Chapter 5

Analytic Trigonometry

5.4 Sum and Difference Formulas

Chapter 5.4

Homework

- Read Sec 5-4
- Do p404 1-73 other odd (1, 5, 9, etc.)

Chapter 5.4

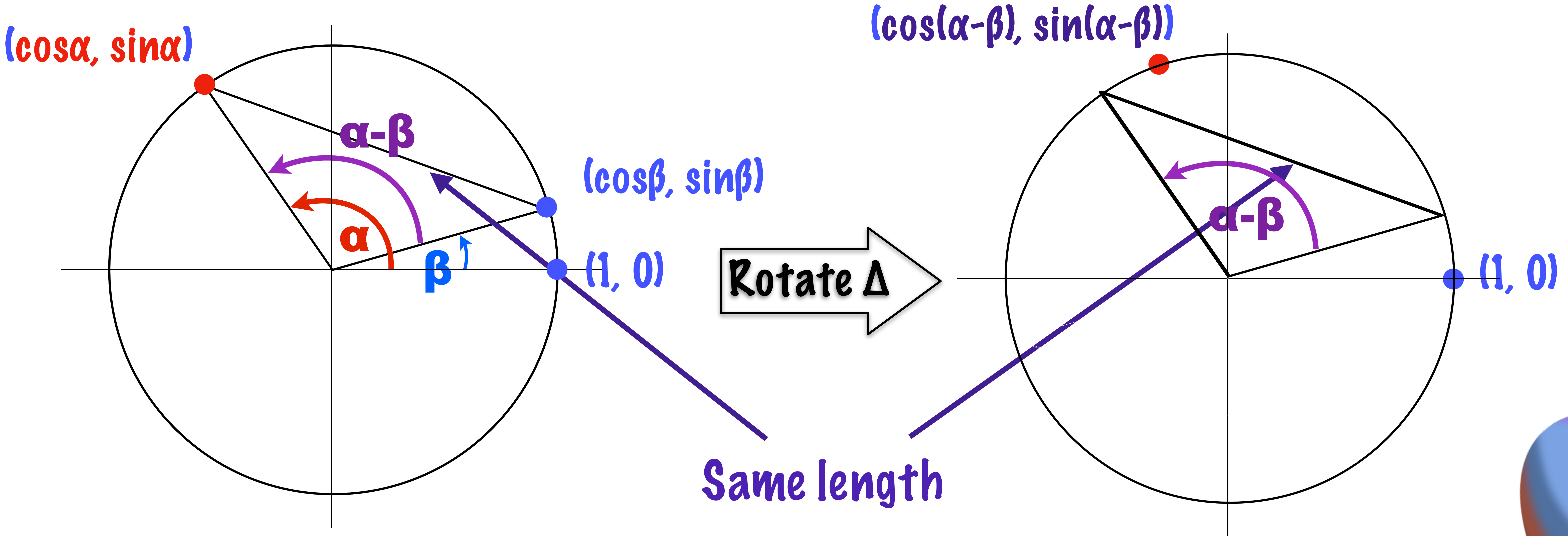
Objectives

Use Sum and Difference Formulae to evaluate trig expressions, verify trig identities, and solve trig equations.

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$



Do you care how we prove this?

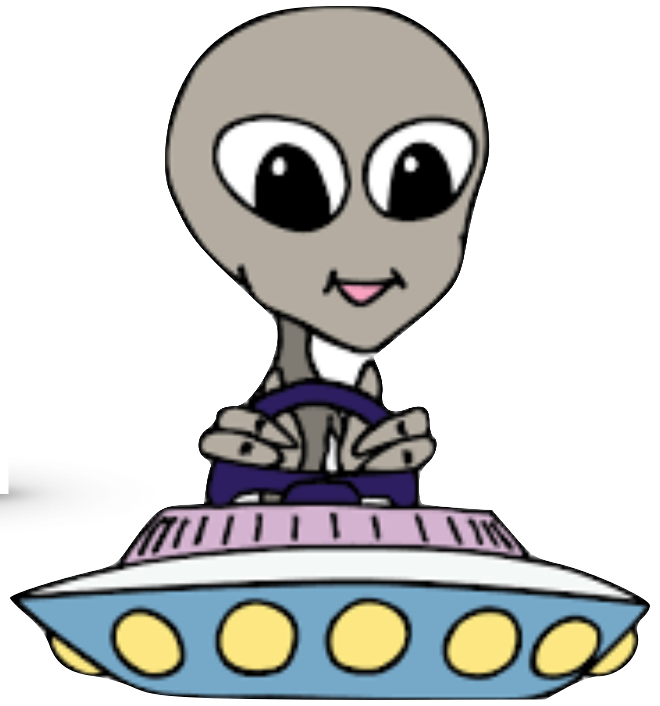
$$AB = \sqrt{(\cos\alpha - \cos\beta)^2 + (\sin\alpha - \sin\beta)^2} = A'B' = \sqrt{(\cos(\alpha-\beta) - 1)^2 + (\sin(\alpha-\beta) - 0)^2}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- We know that $\cos 30^\circ = \frac{\sqrt{3}}{2}$. Using $\cos 30^\circ = \cos(90 - 60)^\circ$ and the difference formula for cosines: obtain that exact value.

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$



$$\cos 30^\circ = \cos(90 - 60)^\circ = \cos 90^\circ \cos 60^\circ + \sin 90^\circ \sin 60^\circ = 0 \bullet \frac{1}{2} + 1 \bullet \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

- Find the exact value of = $\cos 70^\circ \cos 40^\circ + \sin 70^\circ \sin 40^\circ$.

$$\cos 70^\circ \cos 40^\circ + \sin 70^\circ \sin 40^\circ = \cos(70 - 40)^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

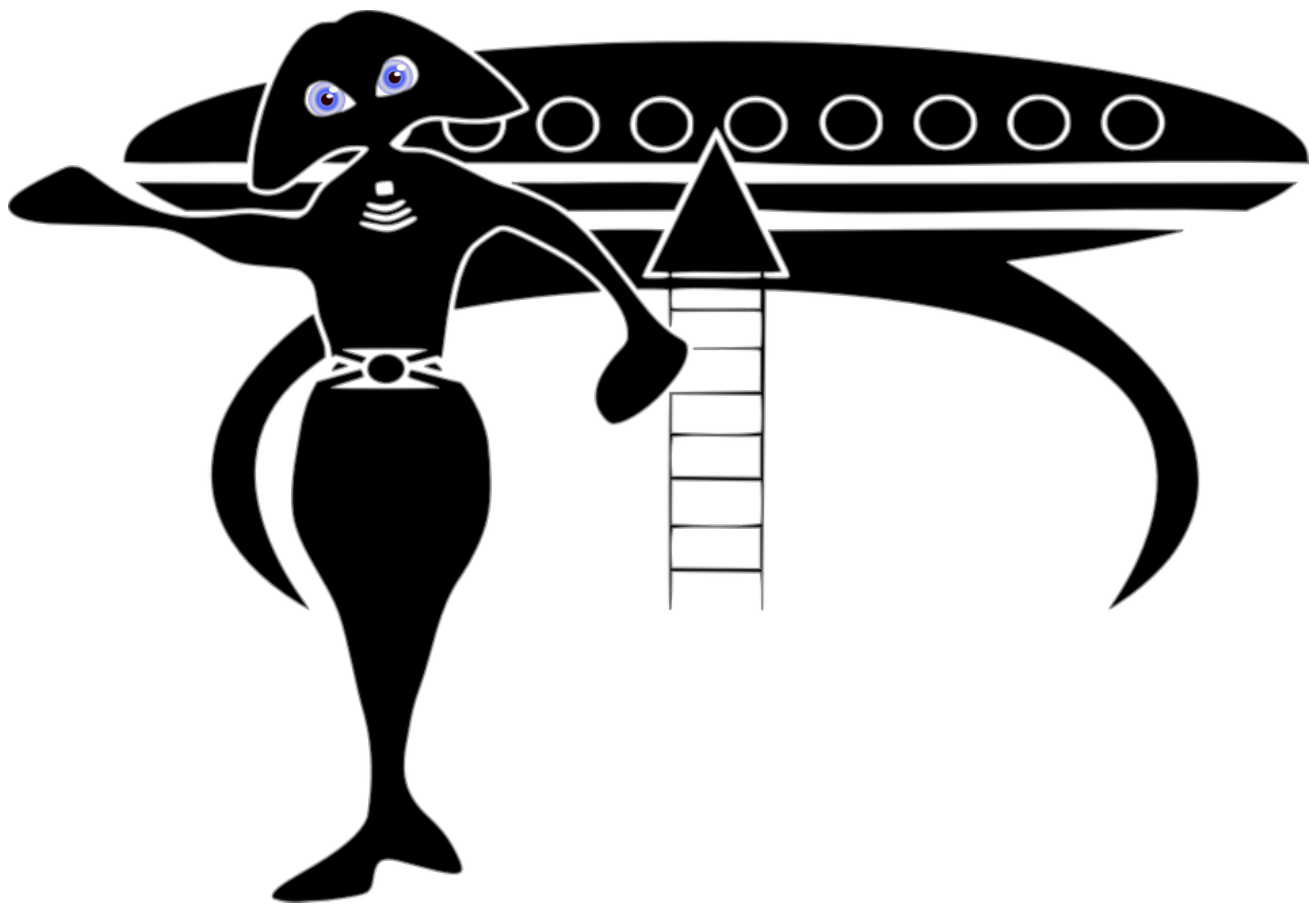
- Verify the identity $\dfrac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta} = 1+\tan\alpha\tan\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$$\frac{\cos(\alpha-\beta)}{\cos\alpha\cos\beta} = \frac{\cos\alpha\cos\beta+\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= \frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}$$

