Chapter 6

6.3 Vectors

Chapter 6.3

Homework

- Read Sec 6.3
- Do p457 1, 3, 9, 11, 21, 25, 33, 39, 43, 47, 49, 53, 59, 65, 71, 85

Chapter 6.3

Objectives:

- Use magnitude and direction to show vectors are equal.
- Visualize scalar multiplication, vector addition, and vector subtraction as geometric vectors.
- Represent vectors in the rectangular coordinate system.
- Perform operations with vectors in terms of **i** and **j**.
- Find the unit vector in the direction of **v**.
- Write a vector in terms of its magnitude and direction.
- Solve applied problems involving vectors.

Reminder

Draw a Picture

Vectors

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

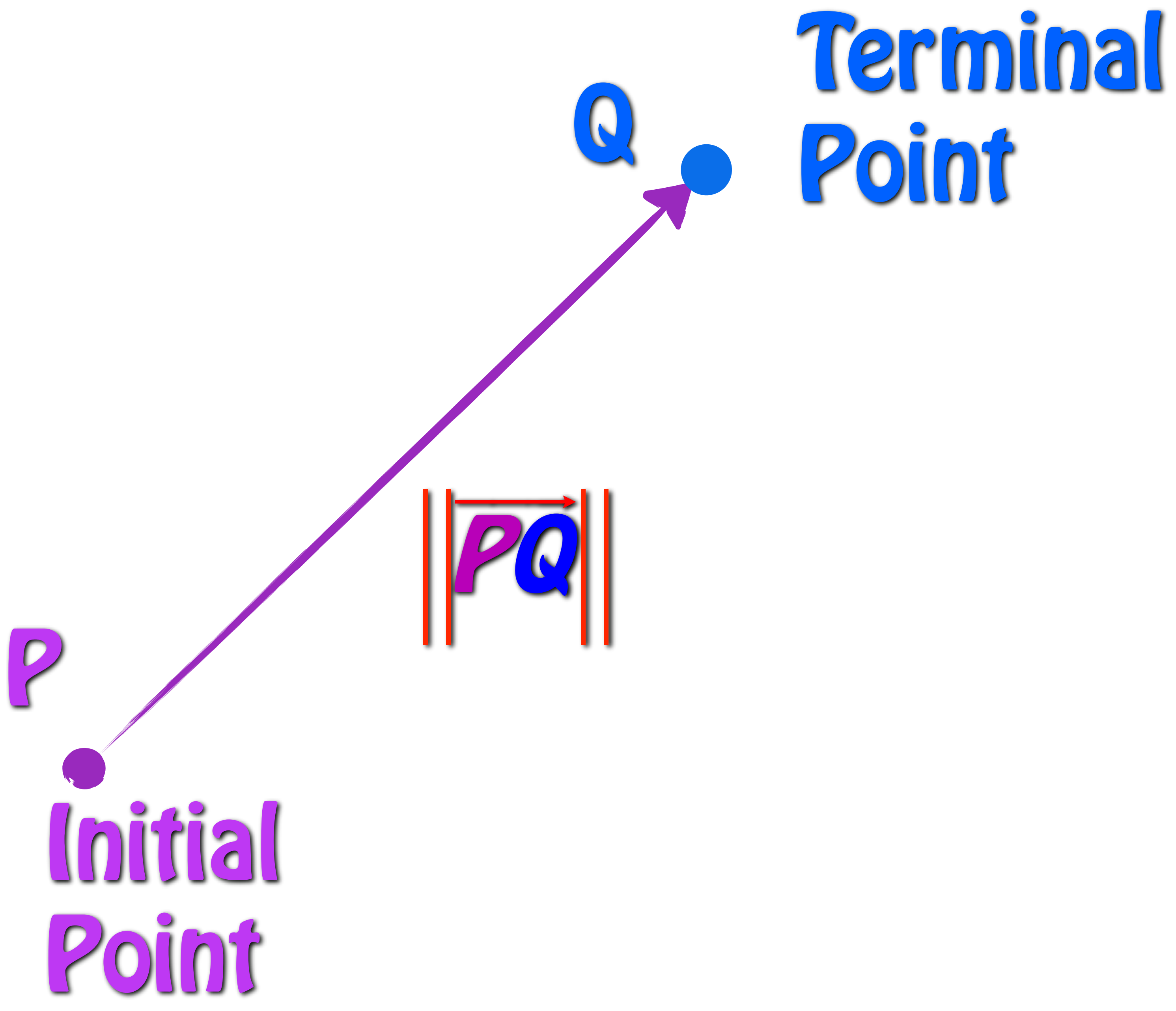
- Quantities that involve both a **magnitude** and a **direction** are called **vector quantities**, or **vectors** for short.

- Quantities that involve magnitude, but **no direction**, are called **scalar quantities**, or **scalars** for short.

Directed Line Segments and Geometric Vectors

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

A line segment to which a **direction** has been assigned is called a **directed line segment**. We call P the initial point and Q the terminal point. We denote this directed line segment by $\overrightarrow{PQ}$.



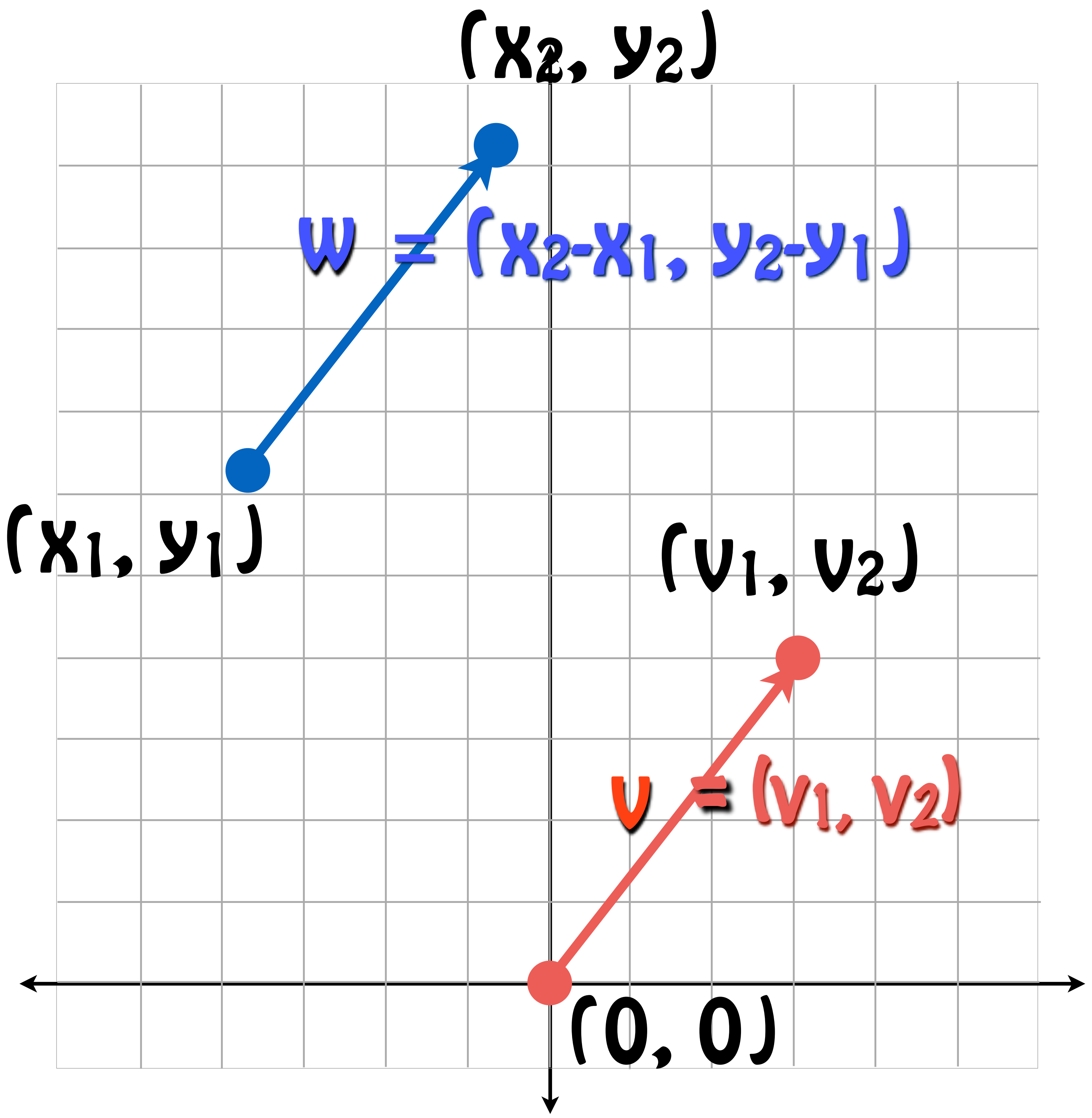
The magnitude of the directed line segment $\overrightarrow{PQ}$ is its length. We denote this by $\|\overrightarrow{PQ}\|$.

Geometrically, a **vector** is a directed line segment.

Standard Position

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- A vector is considered in **"standard position"** when the initial point is located at the origin on the coordinate plane.



A vector **v** in standard position, with terminal point $\langle v_1, v_2\rangle$ can be represented in **component form** as

$$v = \langle v_1, v_2\rangle.$$

Since the notation ⟨ and ⟩ are a pain to use, I will often use ( ).

A vector **w** **NOT** in standard position, with initial point (x_1, y_1) and terminal point (x_2, y_2) can be represented in **component form** as ...

$$w = \langle x_2-x_1, y_2-y_1\rangle = \langle \Delta x, \Delta y\rangle.$$

Magnitude

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- The magnitude of a vector $v = (v_1, v_2)$ in standard position is:

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}$$

- The magnitude of a vector $w = (x_2 - x_1, y_2 - y_1)$ not in standard position is:

$$\|\vec{w}\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

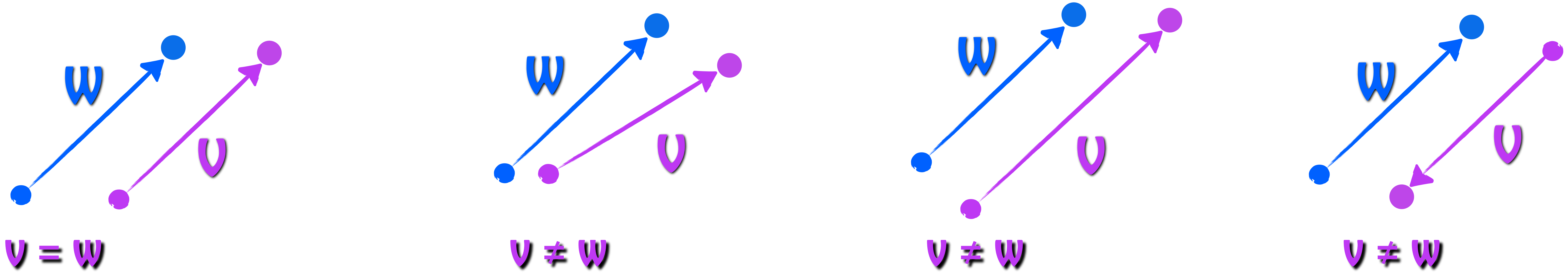
- You should recognize these as nothing more than applications of the distance formula (or Pythagorean Theorem).

Representing Vectors

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- We can also denote vectors with a single letter, like **w** or **v**. You may also see $\vec{w}$ or $\vec{v}$. The magnitude of **w** or **v** will be $\|\vec{w}\|$ or $\|\vec{v}\|$.

- In general, vectors **w** and **v** are equal if they have the **same magnitude and the same direction**. We write this **v** = **w**.



same magnitude and direction	same magnitude different direction	different magnitude same direction	different magnitude different direction

Component Form

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

Component Form of a Vector

The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is given by

$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}.$$

The **magnitude** (or length) of $\mathbf{v}$ is given by

$$\|\mathbf{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}.$$

If $\|\mathbf{v}\| = 1$, $\mathbf{v}$ is a **unit vector.** Moreover, $\|\mathbf{v}\| = 0$ if and only if $\mathbf{v}$ is the zero vector $\mathbf{0}$.

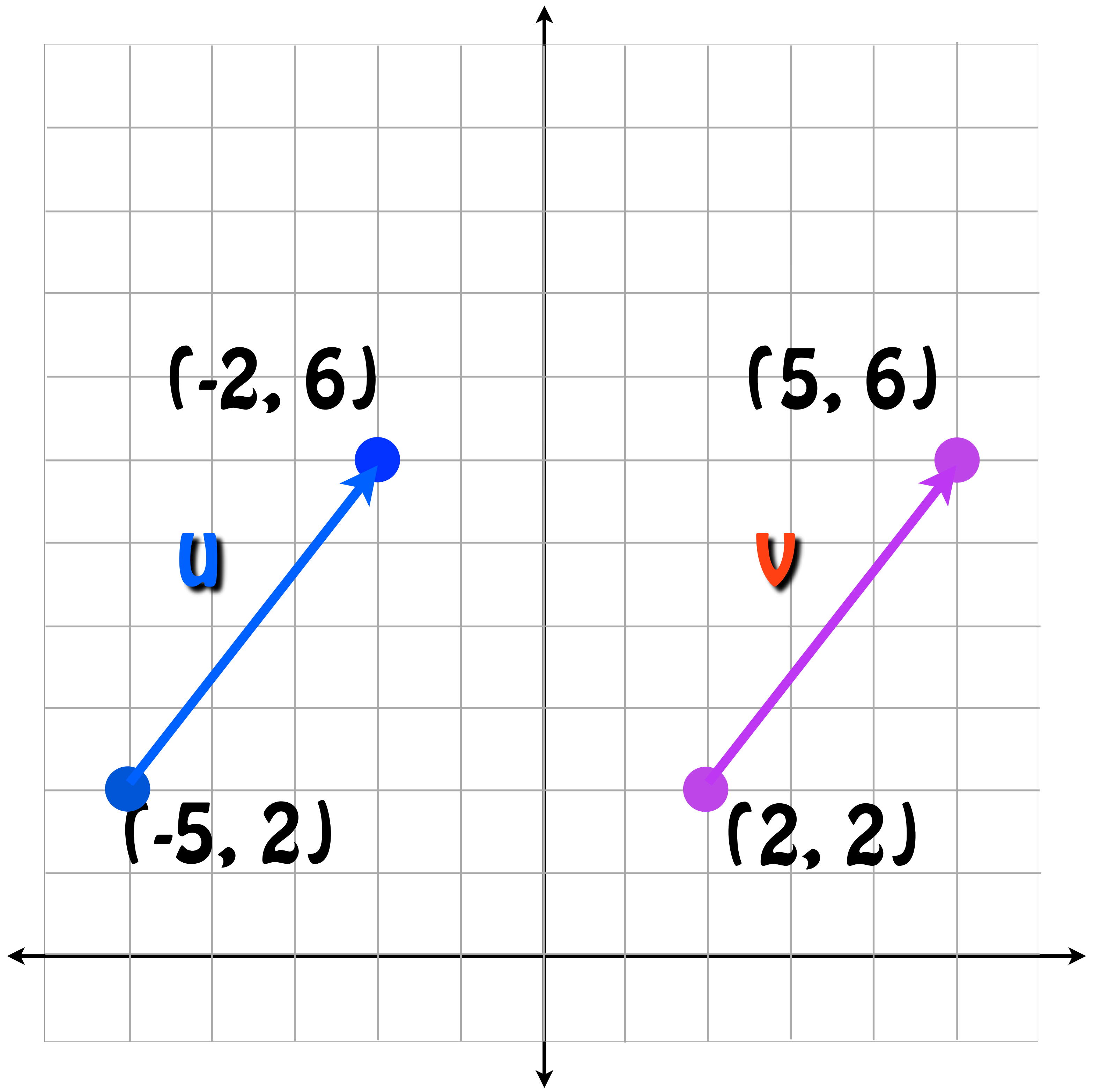
Showing that Two Vectors are Equal

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Show that **u** = **v**.

Equal vectors have the same magnitude and the same direction.

