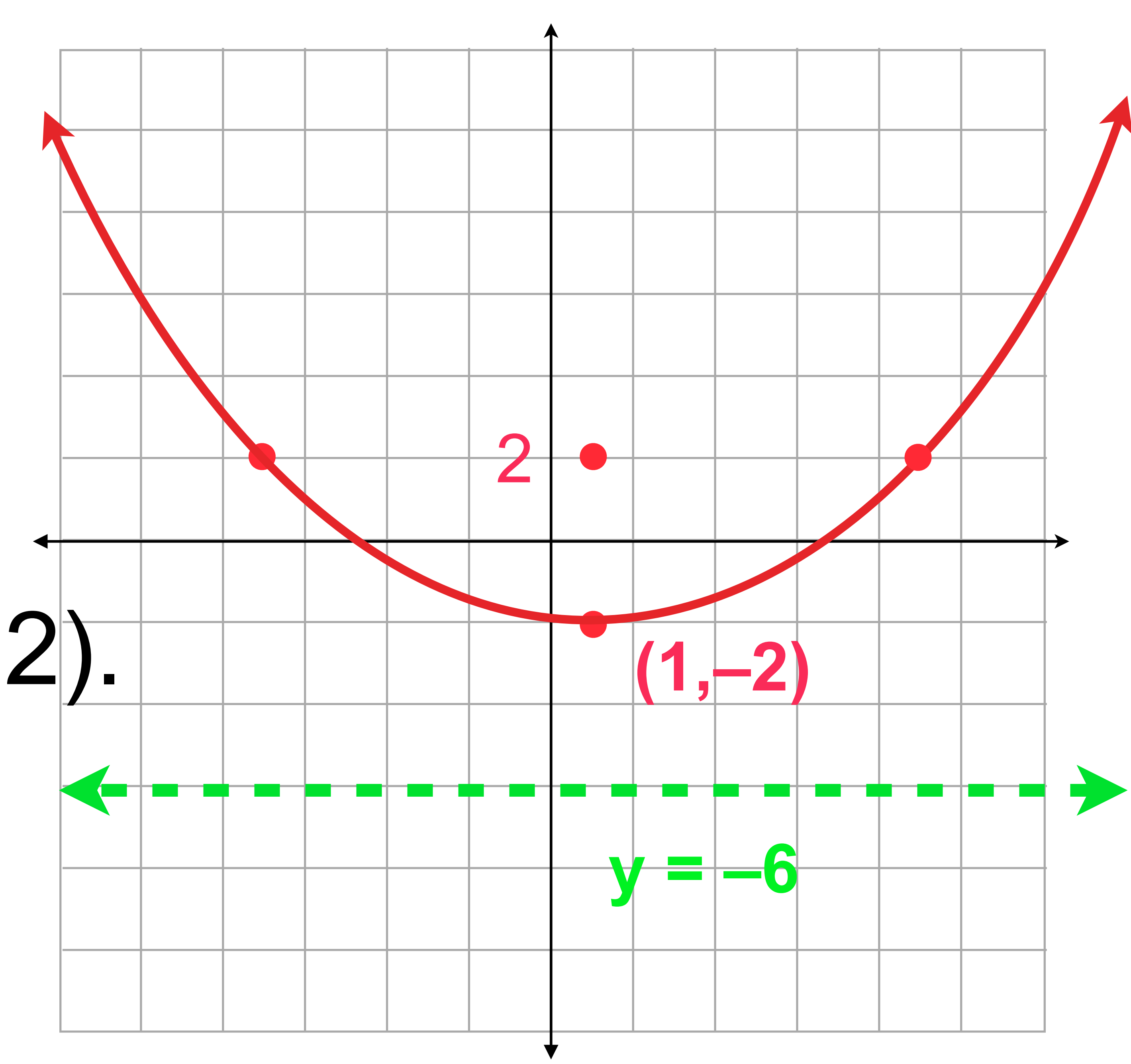
$x^2 - 2x = 16y + 31$ $\quad x^2 - 2x + 1 = 16y + 31 + 1$ $\quad \boldsymbol{(x-1)^2 = 16(y+2)}$

So p = 4, and the vertex is (1, –2).

