Chapter 5

Analytic Trigonometry

5.1 Using Fundamental Identities

Chapter 5.1

Homework

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Chapter 5.1

Objectives

Students will use fundamental trigonometric identities to evaluate trigonometric functions, simplify, and rewrite trigonometric expressions

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

- The fundamental Trigonometric Identities all start with:

$$\sin^2 x + \cos^2 x = 1$$

- We have shown that to be true several times and I am not going to repeat it here. You can verity this easily enough on your own.

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sin x = \frac{1}{\csc x}$$

$$\sec x = \frac{1}{\cos x} \qquad \cos x = \frac{1}{\sec x}$$

$$\cot x = \frac{1}{\tan x} \qquad \tan x = \frac{1}{\cot x}$$

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$

$$1 + \cot^2 x = \csc^2 x$$

$$\csc^2 x - \cot^2 x = 1$$

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$\tan x = \frac{\sin x}{\cos x} \qquad \sec x = \frac{1}{\cos x}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\sec^2 x - \tan^2 x = 1$$

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 - \cos^2 x = \sin^2 x$$

$$1 - \sin^2 x = \cos^2 x$$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Even/Odd Functions

Odd

$\sin(-x) = -\sin x$

$\csc(-x) = -\csc x$

$\tan(-x) = -\tan x$

$\cot(-x) = -\cot x$

Even

$\cos(-x) = \cos x$

$\sec(-x) = \sec x$

The important fact to take from this is the **sign** of the input and the result.

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Co-function Identities

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x \qquad \csc\left(\frac{\pi}{2}-x\right)=\sec x$$

$$\cos\left(\frac{\pi}{2}-x\right)=\sin x \qquad \sec\left(\frac{\pi}{2}-x\right)=\csc x$$

$$\tan\left(\frac{\pi}{2}-x\right)=\cot x \qquad \cot\left(\frac{\pi}{2}-x\right)=\tan x$$

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Summarized in your book

STUDY TIP

You should learn the fundamental trigonometric identities well, because they are used frequently in trigonometry and they will also appear later in calculus. Note that u can be an angle, a real number, or a variable.

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Using Fundamental Identities to Verify Other Identities

- To **verify an identity**, we show that one side of the identity can be simplified so that one side of the identity is **identical** to the other side.
- Each side of the equation is **manipulated independently**.
- Usually start with the side containing the more complicated expression.
- Substitute fundamental identities until you can rewrite the original expression in a form identical to the other side.

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Study Suggestion

Make certain you understand how these identities are derived and then memorize a few of these identities. You will need to use variations of these fundamental identities.

When you see $\sin^2 x + \cos^2 x = 1$, you will want to remember that can also be written...

$1 - \cos^2 x = \sin^2 x$ $\quad$ $1 - \sin^2 x = \cos^2 x$

Know the identities!!!

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Study Suggestion 2

- Verifying an identity is different from solving an equation. We do not use the properties of equality to verify an identity. Each side is **manipulated independently** to a match.

- You cannot be sure an identity is actually an identity until that identity has been verified.

- When working with these identities, do not get lazy (as I do on occasion) and not write the variable (x or θ). **Do not write sin tan when you mean to write sin x tan x.** The functions are meaningless without an argument (variable).

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Example If csc u = –5/3 and cos u > 0, find t the other five trigonometric values.

$$\csc u = -\frac{5}{3}$$

$$\sin u = -\frac{3}{5}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec u = \frac{5}{4}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cos^2 u = 1 - \left(-\frac{3}{5}\right)^2 = 1 - \frac{9}{25} = \frac{16}{25}$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\cos u = \pm\frac{4}{5} = \frac{4}{5}$$

$$\cos u > 0$$

$$\cot u = -\frac{4}{3}$$

$$\cot x = \frac{1}{\tan x}$$

$$\tan u = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}$$

$$\tan x = \frac{\sin x}{\cos x}$$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

We can start by simplifying a trigonometric expression by substitution.