$$= 1 + \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\cos\beta} \quad = 1+\tan\alpha\tan\beta$$



$\tan x = \dfrac{\sin x}{\cos x}$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Verify the identity $\cos(\alpha + \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

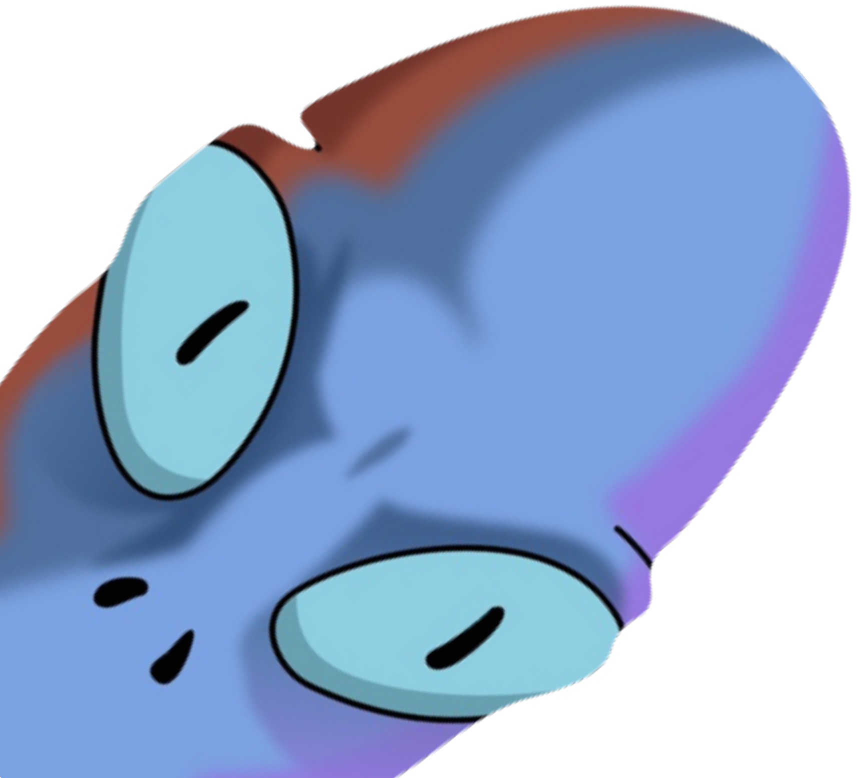
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos(\alpha - (-\beta)) \quad = \cos\alpha\cos(-\beta) + \sin\alpha\sin(-\beta)$$

$$= \cos\alpha\cos\beta + \sin\alpha(-\sin\beta)$$

$\sin(-\beta) = -\sin\beta$

$$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$$



$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Find the exact value of $\cos\frac{7\pi}{12}$

$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$

$$\cos\frac{7\pi}{12}=\cos\left(\frac{4\pi}{12}+\frac{3\pi}{12}\right)=\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

$$=\frac{1}{2}\bullet\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\bullet\frac{\sqrt{2}}{2}$$

$$=\frac{\sqrt{2}-\sqrt{6}}{4}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

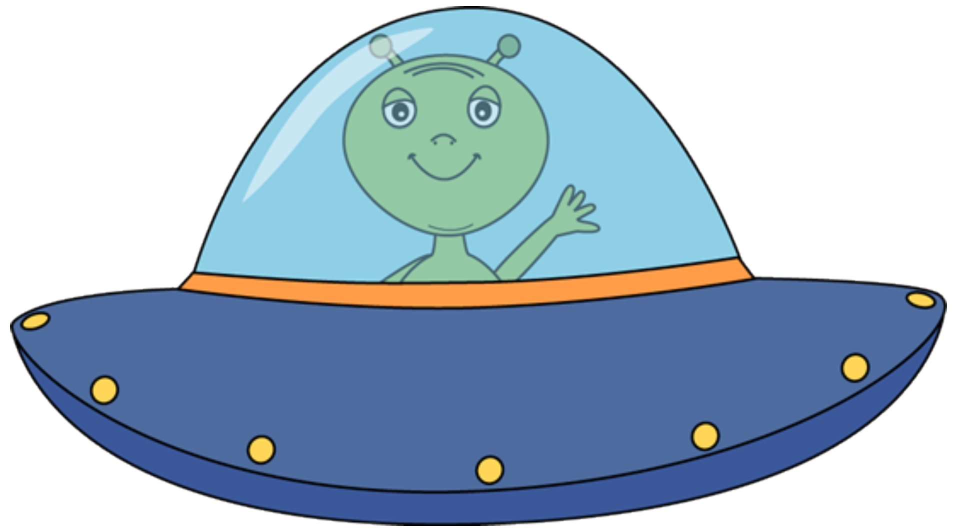
- Verify the identity $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\sin(\alpha-\beta) = \cos\left(\frac{\pi}{2} - (\alpha-\beta)\right) = \cos\left(\frac{\pi}{2} - \alpha + \beta\right)$$

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$

$$= \cos\left(\left(\frac{\pi}{2} - \alpha\right) + \beta\right)$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$



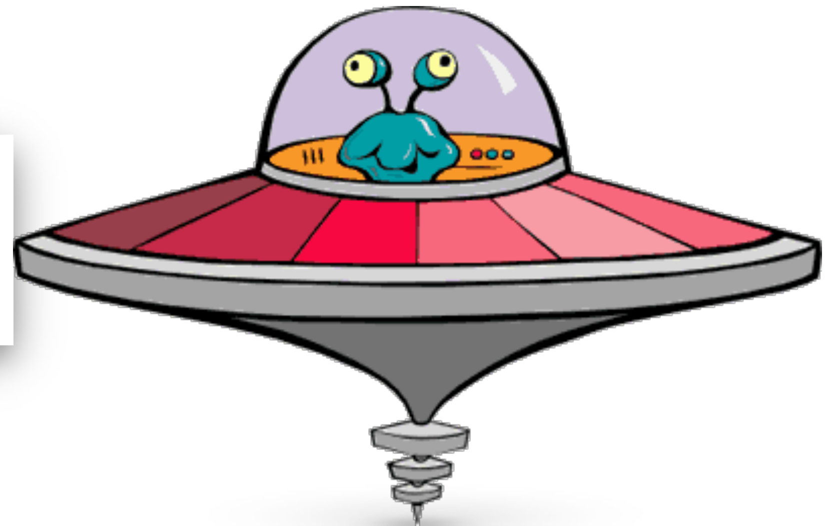
$$= \cos\left(\frac{\pi}{2} - \alpha\right)\cos(\beta) - \sin\left(\frac{\pi}{2} - \alpha\right)\sin(\beta)$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$



Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Verify the identity **sin(α + β) = sin α cos β + cos α sin β**

$$\sin(\alpha+\beta)=\cos\left(\frac{\pi}{2}-(\alpha+\beta)\right)=\cos\left(\frac{\pi}{2}-\alpha-\beta\right)$$

