Chapter 10

Topics in Analytical Trigonometry

10.7 Polar Coordinates

Chapter 10.7

Homework:

Read Sec 10.8

Do p783 3, 9, 11, 21, 39, 47, 55, 61, 69

Chapter 10.7

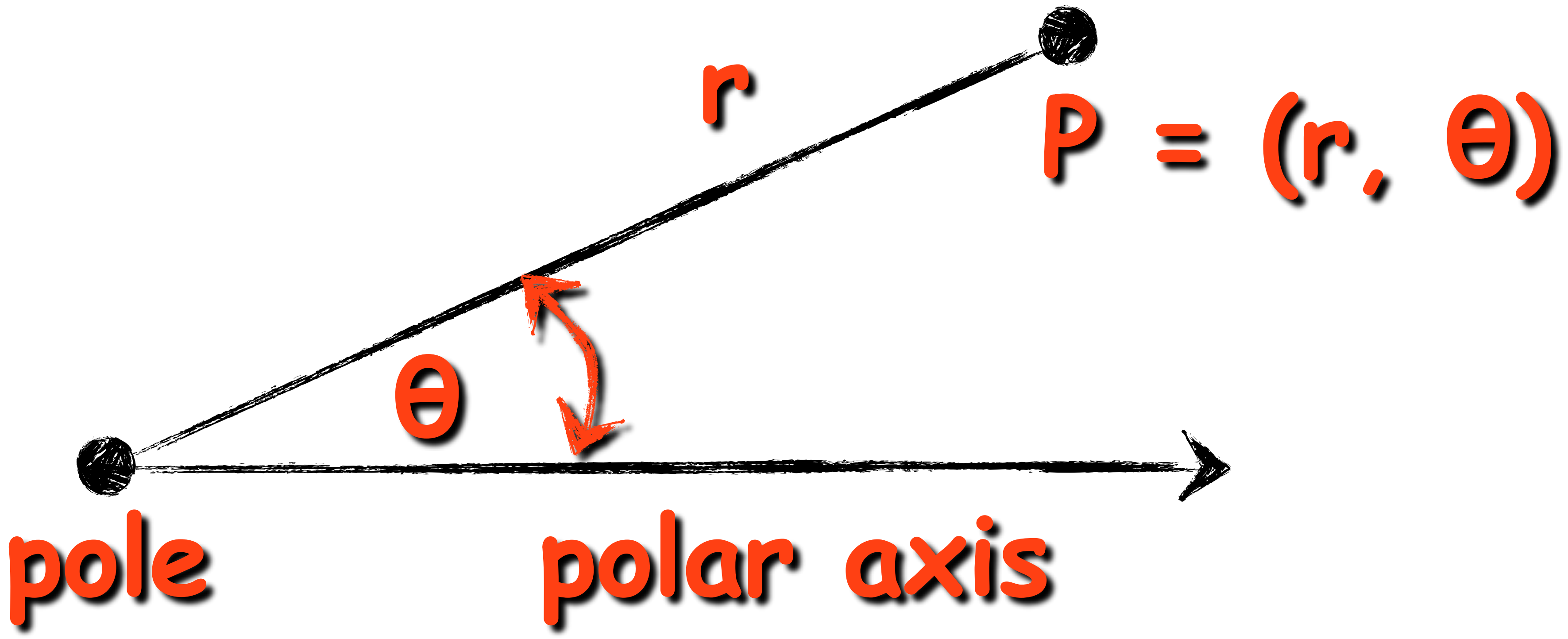
Objectives:

- Plot points in the polar coordinate system.
- Find multiple sets of polar coordinates for a given point.
- Convert a point from polar to rectangular coordinates.
- Convert a point from rectangular to polar coordinates.
- Convert an equation from rectangular form to polar form.
- Convert an equation from polar form to rectangular form.

Plotting Points in the Polar Coordinate System

Objective: Use Polar Coordinates for points and solving equations.

The **basis** of the polar coordinate system is a horizontal ray that extends to the right. The ray is called the **polar axis**. The endpoint (initial point) of the ray is called the **pole**. A point P in the polar coordinate system is represented by an ordered pair **P = (r, θ)**, where r is the **directed** distance of the point from the pole and **θ** is the angle in standard position with terminal side through point P.

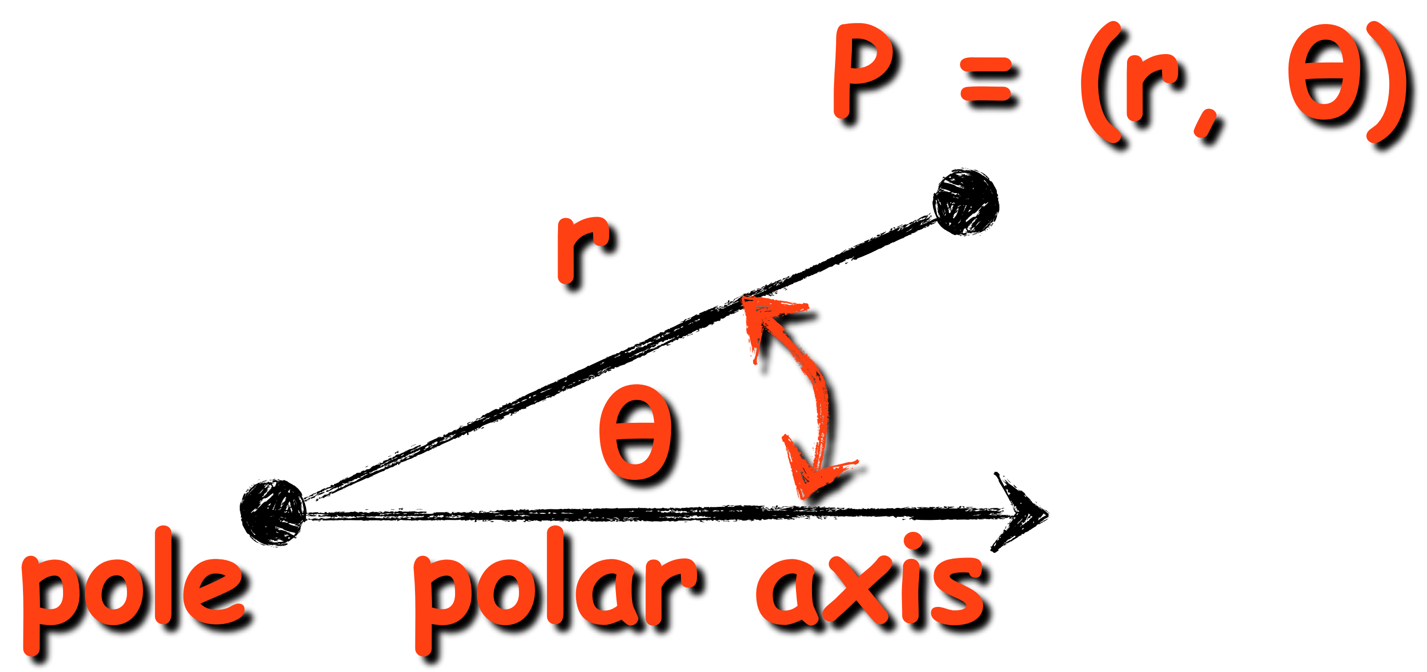


We refer to the ordered pair **(r, θ)** as the **polar coordinates** of P.

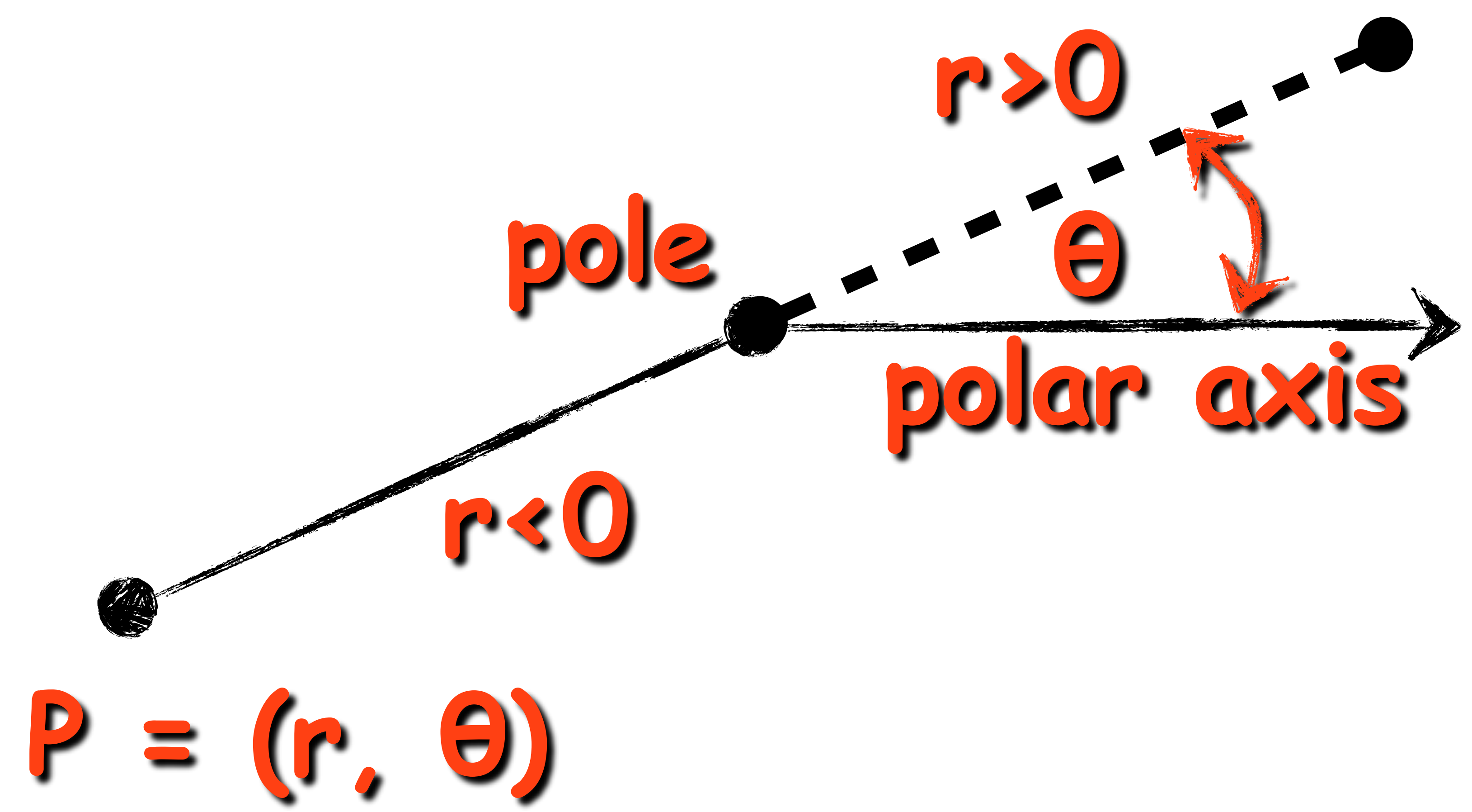
Plotting Points in the Polar Coordinate System

Objective: Use Polar Coordinates for points and solving equations.

- The point $P = (r, \theta)$ is $|r|$ units from the pole.
  - If $r > 0$, the point lies on the the terminal side of θ.



  - If $r < 0$, the point lies on the ray opposite the terminal side.



  - If $r = 0$, the point lies on the pole.

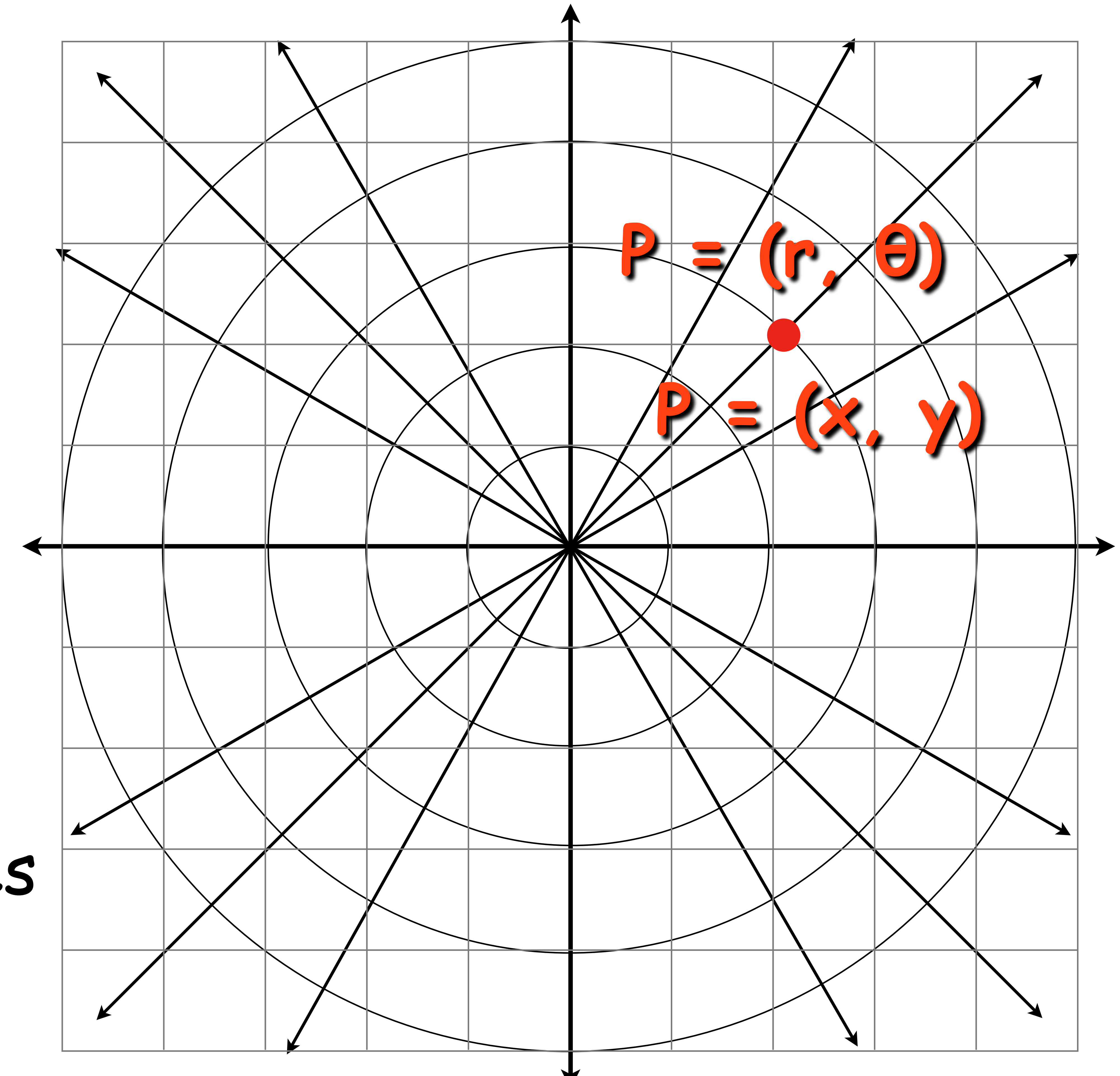
The Polar Coordinate Plane

Look familiar? Just a few extra circles.