Simplify: $\cos x(\sec x - \cos x)$

$$\cos x(\sec x - \cos x) = \cos x\left(\frac{1}{\cos x} - \cos x\right)$$

$$\sec x = \frac{1}{\cos x}$$

$$= \frac{\cos x}{\cos x} - \cos^2 x$$

$$= 1 - \cos^2 x \qquad 1 - \cos^2 x = \sin^2 x$$

$$= \sin^2 x$$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Simplify: $\csc^2 x \cot x - \cot x$

$$\csc^2 x \cot x - \cot x = \left(\csc^2 x - 1\right)\cot x$$

$$= \left(\cot^2 x\right)\cot x \qquad \csc^2 x - 1 = \cot^2 x$$

$$= \cot^3 x$$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Simplify: $\tan x \sin x + \cos x$

$$\tan x \sin x + \cos x = \left(\frac{\sin x}{\cos x}\right)\sin x + \cos x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x} + \cos x$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} = \frac{\sin^2 x + \cos^2 x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$= \frac{1}{\cos x} = \sec x$$

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Verify: $\csc x \tan x = \sec x$

$$\csc x \tan x = \sec x$$

$$\csc x \tan x = \frac{1}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cos x}$$

$$= \frac{1}{\cos x}$$

$$= \sec x$$

$$\csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

The identity be verified.

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Verify: $\cot^2 x + \cos^2 x + \sin^2 x = \csc^2 x$

$$\cot^2 x + \cos^2 x + \sin^2 x = \cot^2 x + 1 \qquad \cos^2 x + \sin^2 x = 1$$

$$= \csc^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

verified.

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Example: Using Factoring to Verify an Identity

Verify: $\sin x - \sin x \cos^2 x = \sin^3 x$

Factor sin x from the two terms.

$$\sin x - \sin x \cos^2 x = \sin x(1 - \cos^2 x)$$ Distributive Property

$$= \sin x(\sin^2 x)$$ $1 - \cos^2 x = \sin^2 x$

$$= \sin^3 x$$ We did it.

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Example: Combining Fractional Expressions to Verify an Identity

Verify: $\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\csc x$

Common denominator

$$=\frac{\sin^2 x}{\sin x(1+\cos x)}+\frac{(1+\cos x)(1+\cos x)}{\sin x(1+\cos x)}$$

$$=\frac{\sin^2 x}{\sin x(1+\cos x)}+\frac{(1+2\cos x+\cos^2 x)}{\sin x(1+\cos x)}$$

$$=\frac{\sin^2 x+1+2\cos x+\cos^2 x}{\sin x(1+\cos x)}$$

$$\sin^2 x+\cos^2 x=1$$

$$=\frac{\sin^2 x+\cos^2 x+1+2\cos x}{\sin x(1+\cos x)}$$

$$=\frac{1+1+2\cos x}{\sin x(1+\cos x)}=\frac{2+2\cos x}{\sin x(1+\cos x)}$$

$$=\frac{2(1+\cos x)}{\sin x(1+\cos x)}=\frac{2}{\sin x}=2\csc x$$

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Example: Working with Both Sides Separately

Verify: $\dfrac{1}{1+\sin\theta}+\dfrac{1}{1-\sin\theta} = 2+2\tan^2\theta$

Common denominator

$$= \frac{(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}+\frac{(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{(1-\sin\theta)+(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta} = \frac{2}{\cos^2\theta}$$