$\sin x = \cos\left(\frac{\pi}{2}-x\right)$

$$=\cos\left(\left(\frac{\pi}{2}-\alpha\right)-\beta\right)$$

cos(α - β) = cos α cos β + sin α sin β

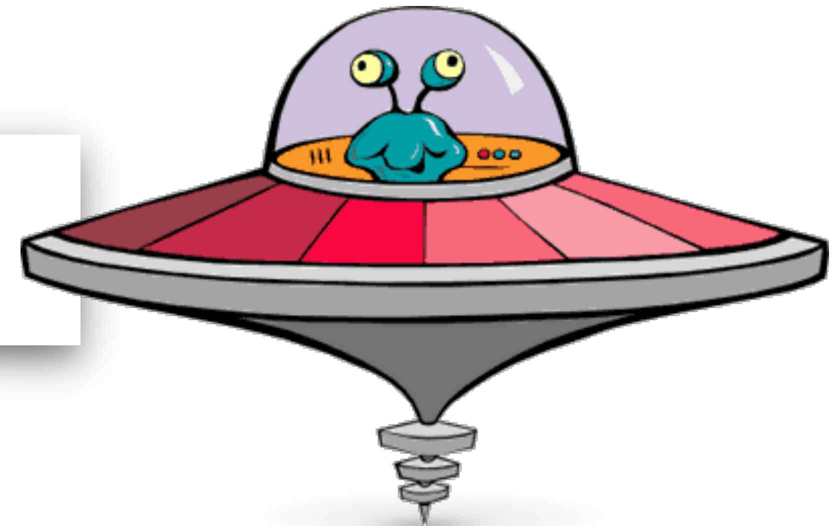
$$=\cos\left(\frac{\pi}{2}-\alpha\right)\cos(\beta)+\sin\left(\frac{\pi}{2}-\alpha\right)\sin(\beta)$$

$\cos\left(\frac{\pi}{2}-x\right)=\sin x$

$\sin\left(\frac{\pi}{2}-x\right)=\cos x$

$$=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$$

sin(α + β) = sin α cos β + cos α sin β

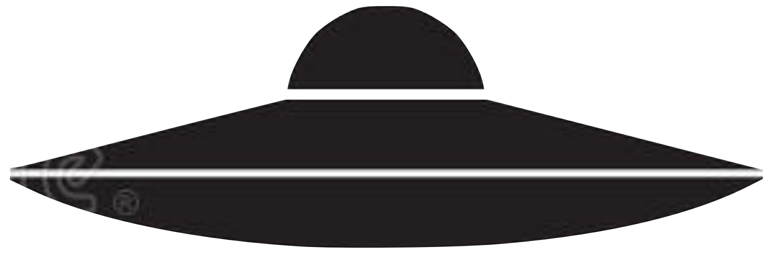


Sum and Difference Formulae

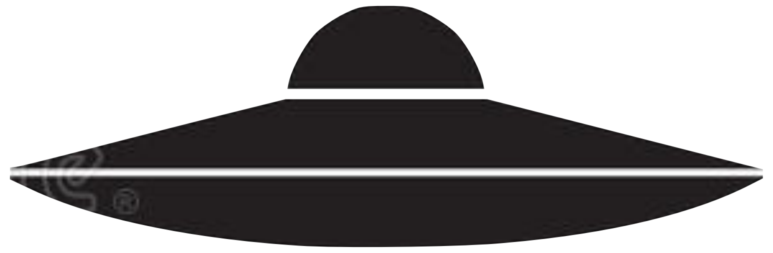
Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- All together now!

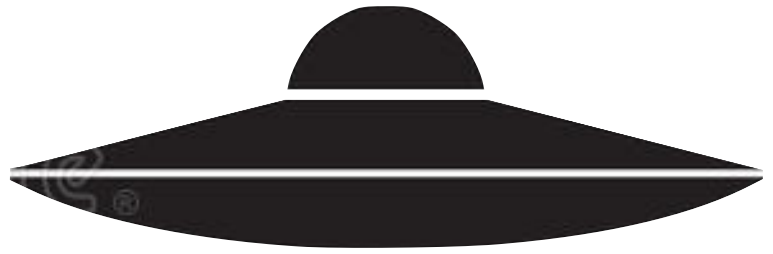
$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$



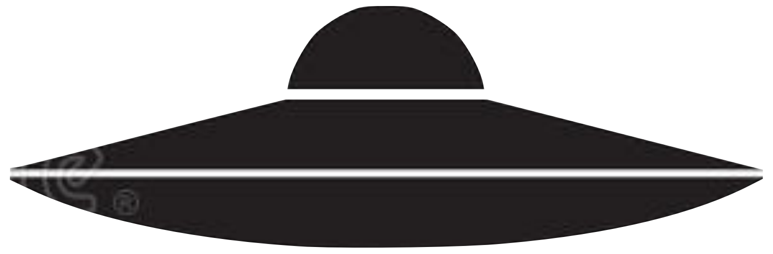
$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$



$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$



$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$



Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- From your book

Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Sum and Difference Formulae

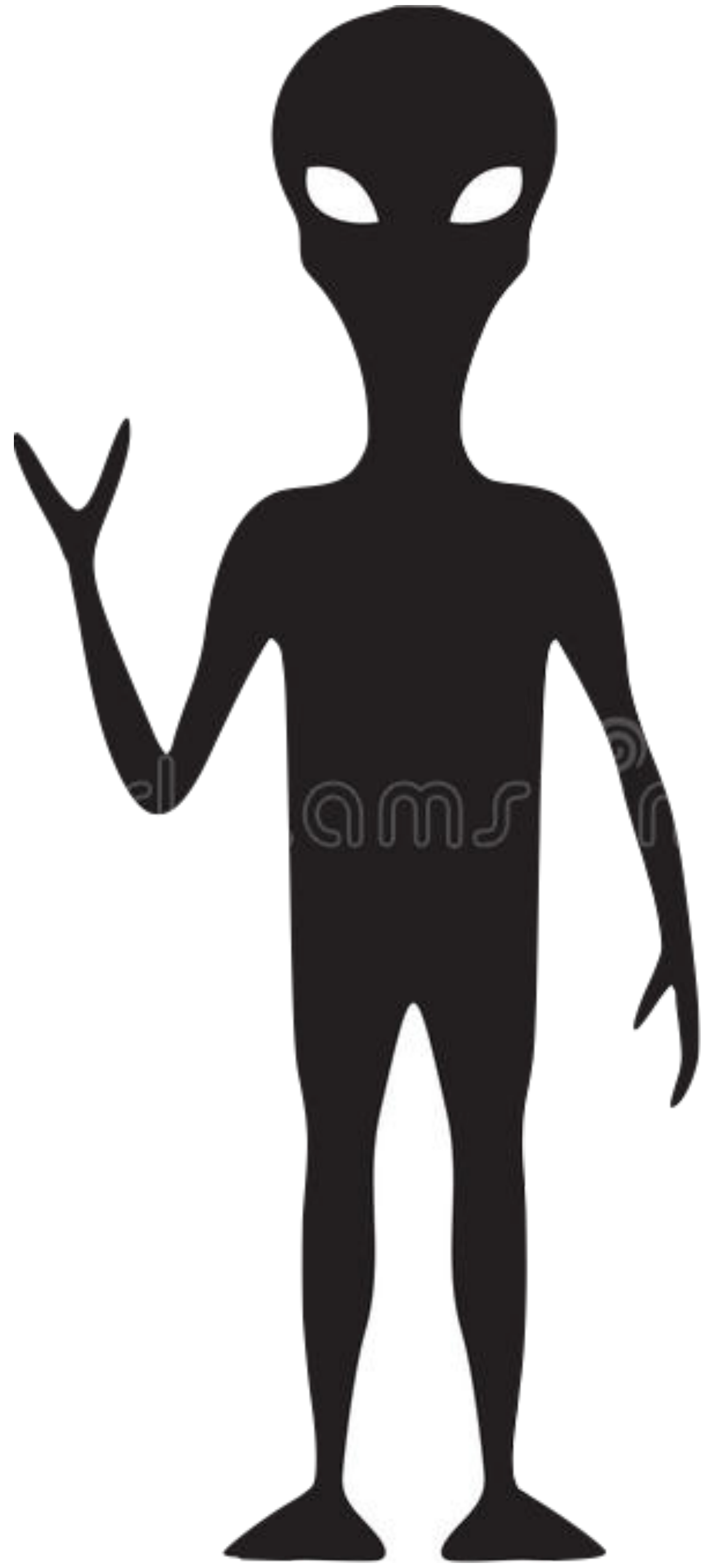
Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Find the exact value of $\sin\frac{5\pi}{12}$ using the fact that $\frac{5\pi}{12} = \frac{3\pi}{12} + \frac{2\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$

$$\sin\frac{5\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{6}\cos\frac{\pi}{4}$$

$$= \frac{\sqrt{2}}{2} \bullet \frac{\sqrt{3}}{2} + \frac{1}{2} \bullet \frac{\sqrt{2}}{2}$$