Use the distance formula to show that **u** and **v** have the same magnitude.



$$\|\vec{u}\| = \sqrt{(-2 - -5)^2 + (6-2)^2}$$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\|\vec{u}\| = 5$$

$$\|\vec{v}\| = \sqrt{(5-2)^2 + (6-2)^2}$$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\|\vec{v}\| = 5$$

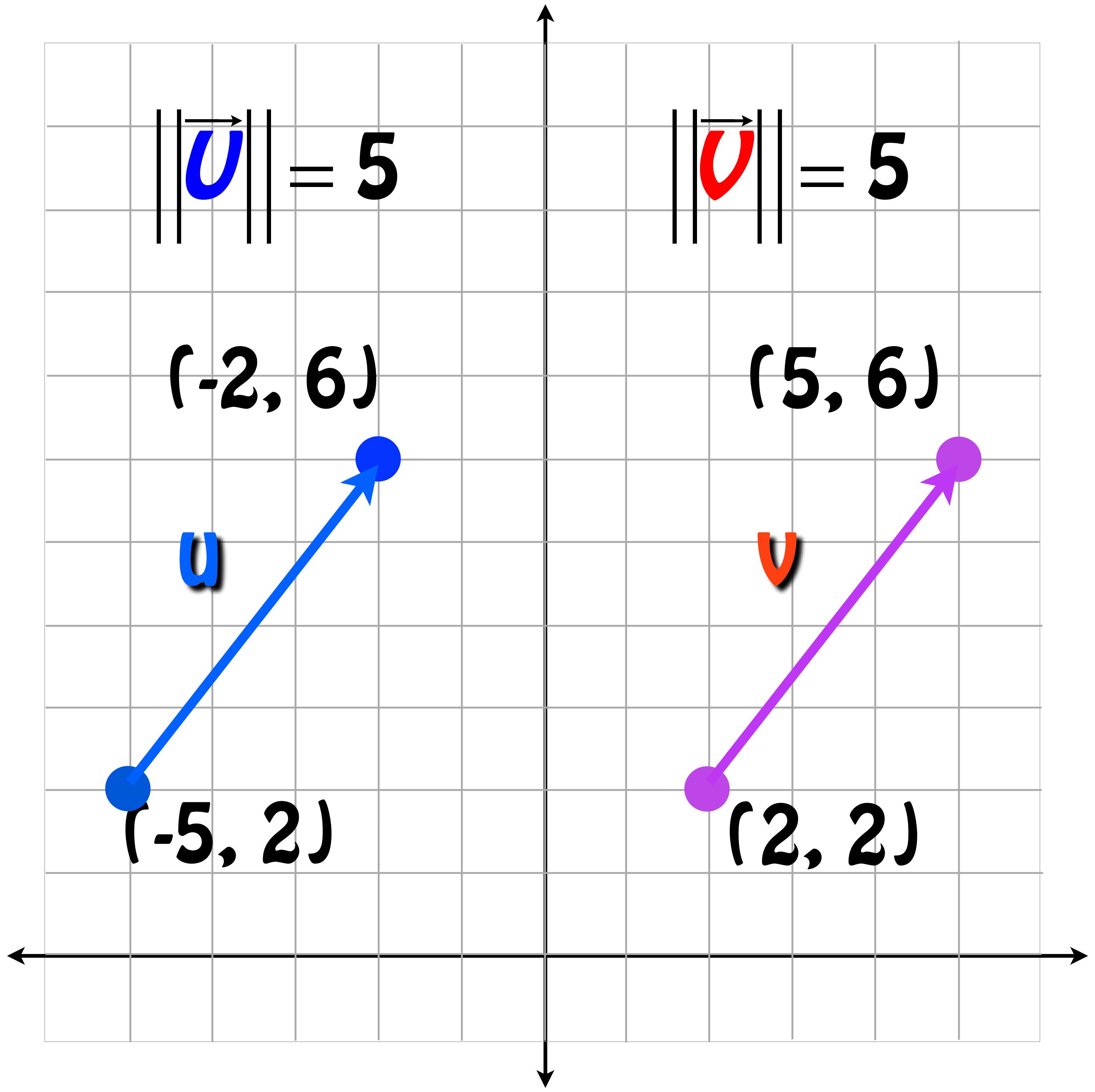
Showing that Two Vectors are Equal

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Show that **u** = **v**.

Equal vectors have the same magnitude and the same direction.

Use the slope formula to show that **u** and **v** have the same direction.



$$m_u = \frac{6-2}{-2--5} = \frac{4}{3}$$

$$m_v = \frac{6-2}{5-2} = \frac{4}{3}$$

- Since **u** and **v** have the same magnitude and direction, **u** = **v**.

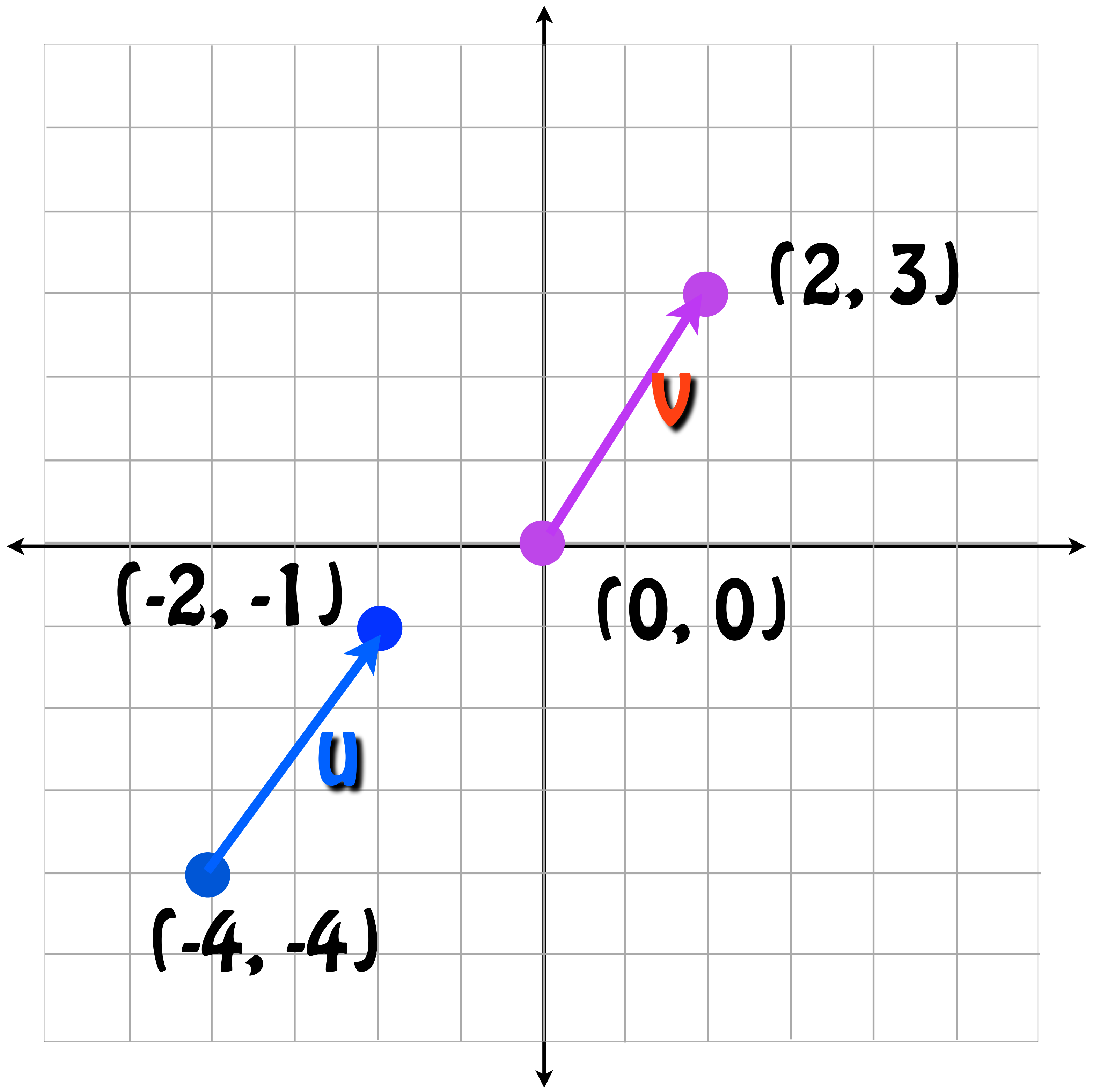
Showing that Two Vectors are Equal

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Show that **u** = **v**.

Equal vectors have the same magnitude and the same direction.

Use the distance formula to show that **u** and **v** have the same magnitude.



$$\|\vec{u}\| = \sqrt{(-2 - -4)^2 + (-1 - -4)^2}$$

$$= \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$\|\vec{u}\| = \sqrt{13}$$

$$\|\vec{v}\| = \sqrt{(2 - 0)^2 + (3 - 0)^2}$$

$$= \sqrt{2^2 + 3^2} = \sqrt{13}$$

$$\|\vec{v}\| = \sqrt{13}$$

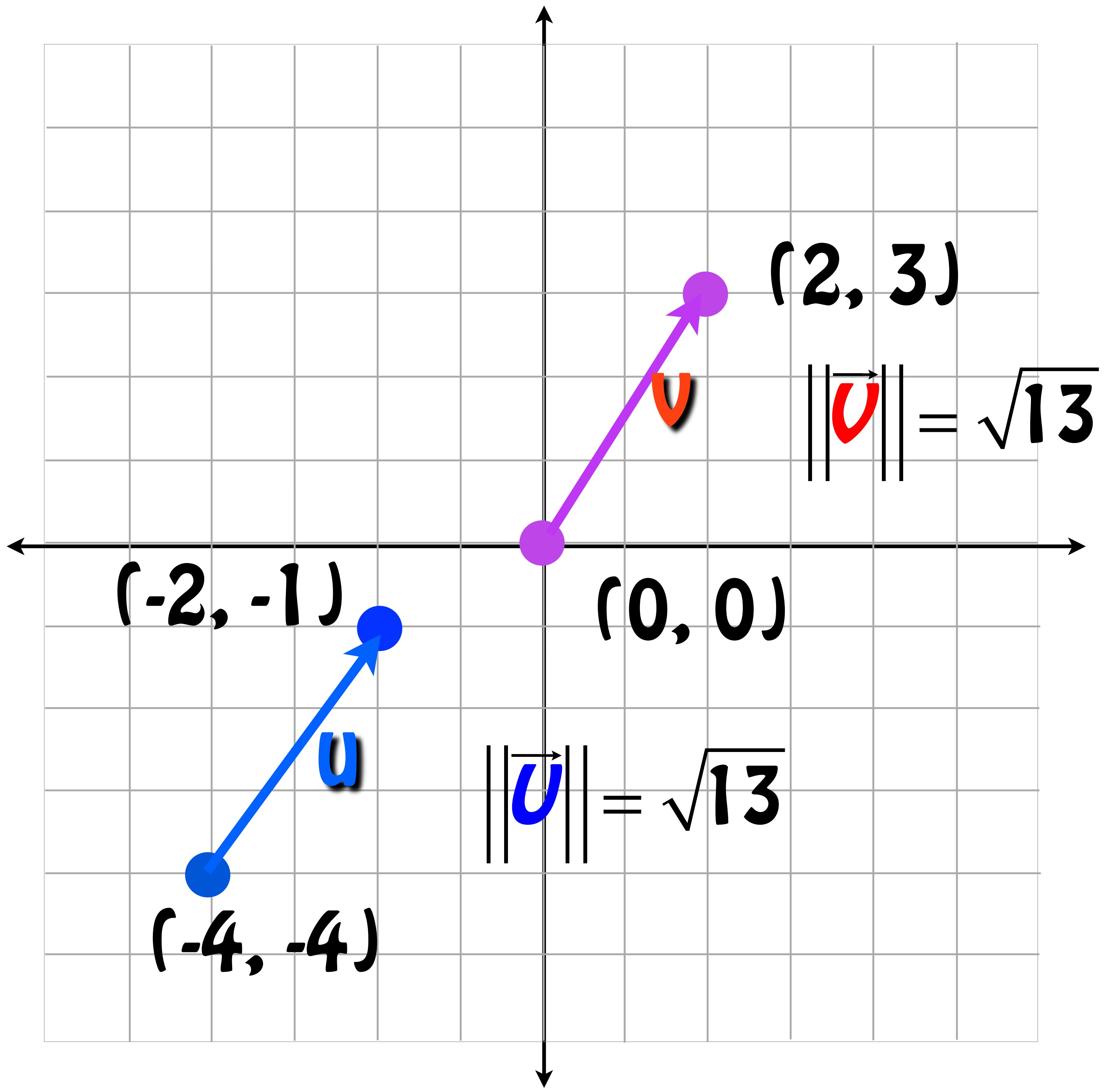
Showing that Two Vectors are Equal

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Show that **u** = **v**.

Equal vectors have the same magnitude and the same direction.

Use the slope formula to show that **u** and **v** have the same direction.



$$m_u = \frac{-1 - -4}{-2 - -4} = \frac{3}{2}$$

$$m_v = \frac{2 - 0}{3 - 0} = \frac{3}{2}$$

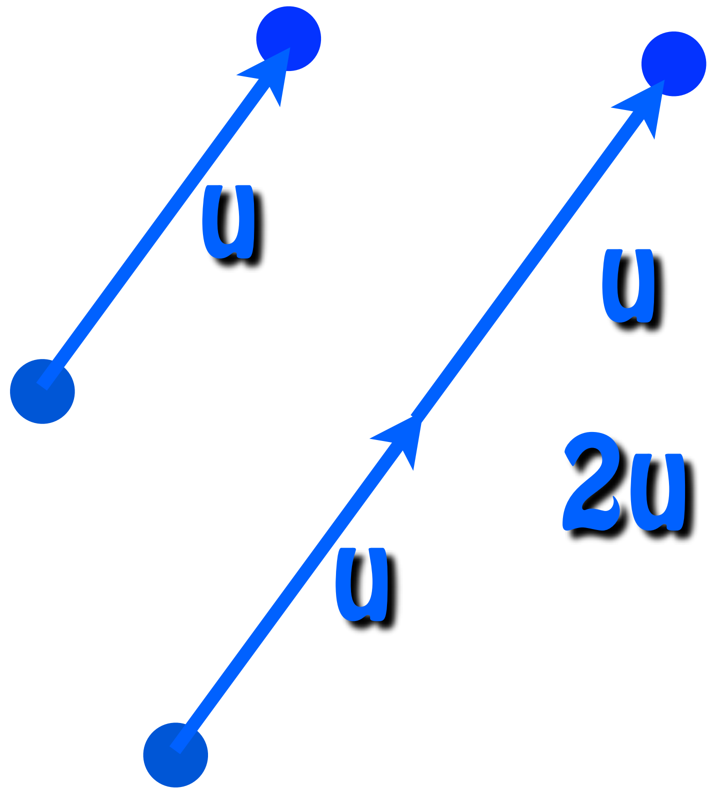
- Since **u** and **v** have the same magnitude and direction, **u** = **v**.

Scalar Multiplication

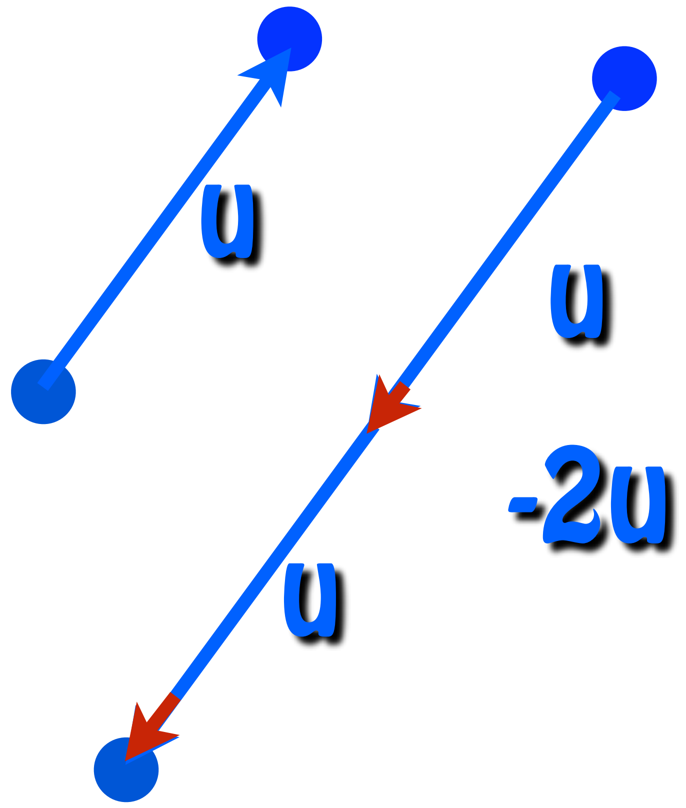
Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- The multiplication of a real number **k** and a vector **v** is called scalar multiplication. We write this product **kv**.