$$= 2+2\tan^2\theta$$

$$= 2(1+\tan^2\theta)$$

$1+\tan^2 x = \sec^2 x$

$$= 2\sec^2\theta$$

$\sec^2 x = \dfrac{1}{\cos^2 x}$

$$= \frac{2}{\cos^2\theta}$$

Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

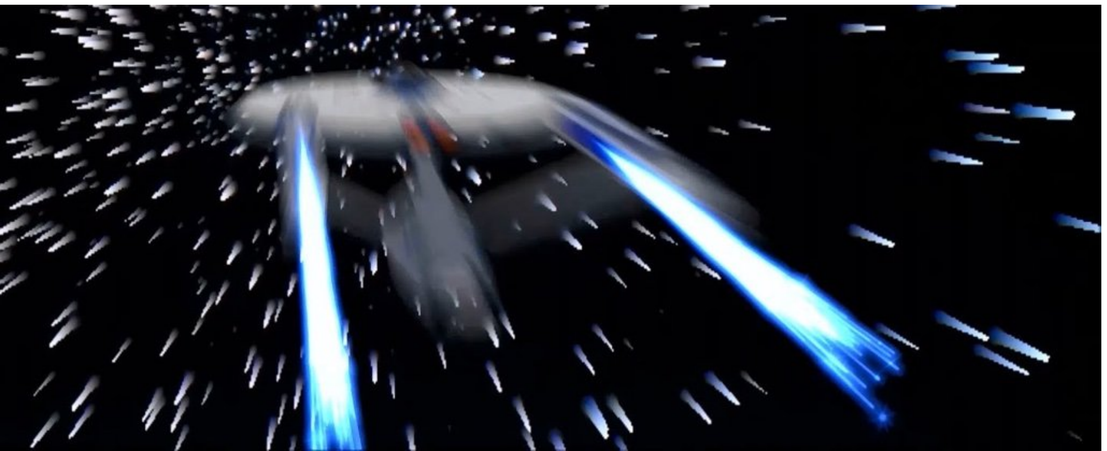
Verify: $\dfrac{1+\cos\theta}{\sin\theta} = \csc\theta + \cot\theta$

$$\frac{1+\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \csc\theta + \cot\theta$$

$$\frac{1}{\sin\theta} = \csc\theta$$

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$



Objective: Use the fundamental trigonometric identities to verify identities.

Fundamental Trigonometric Identities

Verify: $\dfrac{\sec x + \csc(-x)}{\sec x \csc x} = \sin x - \cos x$

$$\frac{\sec x + \csc(-x)}{\sec x \csc x} = \frac{\sec x + -\csc(x)}{\sec x \csc x}$$

$$\csc(-x) = -\csc x$$

$$= \frac{\sec x}{\sec x \csc x} - \frac{\csc x}{\sec x \csc x}$$

$$= \frac{1}{\csc x} - \frac{1}{\sec x}$$

$$= \sin x - \cos x$$

$$\cos x = \frac{1}{\sec x}$$

$$\sin x = \frac{1}{\csc x}$$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Simplify: $\csc^2 x \cot x - \cot x$

$$\csc^2 x \cot x - \cot x = \left(\csc^2 x - 1\right)\cot x$$

$\csc^2 x - 1 = \cot^2 x$

$$= \left(\cot^2 x\right)\cot x$$

$$= \cot^3 x$$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Simplify: $\tan x \sin x + \cos x$

$$\tan x \sin x + \cos x = \left(\frac{\sin x}{\cos x}\right)\sin x + \cos x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$= \frac{\sin^2 x}{\cos x} + \cos x$$

$$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$$

$$\cos^2 x + \sin^2 x = 1$$

$$= \frac{\sin^2 x + \cos^2 x}{\cos x} \quad = \frac{1}{\cos x} = \sec x$$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Combining Fractional Expressions

Simplify: $\dfrac{\sec t}{\tan t} - \dfrac{\tan t}{1+\sec t}$

$$= \frac{\sec t\left(1+\sec t\right)}{\tan t\left(1+\sec t\right)} - \frac{\tan t\left(\tan t\right)}{\tan t\left(1+\sec t\right)}$$

Common denominator

$$= \frac{\sec t + \sec^2 t}{\tan t\left(1+\sec t\right)} - \frac{\tan^2 t}{\tan t\left(1+\sec t\right)}$$

$$= \frac{\sec t + \sec^2 t - \tan^2 t}{\tan t\left(1+\sec t\right)}$$

$\sec^2 x - \tan^2 x = 1$

$$= \frac{\sec t + 1}{\tan t\left(1+\sec t\right)} = \frac{1}{\tan t} = \cot t$$

$\cot x = \dfrac{1}{\tan x}$

Objective: Use the fundamental trigonometric identities to verify identities.

The Fundamental Identities

Factor: $\cos^2\theta - 1$

$\cos^2\theta - 1 = \cos^2\theta - 1^2 = (\cos\theta - 1)(\cos\theta + 1)$

$a^2 - b^2 = (a+b)(a-b)$

Factor: $\sin^2\theta - 3\sin\theta - 10 = (\sin\theta - 5)(\sin\theta + 2)$

Factor: $\sec^2 t - \tan t - 3$

$\sec^2 x = 1 + \tan^2 x$

$= (1 + \tan^2 t) - \tan t - 3 = \tan^2 t - \tan t - 2 = (\tan t - 2)(\tan t + 1)$