Now lets add the Rectangular Coordinate Plane.

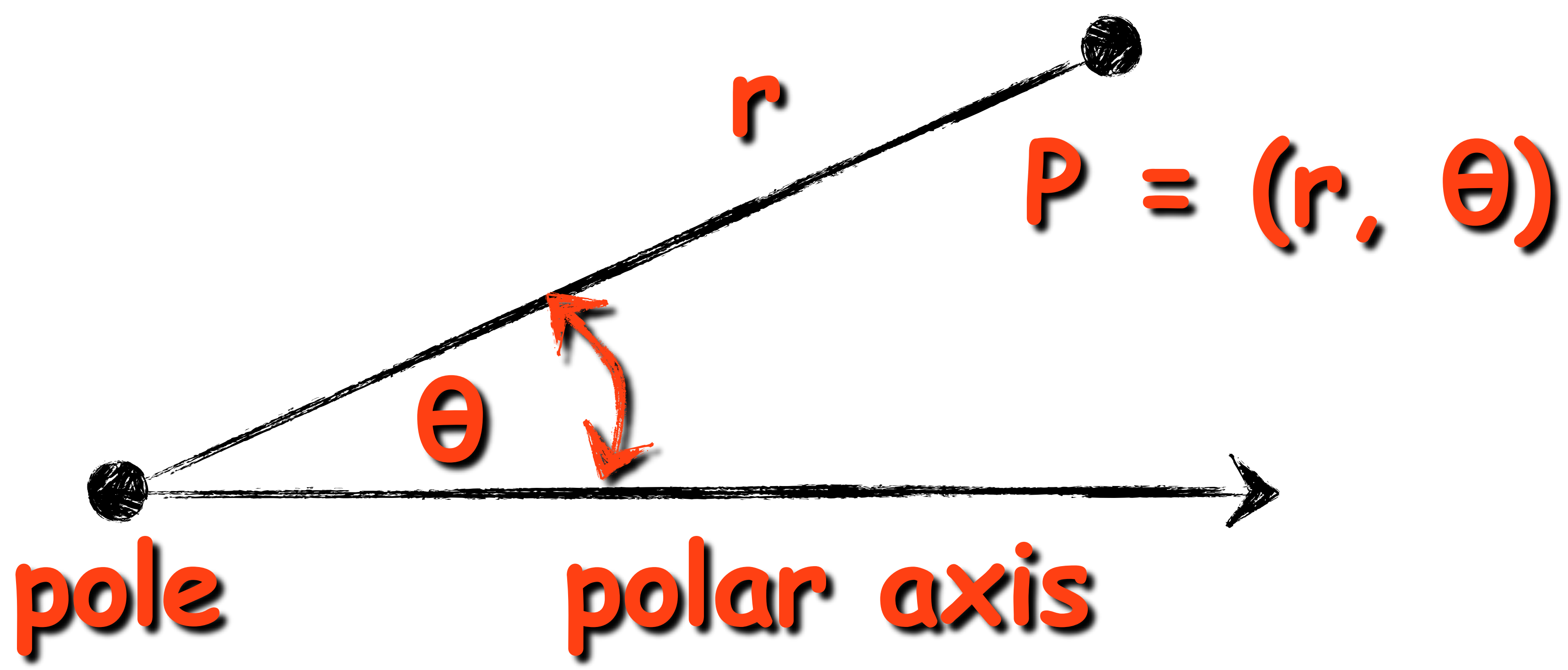
Voila, the Polar Coordinate Plane.

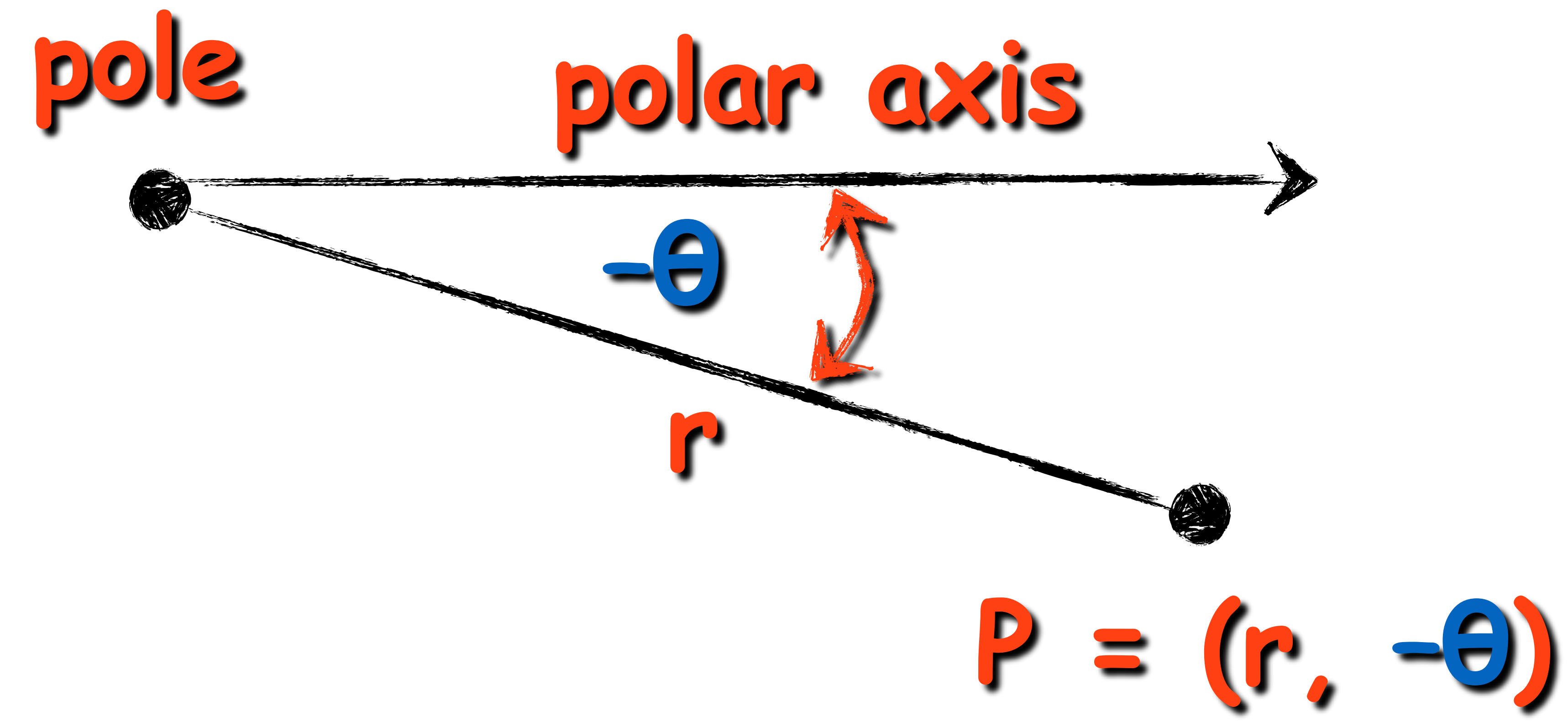
Every point has rectangular coordinates (x, y) and polar coordinates (r, θ)



Plotting Points in the Polar Coordinate System

We refer to the ordered pair **P = (r, θ)** as the **polar coordinates** of **P**. **r** is a directed distance from the pole to **P**. **θ** is an angle from the polar axis to the line segment from the pole to P.





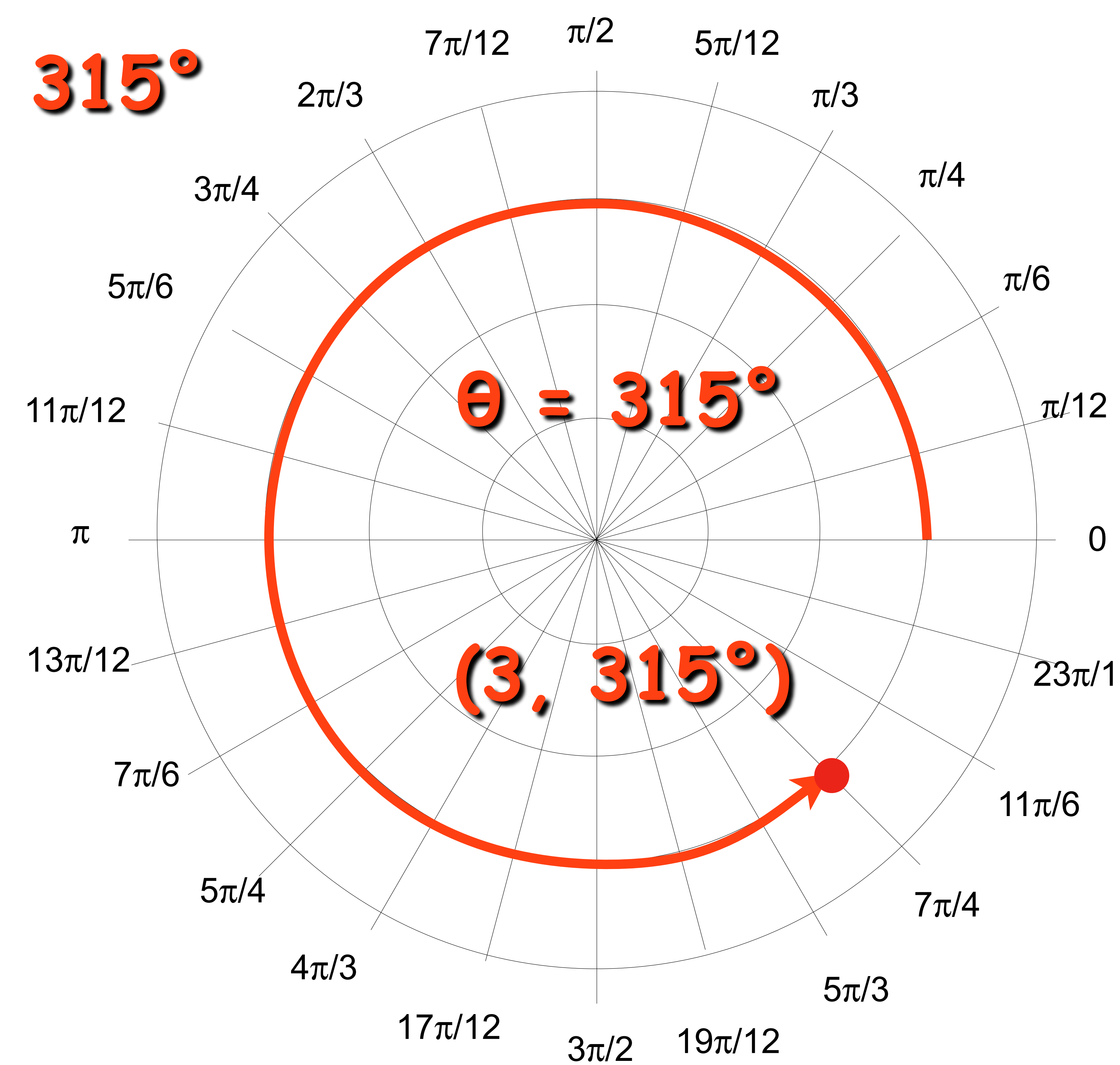
The angle **θ** can be measured in degrees or radians. **Positive** angles are measured **counterclockwise** from the polar axis. **Negative** angles are measured **clockwise** from the polar axis.

Example: Plotting Points in a Polar Coordinate System

Plot the point with the following polar coordinates: **(3, 315°)**

Because 315° is a positive angle,draw **θ = 315°** counterclockwise from the polar axis.

Because r = 3 is positive, plot the point going out three units on the terminal side of **θ**.