Multiplying a vector by any **positive** real number (except 1) changes the magnitude of the vector **but not its direction**.



Multiplying a vector by any **negative** number **reverses the direction** of the vector.



- The vector **kv** is called a scalar multiple of the vector **v**. The direction and magnitude of **kv** are:

Magnitude: $= k(v_1, v_2) = (kv_1, kv_2)$

$= |k| \, \|\vec{v}\|$

Direction: If $k < 0$, opposite direction of **v**,
If $k > 0$, same direction of **v**

Vector Addition

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- To add vectors, simply add the components.
  - If $u = \langle u_1, u_2 \rangle$ and $w = \langle w_1, w_2 \rangle$ then $u + w = \langle u_1 + w_1, u_2 + w_2 \rangle$
  - To subtract vectors, as has always been the case with subtraction, simply add the opposite. If $w = \langle a, b \rangle$ then $-w = -1w = \langle -a, -b \rangle$
    - If $u = \langle u_1, u_2 \rangle$ and $w = \langle w_1, w_2 \rangle$

      then $u - w = u + -w = \langle u_1 + -w_1, u_2 + -w_2 \rangle = \langle u_1 - w_1, u_2 - w_2 \rangle$.

$kv = k\langle v_1, v_2 \rangle = \langle kv_1, kv_2 \rangle \quad = |k| \|\vec{v}\|$

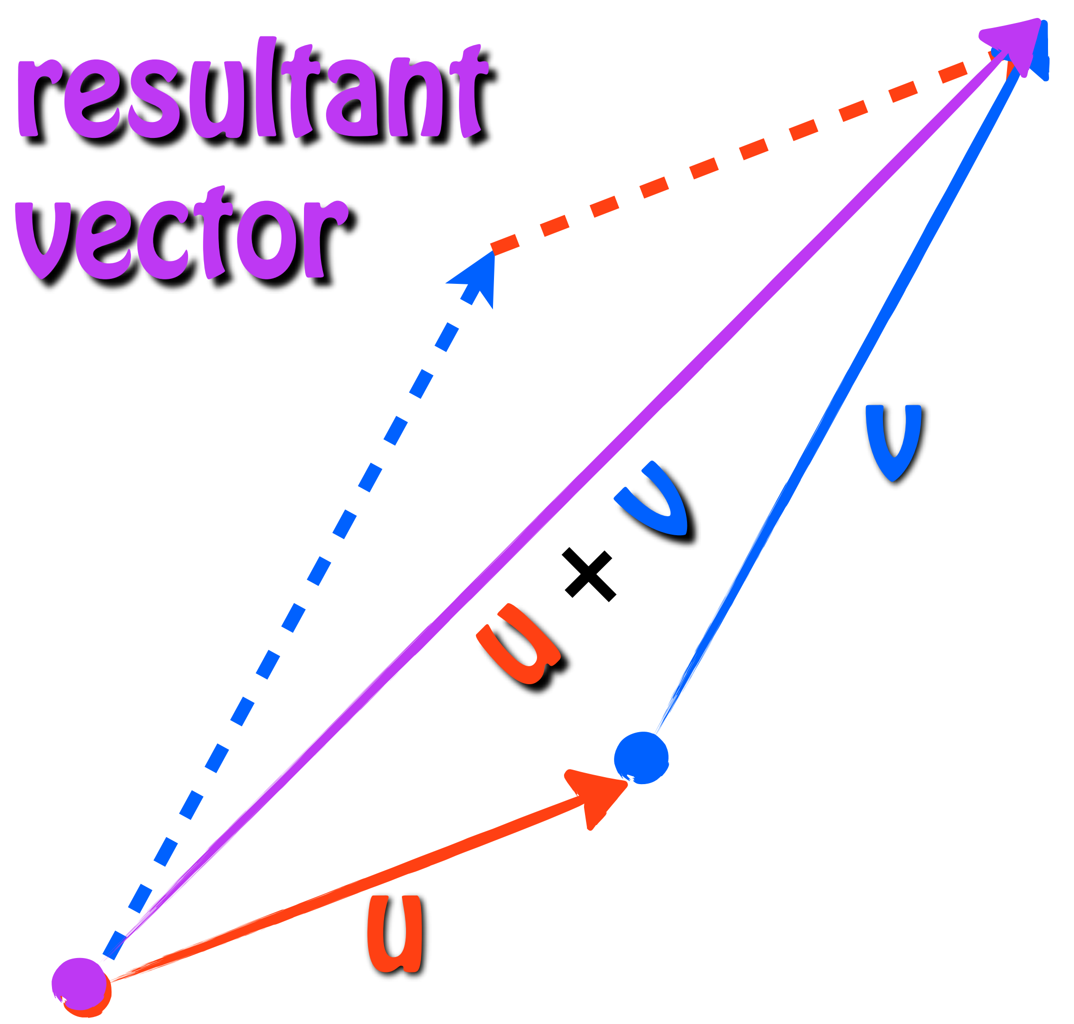
$u + w = \langle u_1 + w_1, u_2 + w_2 \rangle$

$u - w = \langle u_1 - w_1, u_2 - w_2 \rangle$

The Sum of Two Vectors

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- The **sum** of **u** and **v**, denoted **u** + **v** is called the **resultant vector**. A geometric method (nose to tail or triangle method) for adding two vectors is shown in the figure. Here is how we find this vector:



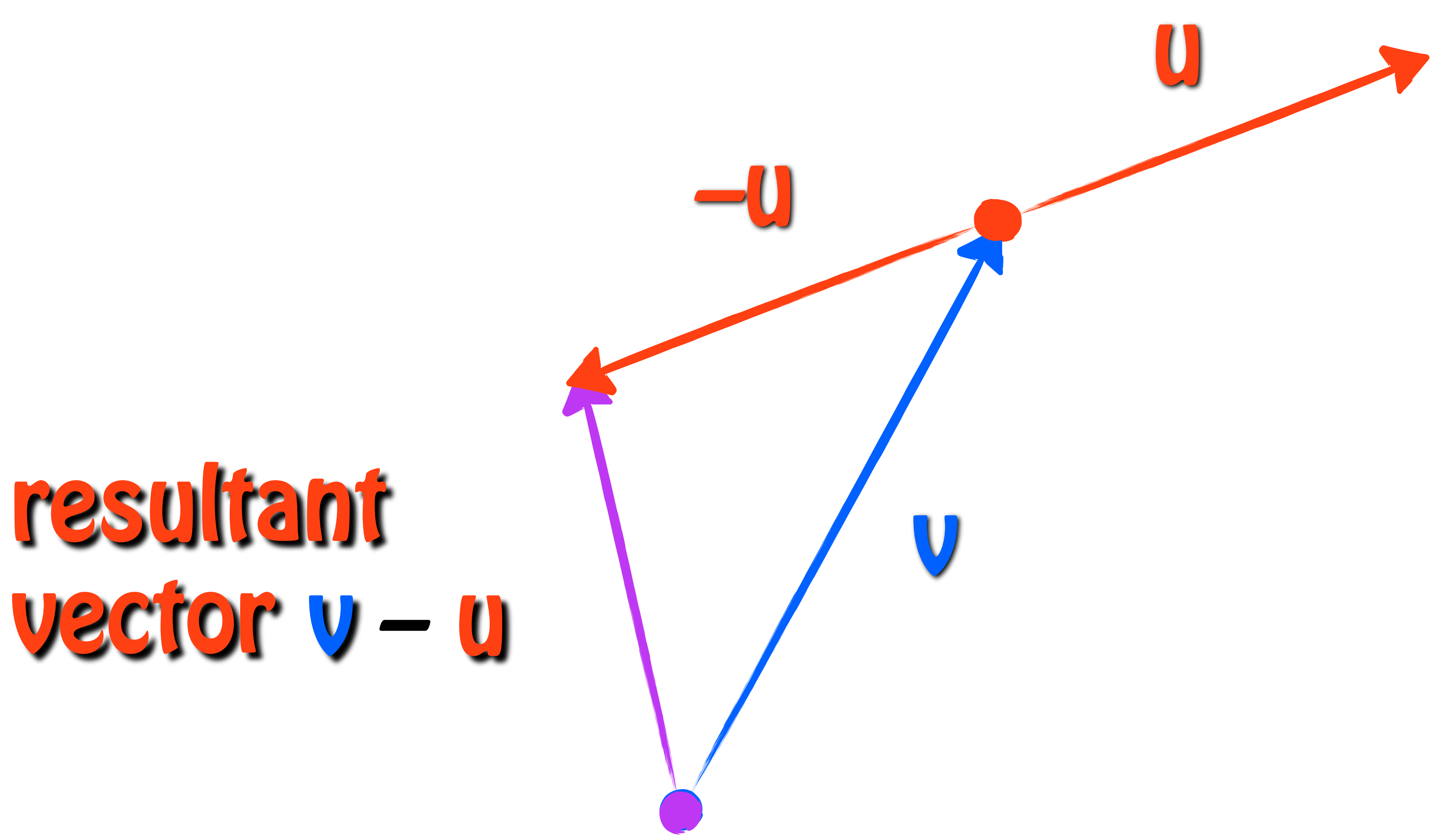
- Position **u** and **v**, so that the terminal point (nose) of **u** coincides with the initial point of **v** (tail).

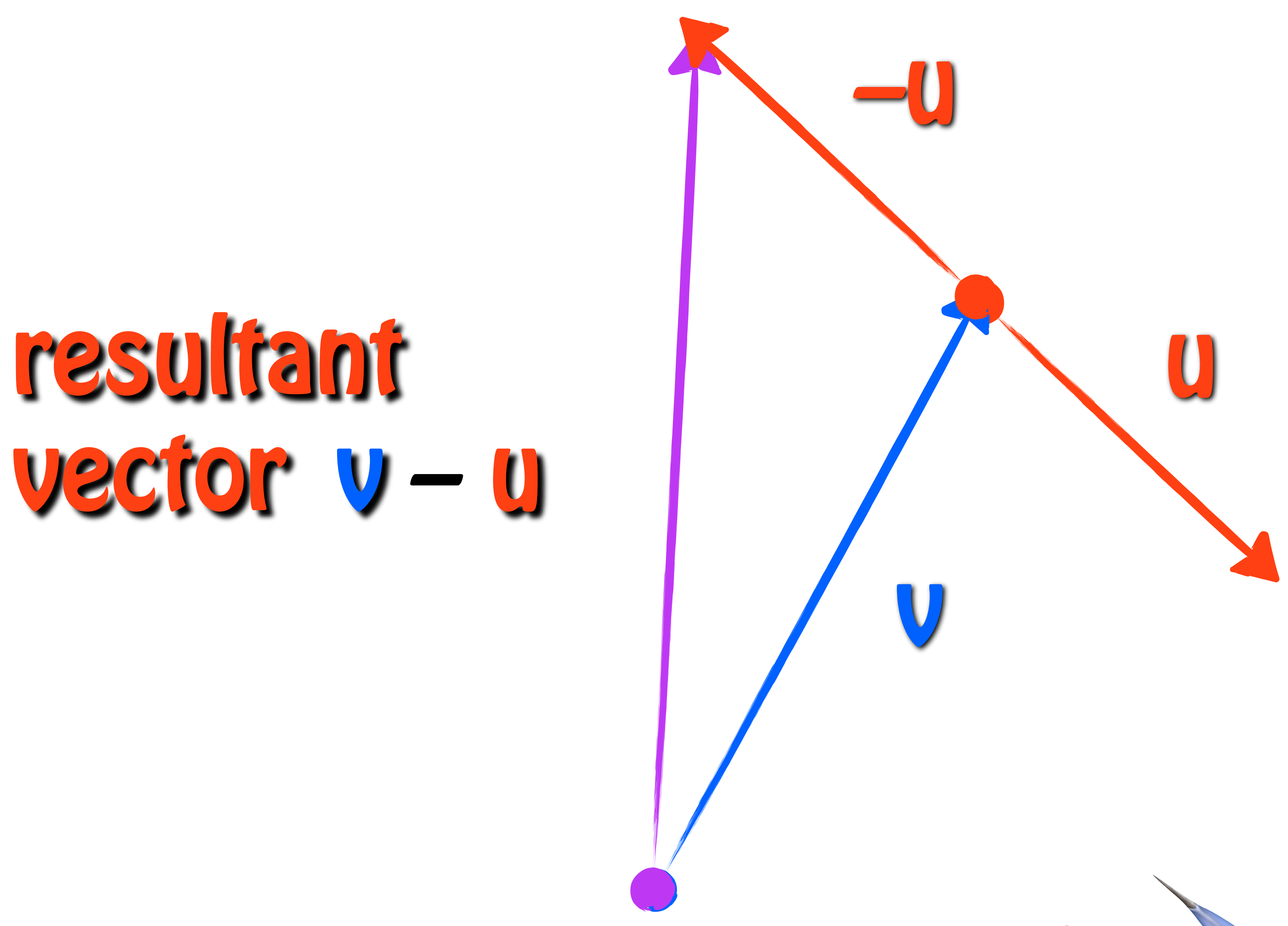
- The resultant vector, **u** + **v**, extends from the initial point of **u** to the terminal point of **v**.

The Difference of Two Vectors

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

The difference of two vectors, **v** – **u**, is defined as **v** – **u** = **v** + (– **u**), where – **u** is the scalar multiplication of **u** and –1, –1**u**. The difference **v** – **u** is shown geometrically in the figure.





Vector Operations

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

Definitions of Vector Addition and Scalar Multiplication

Let $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ be vectors and let k be a scalar (a real number). Then the *sum* of $\mathbf{u}$ and $\mathbf{v}$ is the vector

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2 \rangle \qquad \text{Sum}$$

and the *scalar multiple* of k times $\mathbf{u}$ is the vector

$$k\mathbf{u} = k\langle u_1, u_2 \rangle = \langle ku_1, ku_2 \rangle. \qquad \text{Scalar multiple}$$

Vector Operations

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Let $u = \langle -5, 2 \rangle$ and $v = \langle 6, -3 \rangle$

Keep in mind we are using vector component notation.

