Chapter 6

Additional Topics in Trigonometry

6.1 The Law of Sines

Chapter 6.1

Homework

Read Sec 6.1 Do p436 1-41 odd

Chapter 6.1

Objectives

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.

Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

Solve applied problems using the Law of Sines.

Note

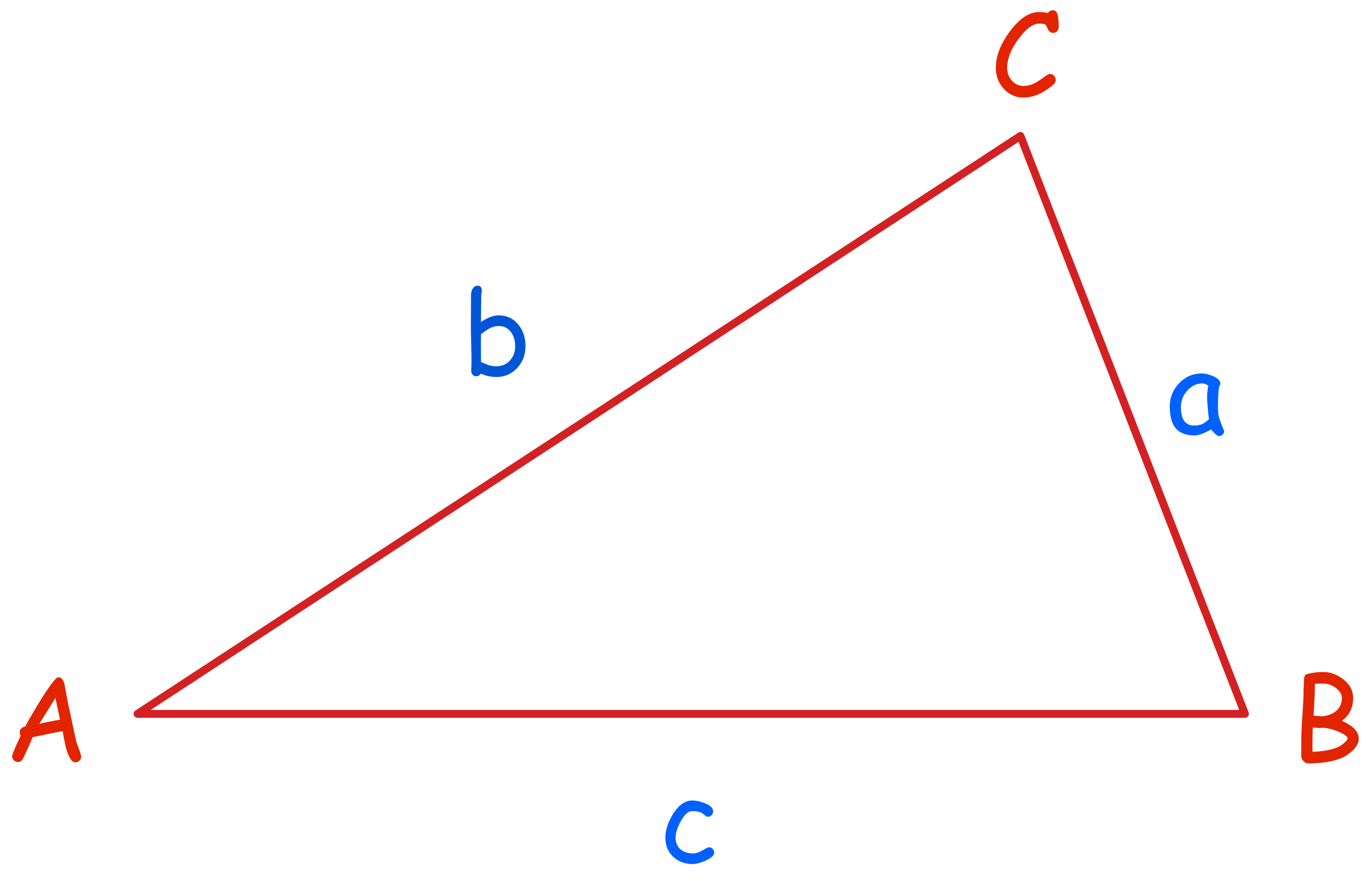
Before we start, I would like to remind you to draw a picture whenever possible to help visualize what you are trying to accomplish.

Draw a Picture!!!

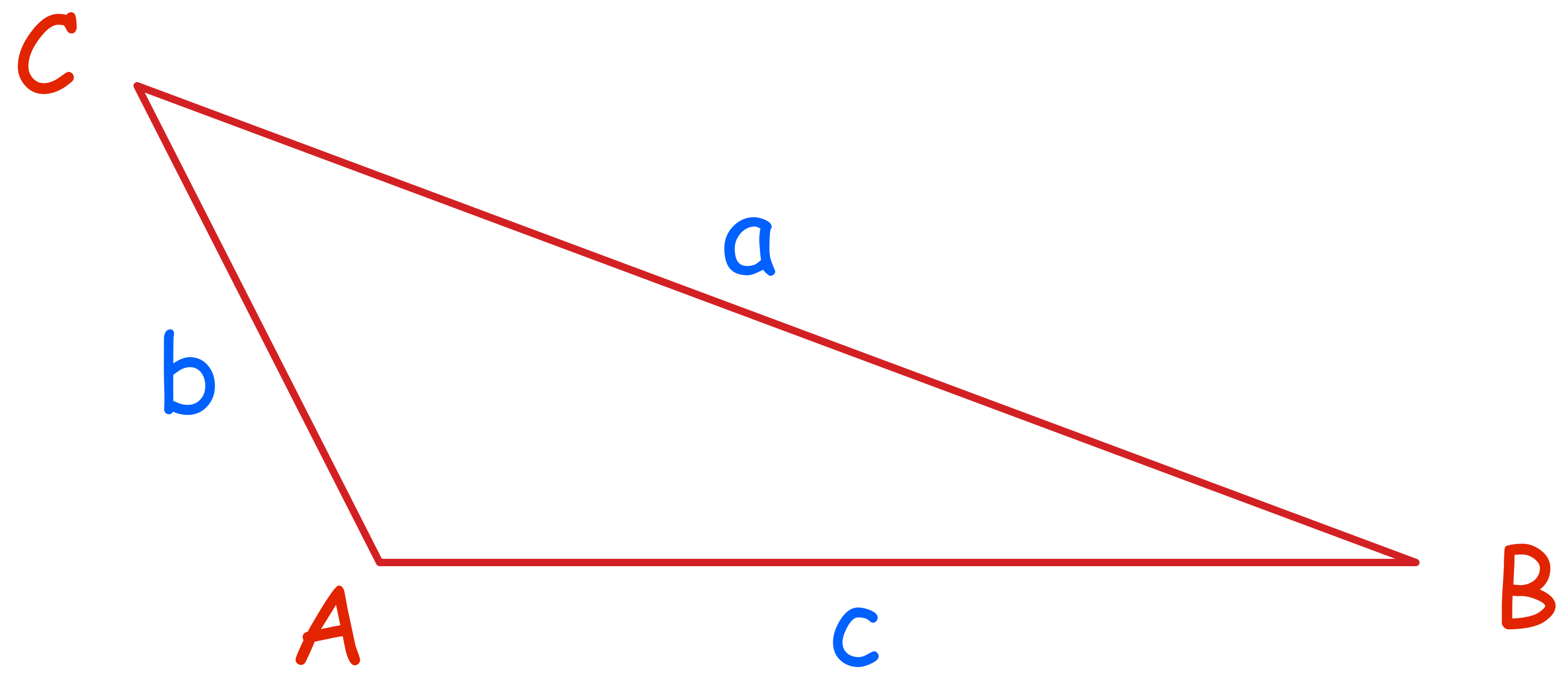
Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

- An **oblique triangle** is a triangle that does **not** contain a right angle.



- An oblique triangle has
- either three acute angles



- or two acute angles and one obtuse angle.

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

- The relationships among the sides and angles of **right triangles** defined by the **trigonometric functions** are **not** valid for oblique triangles.

- To solve an oblique triangle (find the measures of all sides and angles) you need the length of at least one side, and two other measures of the triangle.

- We have a couple of laws to help us solve triangles; the **Law of Sines**, and the **Law of Cosines**. Today we learn the **Law of Sines**.

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

- If A, B, And C are the measures of the angles of a triangle, and a, b, and c, are the lengths of the sides opposite the angles, then:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- In other words; the ratio of length of side of a triangle opposite an angle to the sine of that angle is the same for all three sides.