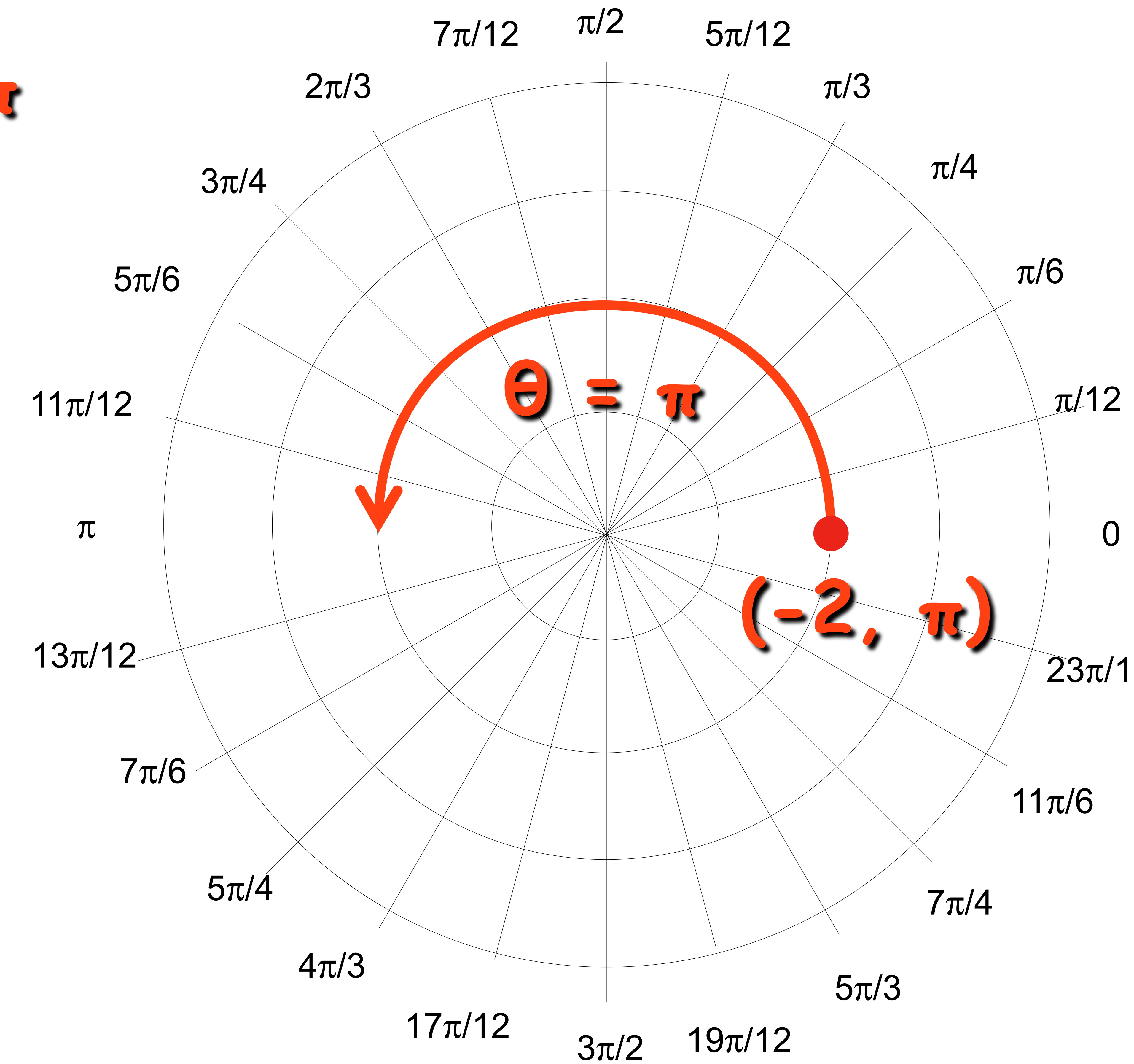


Example: Plotting Points in a Polar Coordinate System

Objective: Use Polar Coordinates for points and solving equations.

Plot the point with the following polar coordinates: **(-2, π)**

Because **π** is a positive angle, draw **θ = π** counterclockwise from the polar axis.



Because r = -2 is negative, plot the point going out two units along the ray **opposite** the terminal side of **θ**.

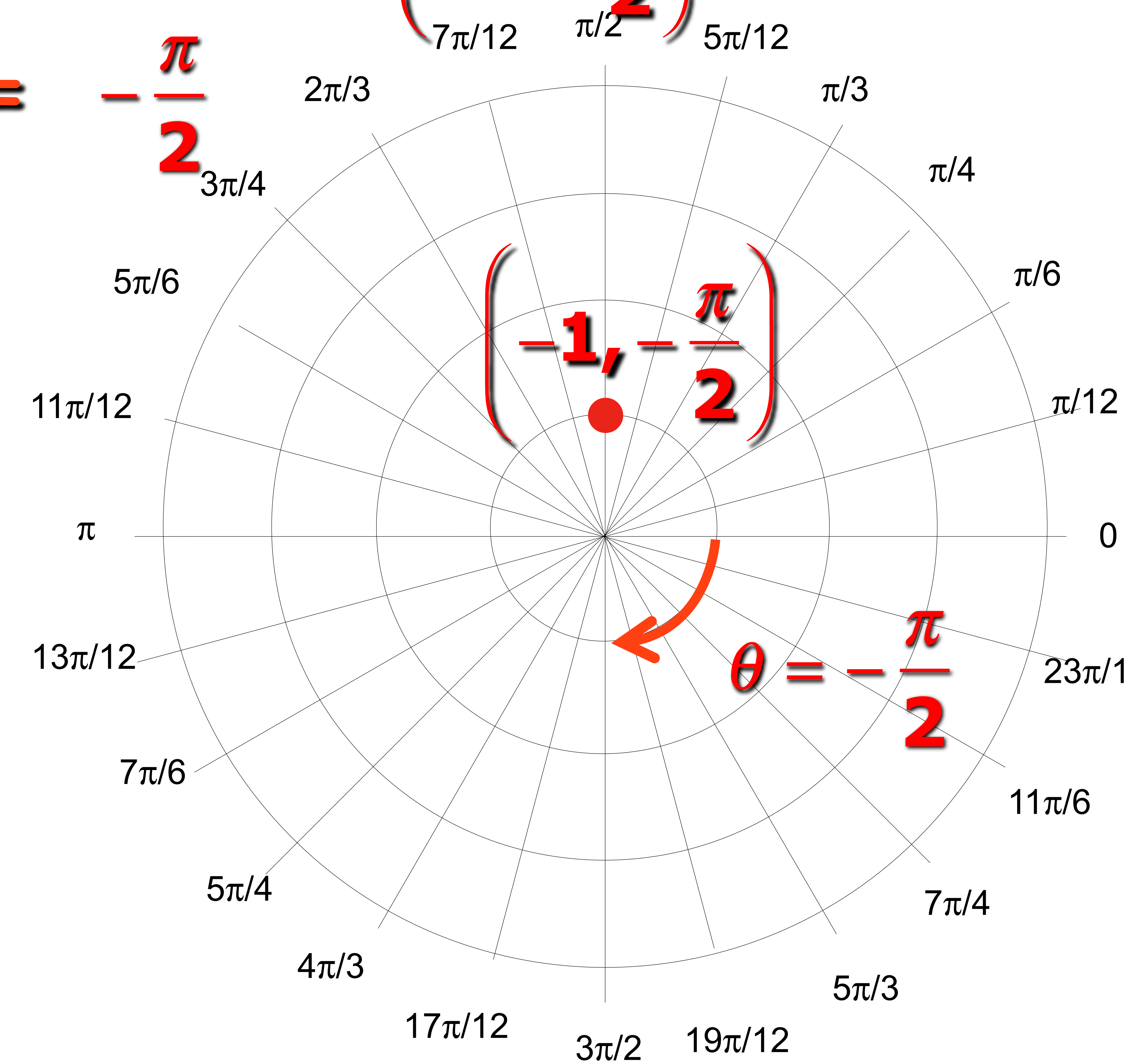
Example: Plotting Points in a Polar Coordinate System

Objective: Use Polar Coordinates for points and solving equations.

Plot the point with the following polar coordinates: $\left(-1, -\frac{\pi}{2}\right)$

Because $-\frac{\pi}{2}$ is a negative angle, draw $\theta = -\frac{\pi}{2}$ clockwise from the polar axis.

Because r = –1 is negative, plot the point going out one unit along the ray **opposite** the terminal side of θ.



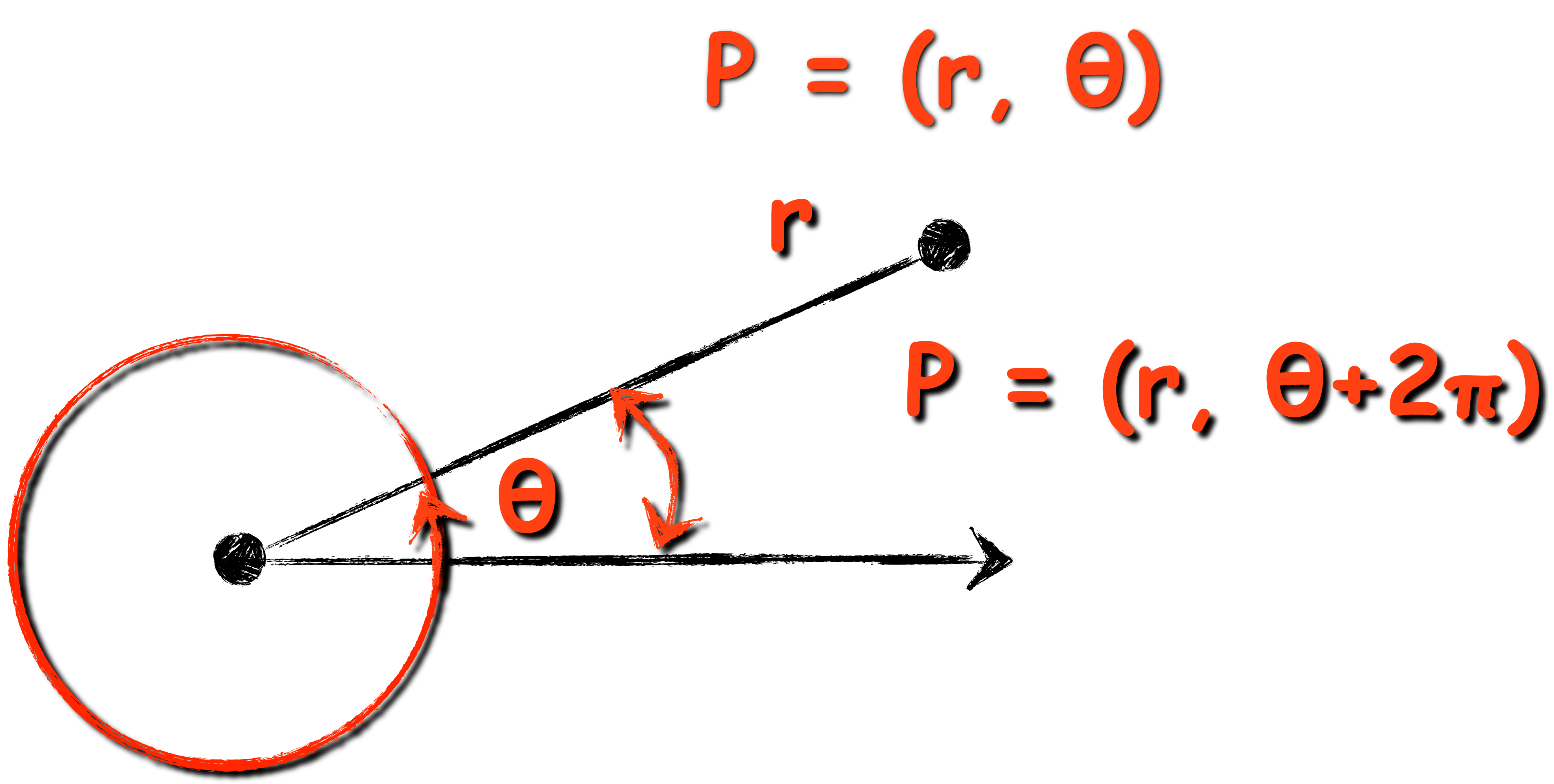
Multiple Representations

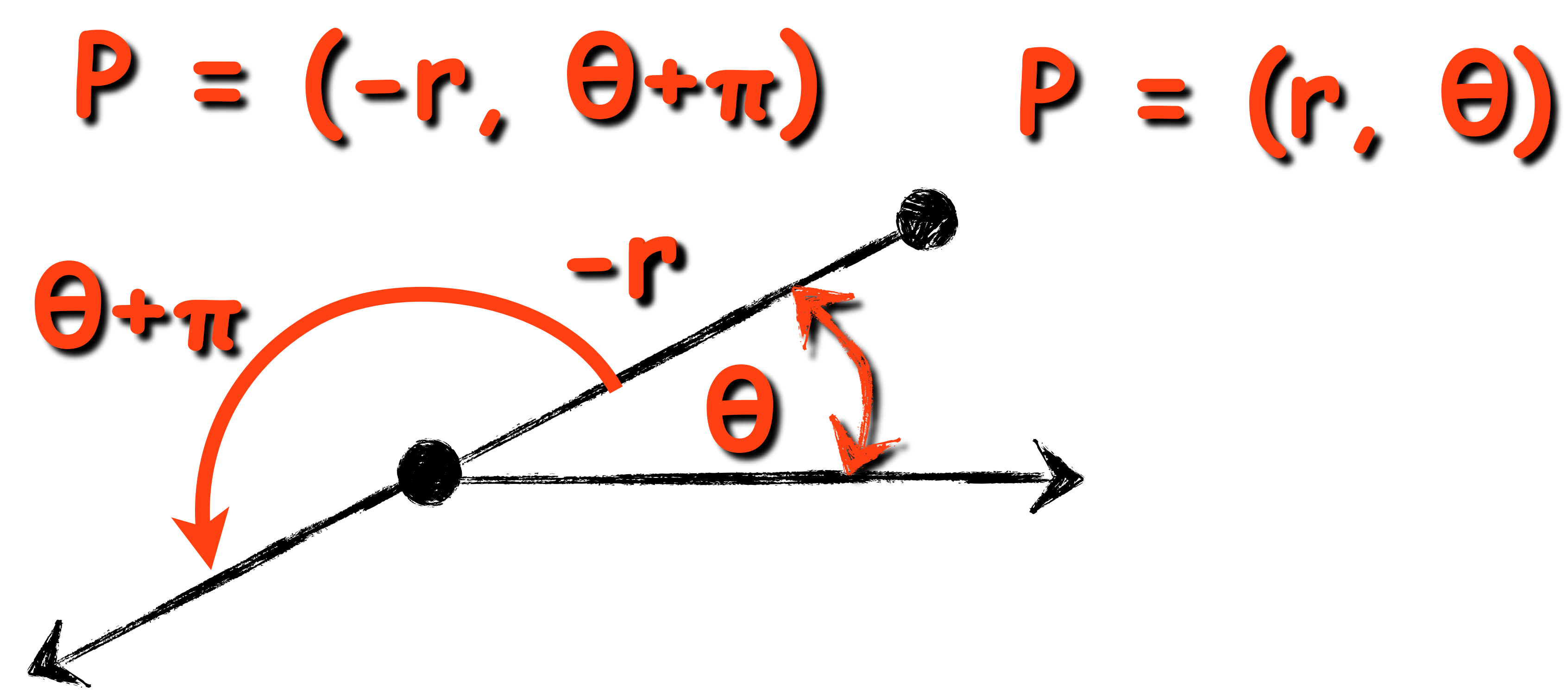
Multiple Representations of Polar Coordinates

Objective: Use Polar Coordinates for points and solving equations.