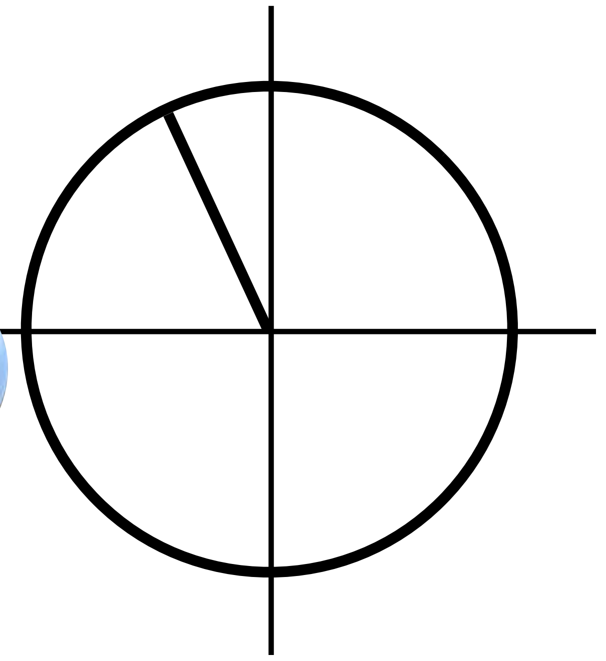
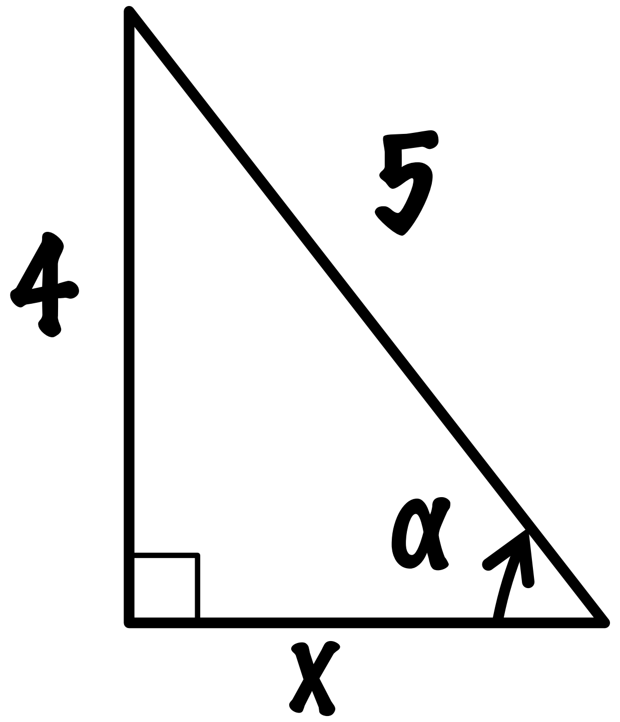
$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}$$



Sum and Difference Formulae

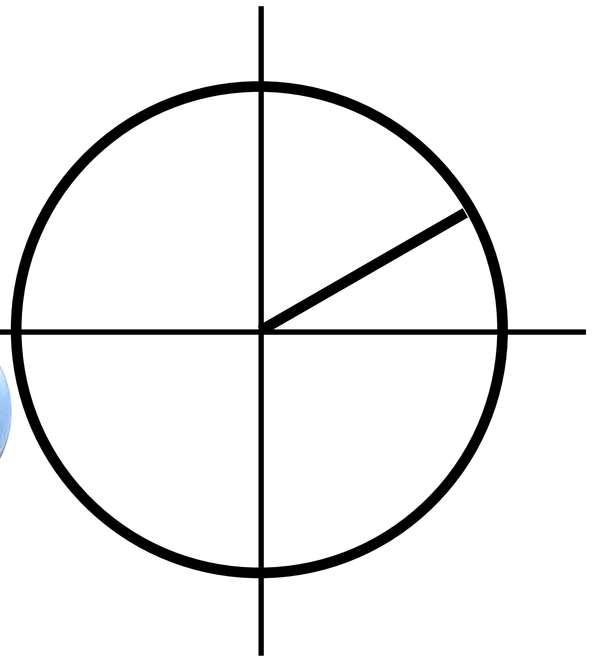
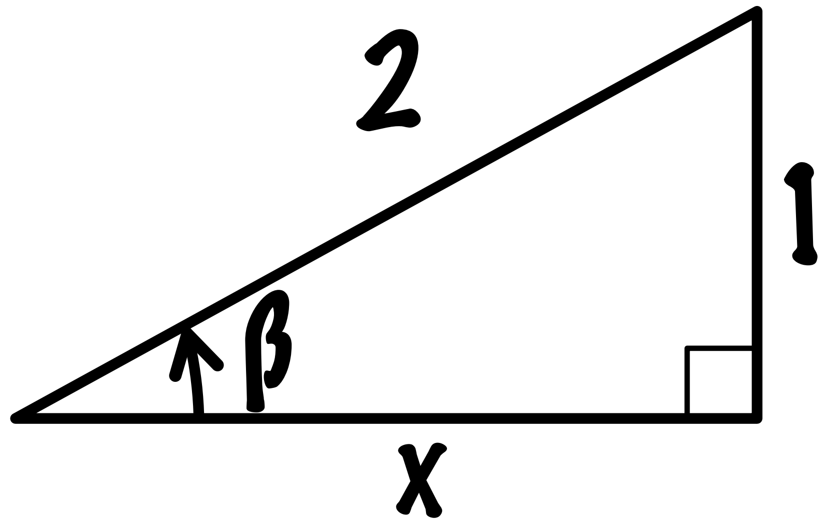
Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Suppose that $\sin\alpha = \dfrac{4}{5}$ for a quadrant II angle α and $\sin\beta = \dfrac{1}{2}$ for a quadrant I angle β. Find the exact value of cosα and cosβ.

$$x^2 + 4^2 = 5^2$$
$$x = -3$$

$$\cos\alpha = -\frac{3}{5}$$

$$x^2 + 1^2 = 5^2$$
$$x = \sqrt{3}$$

$$\cos\beta = \frac{\sqrt{3}}{2}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Suppose that $\sin\alpha = \frac{4}{5}$ for a quadrant II angle α and $\sin\beta = \frac{1}{2}$ for a quadrant I angle β. Find the exact value of cos(α+β).

$$\cos\alpha = -\frac{3}{5} \quad \cos\beta = \frac{\sqrt{3}}{2}$$

cos(α + β) = cos α cos β − sin α sin β

$$= -\frac{3}{5}\bullet\frac{\sqrt{3}}{2} - \frac{4}{5}\bullet\frac{1}{2}$$

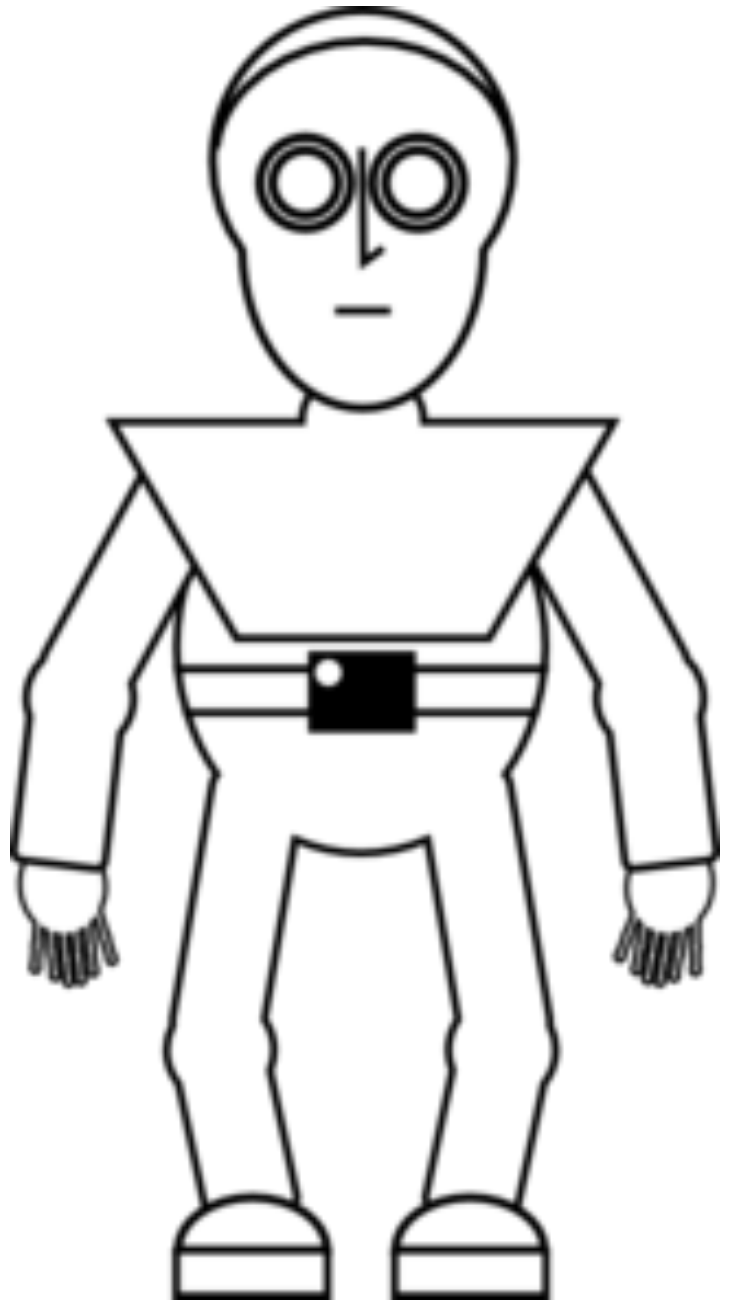
$$= -\frac{3\sqrt{3}}{10} - \frac{4}{10} = -\frac{3\sqrt{3}+4}{10}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Suppose that $\sin\alpha = \dfrac{4}{5}$ for a quadrant II angle α and $\sin\beta = \dfrac{1}{2}$ for a quadrant I angle β. Find the exact value of sin(α+β).