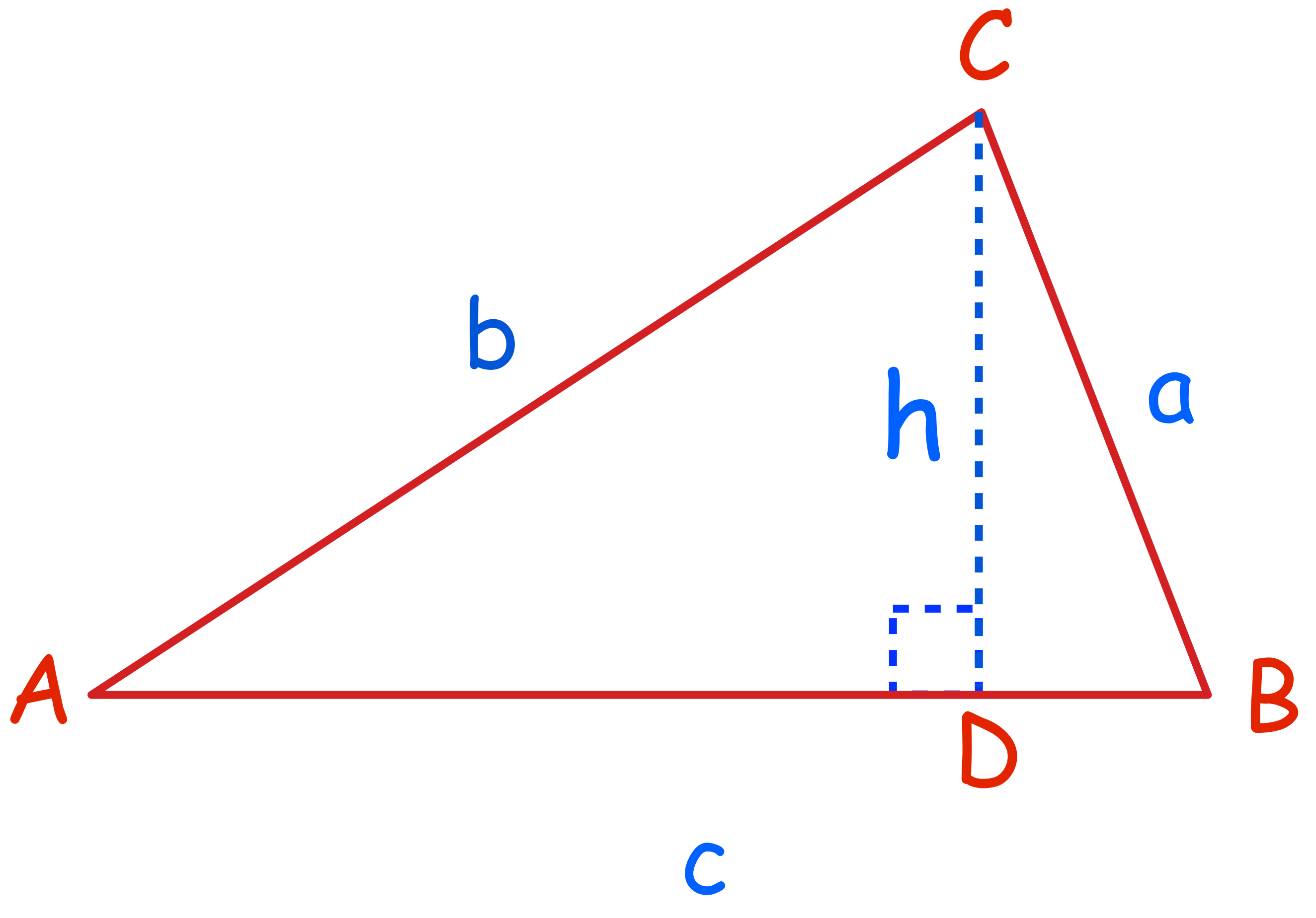
- Of course, it is also true that:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

- If A, B, And C are the measures of the angles of a triangle, and a, b, and c, are the lengths of the sides opposite the angles.



- Let h be the height of the triangle to side c.

$$\sin A = \frac{h}{b} \qquad h = b\sin A \qquad \sin B = \frac{h}{a} \qquad h = a\sin B$$

$$a\sin B = b\sin A \qquad \frac{a}{\sin A} = \frac{b}{\sin B}$$

- We can repeat the process with a new height to get C.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Sines

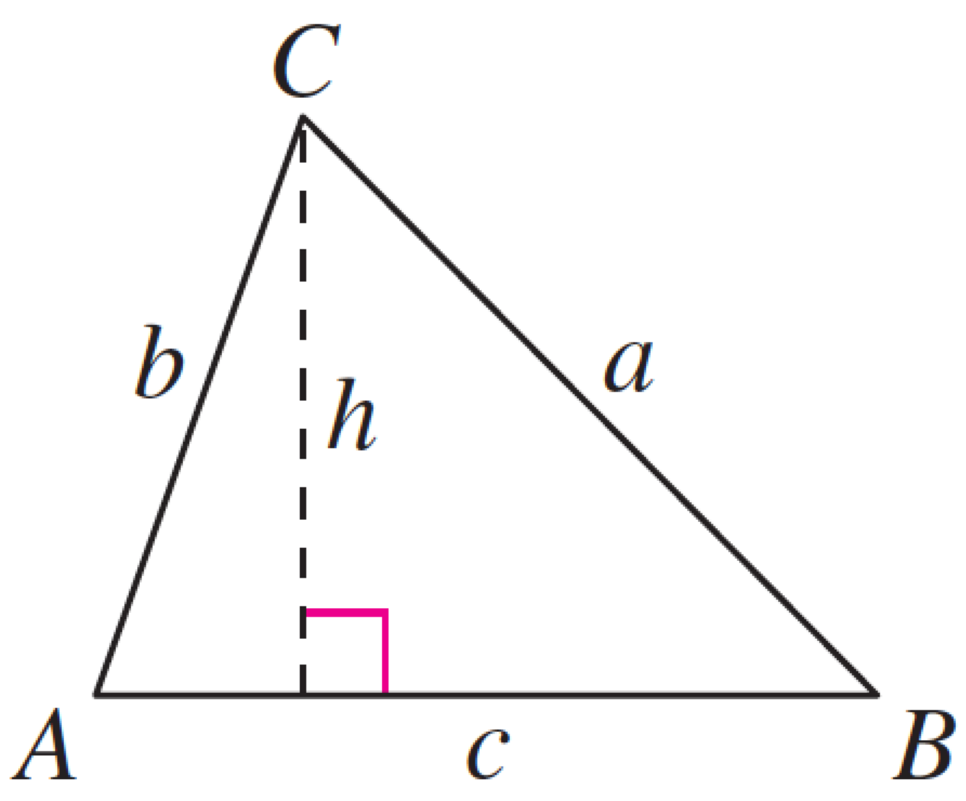
Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

From your book

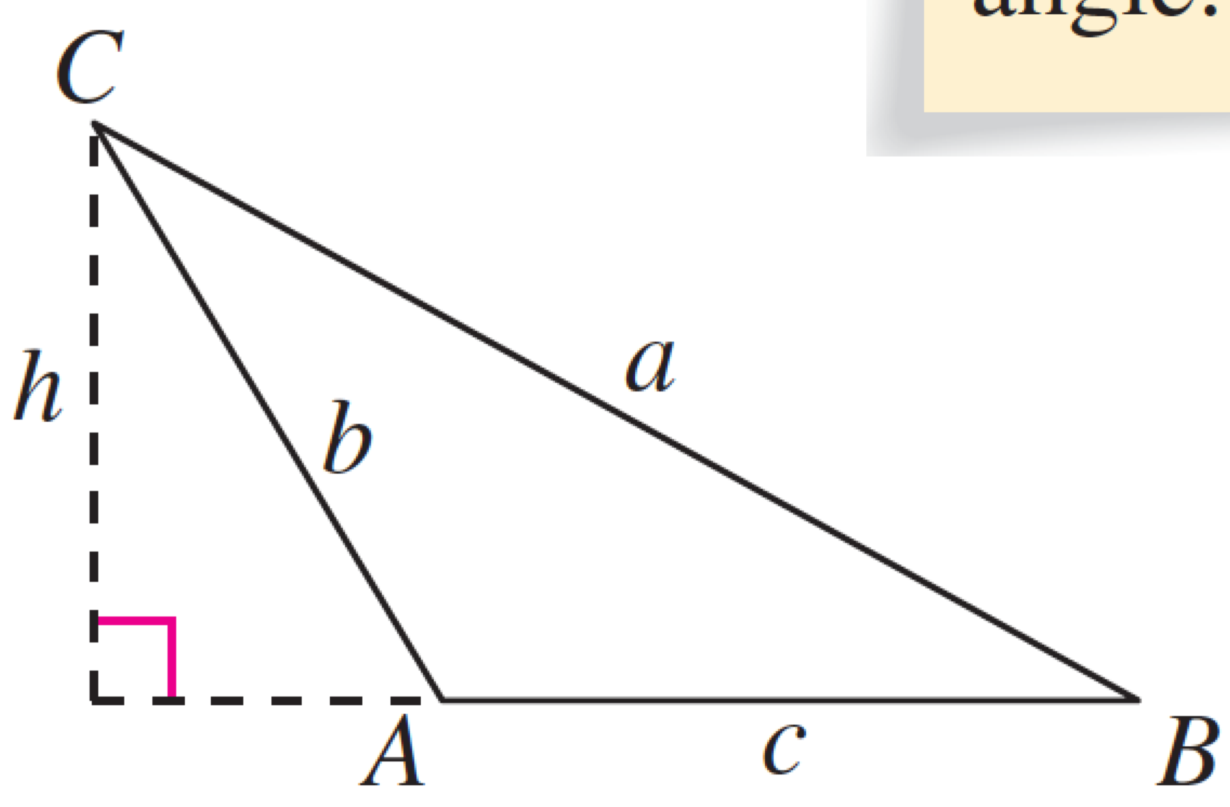
Law of Sines

If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



A is acute.