- Multiple Representations of Points.
  - If n is any integer, the point (r, θ) can be represented as

$$(r, \theta) = (r, \theta \pm n2\pi) \qquad (r, \theta) = (-r, \theta + (2n-1)\pi)$$





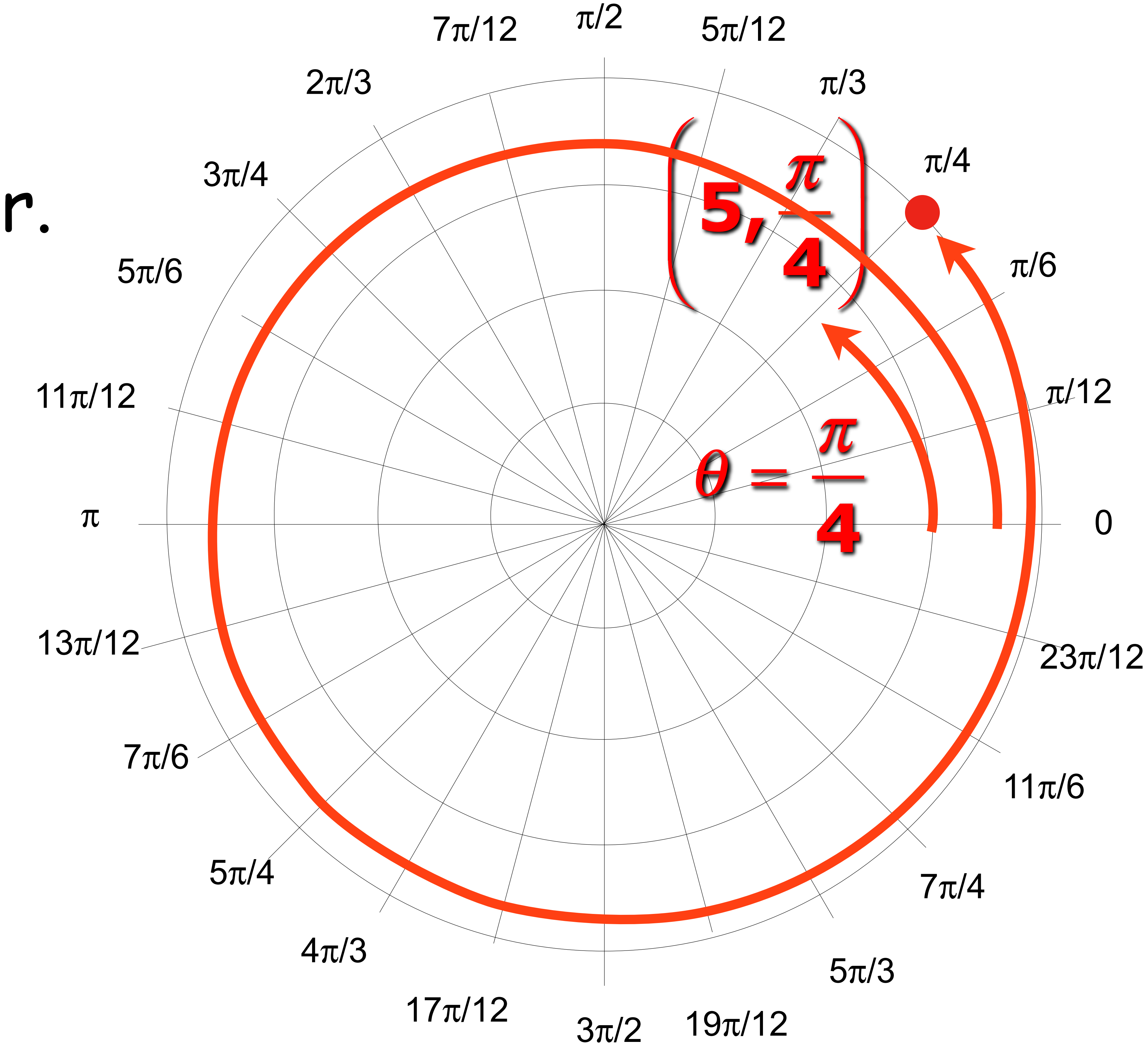
Example: Plotting Points in a Polar Coordinate System

Objective: Use Polar Coordinates for points and solving equations.

Find **another** representation of $\left(5, \frac{\pi}{4}\right)$ in which r is positive and $2\pi < \theta < 4\pi$

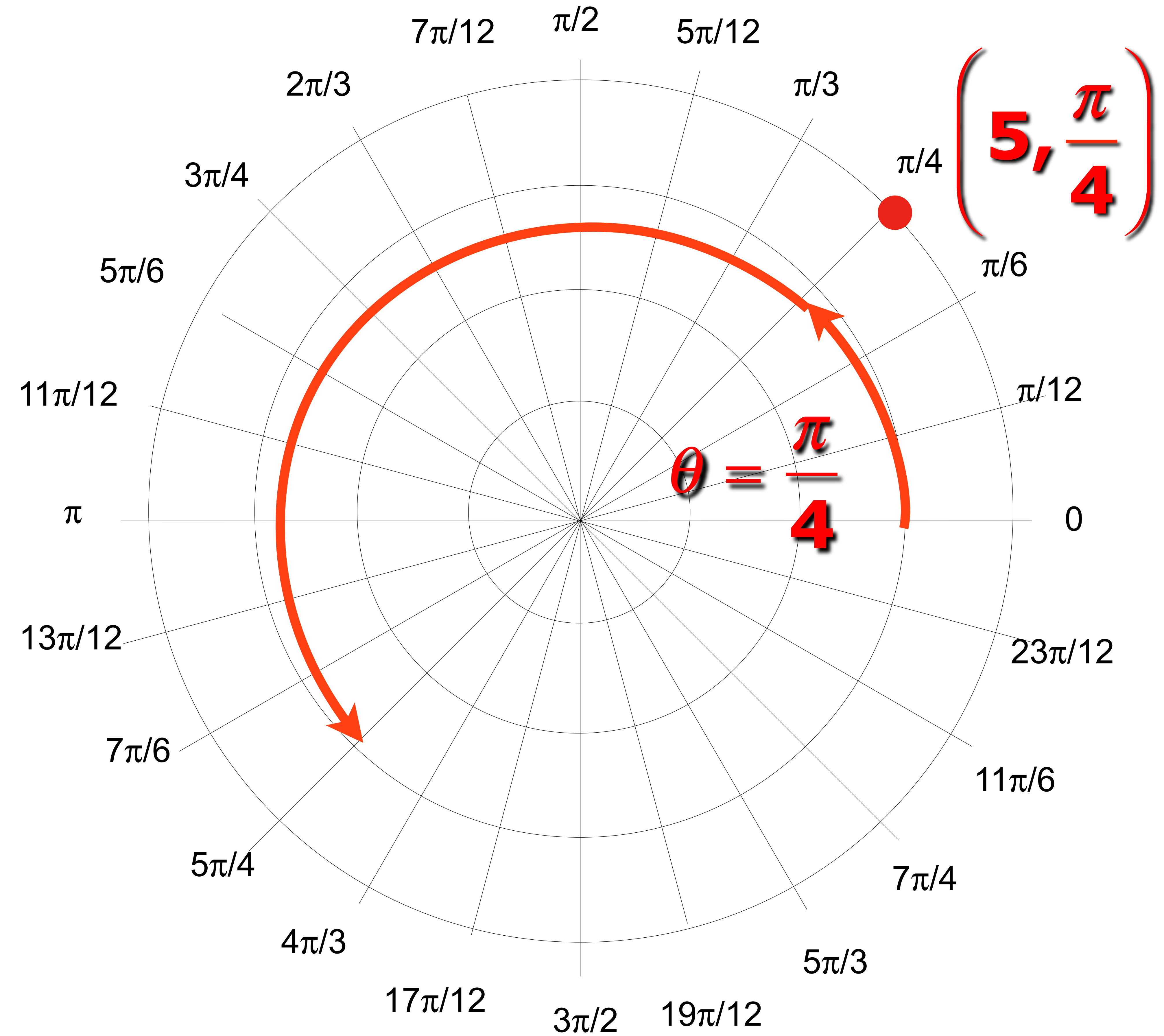
Simply add 2π to $\frac{\pi}{4}$ and do not change r.

$$\left(5, \frac{9\pi}{4}\right)$$



Example: Plotting Points in a Polar Coordinate System

Find another representation of $\left(5, \frac{\pi}{4}\right)$ in which r is negative and $0 < \theta < 2\pi$



Simply add π to $\frac{\pi}{4}$ and change r to -r.

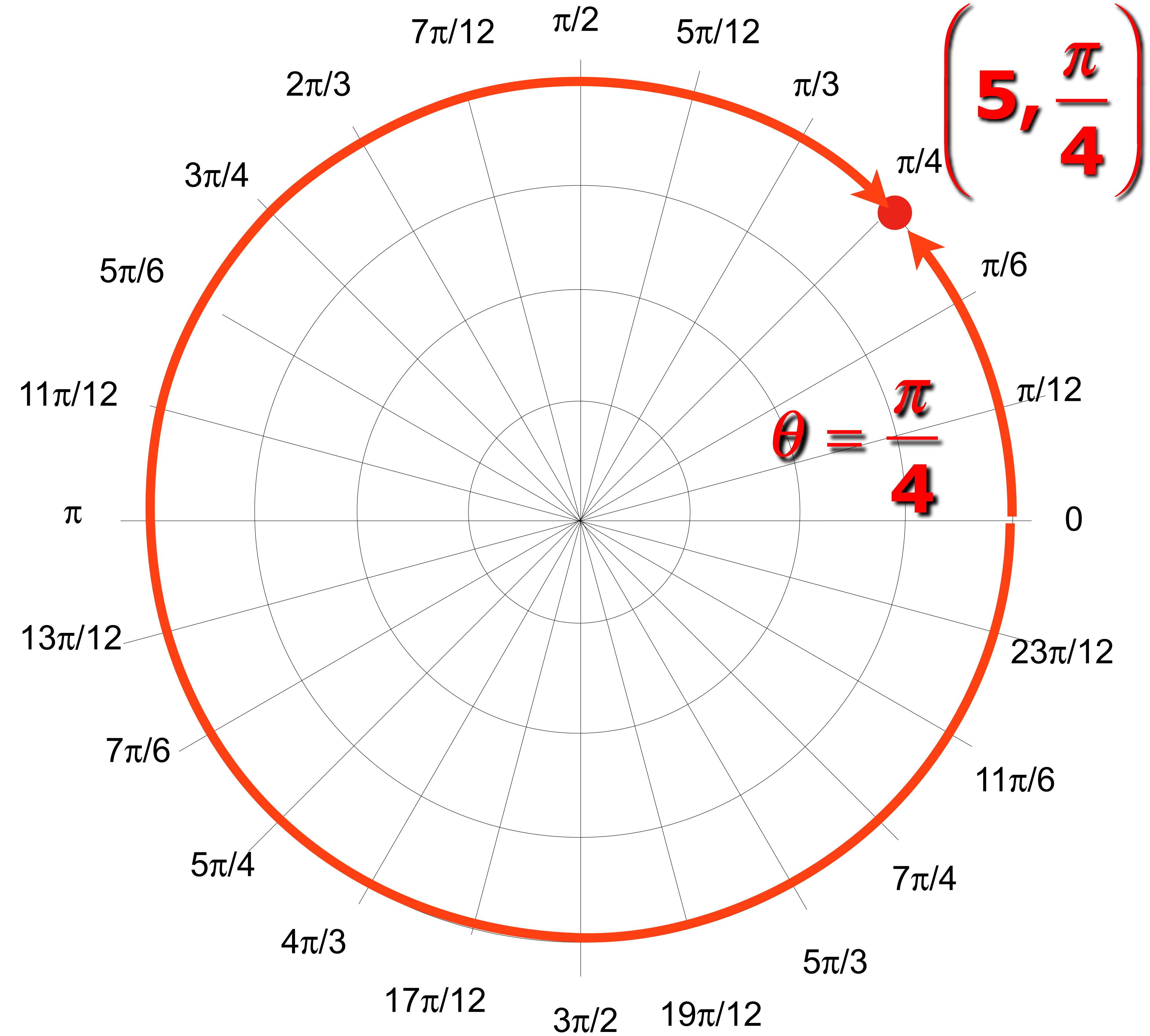
$$\left(-5, \frac{5\pi}{4}\right)$$

Example: Finding Other Polar Coordinates for a Given Point

Find another representation of $\left(5, \frac{\pi}{4}\right)$ in which r is positive and $-2\pi < \theta < 0$