Thus the focus is 4 units above the vertex at (1, 2).

The directrix is 4 units below the vertex at y = –6.

The endpoints of the latus rectum are (-7, 2) and (9, 2).



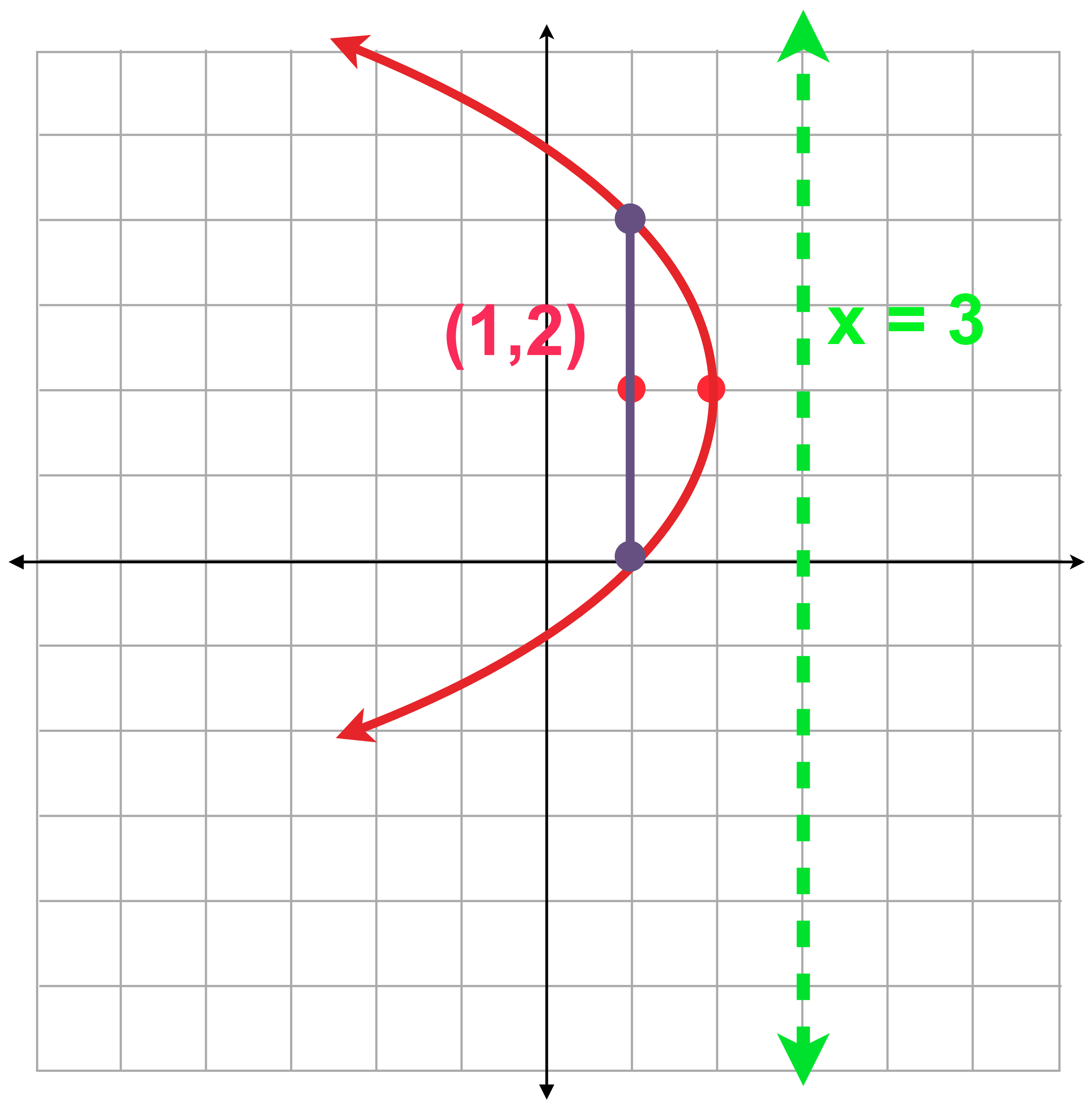
Keep in mind, when all else fails, a table of values will help you out.