Find $4u$ $= 4\langle -5, 2 \rangle = \langle -20, 8 \rangle$

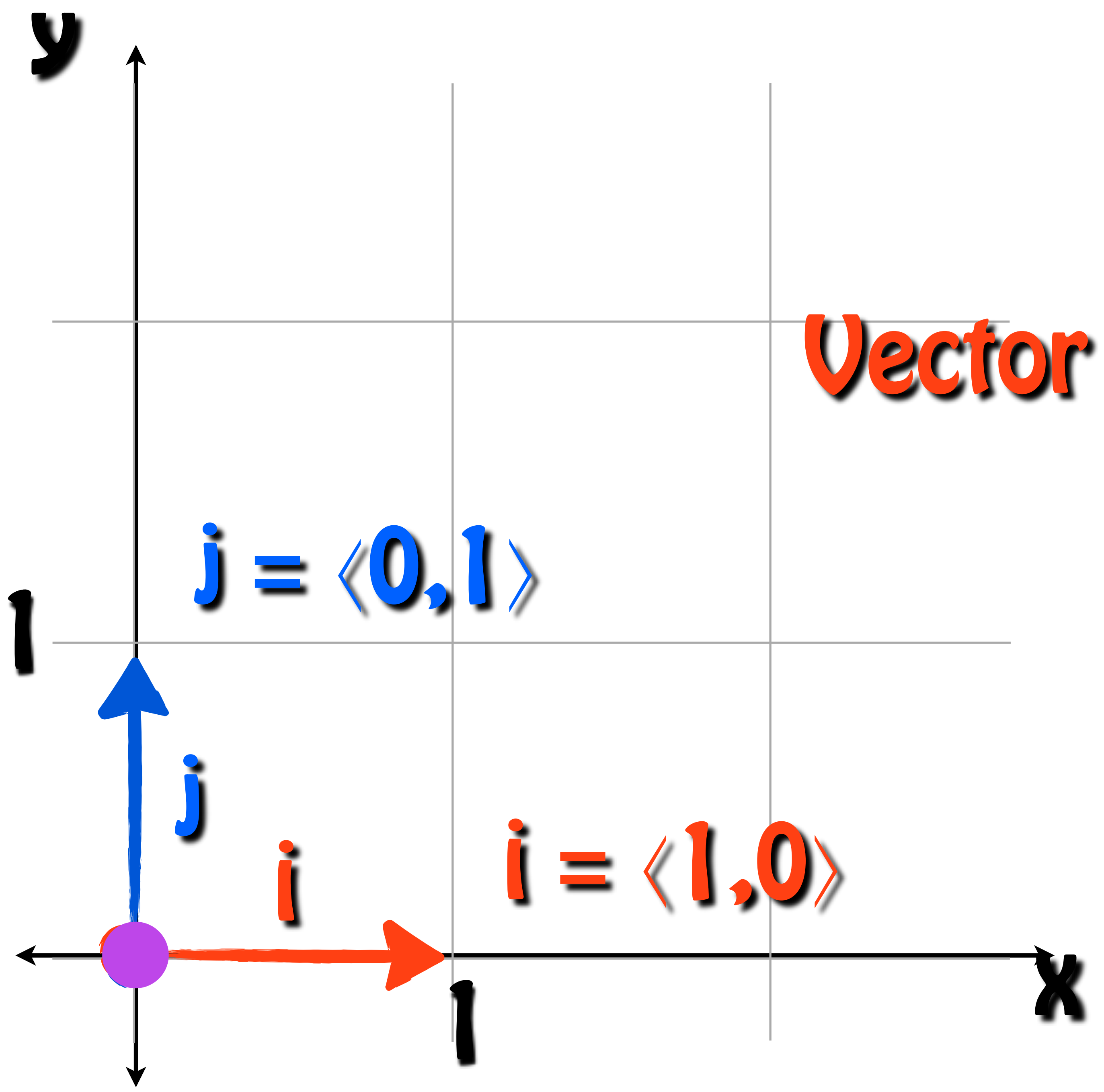
Find $u + v$ $= \langle -5+6, 2+-3 \rangle = \langle 1, -1 \rangle$

Find $2u - v$ $= 2\langle -5, 2 \rangle - \langle 6, -3 \rangle = \langle -10, 4 \rangle - \langle 6, -3 \rangle = \langle -10-6, 4--3 \rangle = \langle -16, 7 \rangle$

The i and j Unit Vectors

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Vectors can be represented using vectors positioned and identified along the axes of the coordinate plane.
  - A vector 1 unit in length along the positive x-axis is called Vector i.
  - A vector 1 unit in length along the positive y-axis is called Vector j.



Vector i and Vector j are unit vectors.

Representing Vectors in Rectangular Coordinates

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- We can now represent vector **u** as a sum of scalar multiples of **i** and **j**.

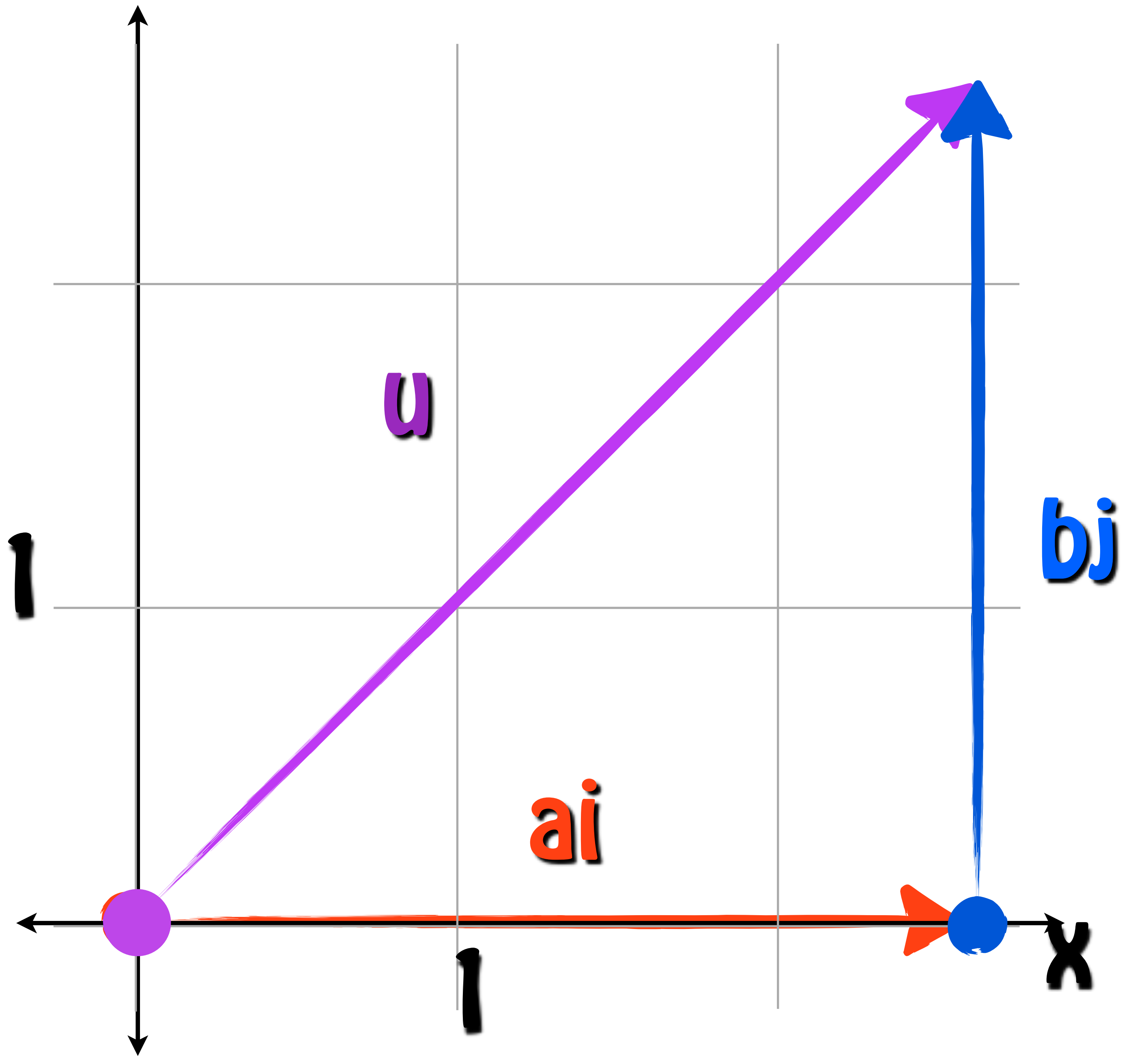
$$\mathbf{u} = \langle a, b \rangle \quad = \langle a, 0 \rangle + \langle 0, b \rangle \quad = a\langle 1, 0 \rangle + b\langle 0, 1 \rangle \quad = a\mathbf{i} + b\mathbf{j}$$

a and **b** are **scalar components** of **u**. $\quad \mathbf{u} = \langle a, b \rangle = a\mathbf{i} + b\mathbf{j}$

ai + bj is a **linear combination** of **i** and **j**.

a is the **horizontal component** of **u**.

b is the **vertical component** of **u**.

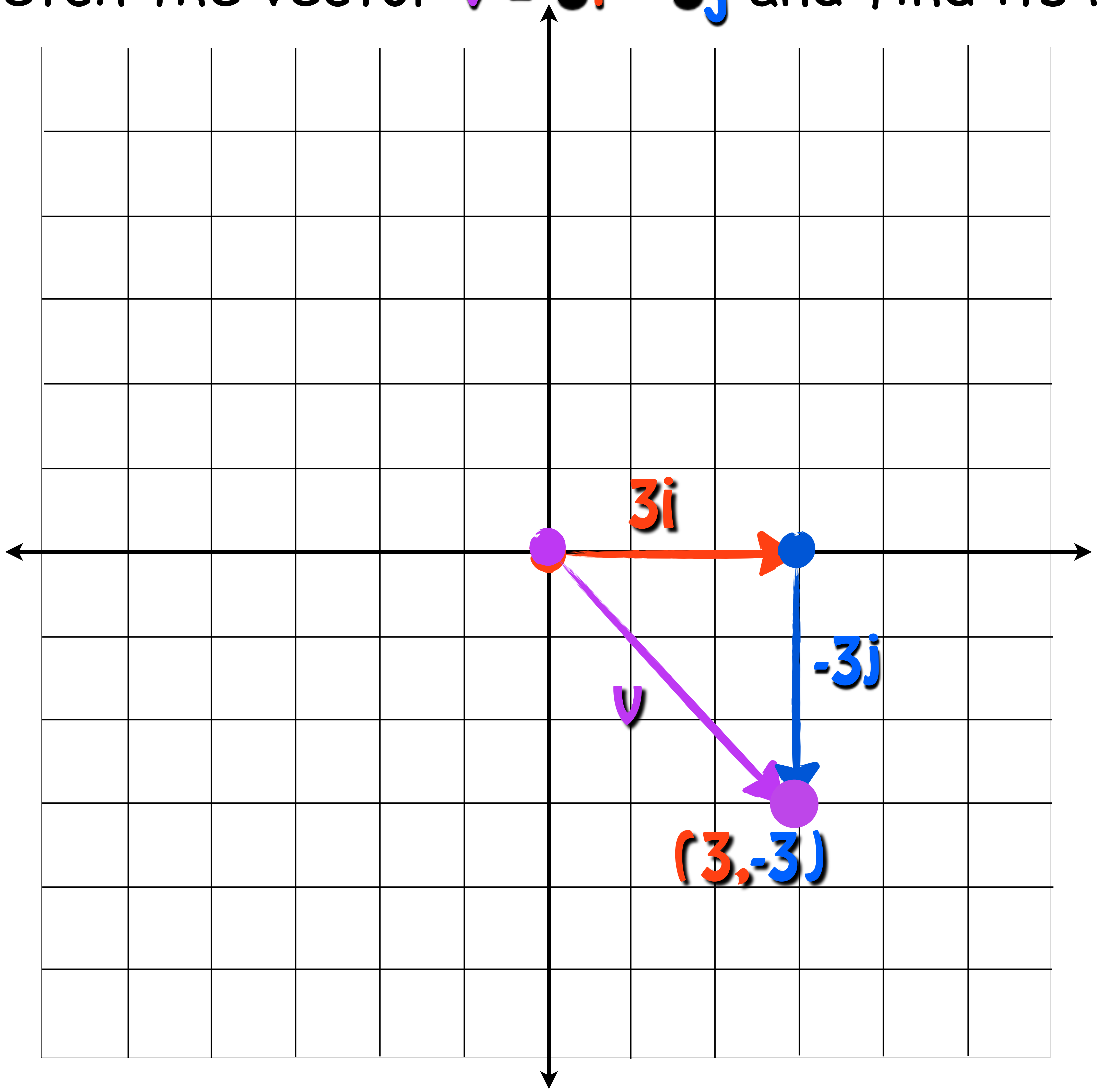


$$\left\|\vec{u}\right\| = \sqrt{a^2 + b^2}$$

Representing Vectors in Rectangular Coordinates

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

Sketch the vector $\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$ and find its magnitude.



$$\|\vec{v}\| = \sqrt{a^2 + b^2}$$

$$= \sqrt{3^2 + (-3)^2}$$

$$= \sqrt{18}$$

Representing Vectors in Rectangular Coordinates

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- We can turn this around by representing vector V with initial point $P_1(x_1,y_1)$ and terminal point $P_2(x_2,y_2)$ as:

$$v = (x_2 - x_1)i + (y_2 - y_1)j$$

- When subtracting, order is important. Be sure to subtract the initial point coordinates from the terminal point coordinates.

Representing Vectors in Rectangular Coordinates

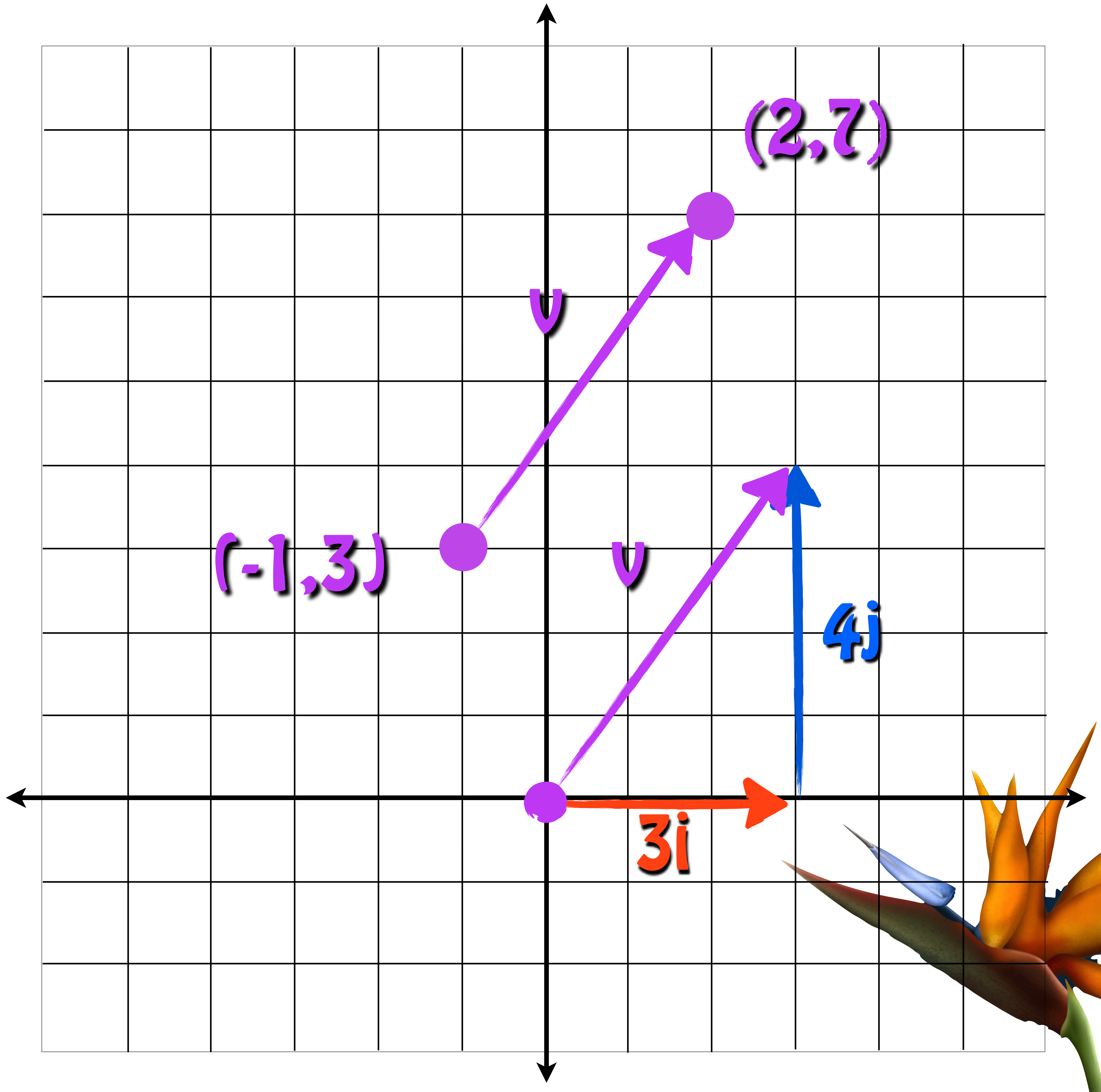
Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Let v be the vector from initial point $P_1 = (-1, 3)$ to terminal point $P_2 = (2, 7)$. Write v in terms of i and j.

$$v = (x_2 - x_1)i + (y_2 - y_1)j$$

$$v = (2 - -1)i + (7 - 3)j$$

$$v = 3i + 4j$$



Adding and Subtracting Vectors in Terms of i and j

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$ then:

$$\mathbf{v} + \mathbf{w} = (a_1 + a_2)\mathbf{i} + (b_1 + b_2)\mathbf{j}$$

$$\mathbf{v} - \mathbf{w} = (a_1 - a_2)\mathbf{i} + (b_1 - b_2)\mathbf{j}$$

- When subtracting, order is important. Be sure to subtract the initial point coordinates from the terminal point coordinates.

