Chapter 7
Systems of Equations and Inequalities

7.1 Systems of Equations in Two Variables

Homework

- Read Sec 7.1 and complete reading notes.
- Do p503 5-27 odd, 35, 41, 43, 45, 47, 49-59 odd, 63, 67
- Complete the worksheet.

Objectives

- Solve linear systems by substitution.
- Solve systems of equations graphically
- Identify systems that do not have exactly one ordered-pair solution.
- Solve problems using systems of linear equations.

Systems of Linear Equations and Their Solutions

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

- All equations in the form **Ax + By = C** are straight lines when graphed. That is why they are called **linear equations.**
  - Combining two, or more, such equations is called a **system of linear equations** or a **linear system**.
    - A solution to a system of linear equations in two variables is an **ordered pair** that satisfies both equations in the system.
      - A linear system that has at least one solution is called a **consistent system.**
      - A linear system with no solution is called an **inconsistent system.**

Determining Whether Ordered Pairs are Solutions of a System

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

- Consider the system: $\begin{cases} 2x - 3y = -4 \\ 2x + y = 4 \end{cases}$

  - Determine if the ordered pair (1, 2) is a solution of the system.

$$2x - 3y = -4$$
$$2(1) - 3(2) = -4$$
$$2 - 6 = -4$$

The ordered pair (1, 2) satisfies both equations.

(1, 2) is a solution of the system.

$$2x + y = 4$$
$$2(1) + 2 = 4$$
$$2 + 2 = 4$$

Determining Whether Ordered Pairs Are Solutions of a System

- Consider the system: $\begin{cases} 2x - 3y = -4 \\ 2x + y = 4 \end{cases}$

  - Determine if the ordered pair (7, 6) is a solution of the system.

$$2x - 3y = -4$$
$$2(7) - 3(6) = -4$$
$$14 - 18 = -4$$

The ordered pair (7, 6) satisfies only one equation.

(7, 6) is not a solution of the system.

$$2x + y = 4$$
$$2(7) + 2 = 4$$
$$14 + 6 \neq 4$$

- Procedure for solving a system of linear equations by **substitution**

1. Solve **either** of the equations for one variable in terms of the other variable.
2. Substitute the expression from step 1 into the **other** equation. This results in a single equation with a single variable.
3. Solve the new equation from step 2 for the remaining variable.
4. Substitute the value found for the variable in step 3 into **either** of the original equations and solve for the remaining variable.
5. Check the final ordered pair in both equations.
6. Be certain to write the solution as an **ordered pair!**

Example: Solving a System by Substitution

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

Solve the system: $\begin{cases} 3x + 2y = 4 \\ 2x + y = 1 \end{cases}$

1. $2x + y = 1$

   $y = -2x + 1$

2. $3x + 2y = 4$

   $3x + 2(-2x + 1) = 4$

3. $3x + -4x + 2 = 4$

   $-x + 2 = 4$

   $x = -2$

4. $2x + y = 1$

   $2(-2) + y = 1$

   $-4 + y = 1$

   $y = 5$

4. $3x + 2y = 4$

   $3(-2) + 2y = 4$

   $-6 + 2y = 4$

   $2y = 10$

   $y = 5$

5. **Check** the solution.

The solution of the system is **(-2, 5)**

Method of Substitution

1. *Solve* one of the equations for one variable in terms of the other.
2. *Substitute* the expression found in Step 1 into the other equation to obtain an equation in one variable.
3. *Solve* the equation obtained in Step 2.
4. *Back-substitute* the value obtained in Step 3 into the expression obtained in Step 1 to find the value of the other variable.
5. *Check* that the solution satisfies *each* of the original equations.

STUDY TIP

Because many steps are required to solve a system of equations, it is very easy to make errors in arithmetic. So, you should always check your solution by substituting it into *each* equation in the original system.

Finding solutions graphically

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

To solve a **linear** system graphically using the TI-84:

1. Solve **both** of the equations for y in terms of x.
2. Enter both equations into y=
3. Zoom 6
4. Ask for the intersection. 2nd TRACE CALC 5:intersect ENTER Set your first line ENTER Set your second line ENTER Guess? ENTER
5. Check the final ordered pair in both equations.
6. Be certain to write the solution as an **ordered pair!**

Graphing Manually

- Of course you can always graph using pencil and graph paper to find the solution.