Find the standard form of the equation of the parabola with focus (1, 2) and directrix $x = 3$.

The focus (1, 2) and directrix x = 3 so p = –1, the vertex is (2, 2), and the graph opens left.

$$(y-k)^2 = 4p(x-h) \qquad (y-2)^2 = 4(-1)(x-2)$$

$$(y-2)^2 = -4(x-2)$$

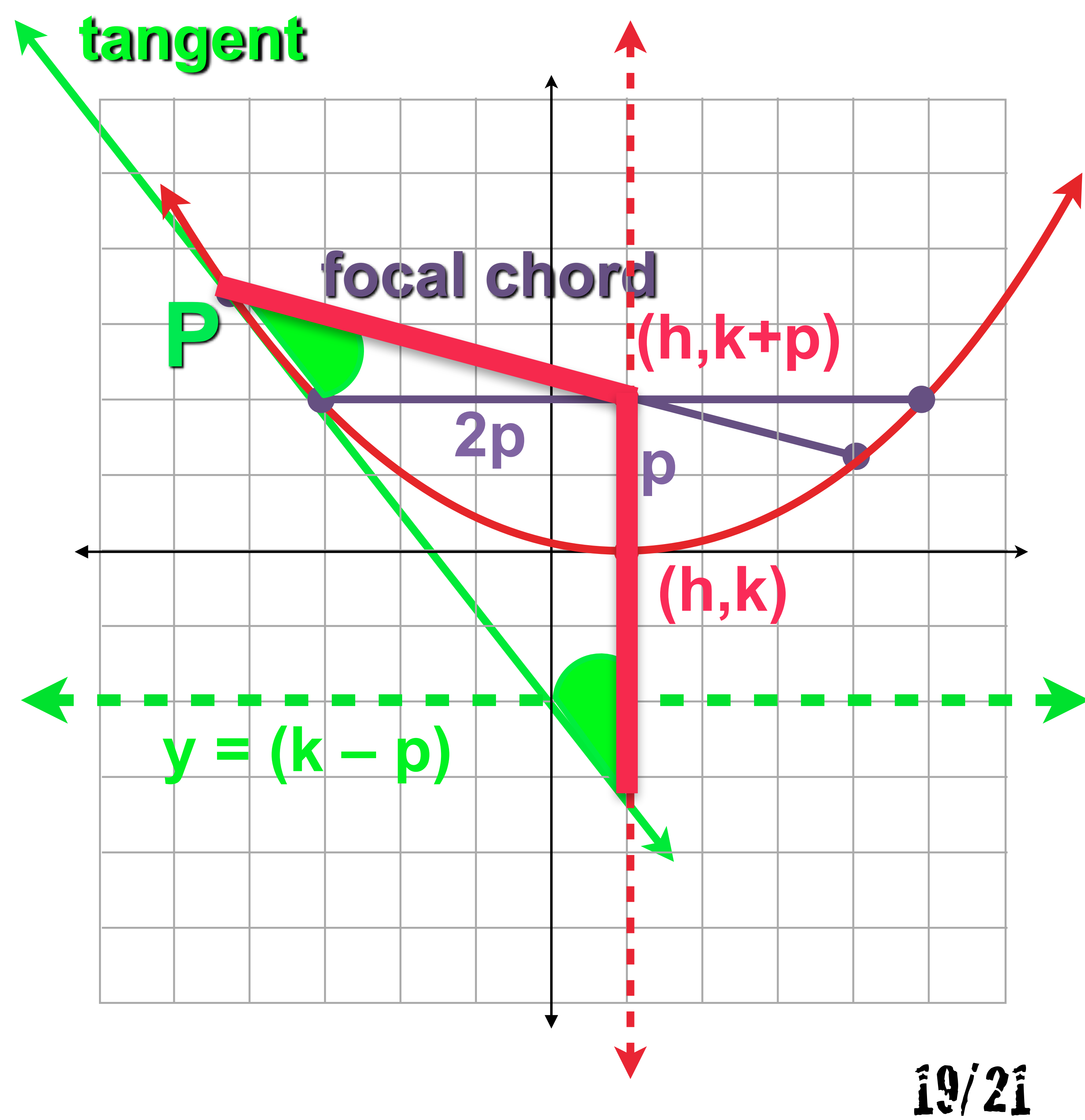


The length of the **latus rectum** is |4(–1)| = 4.