Adding and Subtracting Vectors in Terms of i and j

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

If $v = 7i + 3j$ and $w = 4i - 5j$, find the $v + w$, and $v - w$:

$v + w = (a_1 + a_2)i + (b_1 + b_2)j$

$v + w = (7+4)i + (3 + -5)j$

$v + w = 11i + -2j$

$v - w = (a_1 - a_2)i + (b_1 - b_2)j$

$v - w = (7-4)i + (3 - -5)j$

$v - w = 3i + 8j$

Scalar Multiplication with a Vector in Terms of i and j

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

If **v** = a**i** + b**j**, and k is a real number, then the scalar multiplication of vector **v** and scalar k is:

$$k\mathbf{v} = ka\mathbf{i} + kb\mathbf{j}$$

If **v** = 7**i** + 10**j**, find each of the following vectors:

a. 8**v**

$k\mathbf{v} = ka\mathbf{i} + kb\mathbf{j}$

$8\mathbf{v} = 8 \bullet 7\mathbf{i} + 8 \bullet 10\mathbf{j}$

$8\mathbf{v} = 56\mathbf{i} + 80\mathbf{j}$

b. –5**v**

$k\mathbf{v} = ka\mathbf{i} + kb\mathbf{j}$

$-5\mathbf{v} = -5 \bullet 7\mathbf{i} + -5 \bullet 10\mathbf{j}$

$-5\mathbf{v} = -35\mathbf{i} - 50\mathbf{j}$

The Zero Vector

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- If $\mathbf{v} = a\mathbf{i} + b\mathbf{j}$, and the magnitude of $\mathbf{v}$ is 0, then $\mathbf{v}$ is the zero vector, $\mathbf{0}$.

  The zero vector has no direction and can be written $\mathbf{0} = 0\mathbf{i} + 0\mathbf{j}$.

Properties of Vector Addition

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

Vector Addition Properties

If **u**, **v**, **w** are vectors, and k and l are scalars, then:

1.	$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$	Commutative Property of addition.
2.	$(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$	Associative Property of addition.
3.	$\mathbf{u} + \mathbf{0} = \mathbf{u}$	Additive Identity
4.	$\mathbf{u} + -\mathbf{u} = \mathbf{0}$	Additive Inverse

Properties of Scalar Multiplication

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

Scalar Multiplication Properties

If u, v, w are vectors, and k and l are scalars, then:

1. $k \cdot l \cdot u = k(l\, u) = (k\, l)\, u$ — Associative Property of multiplication.
2. $k(u+v) = k \cdot u + k \cdot v$ — Distributive Property
3. $(k + l)u = k \cdot u + l \cdot u$ — Distributive Property
4. $1u = u$ — Multiplicative Identity
5. $0u = 0$ — Multiplication Property of 0
6. $\|ku\| = |k|\,\|u\|$ — Scalar Magnitude Multiplication

Finding the Unit Vector that Has the Same Direction as a Given Nonzero Vector v

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- To find the unit vector **u**, with the same direction of vector $v = \langle v_1, v_2 \rangle$, but length 1, simply divide vector v by its magnitude.

$$u = \frac{v}{\|v\|} = \frac{1}{\|v\|}\langle v_1, v_2 \rangle = \frac{1}{\|v\|}\left(v_1 i + v_2 j\right)$$

u is the unit vector with the same direction as vector v.

Finding a Unit Vector

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Find the unit vector in the same direction as **v = 4i – 3j**. Then verify that the vector has magnitude 1.

$$\|v\| = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$$

$$\frac{4i - 3j}{5} = \frac{1}{5}(4i - 3j) = \frac{4}{5}i - \frac{3}{5}j$$

is the unit vector with the same direction as vector v.

$$\left\| \frac{4}{5}i - \frac{3}{5}j \right\| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{-3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = \sqrt{\frac{25}{25}} = 1$$

Direction Angle

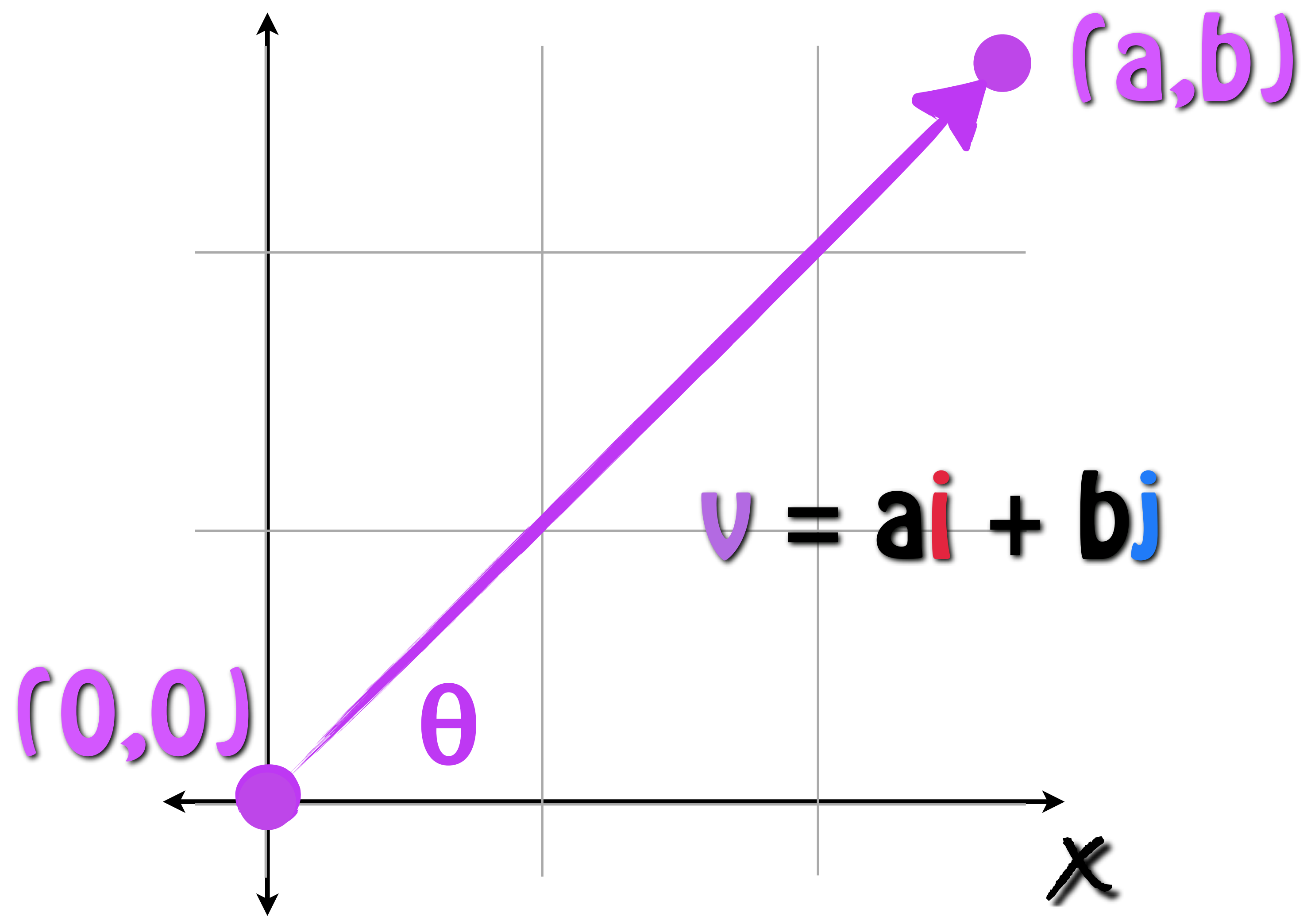
Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Let **v** = ⟨a, b⟩ = a**i** + b**j** be a non-zero vector in standard position. We can represent **v** in terms of magnitude and direction angle.
  - Further, let us stipulate that θ is a positive angle in standard position with terminal side at **v**.

The vector **v** = a**i** + b**j** can be represented as:

$$\mathbf{v} = \|\mathbf{v}\|\cos\theta\,\mathbf{i} + \|\mathbf{v}\|\sin\theta\,\mathbf{j}$$

$$\tan\theta = \frac{b}{a}$$



Direction Angle

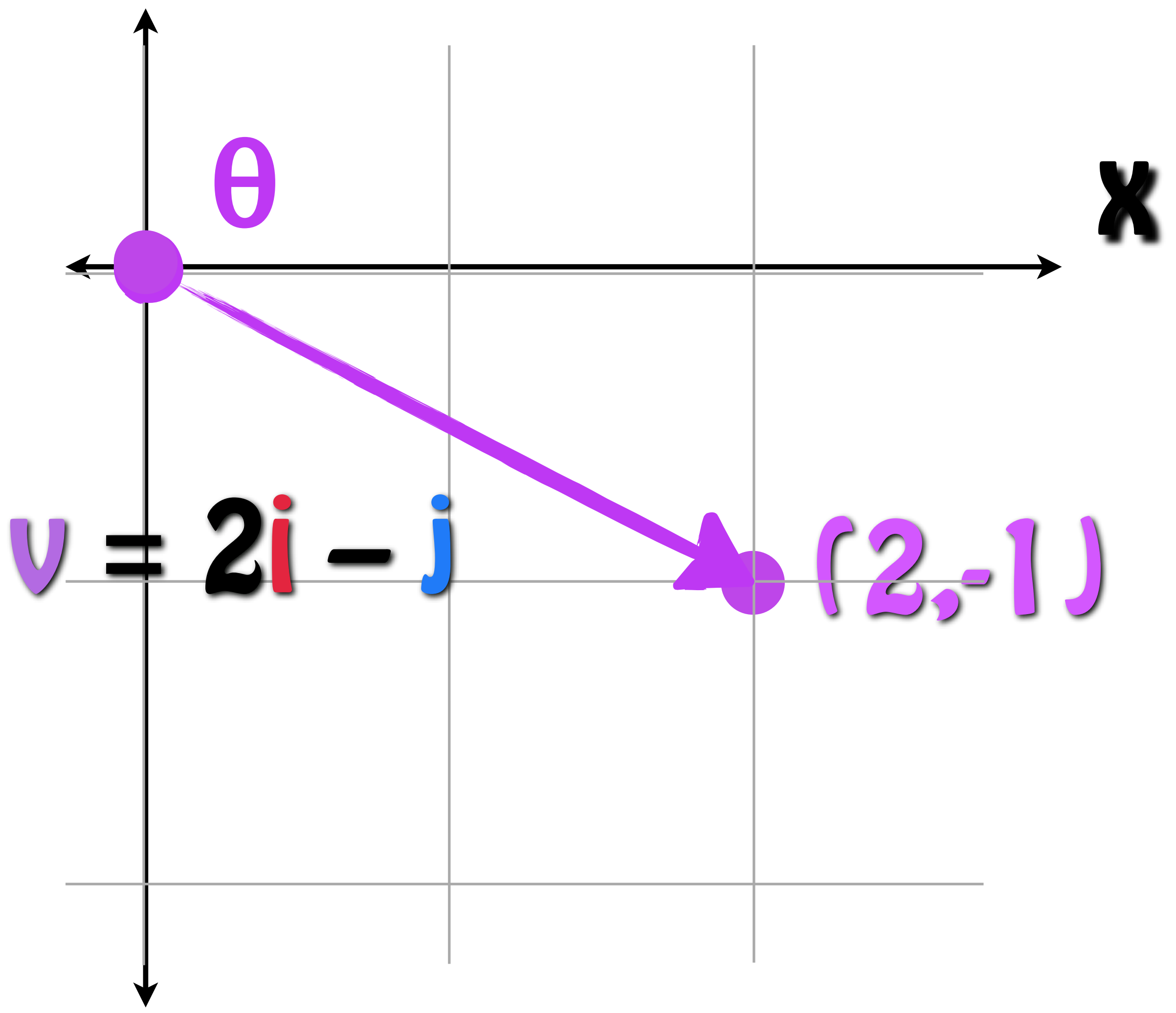
Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Find the direction angle for

$$v = 2i - j$$

$$\tan\theta = \frac{-1}{2} \qquad \tan^{-1}\frac{-1}{2} = \theta$$

$$\tan^{-1}\frac{-1}{2} = -26.5651^\circ$$



We know we are in Quadrant IV and direction angles are > 0;

$$\theta = 360^\circ - 26.5651^\circ = 333.4349^\circ$$

Application

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- The jet stream is blowing at 60 miles per hour in the direction N45°E. Express its velocity as a vector **v** in terms of **i** and **j**.

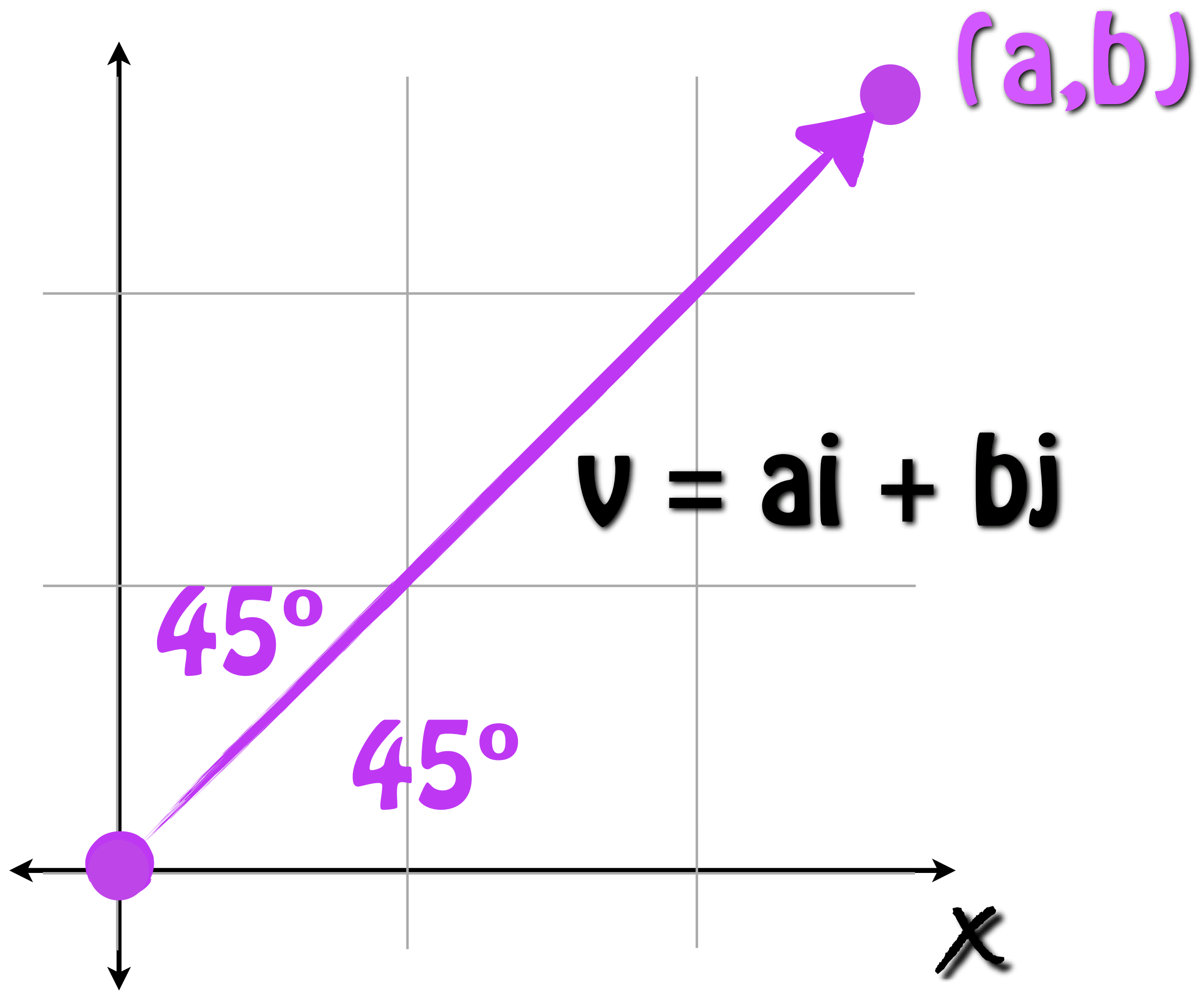
Magnitude: 60 miles per hour

Direction: N45°E

$$v = \|v\|\cos\theta\, i + \|v\|\sin\theta\, j$$

$$v = 60\cos 45^\circ\, i + 60\sin 45^\circ\, j$$

$$v = 60\left(\frac{\sqrt{2}}{2}\right) i + 60\left(\frac{\sqrt{2}}{2}\right) j = 30\sqrt{2}\, i + 30\sqrt{2}\, j$$