Simply subtract 2π from $\frac{\pi}{4}$ and do not change r.

$$\left(5, -\frac{7\pi}{4}\right)$$

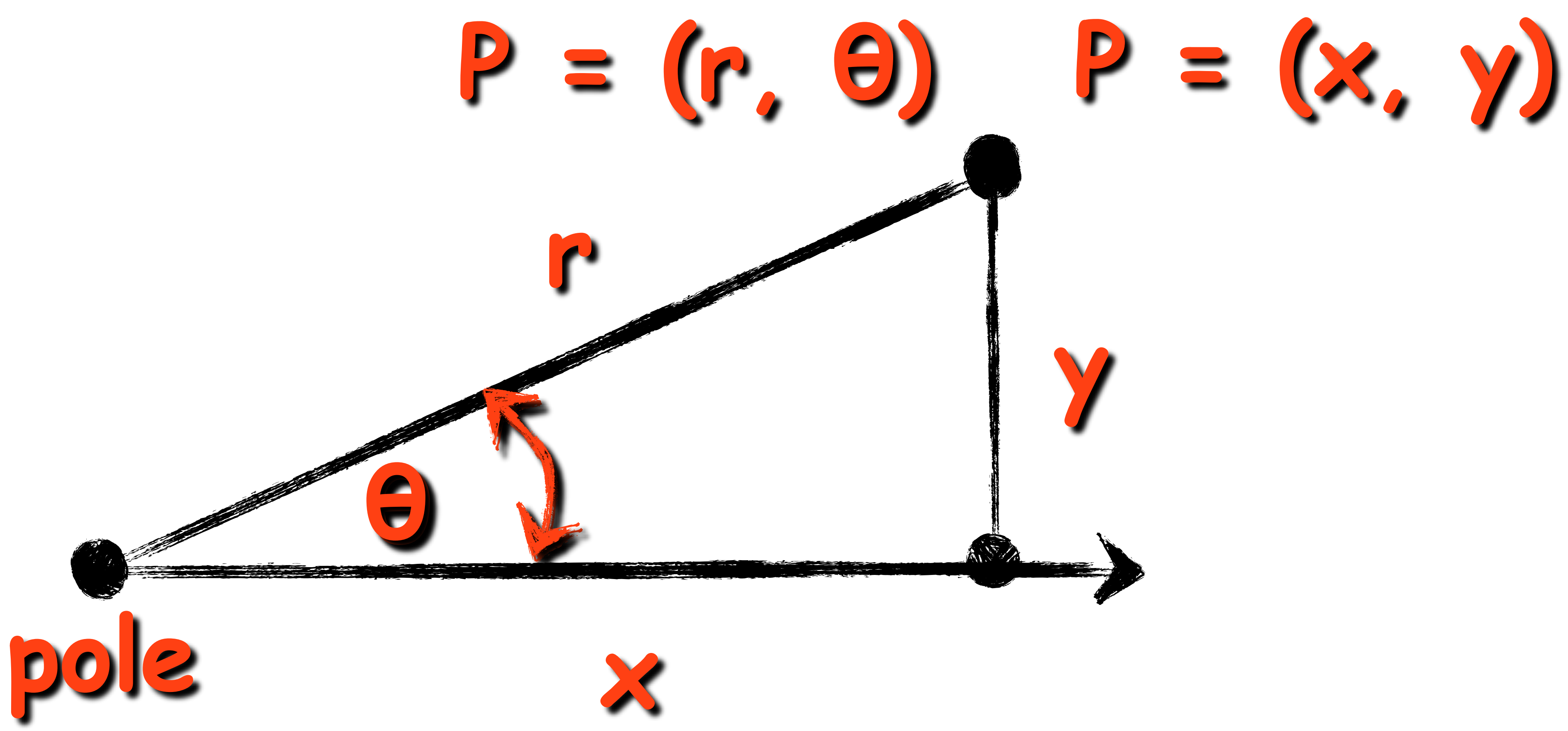


- Converting Coordinates

Relations between Polar and Rectangular Coordinates

Objective: Use Polar Coordinates for points and solving equations.

We can convert polar coordinates (r, θ) to rectangular (x, y) coordinates and rectangular coordinates to polar coordinates.



polar to rectangular

$$x = r\cos\theta$$

$$y = r\sin\theta$$

rectangular to polar

$$r = \sqrt{x^2 + y^2} \quad r > 0$$

$$\tan\theta = \frac{y}{x} \quad 0 < \theta < 2\pi$$

Find the rectangular coordinates of the point with the following polar coordinates: **(3, π)**

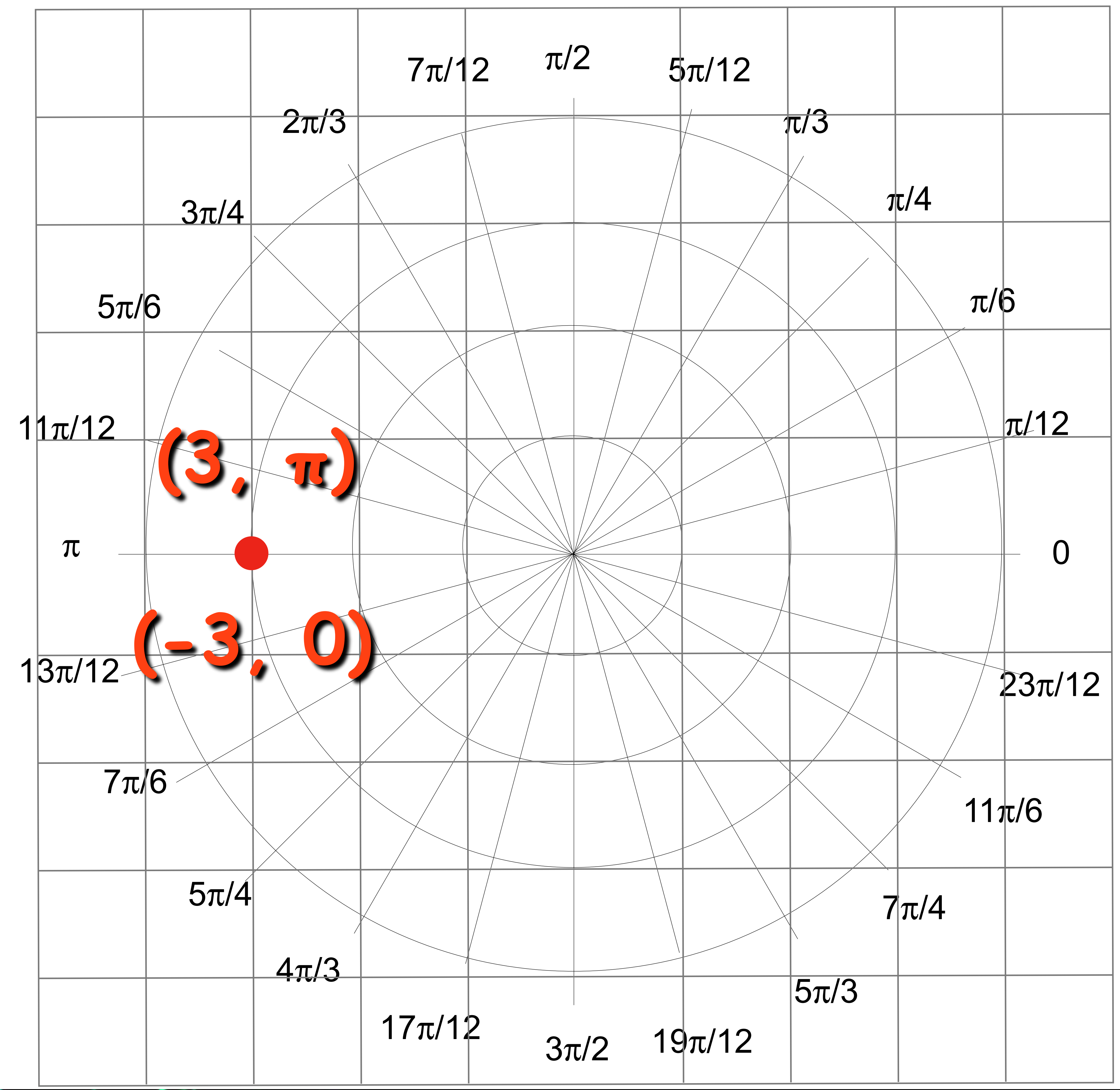
polar to rectangular

$x = r\cos\theta$

$y = r\sin\theta$

$x = 3\cos\pi \quad x = -3$

$y = 3\sin\pi \quad y = 0$



Point Conversion from Polar to Rectangular Coordinates

Objective: Use Polar Coordinates for points and solving equations.

Find the rectangular coordinates of the point with the following polar coordinates: $\left(-10, \frac{\pi}{6}\right)$

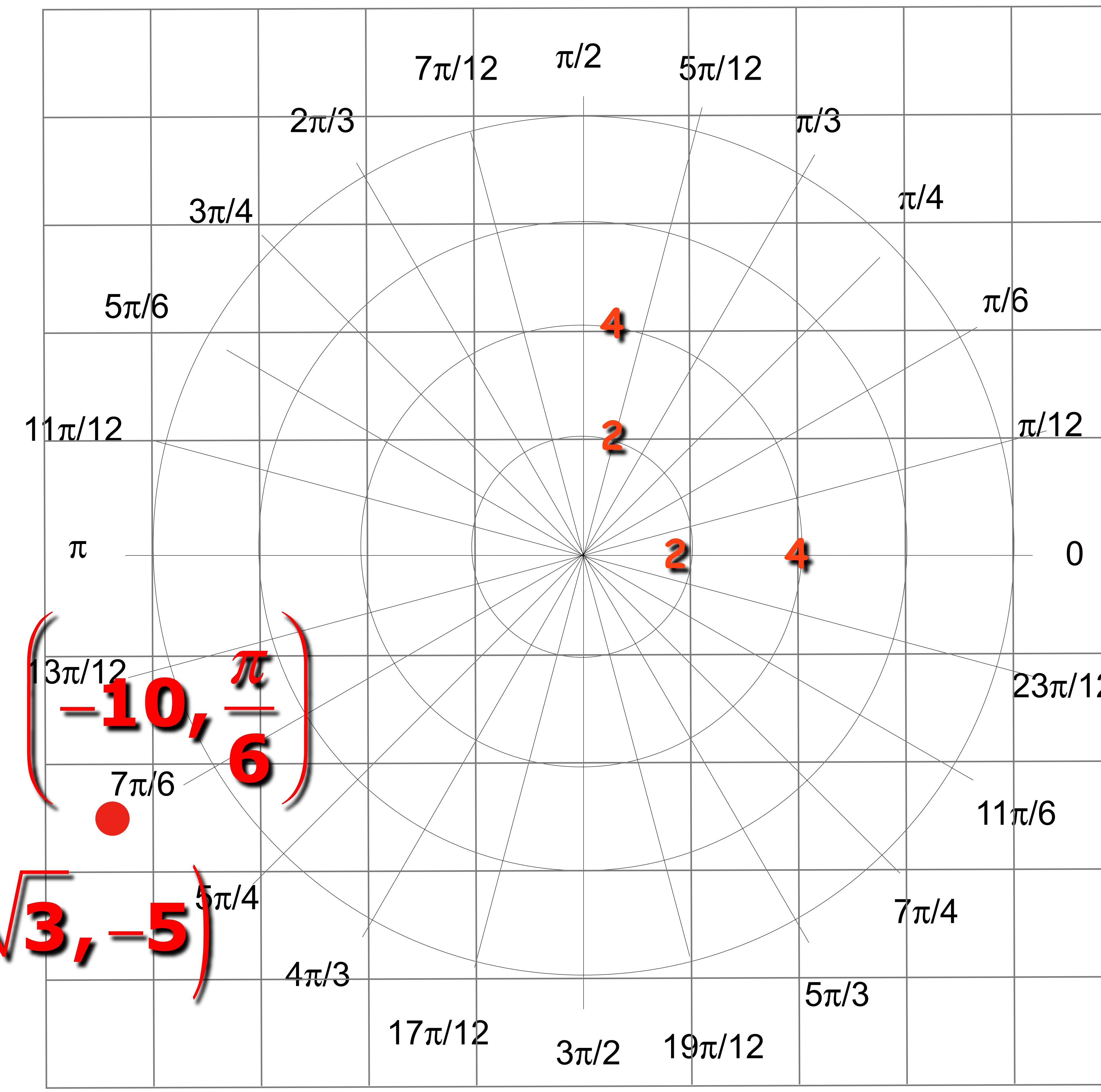
polar to rectangular

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$x = -10\cos\frac{\pi}{6} \qquad x = -5\sqrt{3} \approx -8.66$$

$$y = -10\sin\frac{\pi}{6} \qquad y = -5$$



$\left(-10, \frac{\pi}{6}\right)$

$\left(-5\sqrt{3}, -5\right)$

Point Conversion from Polar to Rectangular Coordinates

Find the rectangular coordinates of the point with the following polar coordinates: $\left(4, \frac{5\pi}{6}\right)$