Chords and Tangents

Remember, a line segment with endpoints on a parabola and containing the focus of the parabola is called a **focal chord**.

A line is **tangent** to a parabola if it intersects the parabola in exactly one point.



The tangent line to a parabola at a point *P* makes equal angles with the following two lines.

1. The line containing the focal chord at *P*.
2. The axis of the parabola.

In other words, the distance from the focus to P, and the distance from the focus to the intersection of **tangent** and axis of symmetry are equal.

This is relatively simple to prove using geometry.

Chords and Tangents

Find an equation of the tangent line to the parabola given by $x^2 - 4y = 0$ at the point (4, 4).

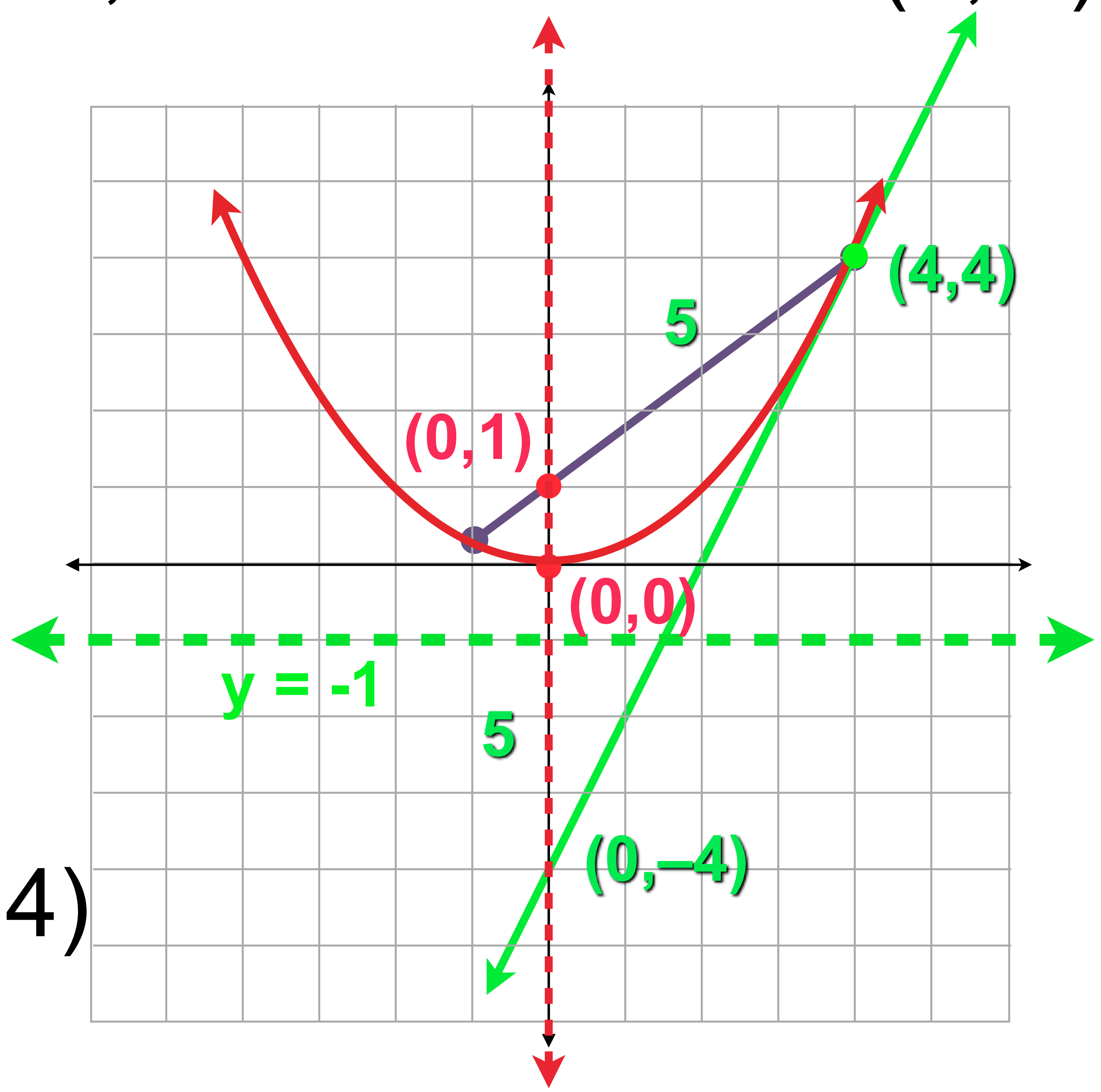
$(x-h)^2 = 4p(y-k)$ $\quad (x-0)^2 = 4(1)(y-0)$ $\quad$ So p = 1, and the vertex is (0, 0).