A is obtuse.

STUDY TIP

When solving triangles, a careful sketch is useful as a quick test for the feasibility of an answer. Remember that the longest side lies opposite the largest angle, and the shortest side lies opposite the smallest angle.

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

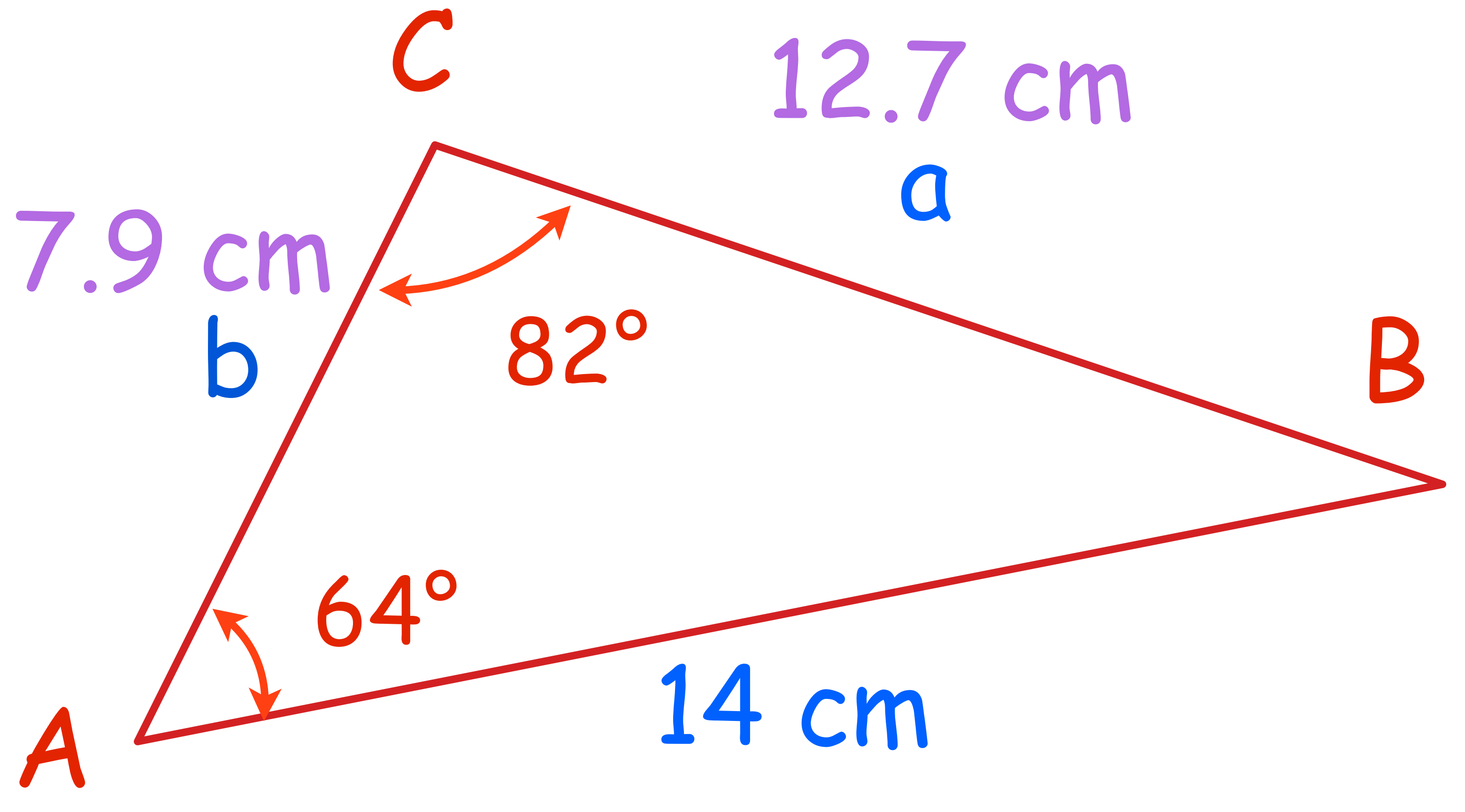
- **Solving** an oblique triangle means finding the lengths of its sides and the measurements of its angles.
- The **Law of Sines** can be used to solve a triangle in which one side and two angles are known, or two sides and one angle opposite are known.
- The three known measurements can be abbreviated using
  - *AAS* (a side and two angles are known),
  - *SSA* (two sides and an angle opposite are known) or
  - *ASA* (two angles and the **side between them** are known).

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

Solve the triangle with $A = 64°$, $C = 82°$, and $c = 14$ centimeters. Round lengths of sides to the nearest tenth.

AAS



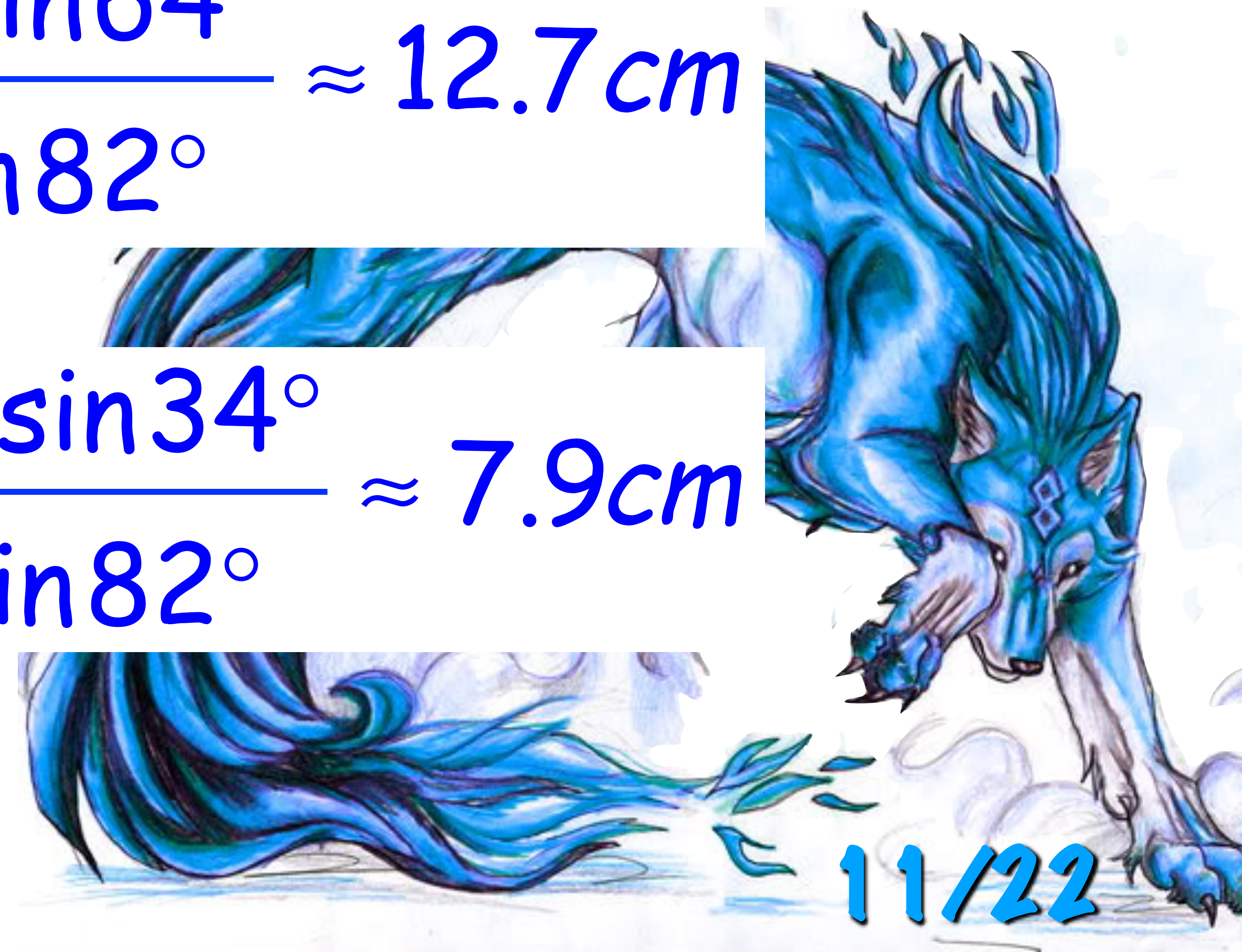
$$B = 180 - 82 - 64 = 34°$$

$$\frac{a}{\sin 64°} = \frac{b}{\sin 34°} = \frac{14}{\sin 82°}$$

$$\frac{a}{\sin 64°} = \frac{14}{\sin 82°} \qquad a = \frac{14 \sin 64°}{\sin 82°} \approx 12.7cm$$

$$\frac{b}{\sin 34°} = \frac{14}{\sin 82°} \qquad b = \frac{14 \sin 34°}{\sin 82°} \approx 7.9cm$$

Please note the appropriate lengths of the sides.