$$\cos\alpha = -\frac{3}{5} \qquad \cos\beta = \frac{\sqrt{3}}{2}$$



sin(α + β) = sin α cos β + cos α sin β

$$= \frac{4}{5}\bullet\frac{\sqrt{3}}{2} + -\frac{3}{5}\bullet\frac{1}{2}$$

$$= \frac{4\sqrt{3}}{10} - \frac{3}{10} = -\frac{4\sqrt{3}-3}{10}$$

Sum and Difference Formulae

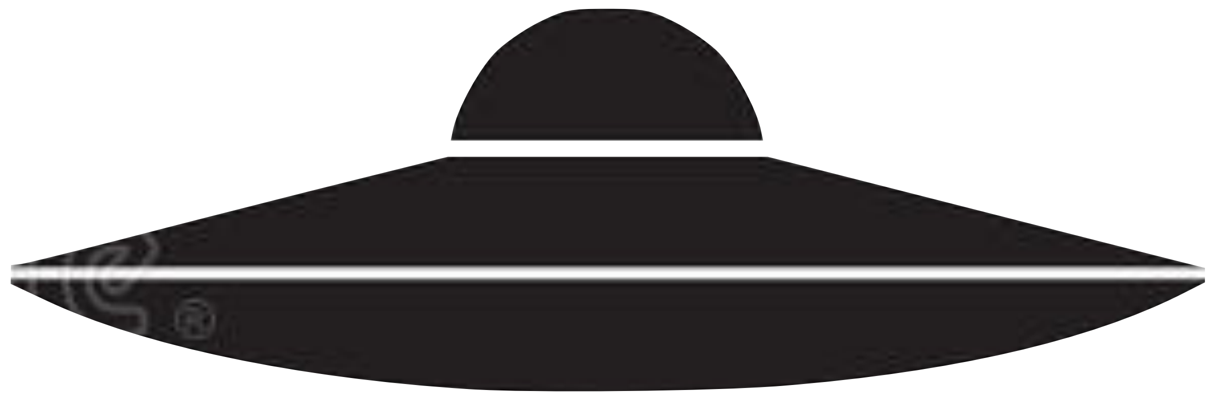
Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

$\tan(\alpha+\beta)$

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}=\frac{\dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta-\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$

$$=\frac{\dfrac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta}+\dfrac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\dfrac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}=\frac{\dfrac{\sin\alpha}{\cos\alpha}+\dfrac{\sin\beta}{\cos\beta}}{1-\dfrac{\sin\alpha}{\cos\alpha}\bullet\dfrac{\sin\beta}{\cos\beta}}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\bullet\tan\beta}$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$



Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

$\tan(\alpha-\beta)$

$$\tan(\alpha-\beta)=\tan(\alpha+(-\beta))$$

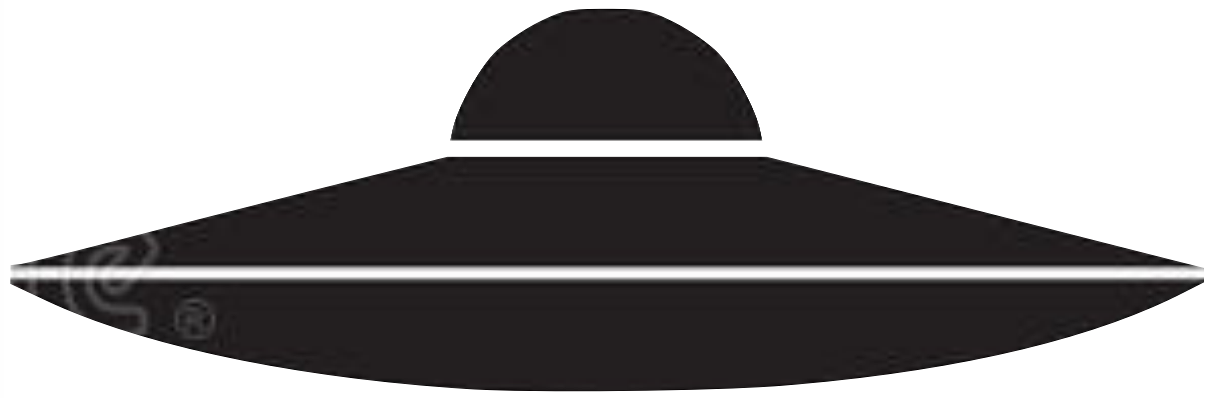
$$=\frac{\tan\alpha+\tan(-\beta)}{1-\tan\alpha\tan(-\beta)}$$