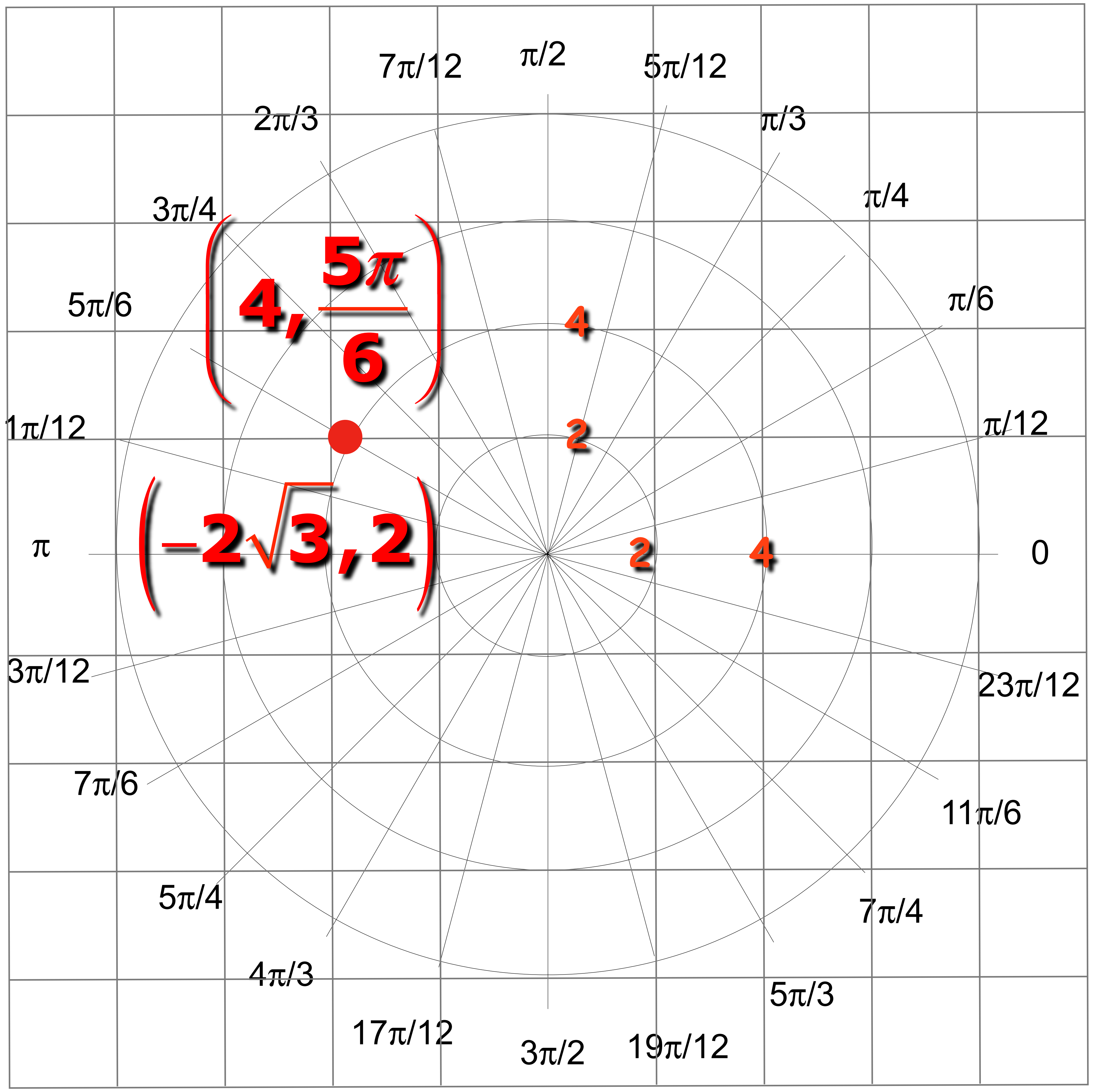
polar to rectangular

$x = r\cos\theta$

$y = r\sin\theta$

$$x = 4\cos\frac{5\pi}{6} \qquad x = -2\sqrt{3} \approx -3.4641$$

$$y = 4\sin\frac{5\pi}{6} \qquad y = 2$$



Objective: Use Polar Coordinates for points and solving equations.

Find polar coordinates of the point whose rectangular coordinates are $\left(1, -\sqrt{3}\right)$

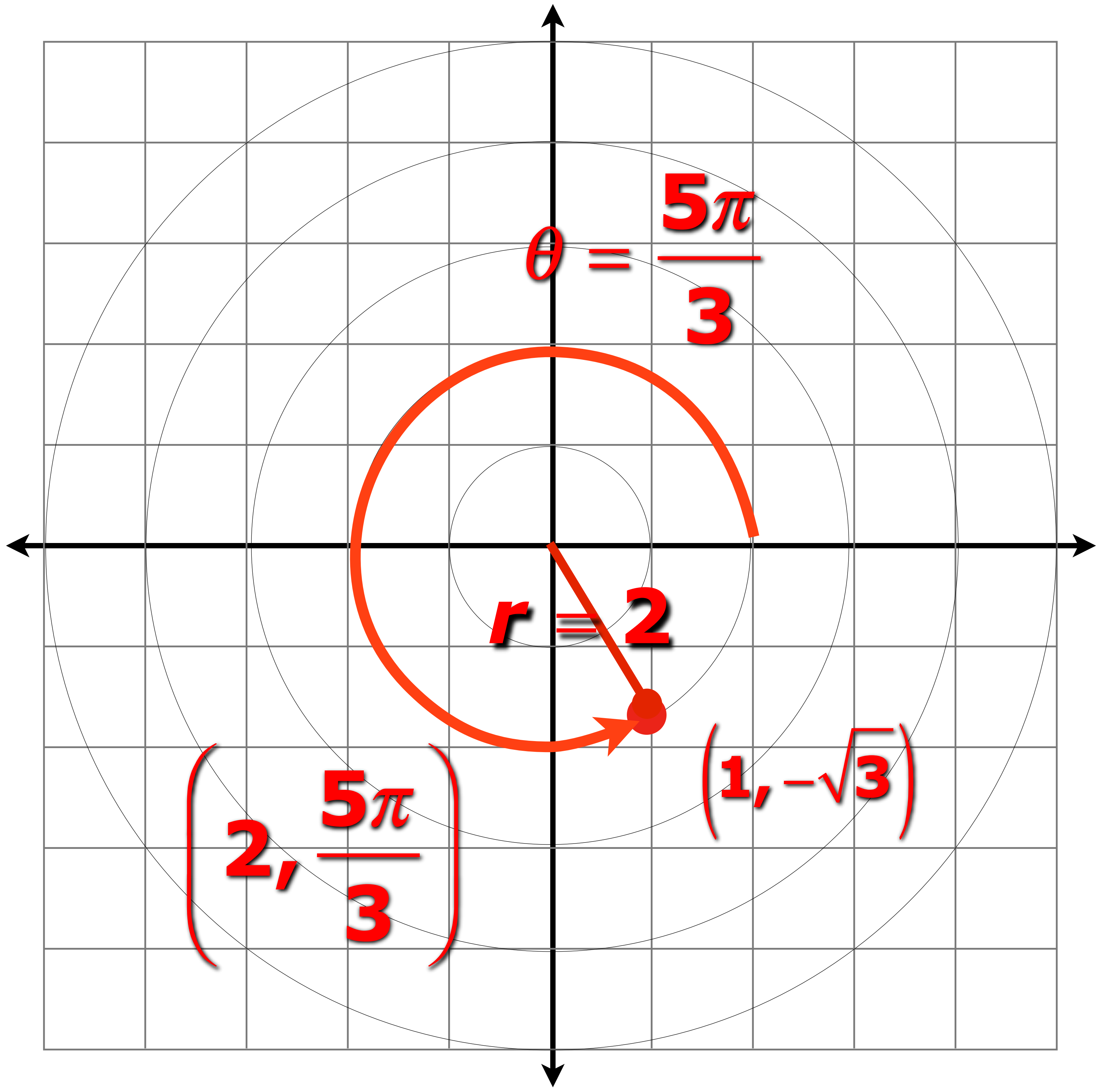
rectangular to polar

$$r = \sqrt{x^2 + y^2} \qquad r > 0$$

$$\tan\theta = \frac{y}{x} \qquad 0 < \theta < 2\pi$$

$$r = \sqrt{1^2 + \left(-\sqrt{3}\right)^2} \qquad r = 2$$

$$\tan\theta = \frac{-\sqrt{3}}{1} \qquad \theta = \frac{5\pi}{3} \qquad \text{QIV}$$