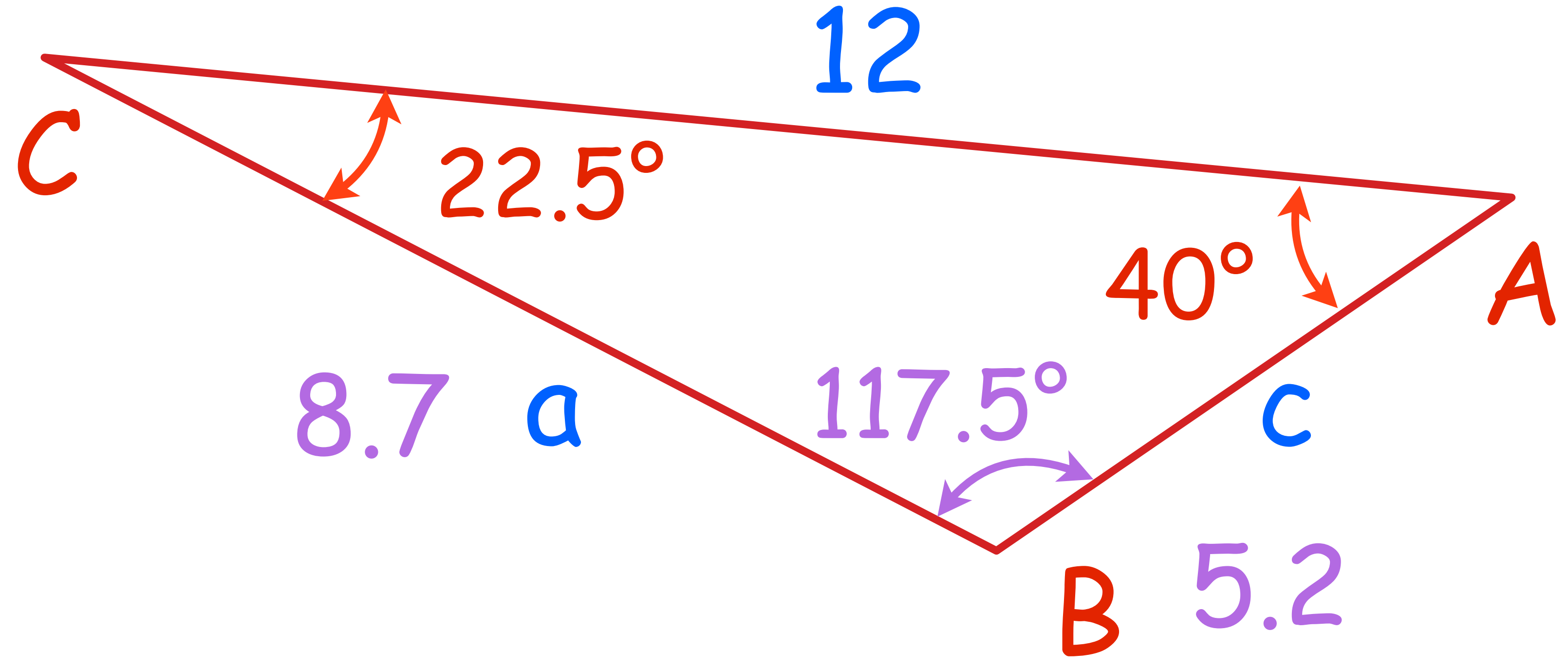


Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

Solve ΔABC if $A = 40°$, $C = 22.5°$, and $b = 12$. Round measures to the nearest tenth.

ASA



$$\frac{a}{\sin 40°} = \frac{12}{\sin 117.5°} = \frac{c}{\sin 22.5°}$$

$B = 180 - 40 - 22.5 = 117.5°$

$$\frac{a}{\sin 40°} = \frac{12}{\sin 117.5°} \qquad a = \frac{12\sin 40°}{\sin 117.5°} = 8.7$$

$$\frac{c}{\sin 22.5°} = \frac{12}{\sin 117.5°} \qquad c = \frac{12\sin 22.5°}{\sin 117.5°} = 5.2$$

Please note the appropriate lengths of the sides.

Ambiguous Case

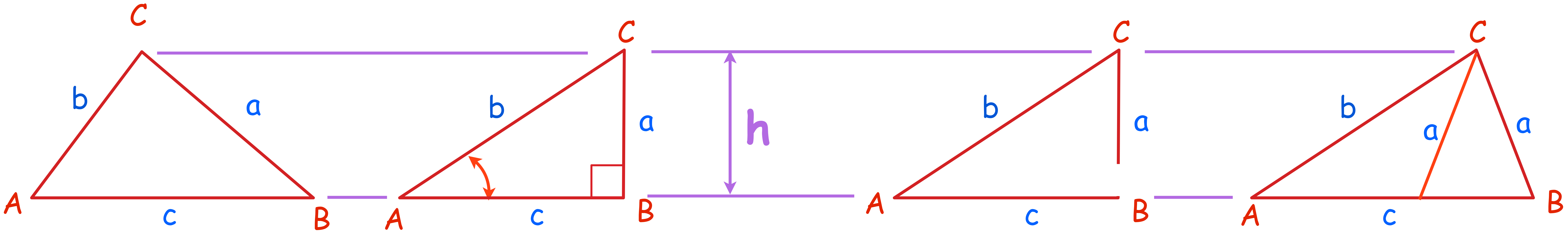
- If we are given two sides and an angle opposite one of the two sides (**SSA**), the given information may result in one triangle, two triangles, or no triangle at all.

  - **SSA** is known as the **ambiguous case** when using the Law of Sines because the given information may result in one triangle, two triangles, or no triangle at all.

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.

Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.



$$h = b \sin A$$

ambiguous case

<u>One Triangle</u>	<u>One Triangle</u>	<u>No Triangle</u>	<u>Two possible Triangles</u>
a > h a > b	a = h	a < h too short to form a triangle	a > h a < b

Note: we are discussing **SSA** and **the side in question is opposite the angle.**