$$=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(-\beta)=-\tan\beta$$

$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$



Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

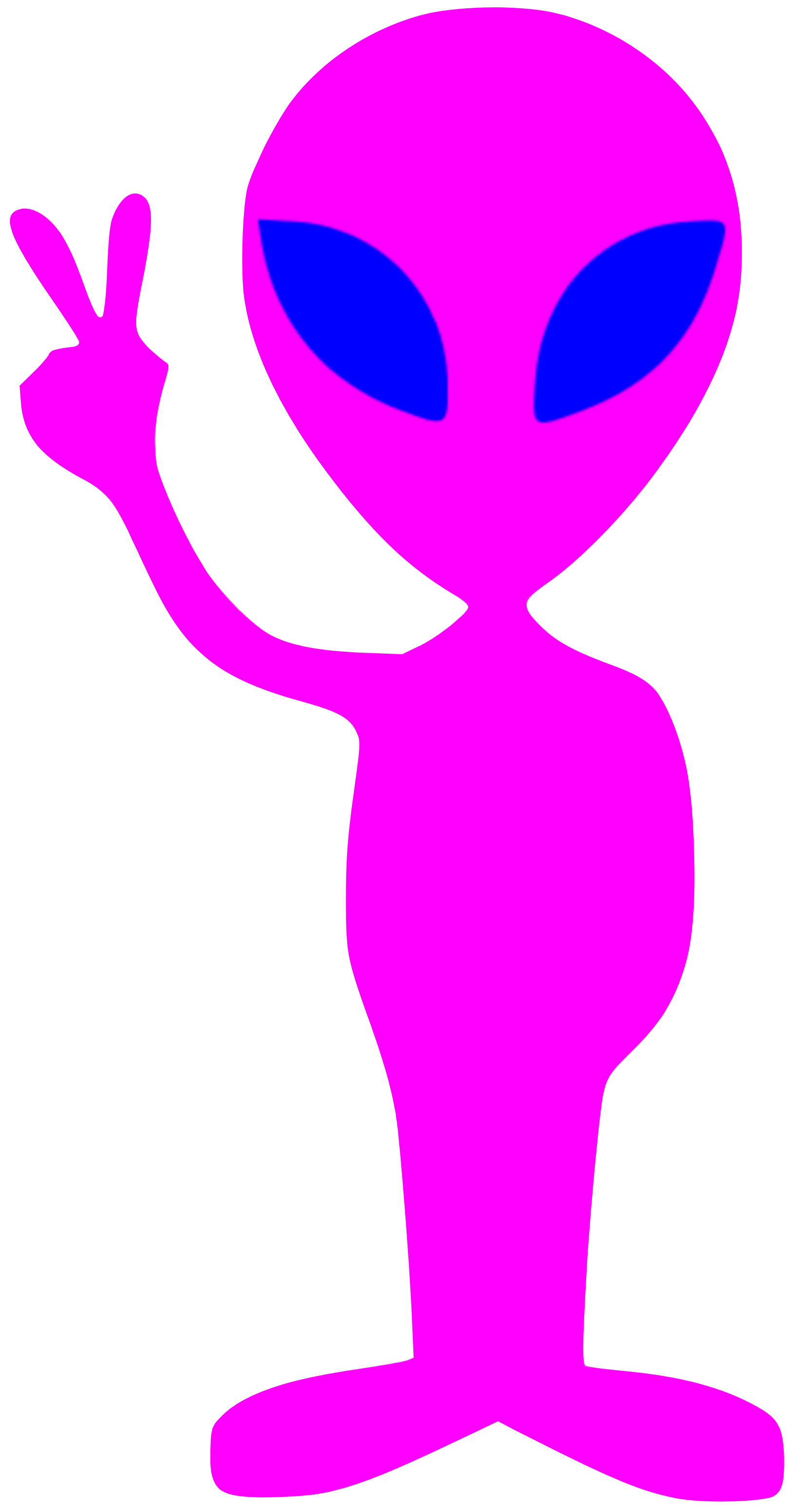
- Once more, all together

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$



$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

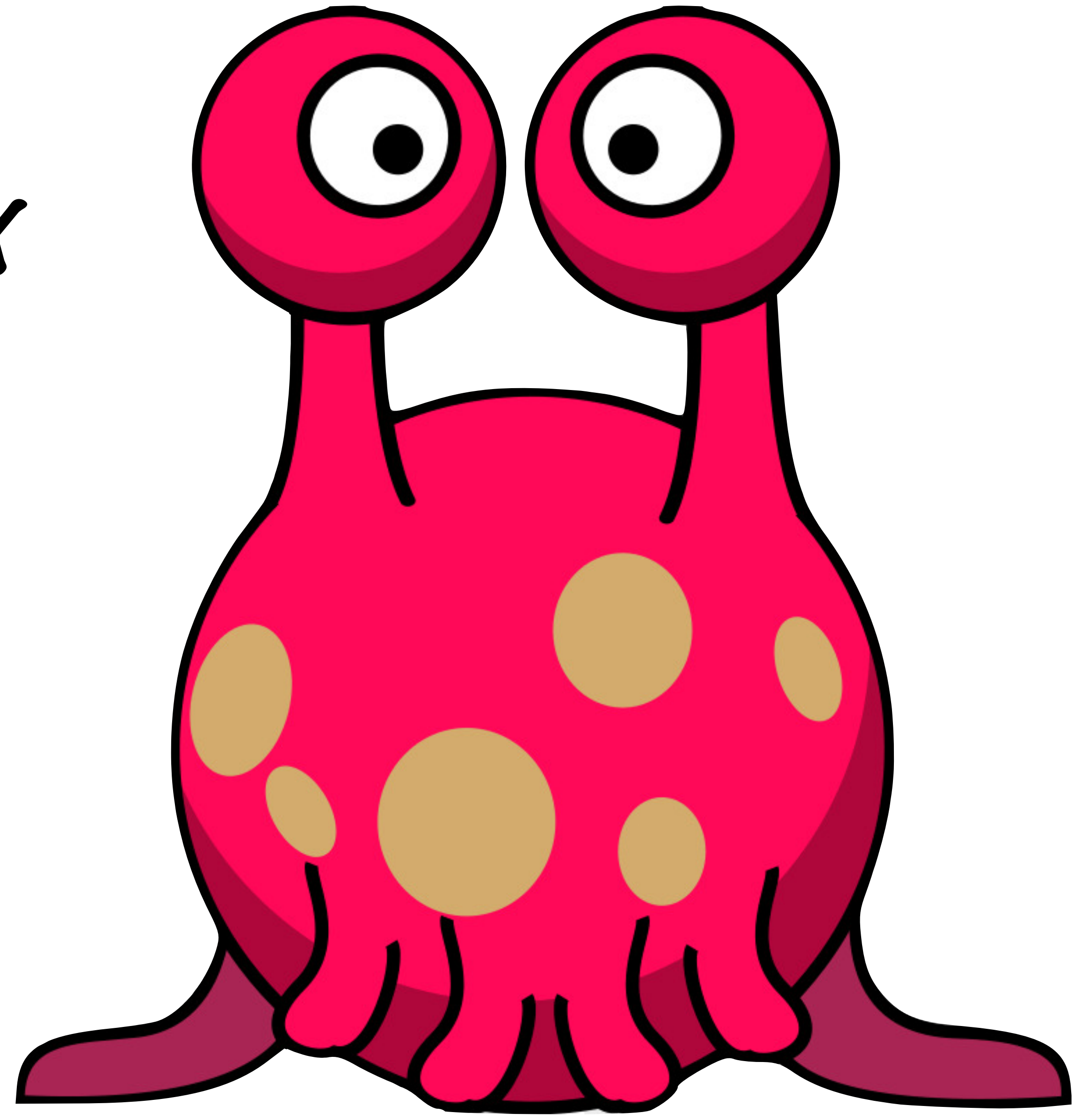
$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Verify the identity $\tan(x+\pi) = \tan x$

$$\tan(x+\pi) = \frac{\tan x + \tan \pi}{1 - \tan x \tan \pi} = \frac{\tan x + 0}{1 - \tan x \cdot 0} = \frac{\tan x}{1} = \tan x$$



- Find the exact value of $\cos 100^{\circ} \cos 55^{\circ} + \sin 100^{\circ} \sin 55^{\circ}$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\cos 100^{\circ} \cos 55^{\circ} + \sin 100^{\circ} \sin 55^{\circ} = \cos(100 - 55)^{\circ}$$

$$= \cos 45^{\circ} = \frac{\sqrt{2}}{2}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Verify the identity $\cos(\pi-\theta)=-\cos\theta$

$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$

$$\cos(\pi-\theta)=\cos\pi\cos\theta+\sin\pi\sin\theta$$

$$=-1\cos\theta+0\sin\theta$$

$$=-\cos\theta$$

- Simplify $\sin(x+3\pi)$

$$\sin(x+3\pi)=\sin(x)\cos(3\pi)+\cos(x)\sin(3\pi)$$

$$=\sin(x)(-1)+\cos(x)(0)=-\sin(x)$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Find the exact value of $\sin 75^\circ$

$$\sin 75^\circ = \sin(45 + 30)^\circ$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2}\bullet\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\bullet\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

- Find the exact value of $\sin 15^\circ$

$$\sin 15^\circ = \sin(45 - 30)^\circ$$

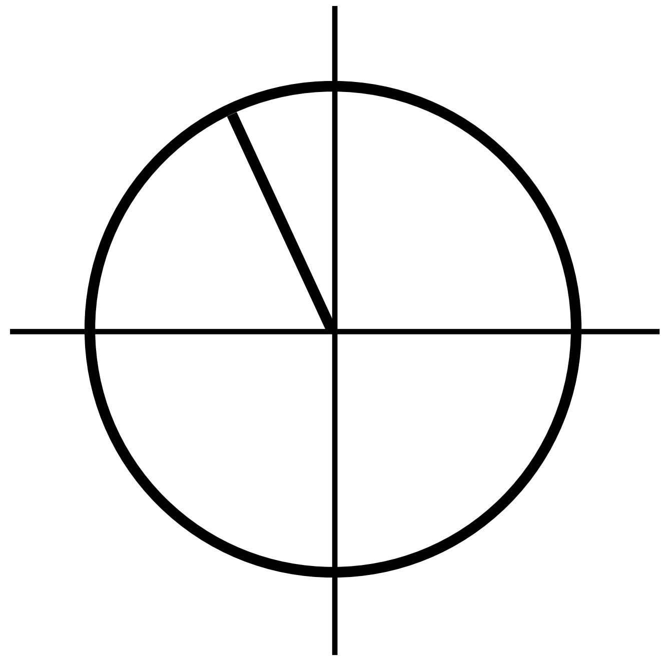
$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