Applications

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

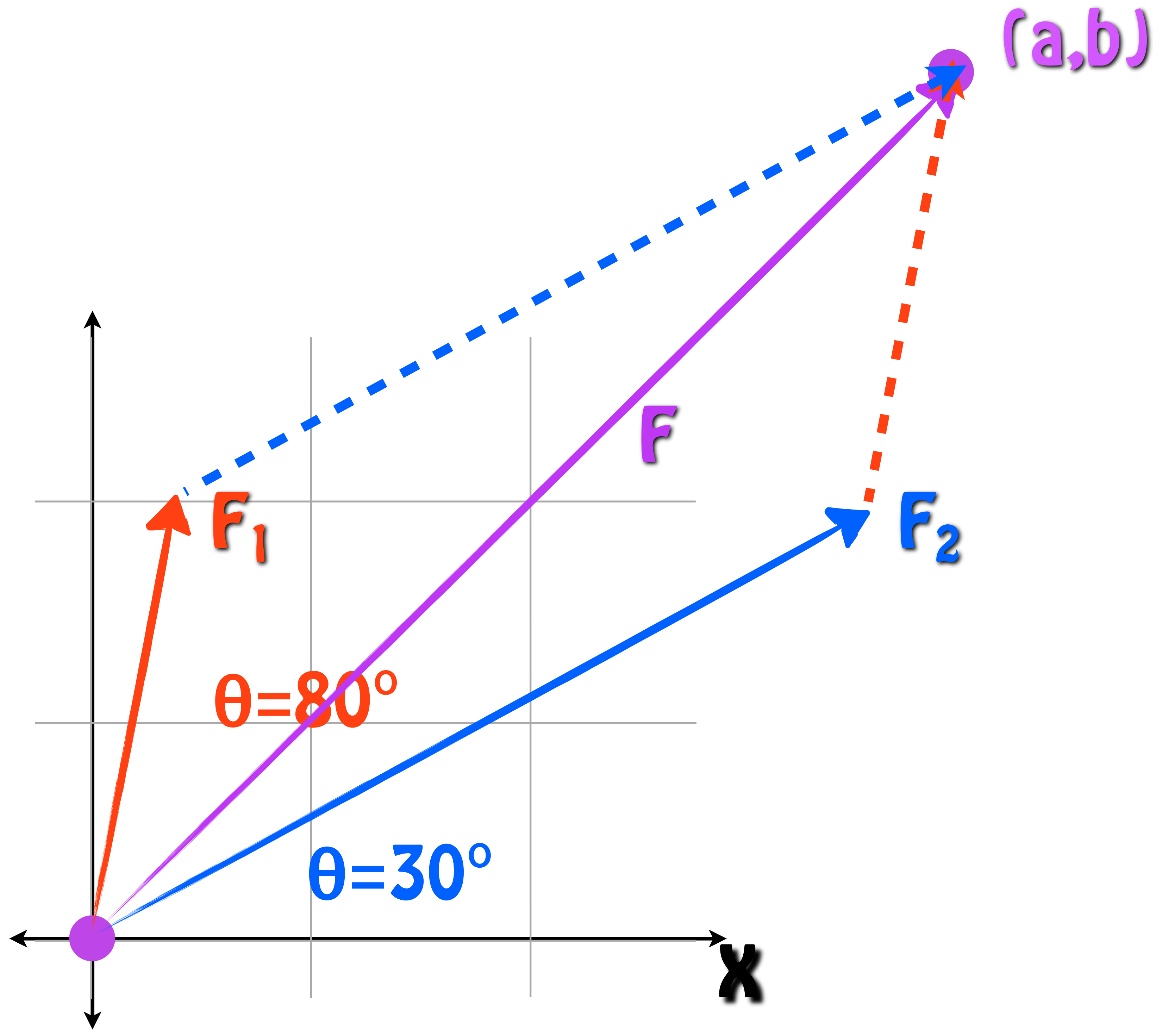
- Vectors can be used to represent many attributes of the physical world. An airplane flying with a headwind (or tailwind) is the result of two force vectors, the force of the plane's engines, and the force of the wind. A sailboat tacking into the wind is working with two forces, the wind, and the resistance of the water. If you are holding your books in your backpack there are also two forces working, gravity and your back muscles. These are examples of **force vectors**.

    - The net result of the two forces working is equal to a single force that is the **sum of the two force vectors**. If the forces reach equilibrium (ain't nuthin' movin') the result is the zero vector.

Example: Application

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- Two forces, F_1 and F_2, of magnitude 30 and 60 pounds, respectively, act on an object. The direction of F_1 is N10°E and the direction of F_2 is N60°E. Find the magnitude, to the nearest hundredth of a pound, and the direction angle, to the nearest tenth of a degree, of the resultant force.



$$v = \|v\|\cos\theta\, i + \|v\|\sin\theta\, j$$

$$F_1 = 30\cos 80^\circ i + 30\sin 80^\circ j$$

$$F_1 = 30(.1736)i + 30(.9848)j$$

$$F_1 = 5.2094i + 29.5442j$$

$$F_2 = 60\cos 30^\circ i + 60\sin 30^\circ j$$

$$F_2 = 60(.8660)i + 60(.5)j$$

$$F_2 = 51.9615i + 30j$$

Example: Application

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$$F_1 = 5.2094i + 29.5442j$$

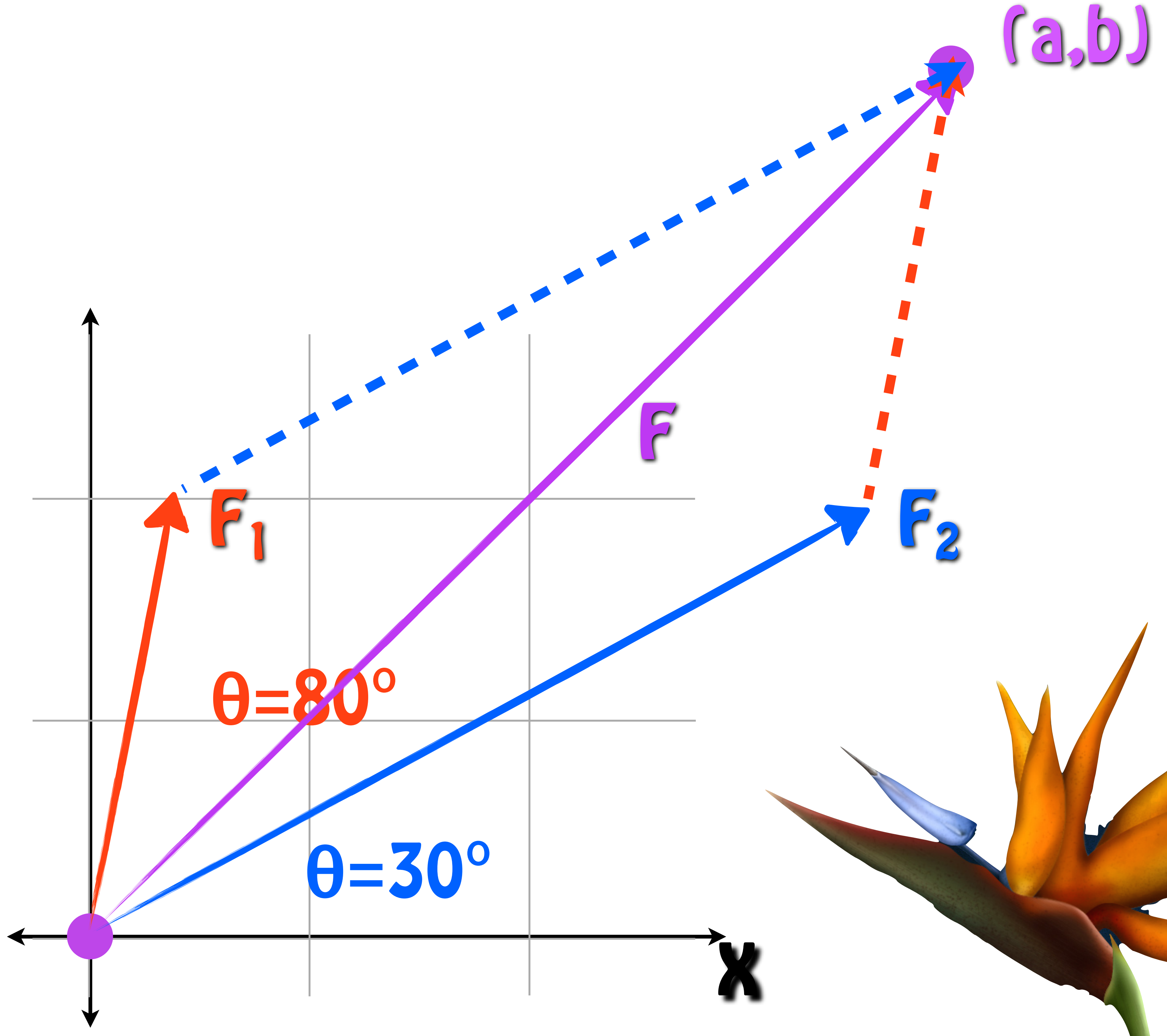
$$F_2 = 60(.8660)i + 60(.5)j$$

$$F = F_1 + F_2$$

$$= 5.2094i + 29.5442j + 51.9615i + 30j$$

$$= (5.2094 + 51.9615)i + (29.5442 + 30)j$$

$$= 57.1709i + 59.5442j$$



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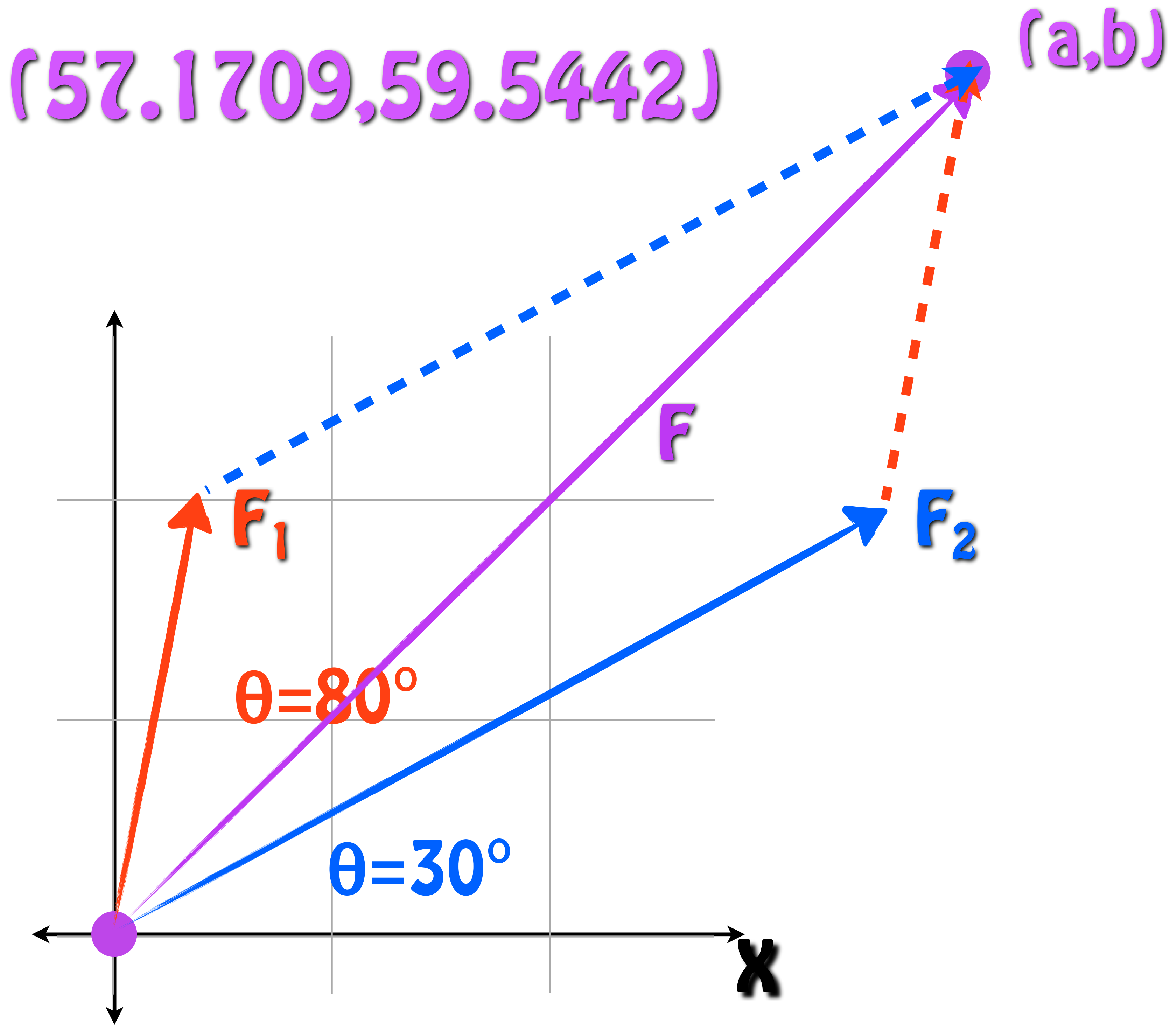

$$F = 57.1709i + 59.5442j$$

$$\|F\| = \sqrt{57.1709^2 + 59.5442^2}$$

$$\|F\| = 82.5471$$

$$\tan^{-1}\frac{59.5442}{57.1709} = \theta_F$$

$$\theta_F \approx 46.1157^\circ$$

Example: Application

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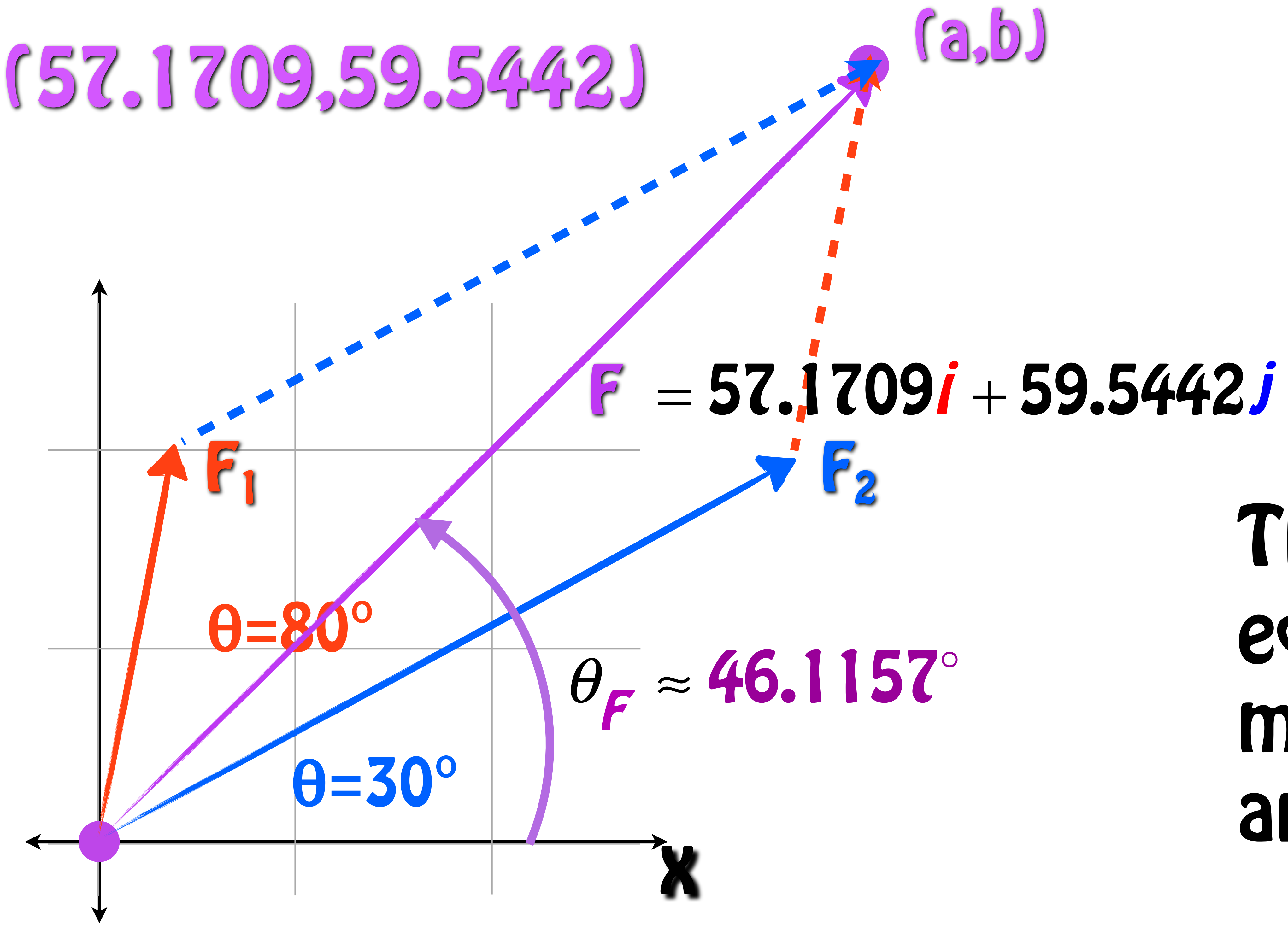