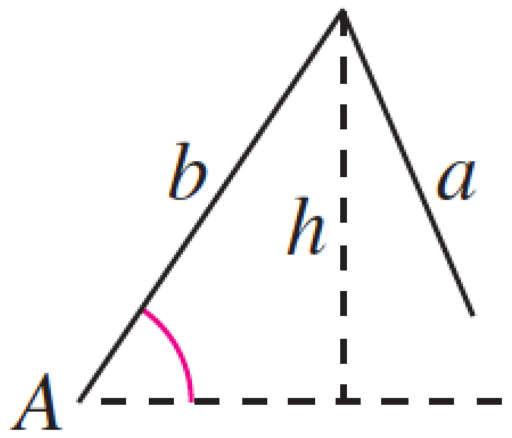
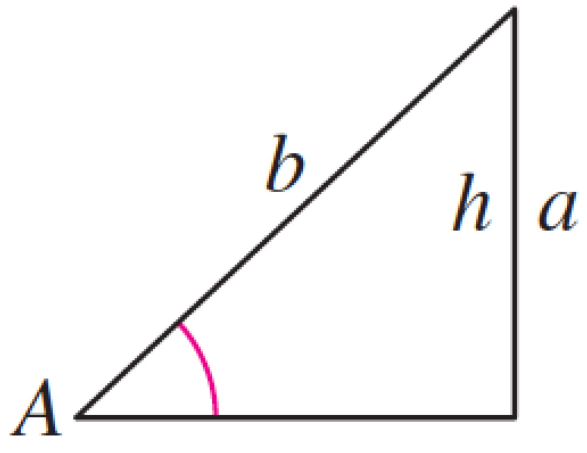
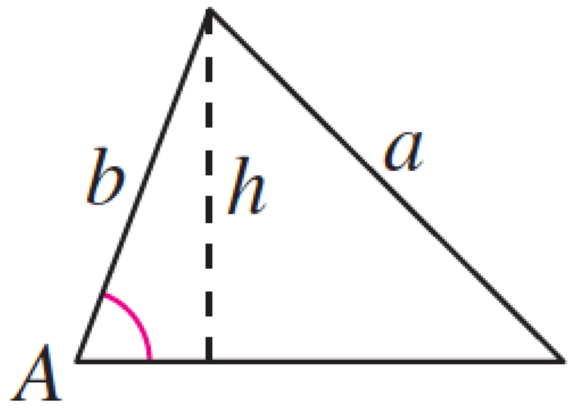
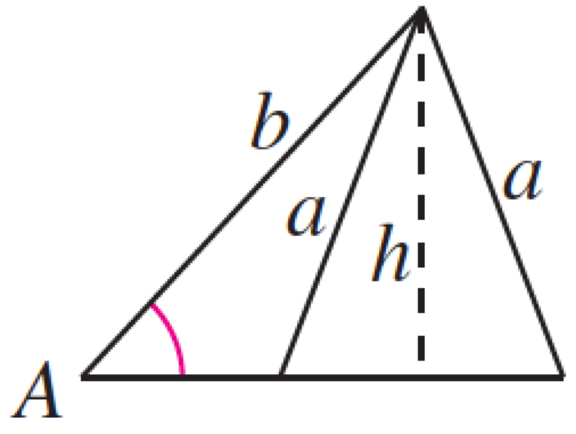
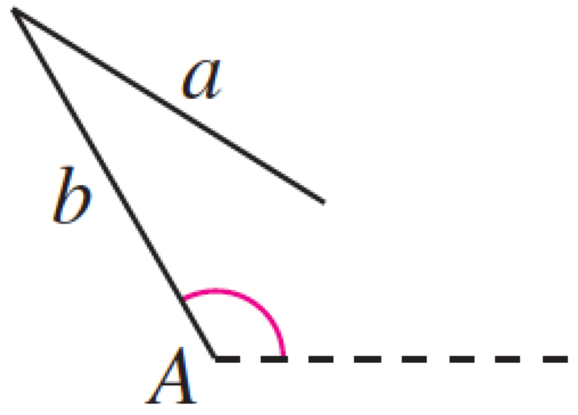
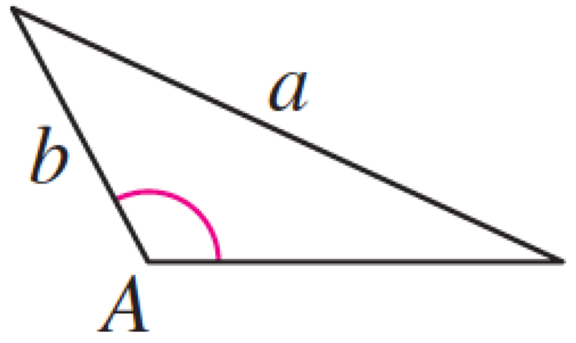
Law of Sines

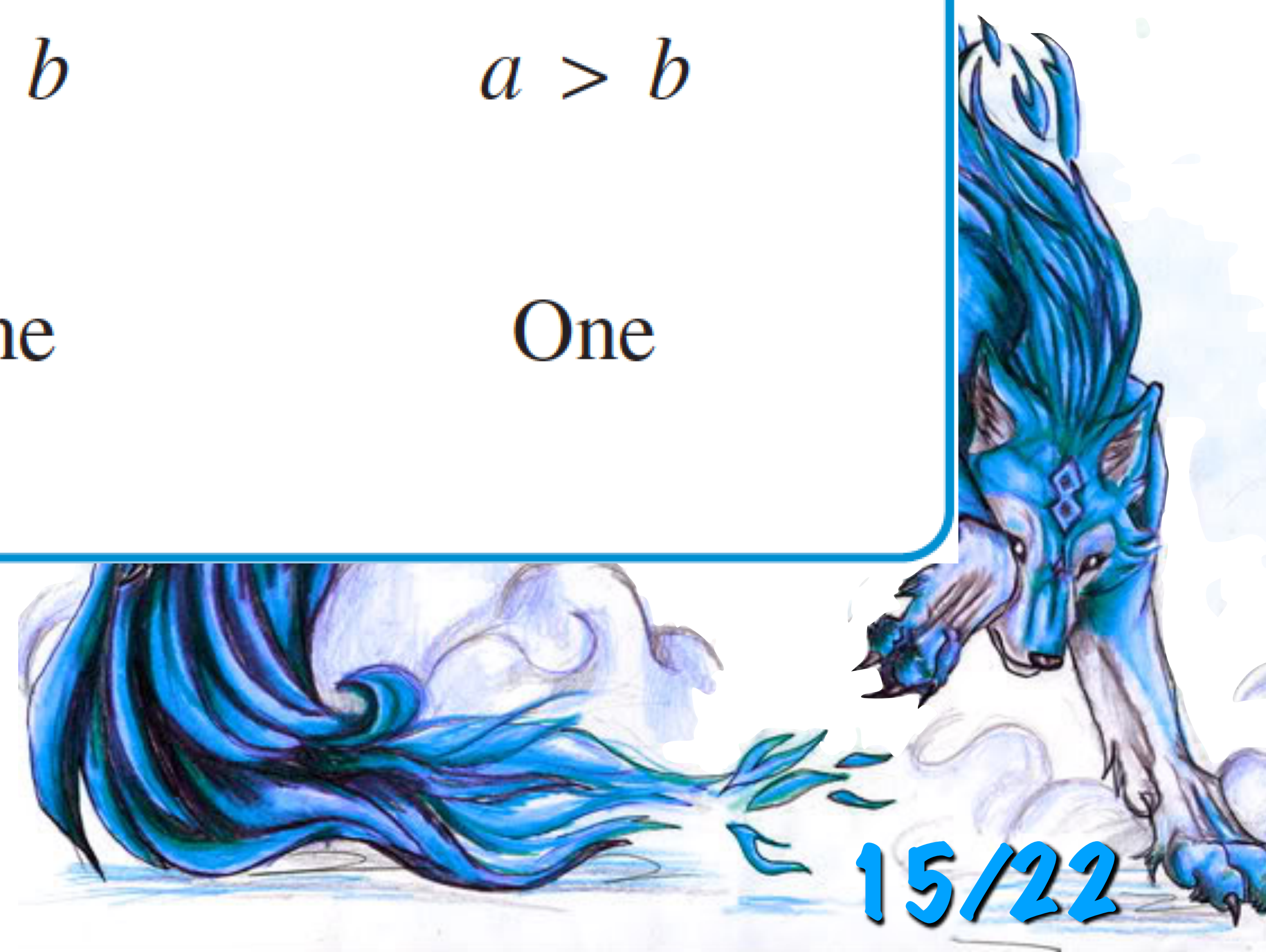
Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

- From your book

The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. $(h = b \sin A)$

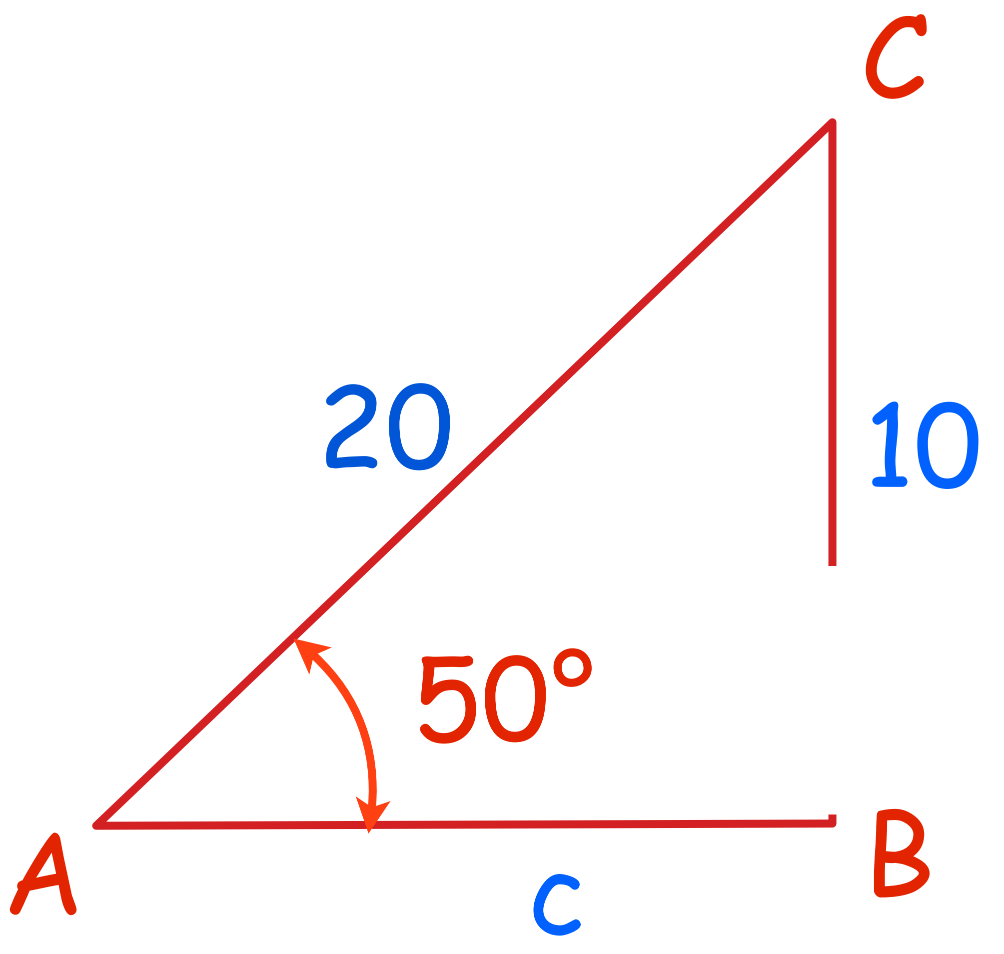
	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a > b$	$h < a < b$	$a \le b$	$a > b$
Triangles possible	None	One	One	Two	None	One