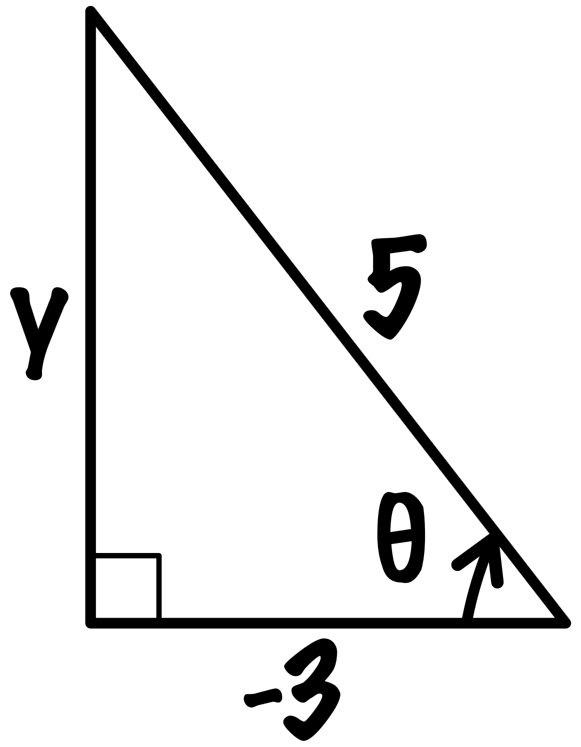
$$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ = \frac{\sqrt{2}}{2}\bullet\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\bullet\frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

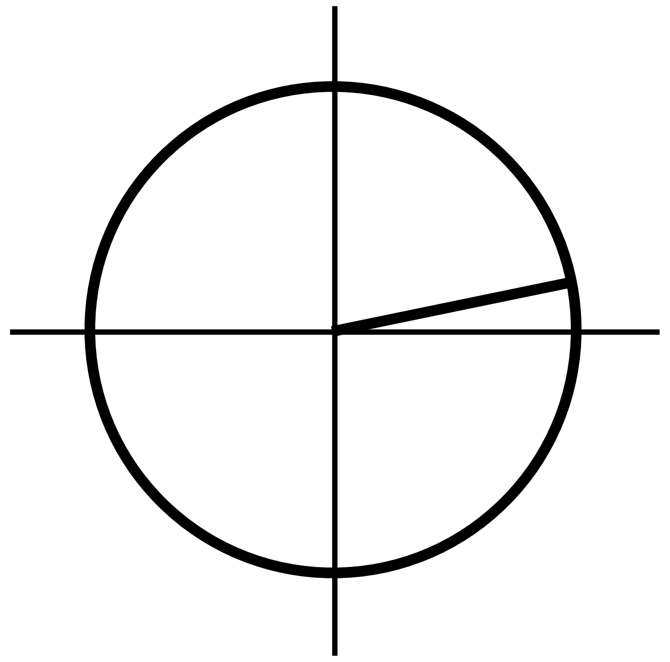
- Suppose that $\cos\theta = -\frac{3}{5}$ for a quadrant II angle θ and $\cos\beta = \frac{12}{13}$ for a quadrant I angle β. Find the exact value of sinθ, sinβ, sin(θ+β), cos (θ+β)

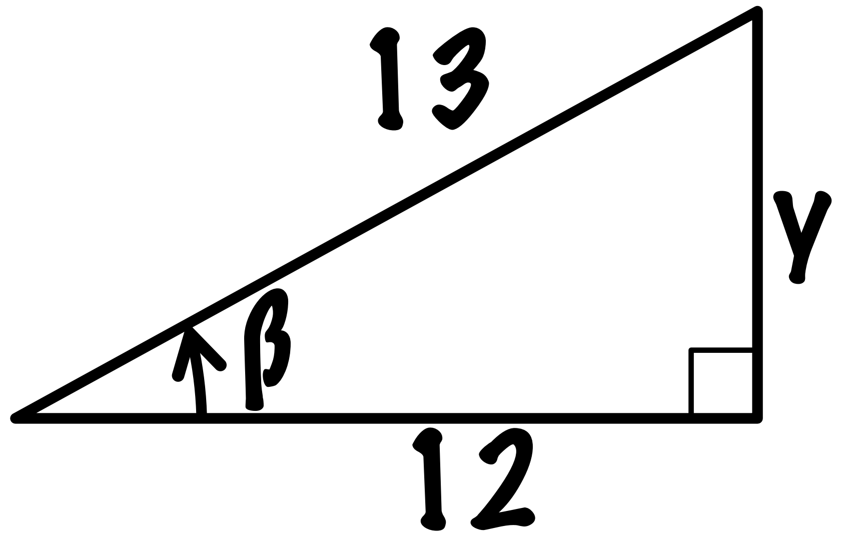




$$y^2 + (-3)^2 = 5^2$$
$$y = 4$$

$$\sin\theta = \frac{4}{5}$$





$$y^2 + 12^2 = 13^2$$
$$y = 5$$

$$\sin\beta = \frac{5}{13}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

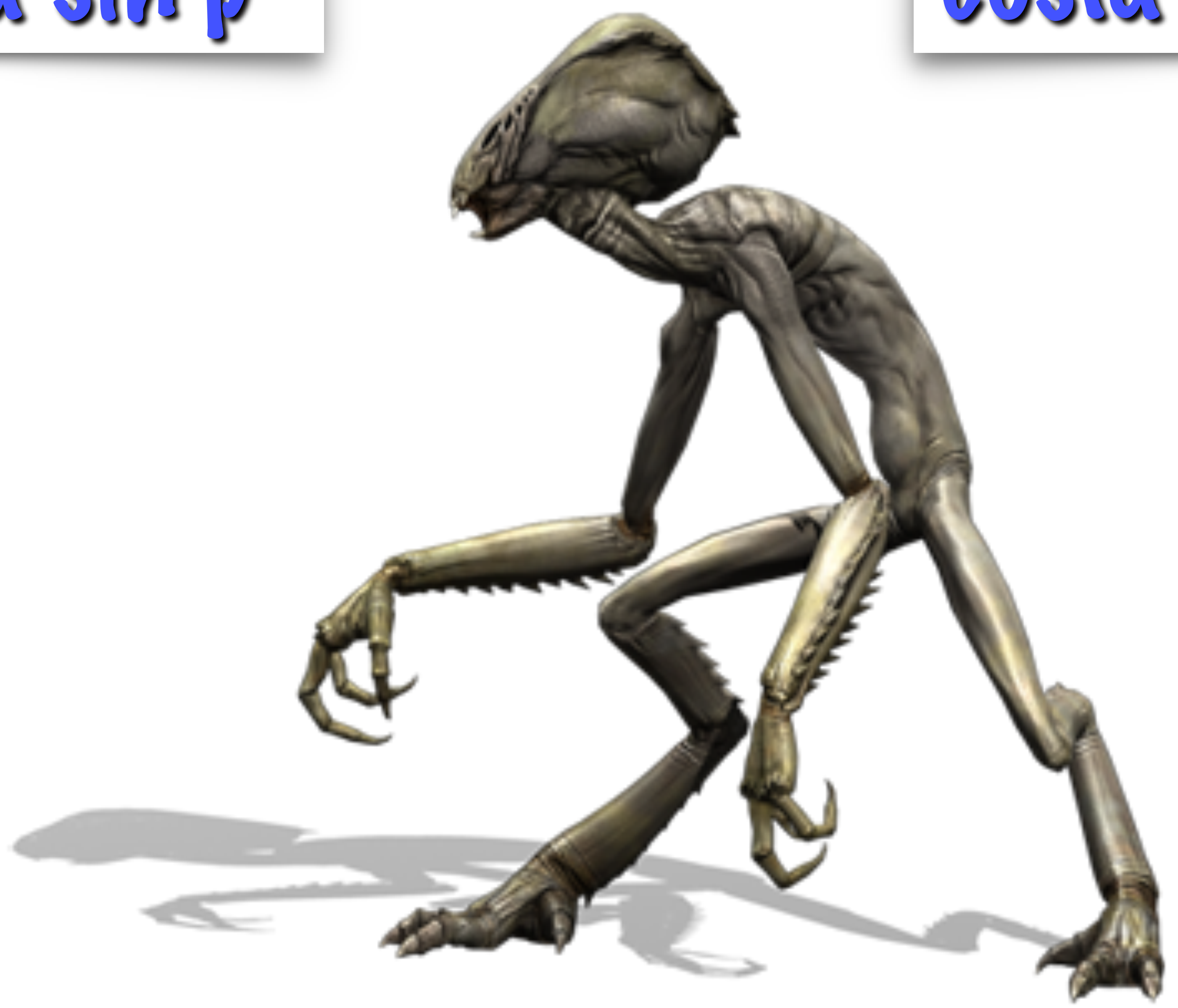
- Suppose that $\cos\theta = -\frac{3}{5}$ for a quadrant II angle θ and $\cos\beta = \frac{12}{13}$ for a quadrant I angle β. Find the exact value of sinθ, sinβ, sin(θ+β), cos (θ+β)