$$F = 57.1709i + 59.5442j$$

$$\|F\| = 82.5471$$

$$\theta_F \approx 46.1157^\circ$$

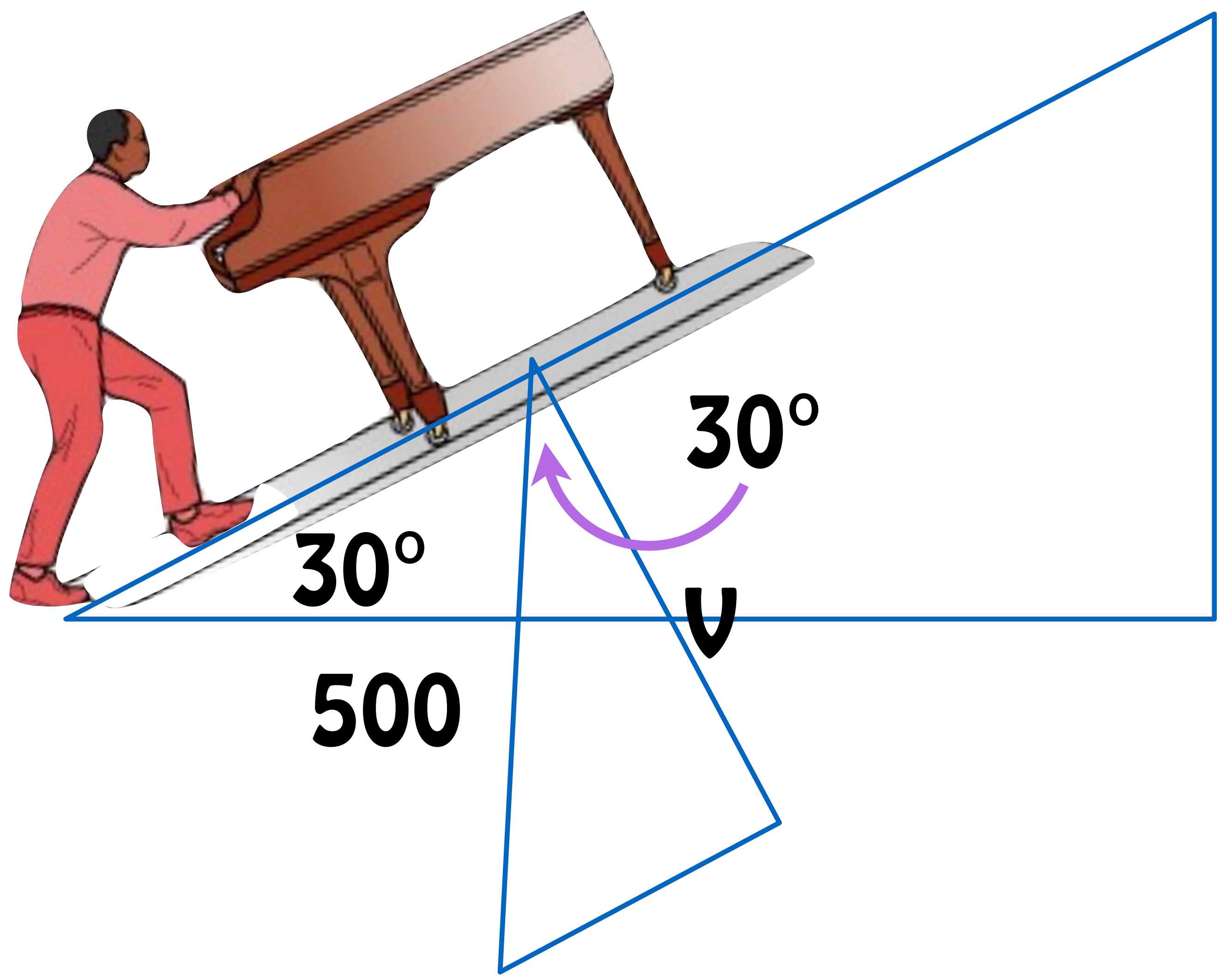
The combined forces F_1 and F_2 are equivalent to a single force, F, with magnitude 82.8471 applied at an approximate angle of 46.1157°.

Application

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- A piano weighing 500 lb is being pushed up a ramp into the back of a truck. The ramp is a board that can support 450 lb and makes a 30° angle with the horizontal. Will the ramp support the piano?

The key here is that the maximum capacity of the ramp refers to perpendicular to the ramp. The 500 lb piano is being pushed at 30° and that force is perpendicular to the ground.



We want to find the component force, v, perpendicular to the ramp.

$$\|v\| = 500\cos 30^{\circ} = 433.0127$$

No problem, the ramp will hold as long as the person pushing weighs less than 17 pounds.

Application

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- A plane leaving Seattle sets a bearing of S 60° E at 600 mph, but there is a wind blowing in the direction N 45° E at 80 mph. What is the resultant speed and direction of the plane? If you get this wrong you may end up in Chicago instead of Miami.

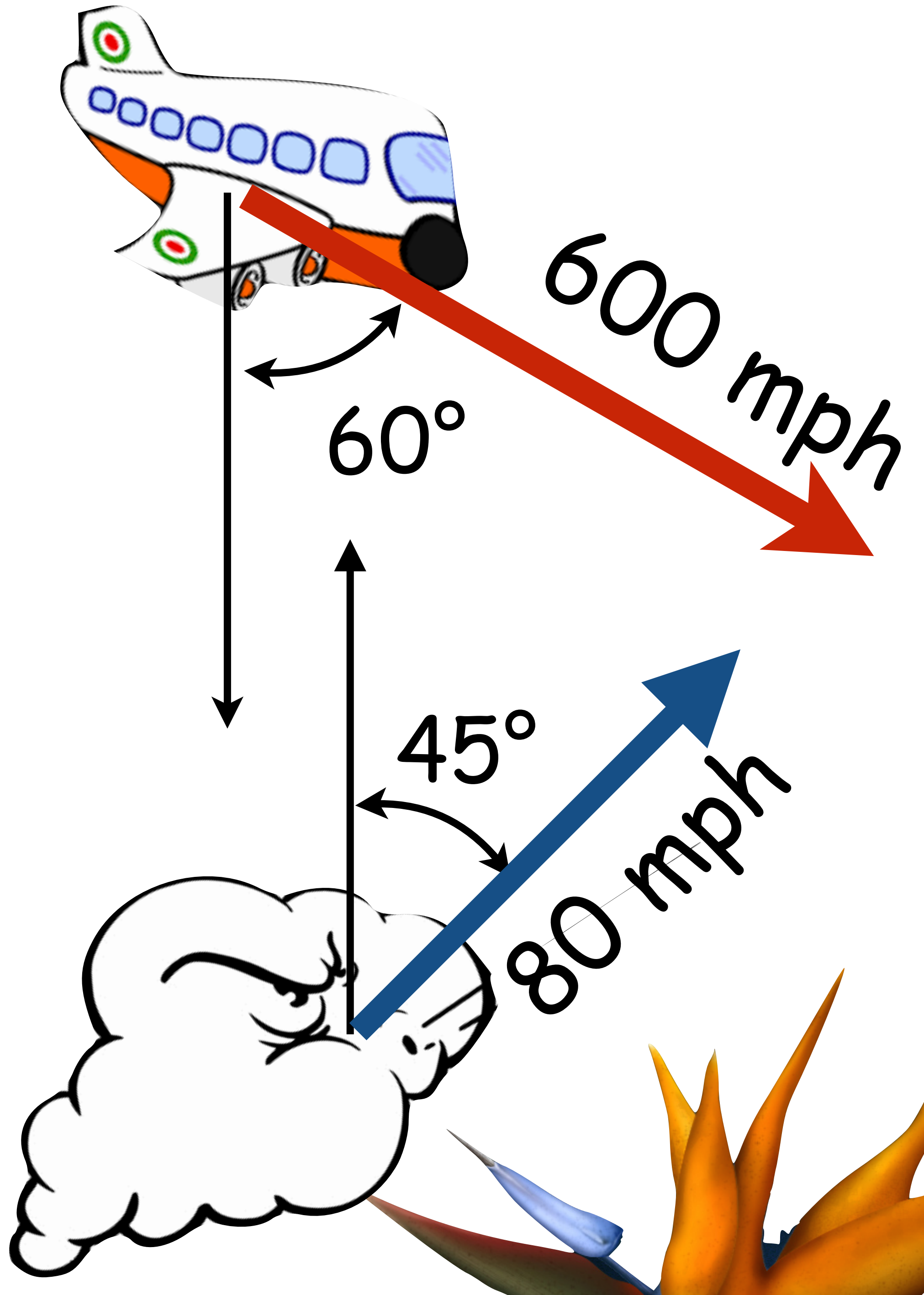
Let the plane vector be p = 600cos330°i + 600sin330°j

Let the air vector be a = 80cos45°i + 80sin45°j

The resultant vector will be the sum of p + a

$$p = 600\cos 330^\circ i + 600\sin 330^\circ j = 600\left(\frac{\sqrt{3}}{2}\right)i + 600\left(-\frac{1}{2}\right)j$$

$$a = 80\cos 45^\circ i + 80\sin 45^\circ j = 80\left(\frac{\sqrt{2}}{2}\right)i + 80\left(\frac{\sqrt{2}}{2}\right)j$$



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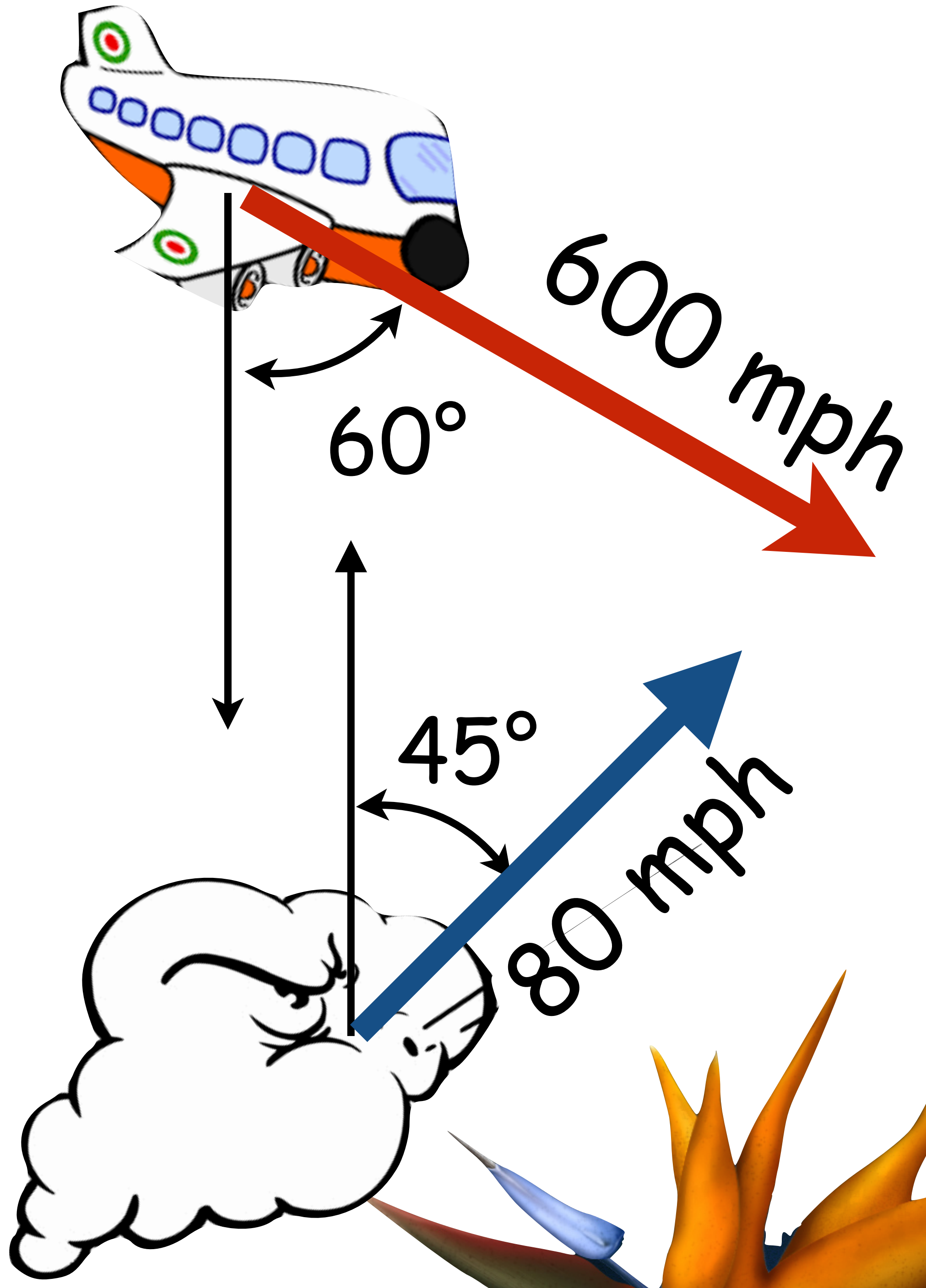
$$\mathbf{p} = 600\left(\frac{\sqrt{3}}{2}\right)\mathbf{i} + 600\left(-\frac{1}{2}\right)\mathbf{j}$$
$$= 300\sqrt{3}\mathbf{i} - 300\mathbf{j}$$

$$\mathbf{a} = 80\left(\frac{\sqrt{2}}{2}\right)\mathbf{i} + 80\left(\frac{\sqrt{2}}{2}\right)\mathbf{j}$$
$$= 40\sqrt{2}\mathbf{i} + 40\sqrt{2}\mathbf{j}$$



$$\mathbf{p} + \mathbf{a} = 300\sqrt{3}\mathbf{i} - 300\mathbf{j} + 40\sqrt{2}\mathbf{i} + 40\sqrt{2}\mathbf{j}$$
$$= 300\sqrt{3}\mathbf{i} + 40\sqrt{2}\mathbf{i} + 40\sqrt{2}\mathbf{j} - 300\mathbf{j}$$
$$= \left(300\sqrt{3} + 40\sqrt{2}\right)\mathbf{i} + \left(40\sqrt{2} - 300\right)\mathbf{j}$$