The focus is (0, 1) and directrix y = –1

From (0,1) to (4,4) d = 5

From (0,1) to where the tangent intersects the axis of symmetry must also be d = 5.
The point of intersection is (0,–4).

The equation of the line through (0,–4) and (4,4) is simple, y = 2x – 4.



Application

An engineer is designing a flashlight using a parabolic reflecting mirror and a light source (like a flashlight or auto headlight). The casting has a diameter of 6 inches and a depth of 4 inches.

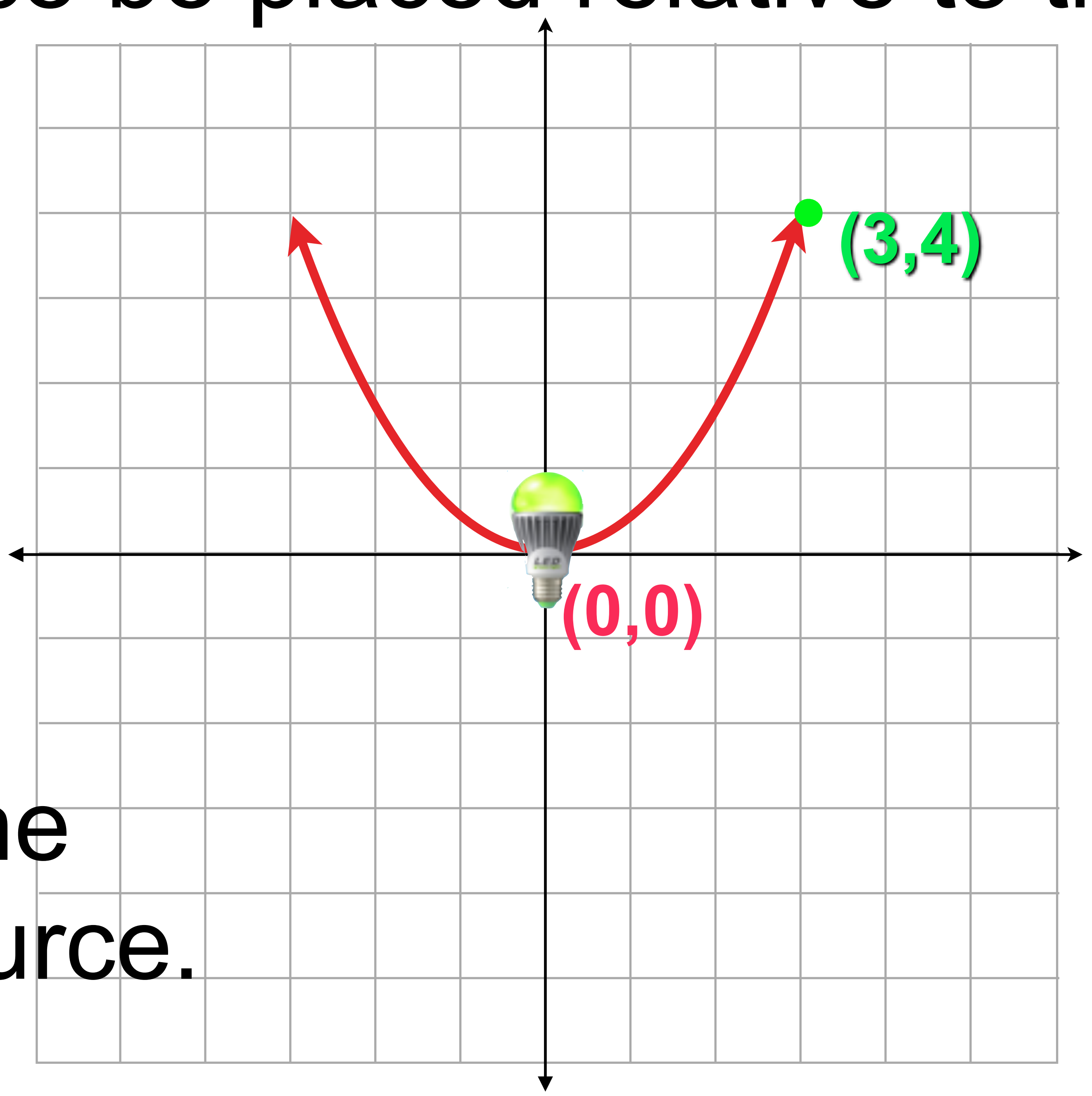
What is the equation of the parabola used to shape the mirror?