Find polar coordinates of the point whose rectangular coordinates are $(-1,-1)$

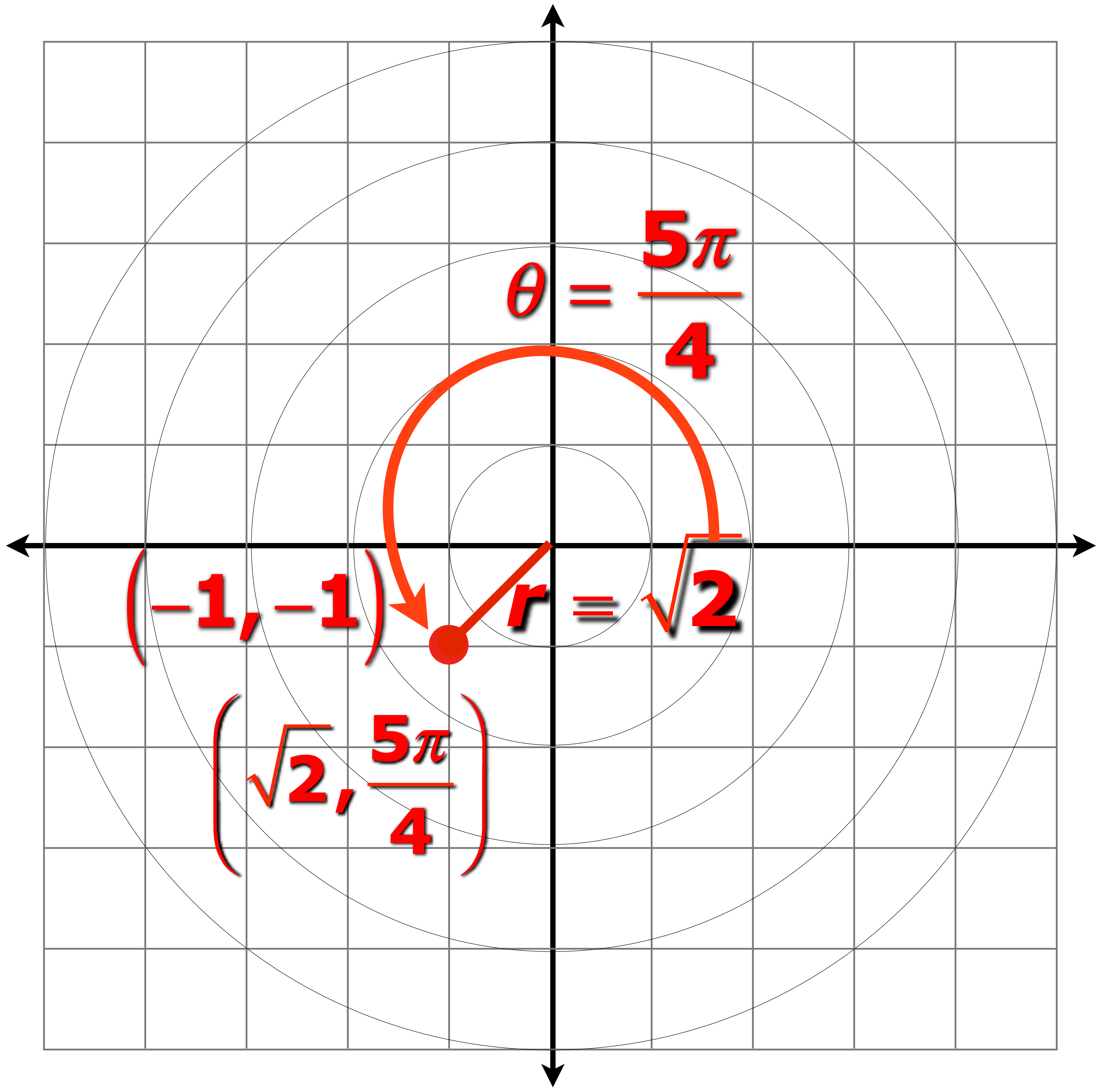
rectangular to polar

$r = \sqrt{x^2 + y^2} \quad r > 0$

$\tan\theta = \dfrac{y}{x} \quad 0 < \theta < 2\pi$

$r = \sqrt{(-1)^2 + (-1)^2} \quad r = \sqrt{2}$

$\tan\theta = \dfrac{-1}{-1} \quad \theta = \dfrac{5\pi}{4} \quad$ QIII



Point Conversion; Rectangular to Polar Coordinates

Find polar coordinates of the point whose rectangular coordinates are $(0, 4)$

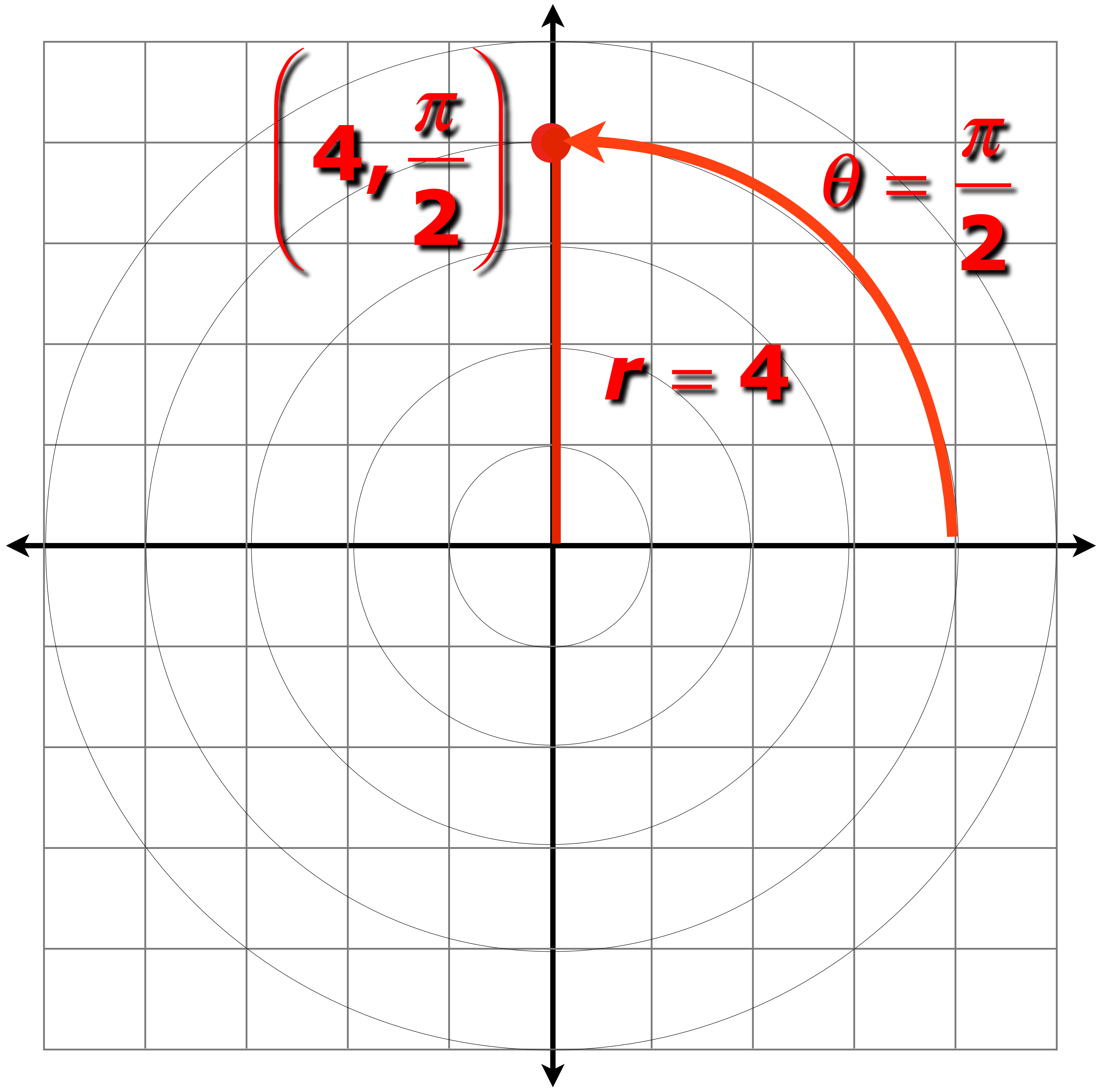
rectangular to polar

$r = \sqrt{x^2 + y^2} \quad r > 0$

$\tan\theta = \dfrac{y}{x} \quad 0 < \theta < 2\pi$

$r = \sqrt{0^2 + 4^2} \qquad r = 4$

$\tan\theta = \dfrac{4}{0} \qquad \theta = \dfrac{\pi}{2} \qquad$ +y-axis



Converting Equations

Example: Converting Equations from Rectangular to Polar

Convert the rectangular equation **$3x - y = 6$** to a polar equation that expresses **r in terms of θ)**.

$$3x - y = 6$$

$$3r\cos\theta - r\sin\theta = 6$$

$$r(3\cos\theta - \sin\theta) = 6$$

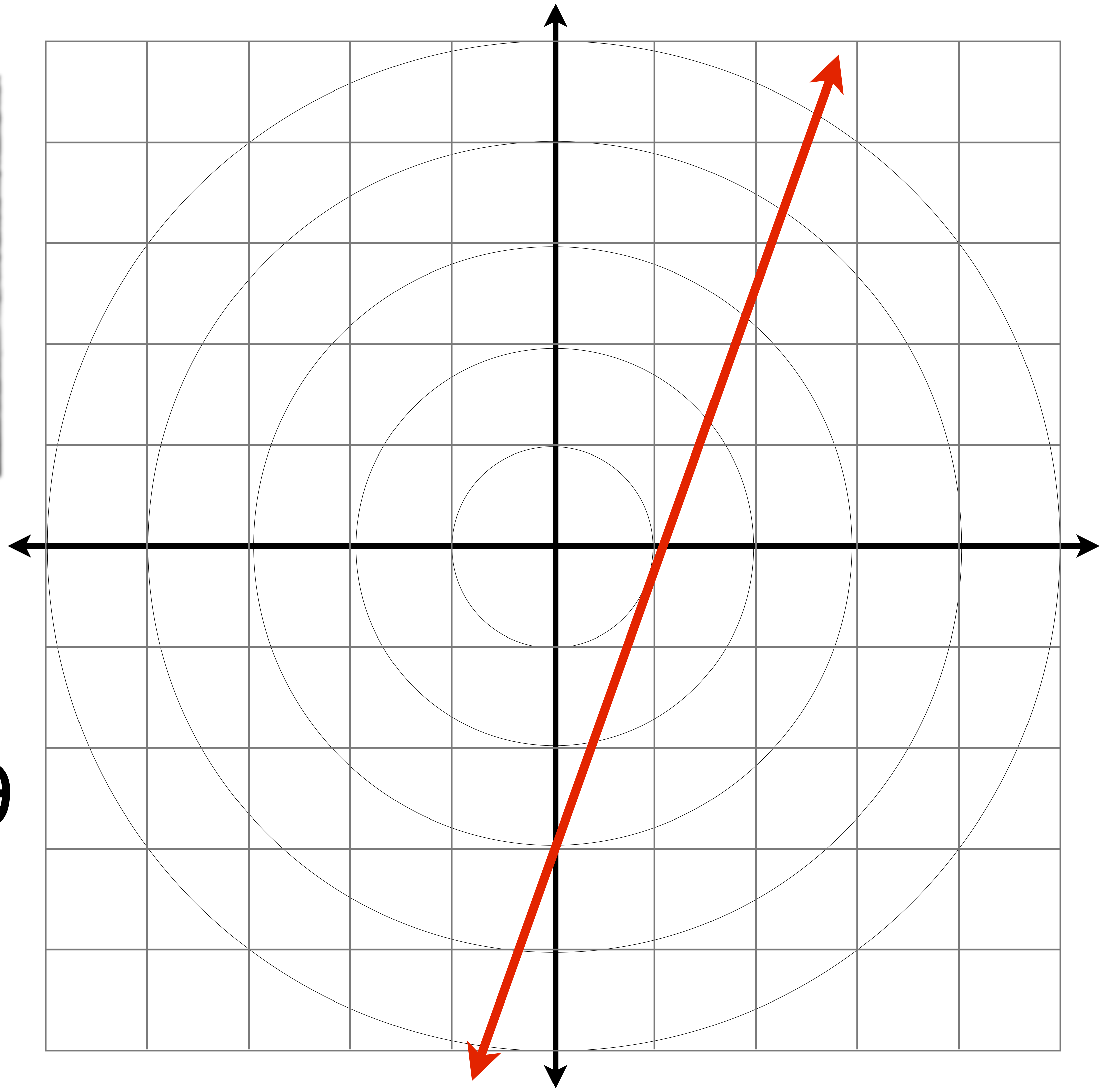
$$r = \frac{6}{3\cos\theta - \sin\theta}$$

$$r = \frac{6}{3\cos\theta - \sin\theta}$$ is a polar equation in r and θ

polar to rectangular

x = r cos θ

y = r sin θ



Example: Converting Equations from Rectangular to Polar

Convert the following rectangular equation to a polar equation that expresses r in terms of θ. $x^2 + (y+1)^2 = 1$

$$x^2 + (y+1)^2 = 1$$

$$(r\cos\theta)^2 + (r\sin\theta + 1)^2 = 1$$

$$r^2\cos^2\theta + r^2\sin^2\theta + 2r\sin\theta + 1 = 1$$

$$r^2(\cos^2\theta + \sin^2\theta) + 2r\sin\theta + 1 = 1$$

$$r^2 + 2r\sin\theta + 1 = 1$$

$$r^2 + 2r\sin\theta = 0$$

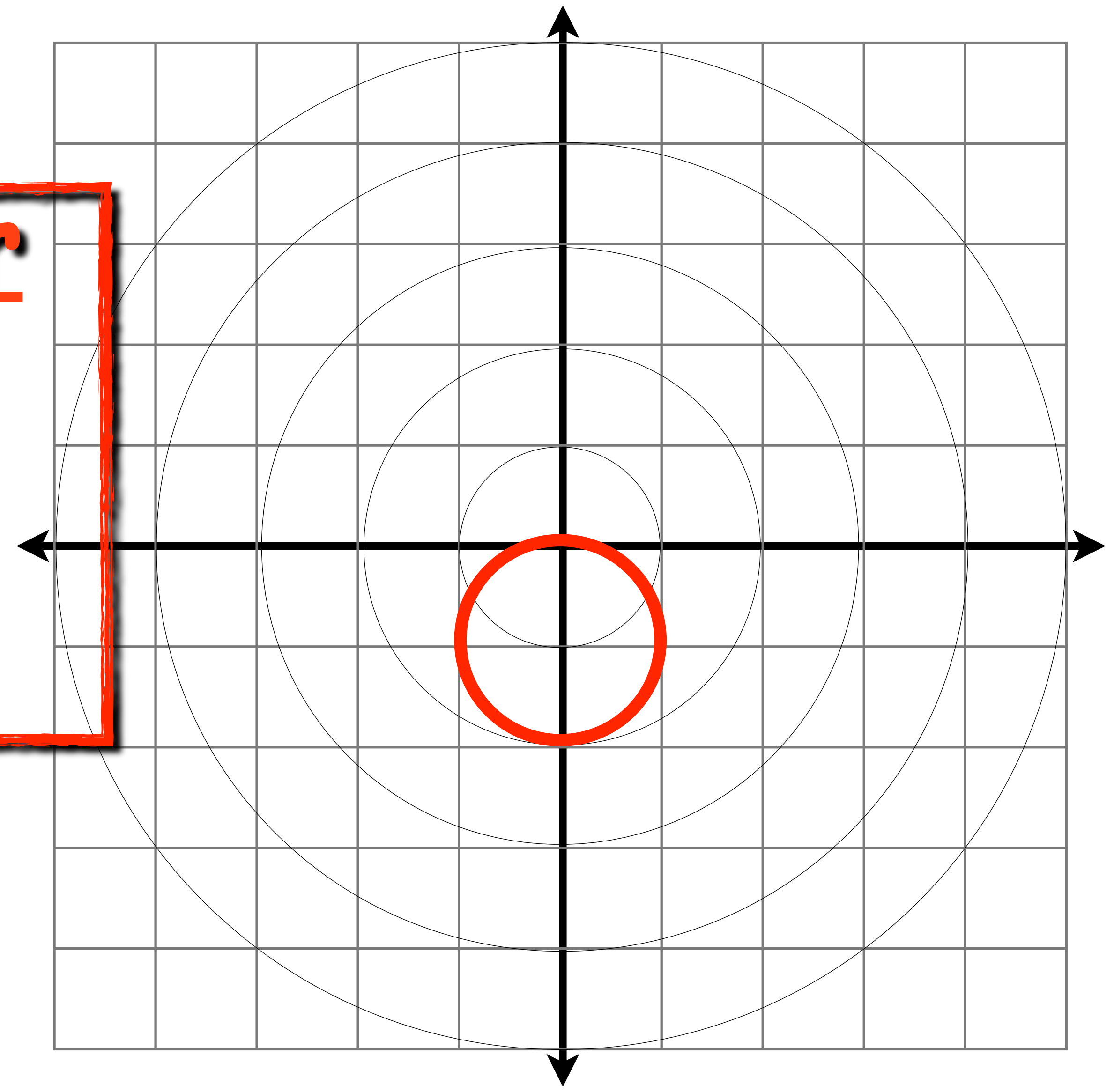
$$r(r + 2\sin\theta) = 0$$

$$r = 0 \text{ or } r = -2\sin\theta$$

polar to rectangular

$x = r\cos\theta$

$y = r\sin\theta$



Obviously r cannot be 0.

$r = -2\sin\theta$ is a polar equation in r and θ

Equation Conversion from Polar to Rectangular Coordinates

Objective: Use Polar Coordinates for points and solving equations.