Application

Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

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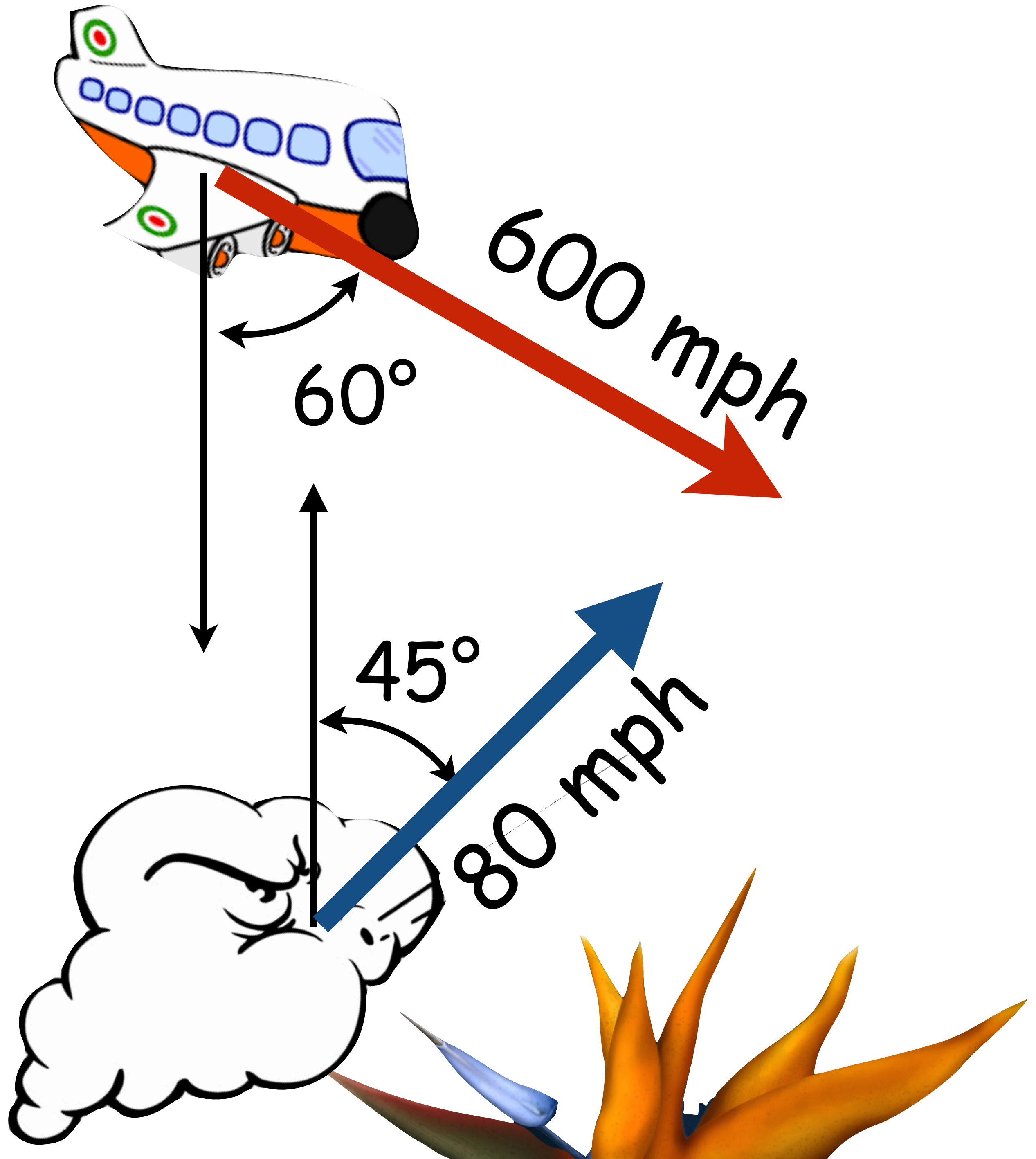
$$\mathbf{p} + \mathbf{a} = \left(300\sqrt{3} + 40\sqrt{2}\right)\boldsymbol{i} + \left(40\sqrt{2} - 300\right)\boldsymbol{j}$$

$$\approx (576.1838)\boldsymbol{i} + (-243.4315)\boldsymbol{j}$$

$$\text{Speed} \approx \left\|(576.1838)\boldsymbol{i} + (-243.4315)\boldsymbol{j}\right\|$$

$$\approx \sqrt{(576.1838)^2 + (-243.4315)^2}$$

$$\approx 625.4971 \ mph$$



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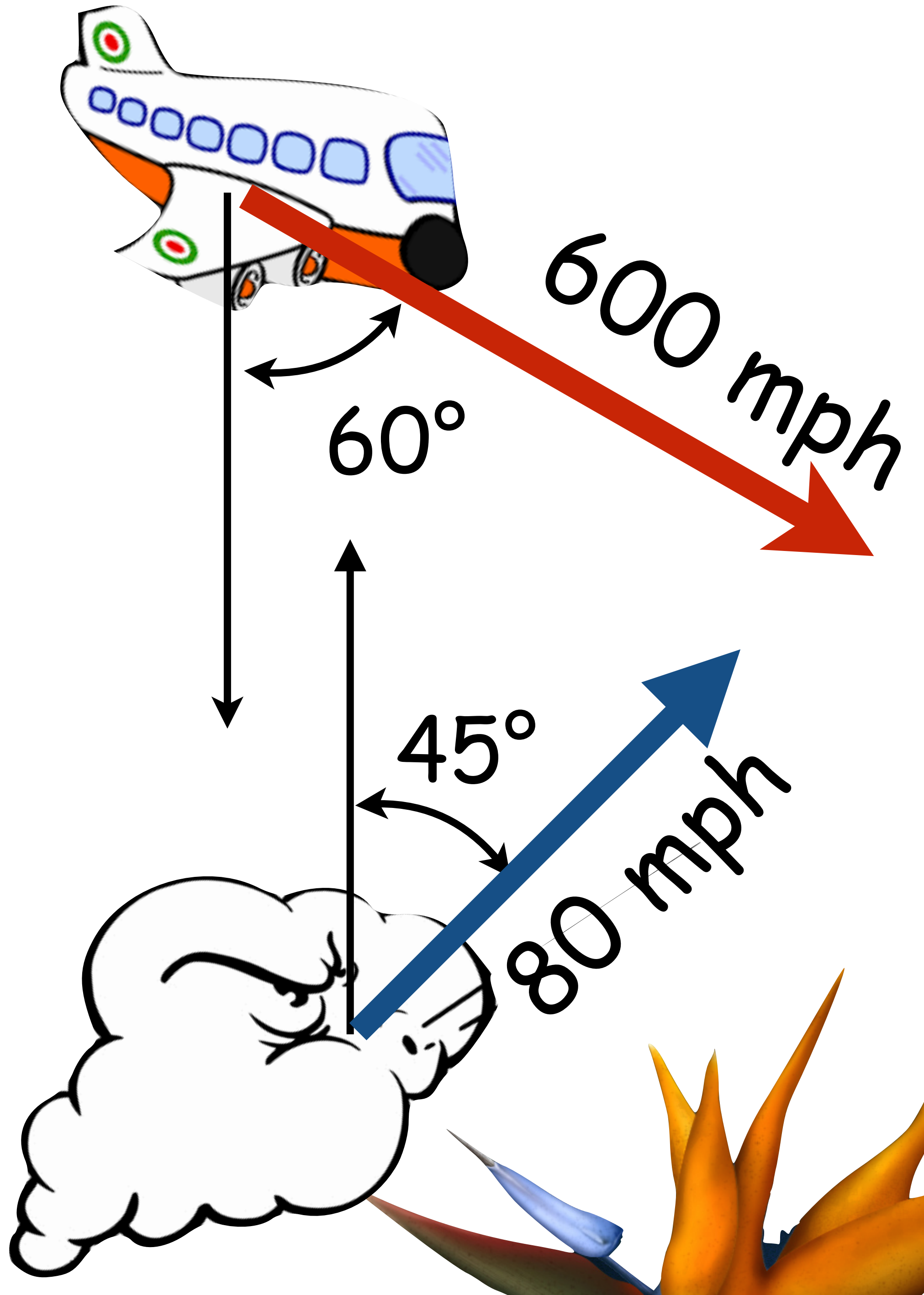
$$\mathbf{p} + \mathbf{a} \approx (576.1838)\mathbf{i} + (-243.4315)\mathbf{j}$$

direction $\quad \tan\theta = \dfrac{-243.4315}{576.1838} \qquad \tan^{-1}\dfrac{-243.4315}{576.1838} = \theta$

$\theta \approx -22.9036^\circ = 337.0965^\circ$ OR S67.0965° E



Application

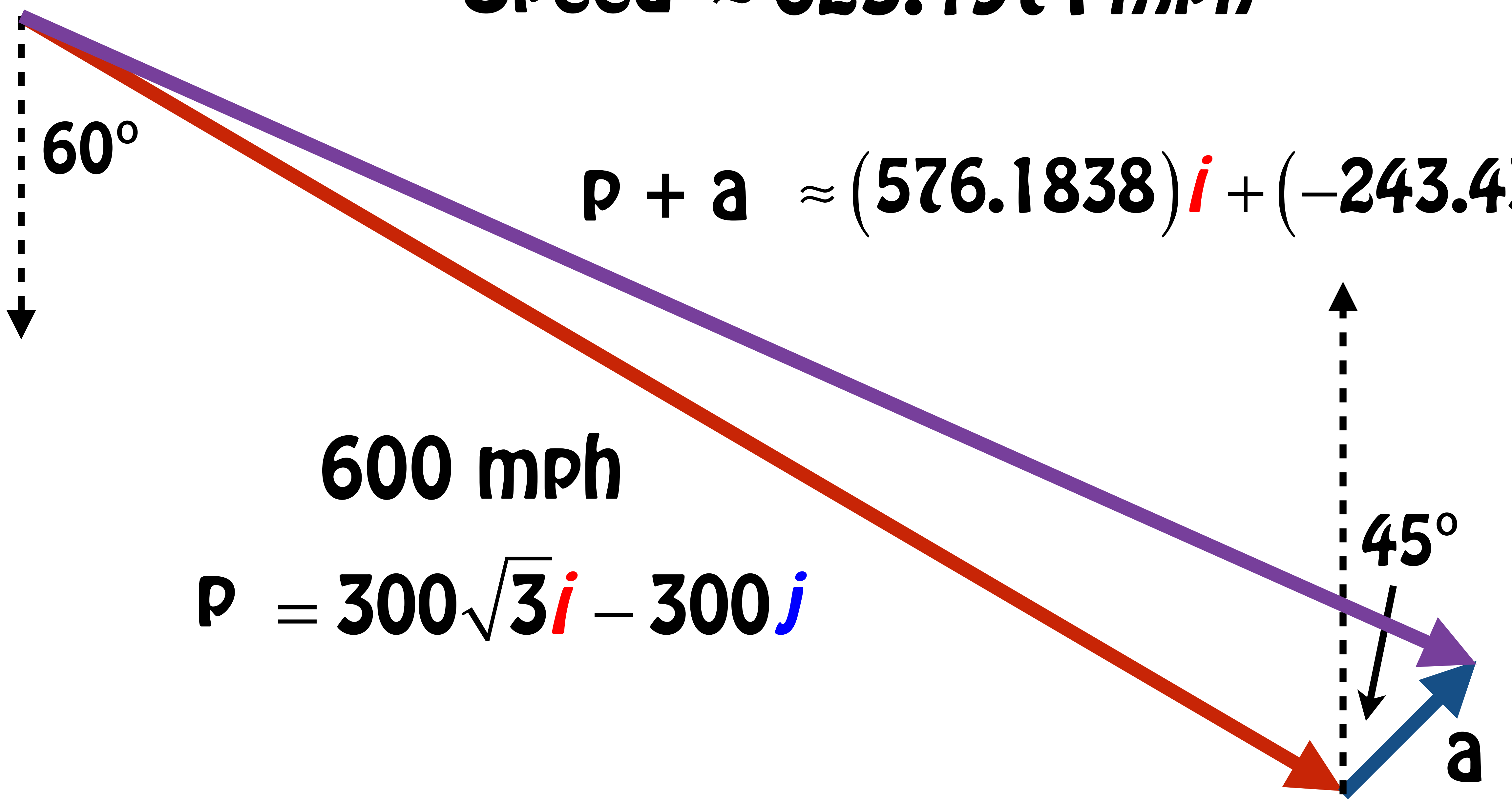
Objective: Write the component forms of vectors, perform basic vector operations, write vectors as linear combinations of unit vectors, and find the direction angles of vectors.

- We can show this geometrically

$\theta \approx -22.9036^\circ = 337.0965^\circ$ OR S67.0965° E

Speed ≈ 625.4971 *mph*

$\mathbf{p} + \mathbf{a} \approx (576.1838)\mathbf{i} + (-243.4315)\mathbf{j}$

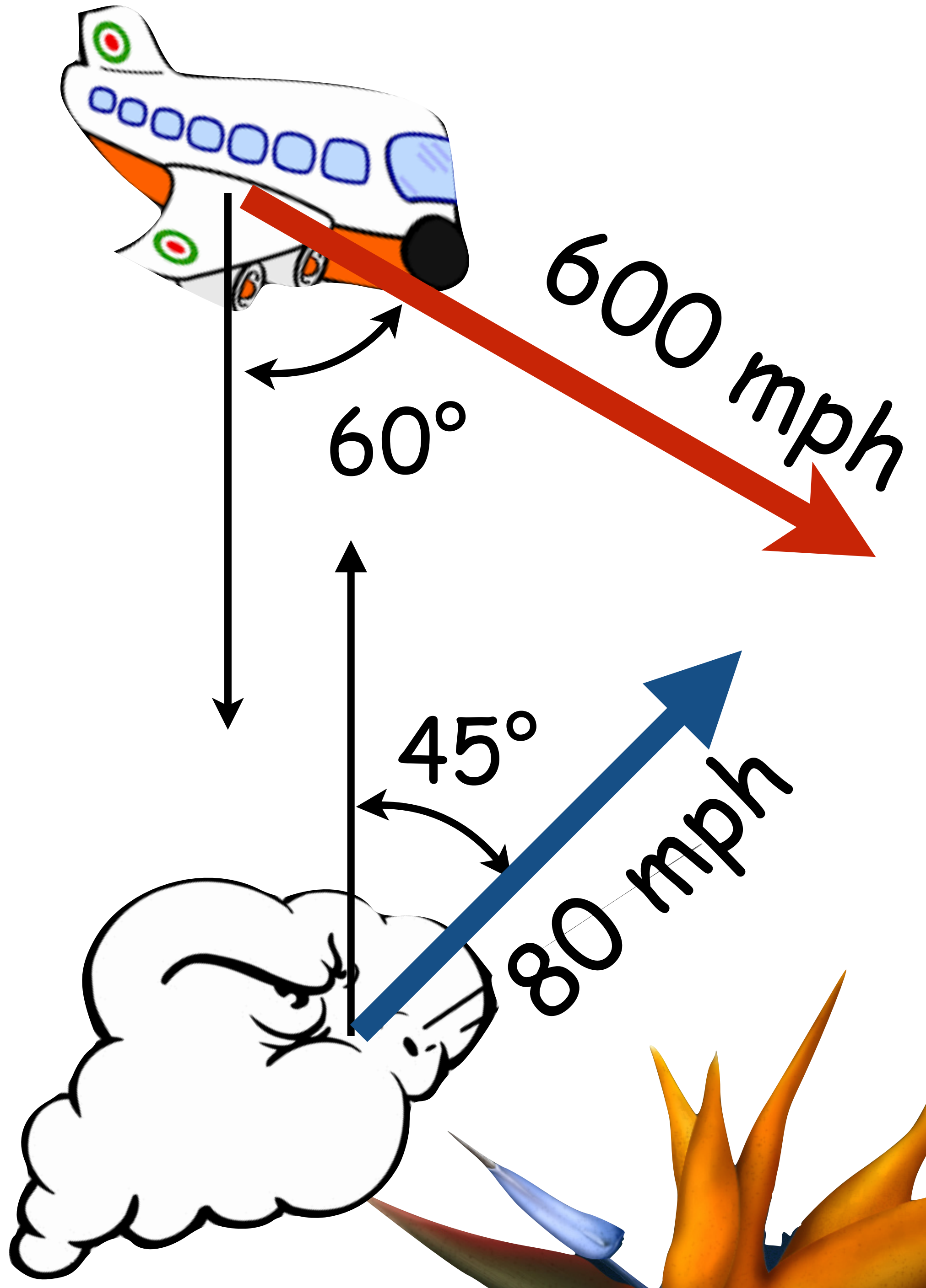


600 mph

$\mathbf{p} = 300\sqrt{3}\mathbf{i} - 300\mathbf{j}$

$\mathbf{a} = 40\sqrt{2}\mathbf{i} + 40\sqrt{2}\mathbf{j}$

80 mph



You can verify your results using the laws of sines and cosines.