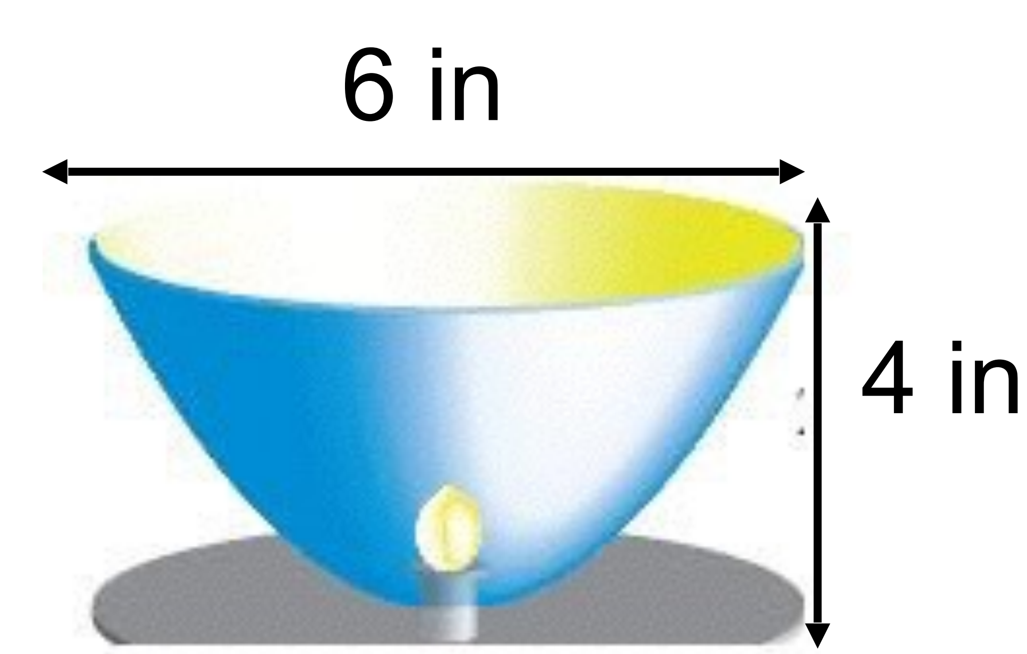
At what point should the light source be placed relative to the mirror's vertex?

Place the shape with vertex (0,0)

$$(x-h)^2 = 4p(y-k) \quad (3-0)^2 = 4p(4-0)$$

$$p = \frac{9}{16} \quad x^2 = \frac{9}{4}y$$





The focus is (0, 9/16) and that is the appropriate position of the light source.