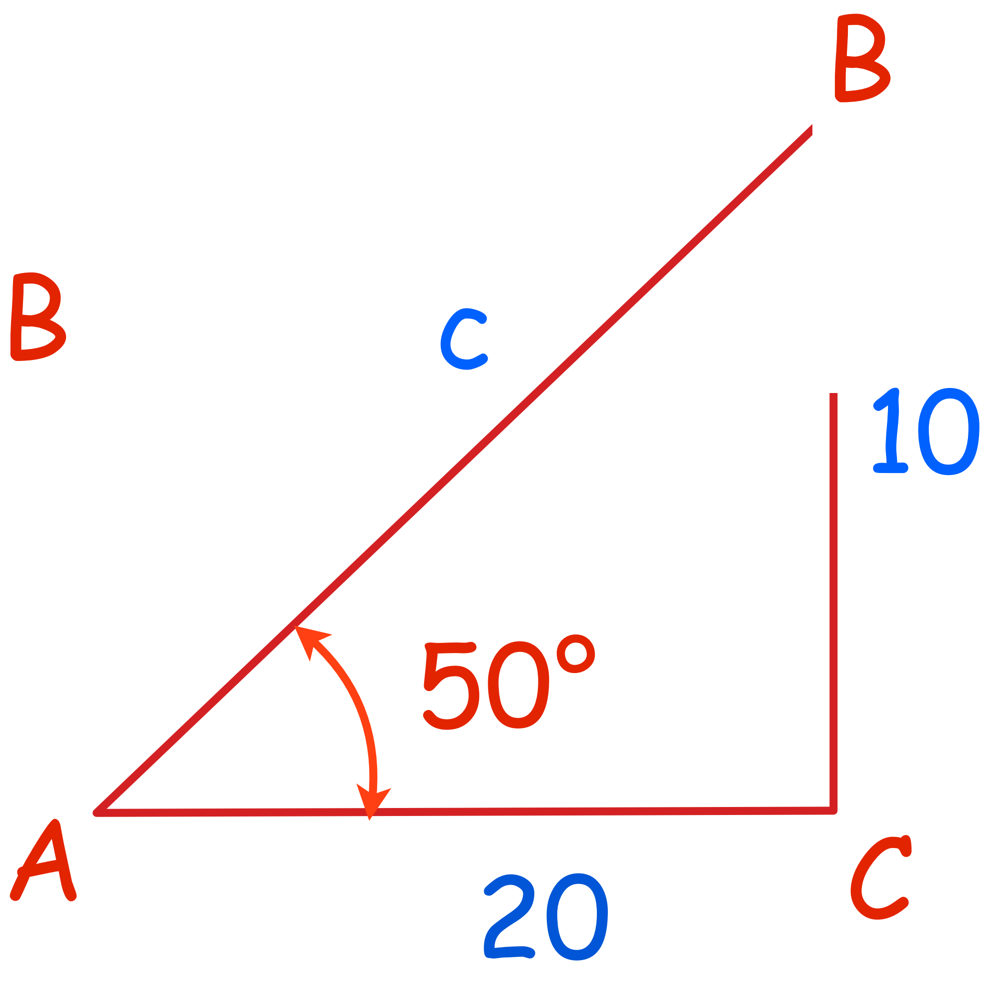


Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

- Solve triangle ABC if $A = 50°$, $a = 10$, and $b = 20$.
  - If you draw the figure accurately with a protractor you will discover there is no possible triangle.





$$\frac{10}{\sin 50^\circ} = \frac{20}{\sin B} = \frac{c}{\sin c}$$

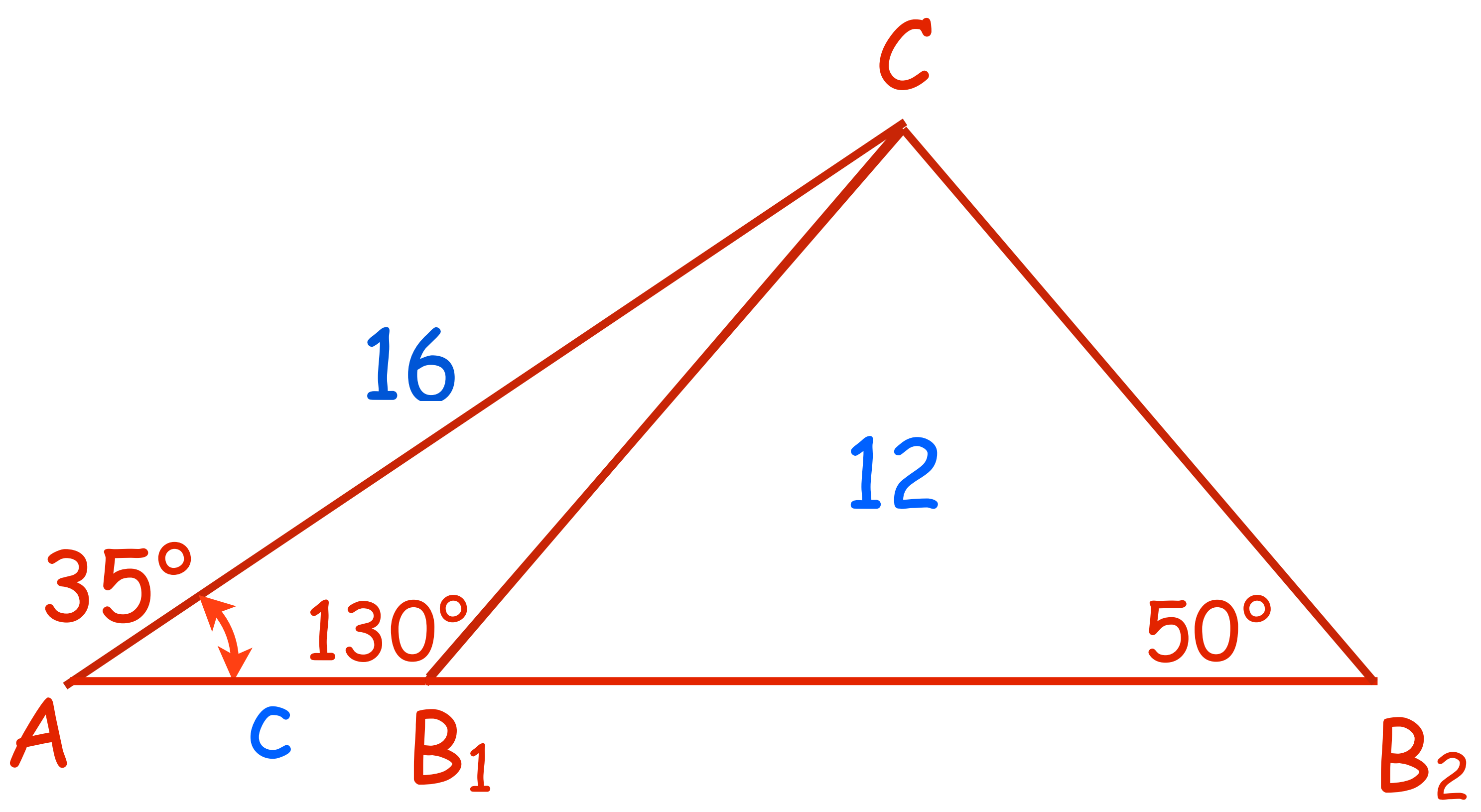
$$\frac{10}{\sin 50^\circ} = \frac{20}{\sin B}$$

$$\sin B = \frac{20 \sin 50^\circ}{10} = 1.532$$

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

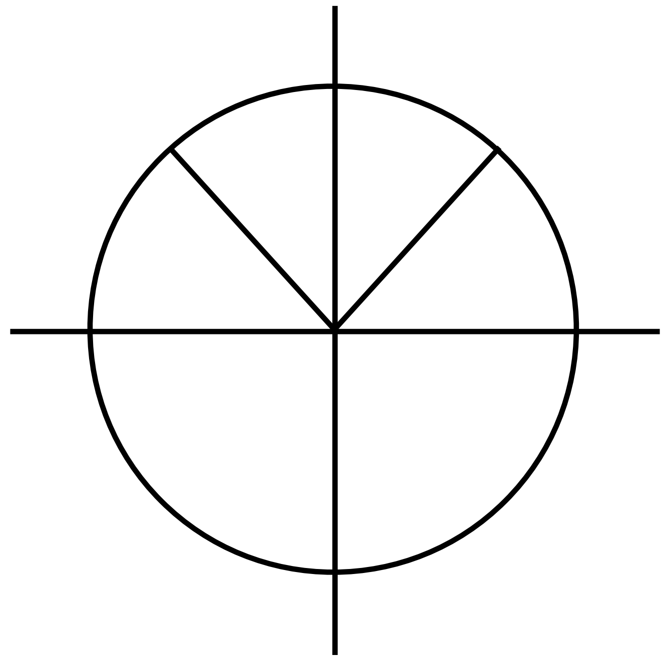
Solve ΔABC if $A = 35°$, $a = 12$, and $b = 16$. (SSA) Round lengths of sides to the nearest tenth and angle measures to the nearest degree.