To convert an equation from polar to rectangular coordinates, the goal is to obtain an equation in which the variables are x and y rather than r and θ. We use one or more of the following equations:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad r = \sqrt{x^2 + y^2} \qquad \tan\theta = \frac{y}{x}$$

To use these equations, it is sometimes necessary to…

square both sides

use an identity

take the tangent of both sides

multiply both sides by r

Example: Converting Equations from Polar to Rectangular Form

Convert the polar equation to a rectangular equation in x and y: $r = 4$

$$r = \sqrt{x^2 + y^2}$$

$$4 = \sqrt{x^2 + y^2}$$

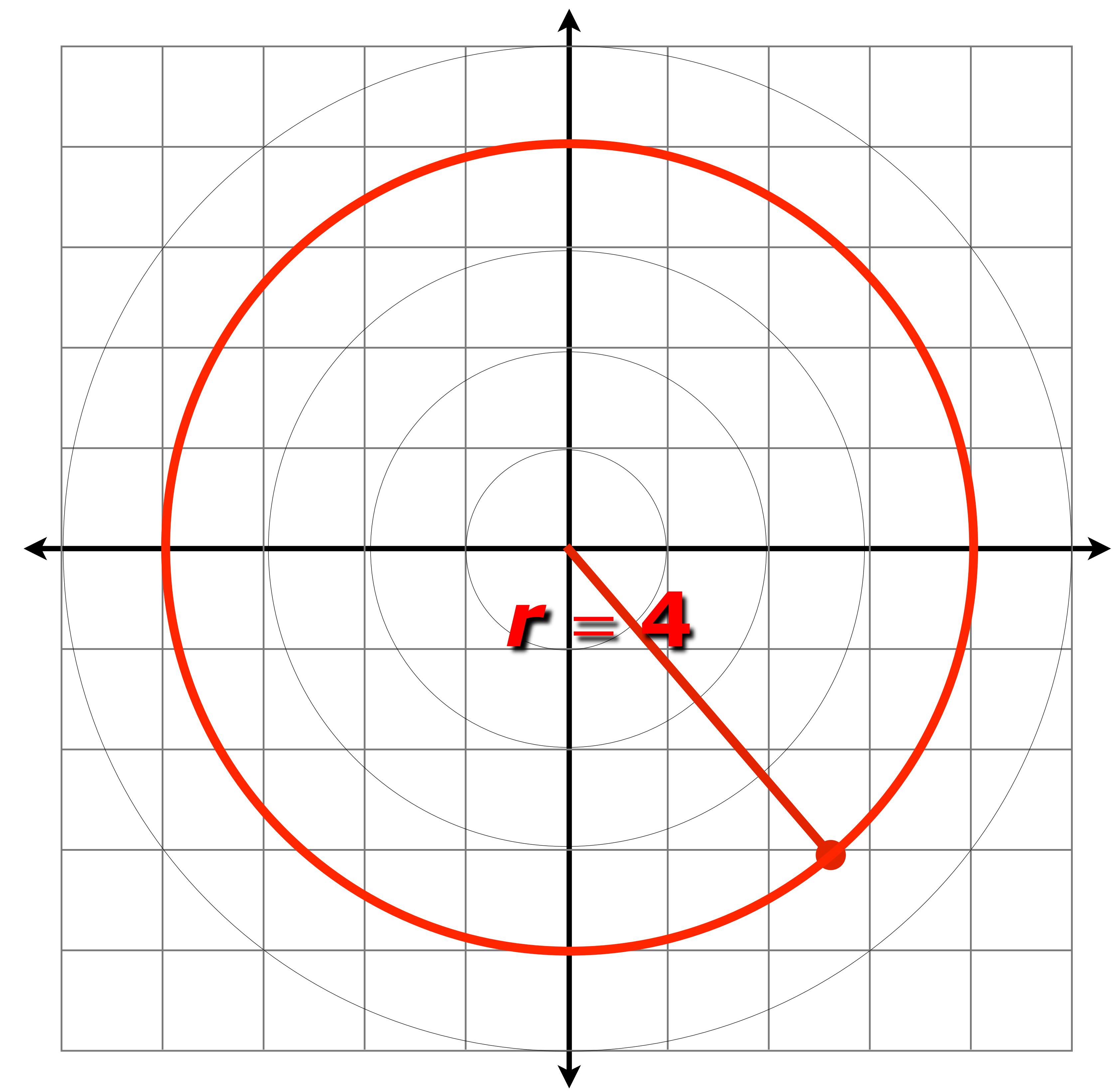
$$16 = x^2 + y^2$$

$$x^2 + y^2 = 16$$

rectangular to polar

$$r = \sqrt{x^2 + y^2} \quad r > 0$$

$$\tan\theta = \frac{y}{x} \quad 0 < \theta < 2\pi$$



The rectangular equation for r = 4 is $x^2 + y^2 = 16$

Convert the polar equation to a rectangular equation in x and y: $\theta = \frac{3\pi}{4}$

$$\tan\theta = \frac{y}{x}$$

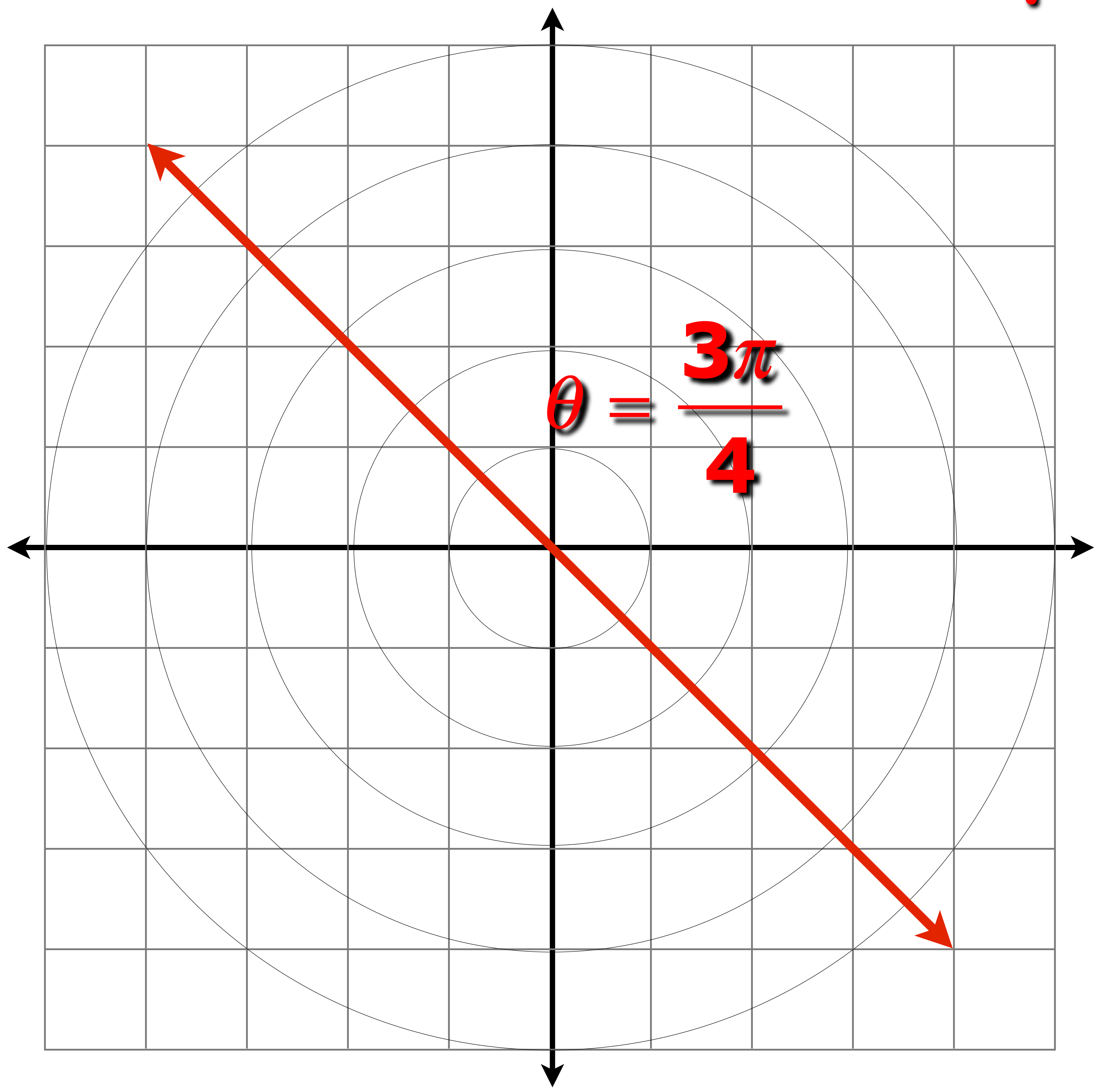
$$\tan\frac{3\pi}{4} = \frac{y}{x}$$

$$y = x\tan\frac{3\pi}{4} = x(-1)$$

rectangular to polar

$$r = \sqrt{x^2 + y^2} \quad r > 0$$

$$\tan\theta = \frac{y}{x} \quad 0 < \theta < 2\pi$$



The rectangular equation for θ = $\frac{3\pi}{4}$ is y = -x

Example: Converting Equations from Polar to Rectangular Form

Objective: Use Polar Coordinates for points and solving equations.

Convert the polar equation to a rectangular equation in x and y: $r = -2\sec\theta$

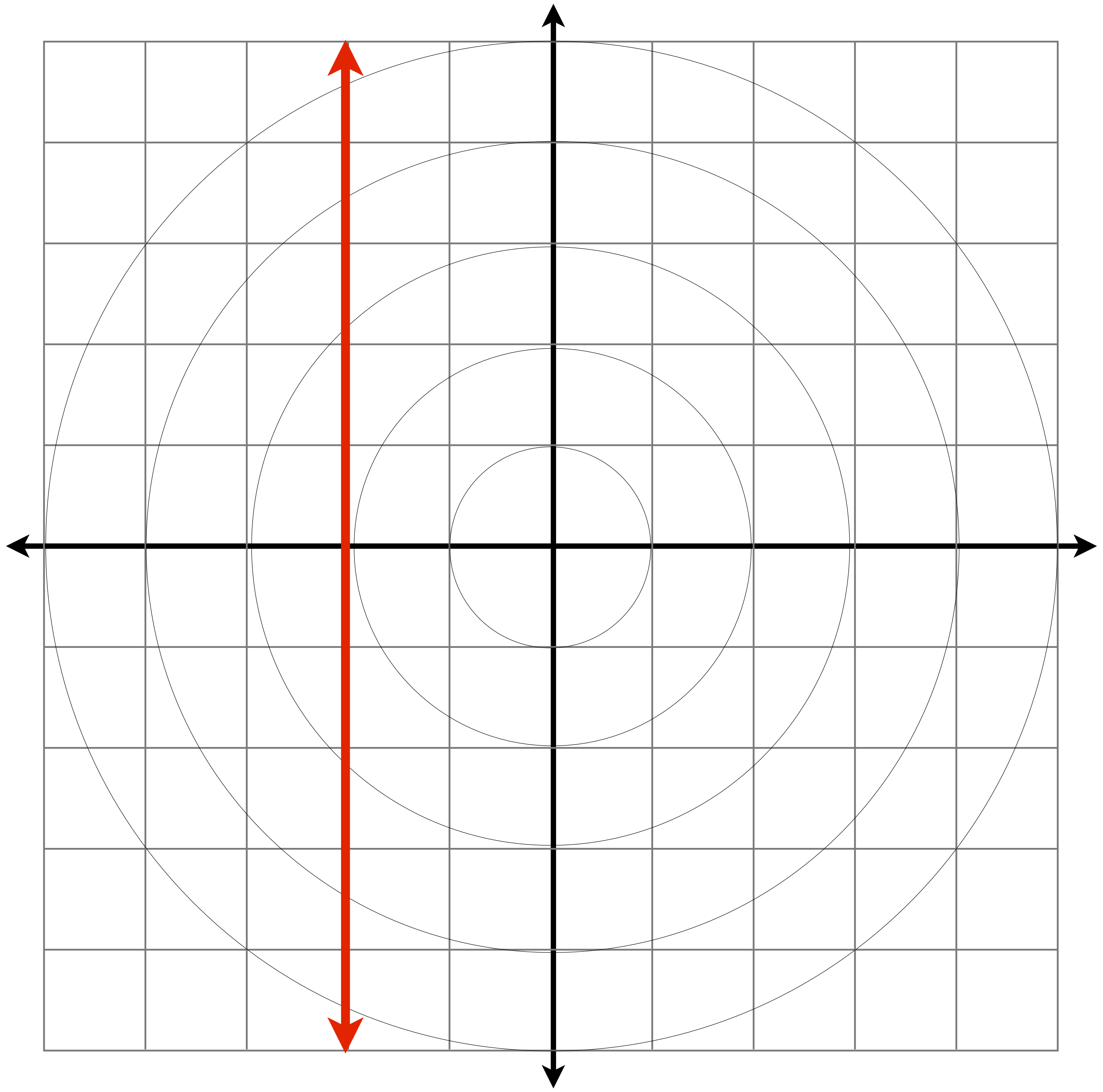
$$r = -2\sec\theta$$

$$r = \frac{-2}{\cos\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$r\cos\theta = -2$$

$$x = -2$$

The rectangular equation for r = -2secθ is x = -2



Example: Converting Equations from Polar to Rectangular Form

Objective: Use Polar Coordinates for points and solving equations.

Convert the polar equation to a rectangular equation in x and y: $r = 10\sin\theta$

$$r = 10\sin\theta$$

$$r^2 = r(10\sin\theta)$$

$$r^2 = 10r\sin\theta \qquad r = \sqrt{x^2 + y^2}$$

$$x^2 + y^2 = 10y \qquad y = r\sin\theta$$

$$x^2 + y^2 - 10y = 0$$

$$x^2 + y^2 - 10y + 25 = 25$$

$$x^2 + (y-5)^2 = 25$$

The rectangular equation for r = 10sinθ is $x^2 + (y - 5)^2 = 25$