$$\sin\theta = \frac{4}{5} \qquad \sin\beta = \frac{5}{13}$$

sin(α + β) = sin α cos β + cos α sin β

$$= \frac{4}{5}\bullet\frac{12}{13} + \frac{-3}{5}\bullet\frac{5}{13}$$

$$= \frac{48}{65} + \frac{-15}{65} = \frac{33}{65}$$



cos(α + β) = cos α cos β − sin α sin β

$$= \frac{-3}{5}\bullet\frac{12}{13} - \frac{4}{5}\bullet\frac{5}{13}$$

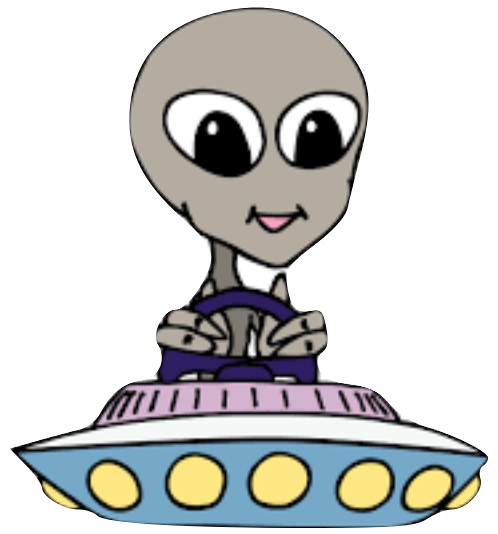
$$= \frac{-36}{65} - \frac{20}{65} = -\frac{56}{65}$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Find the exact value of $\sin\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

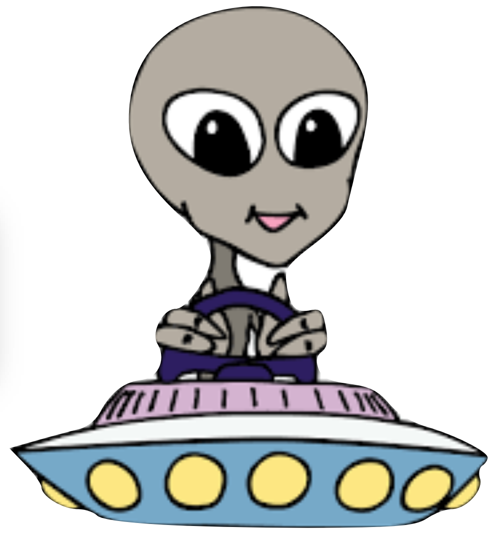


$$\sin\left(\frac{2\pi}{3}-\frac{\pi}{4}\right)=\sin\frac{2\pi}{3}\cos\frac{\pi}{4}-\cos\frac{2\pi}{3}\sin\frac{\pi}{4}=\frac{\sqrt{3}}{2}\bullet\frac{\sqrt{2}}{2}-\frac{-1}{2}\bullet\frac{\sqrt{2}}{2}$$

$$=\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4}=\frac{\sqrt{6}+\sqrt{2}}{4}$$

- Simplify $\sin(x+3\pi)$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$



$$\sin(x+3\pi)=\sin x\cos 3\pi+\cos x\sin 3\pi = \sin x(-1)+\cos x(0) = -\sin x$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Simplify $\tan\left(\theta+\frac{5\pi}{4}\right)$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan\left(\theta+\frac{5\pi}{4}\right)=\frac{\tan\theta+\tan\frac{5\pi}{4}}{1-\tan\theta\tan\frac{5\pi}{4}}=\frac{\tan\theta+1}{1-\tan\theta(1)}=\frac{\tan\theta+1}{1-\tan\theta}$$

- Verify the identity $\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$

$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$

$$\sin\left(\frac{\pi}{2}-\theta\right)=\sin\frac{\pi}{2}\cos\theta-\cos\frac{\pi}{2}\sin\theta=1\cos\theta-0\sin\theta=\cos\theta$$

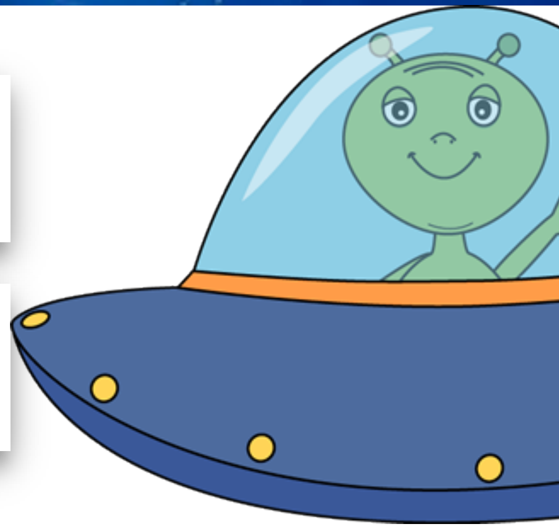
Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

- Verify $\sin(x+y)\sin(x-y) = \cos^2 y - \cos^2 x$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$



$$\sin(x+y)\sin(x-y) = (\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$$

$$= (\sin x\cos y)^2 - (\cos x\sin y)^2$$

$(a-b)(a+b) = a^2 - b^2$

$$= \sin^2 x\cos^2 y - \cos^2 x\sin^2 y$$

$$= (1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)$$

$$= (\cos^2 y - \cancel{\cos^2 y\cos^2 x}) - (\cos^2 x - \cancel{\cos^2 x\cos^2 y})$$

$$= \cos^2 y - \cos^2 x$$

Sum and Difference Formulae

Objective: Use Sum and Difference Formulae to evaluate trig expressions and verify trig identities.

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

- Solve $\cos\left(x+\frac{\pi}{4}\right)-\cos\left(x-\frac{\pi}{4}\right)=1;\ [0,2\pi)$

$$\left(\cos x\cos\frac{\pi}{4}-\sin x\cos\frac{\pi}{4}\right)-\left(\cos x\cos\frac{\pi}{4}+\sin x\cos\frac{\pi}{4}\right)=1$$

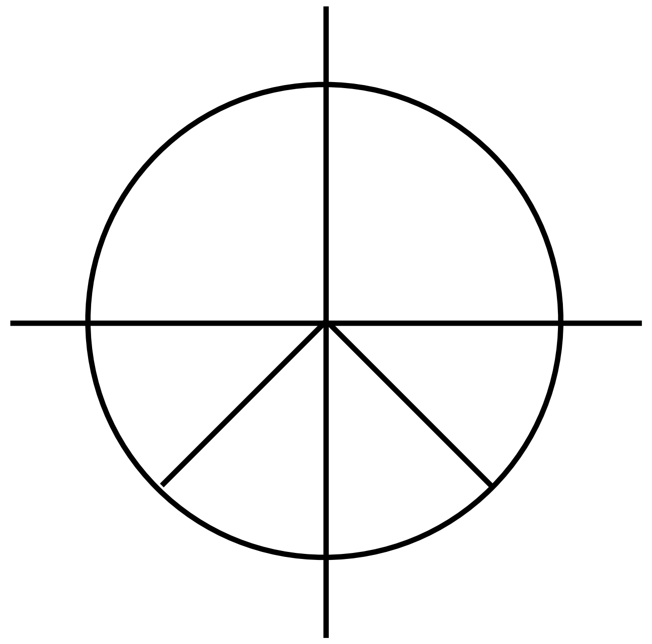
$$\left(\frac{\sqrt{2}}{2}\cos x-\frac{\sqrt{2}}{2}\sin x\right)-\left(\frac{\sqrt{2}}{2}\cos x+\frac{\sqrt{2}}{2}\sin x\right)=1$$

$$\frac{\sqrt{2}}{2}\left(\cos x-\sin x\right)-\left(\cos x+\sin x\right)=1$$

$$\frac{\sqrt{2}}{2}\left(-2\sin x\right)=1$$

$$-\sqrt{2}\left(\sin x\right)=1$$

$$\sin x=-\frac{1}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$



$$x=\frac{5\pi}{4}$$

$$x=\frac{7\pi}{4}$$