5.4

20.8

$$\frac{12}{\sin 35°} = \frac{16}{\sin B_{1/2}} = \frac{c}{\sin C} \qquad \frac{12}{\sin 35°} = \frac{16}{\sin B_{1/2}}$$



$$\sin B_{1/2} = \frac{16 \sin 35°}{12} = .76476858$$

$$B_{1/2} = \sin^{-1} .76476858 = 49.8864 \approx 50° \text{ or } 180 - 50 = 130°$$

$C = 180 - 35 - 130 = 15°$

$$\frac{12}{\sin 35°} = \frac{c}{\sin 15°} \qquad c = \frac{12 \sin 15°}{\sin 35°} \approx 5.4$$

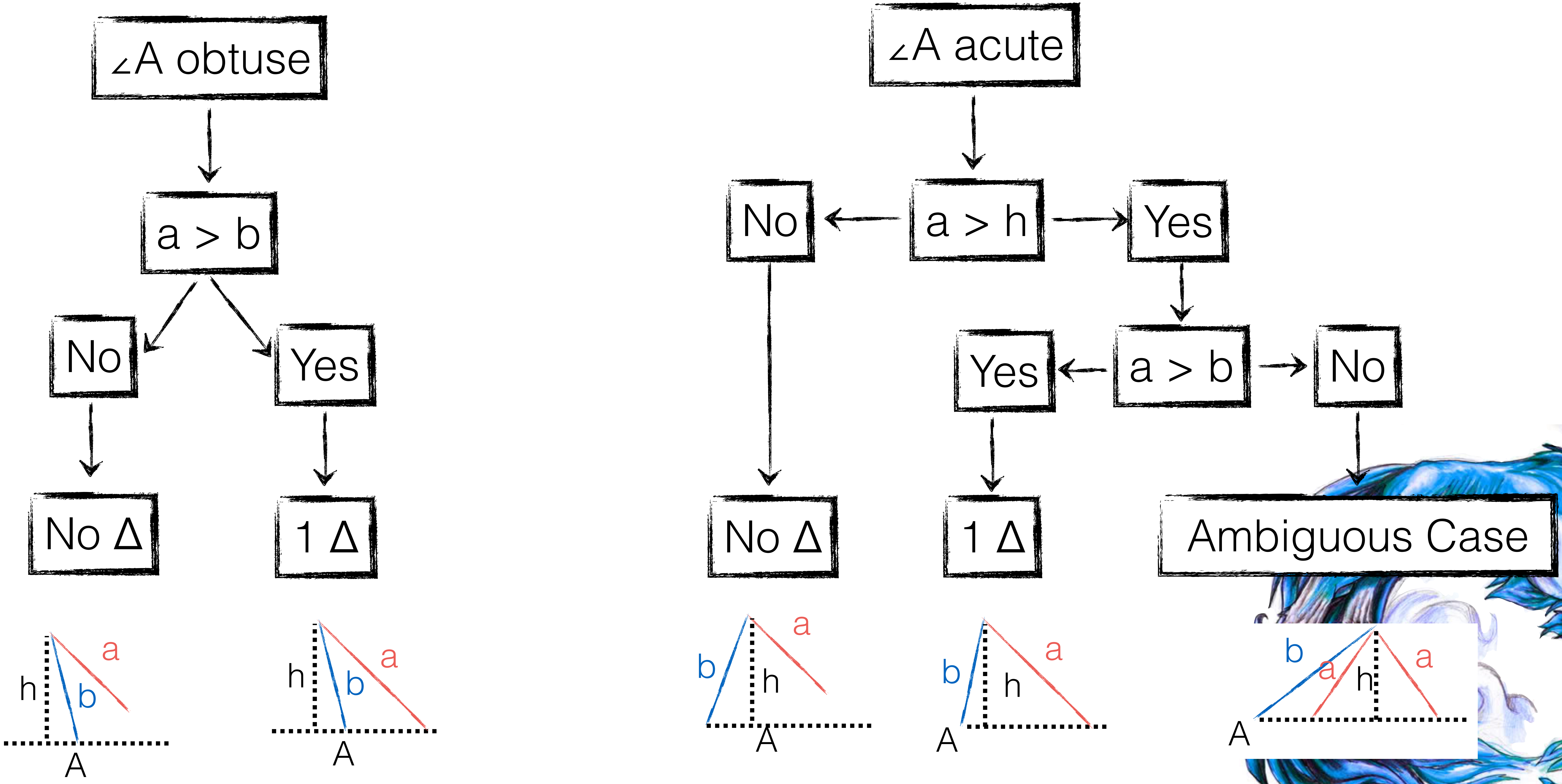
$C = 180 - 35 - 50 = 95°$

$$\frac{12}{\sin 35°} = \frac{c}{\sin 95°} \qquad c = \frac{12 \sin 95°}{\sin 35°} \approx 20.8$$

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

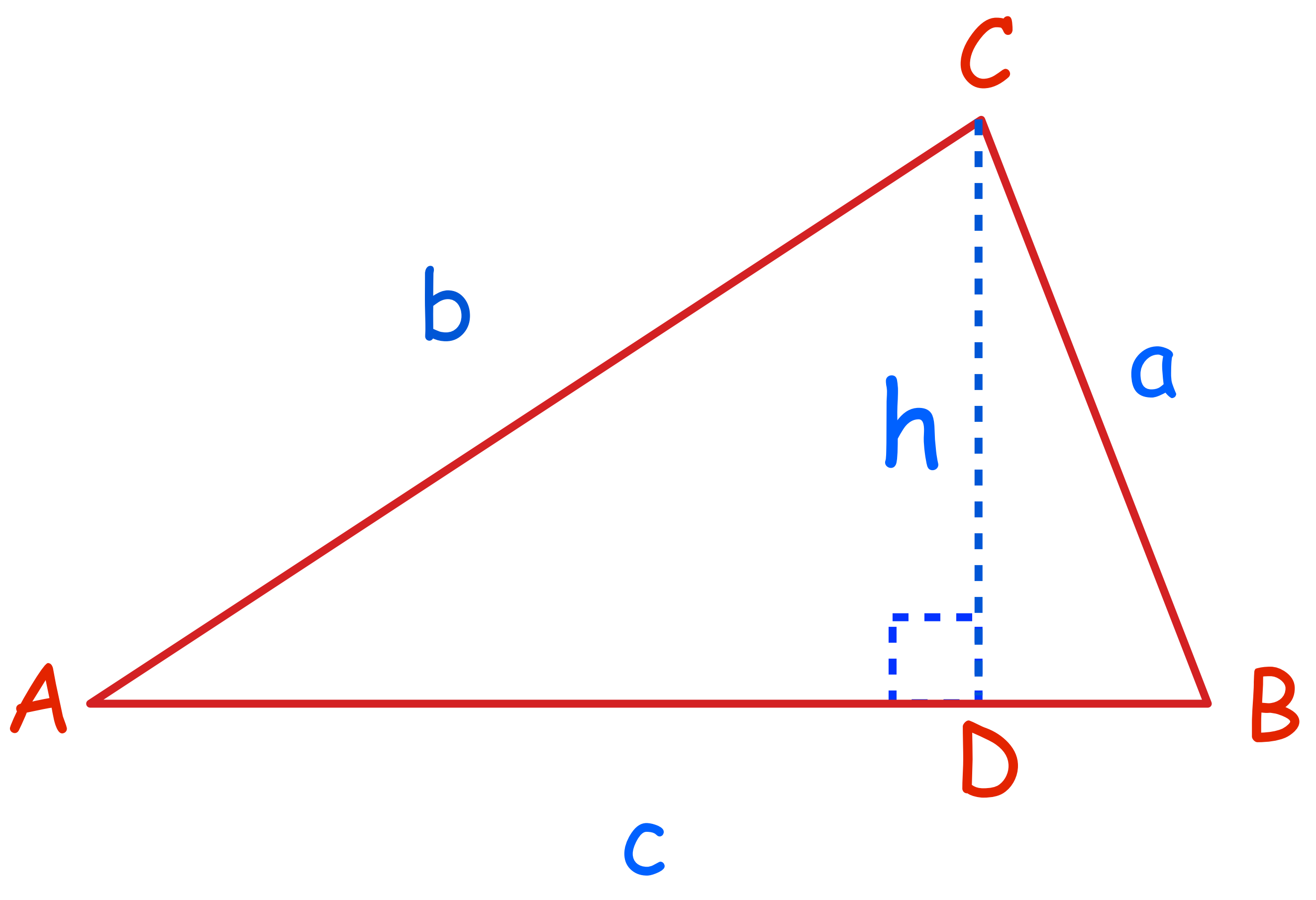
Solving an Oblique Triangle Using Law of Sines - SSA



Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

- The area of a triangle equals one-half the product of the lengths of two sides times the sine of their included angle. In the figure, this wording can be expressed by the formulas



$$Area = \frac{1}{2}bcSinA = \frac{1}{2}acSinB = \frac{1}{2}abSinC$$

$$\frac{h}{\sin A} = \frac{b}{\sin 90°} \qquad h = \frac{b\sin A}{1}$$

$$Area = \frac{1}{2} \bullet \frac{b\sin A}{1} \bullet c = \frac{1}{2}bc\sin A$$

$$Area = \frac{1}{2}hc$$

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

- From your book

Area of an Oblique Triangle

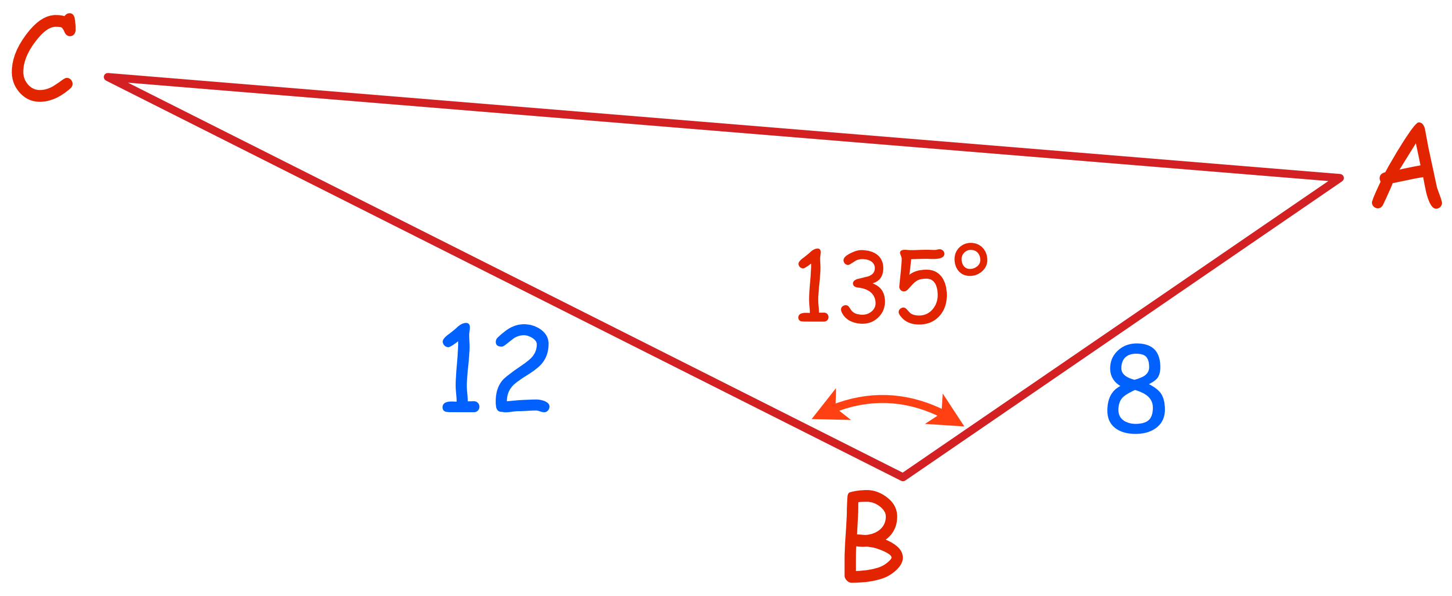
The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,

$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B.$$

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

Find the area of a triangle having two sides of length 8 meters and 12 meters and an included angle of 135°. Round to the nearest square meter.



$$Area = \frac{1}{2}acSinB = \frac{1}{2} \bullet 12 \bullet 8 \bullet \sin 135$$

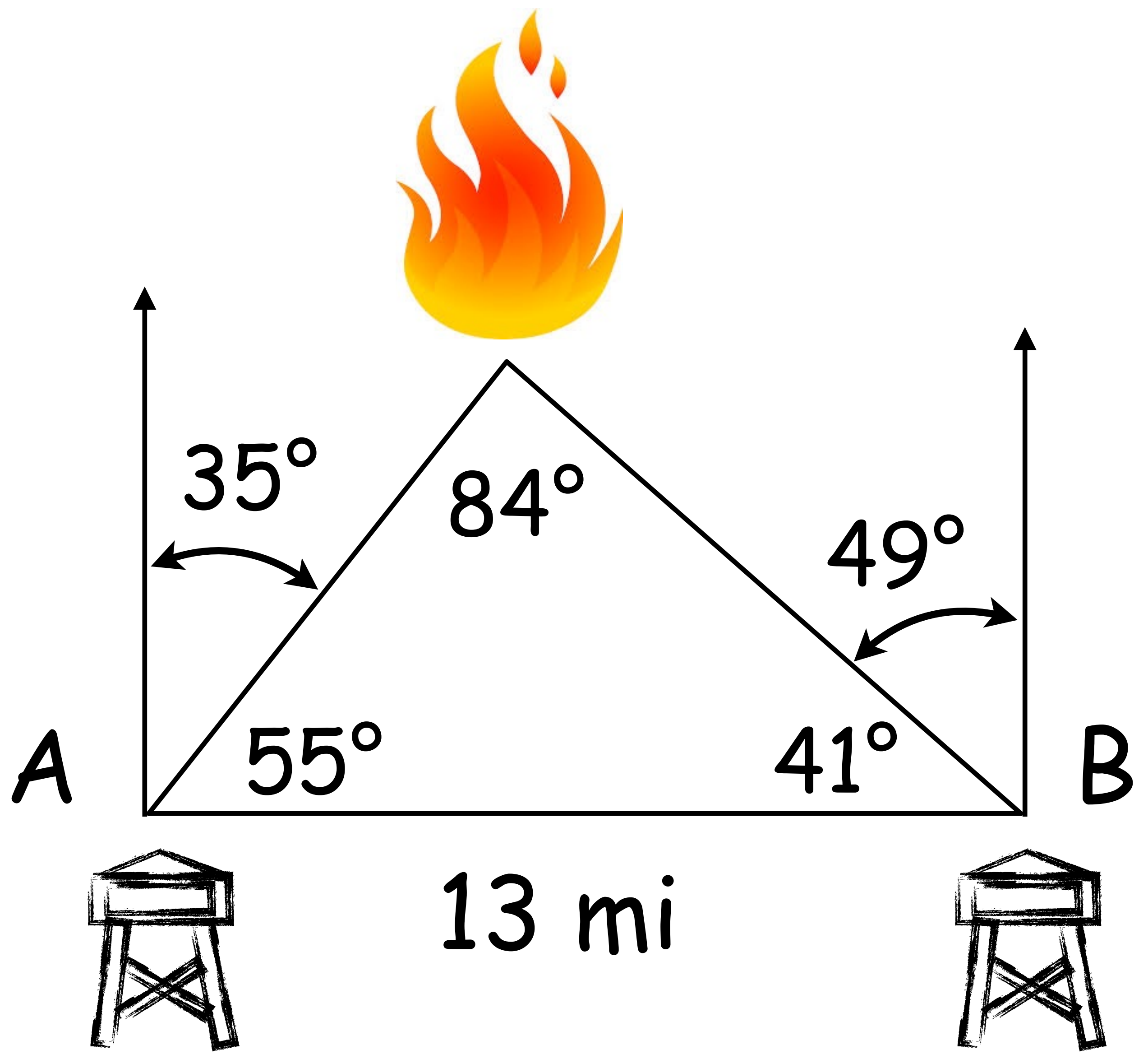
$$\approx \frac{1}{2} \bullet 12 \bullet 8 \bullet .7071$$

$$\approx 33.94 \approx 34\,m^2$$

Law of Sines

Students will know how to use the Law of Sines to solve and find the areas of oblique triangles.
Use the Law of Sines to solve, if possible, the triangle or triangles in the ambiguous case.

Two fire-lookout stations are 13 miles apart, with station B directly east of station A. Both stations spot a fire. The bearing of the fire from station A is N 35° E and the bearing of the fire from station B is N 49° W. How far, to the nearest tenth of a mile, is the fire from station B?



$C = 180 - 55 - 41 = 84°$

$$\frac{a}{\sin 55^\circ} = \frac{13}{\sin 84}$$

$$a = \frac{13 \sin 55^\circ}{\sin 84} \approx 10.7$$

The fire is about 11 miles from station B.