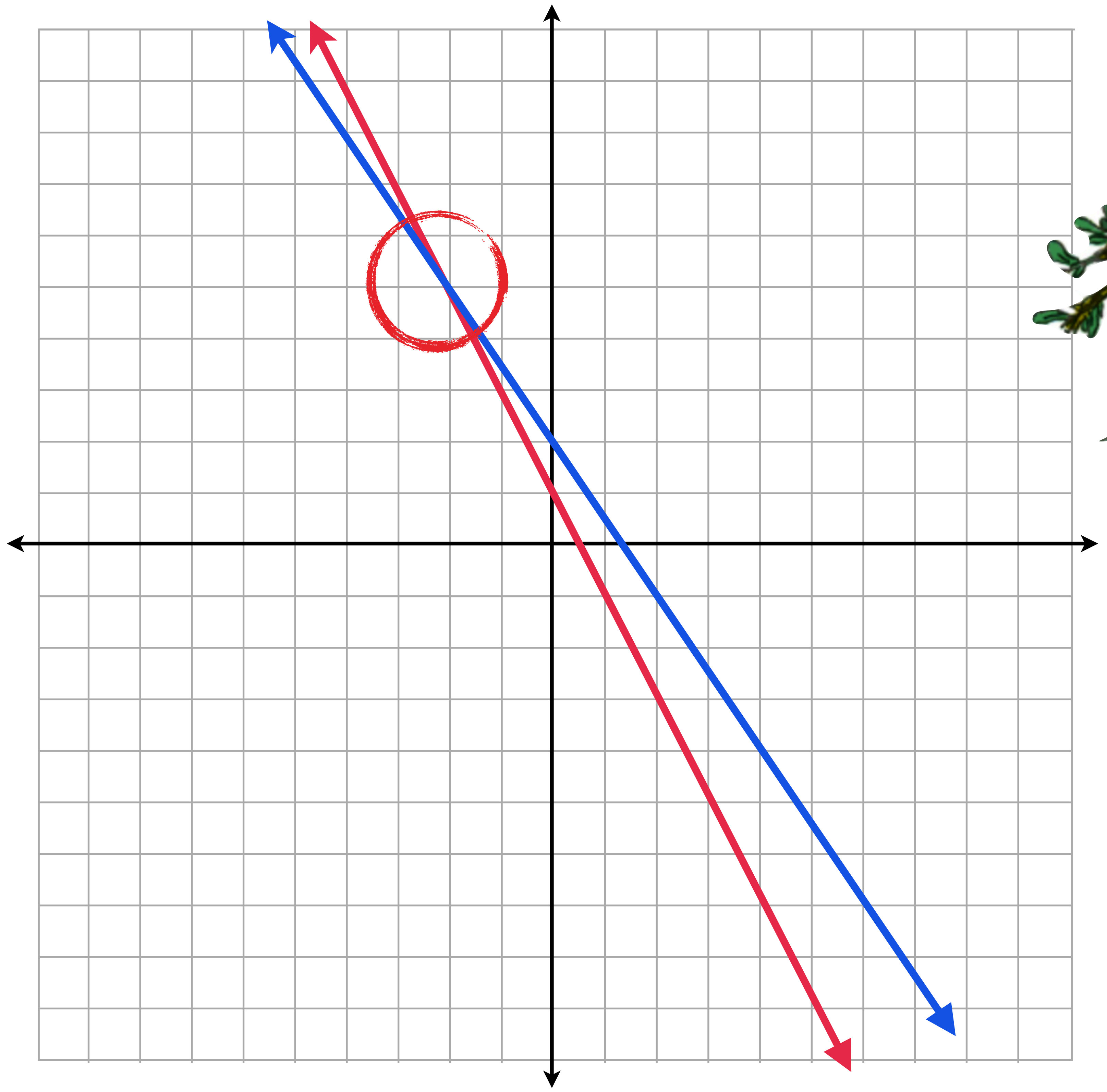
Finding Solutions Graphically

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

Solve the system: $\begin{cases} 3x+2y=4 \\ 2x+y=1 \end{cases}$

1. $y=-\dfrac{3}{2}x+2$

$y=-2x+1$



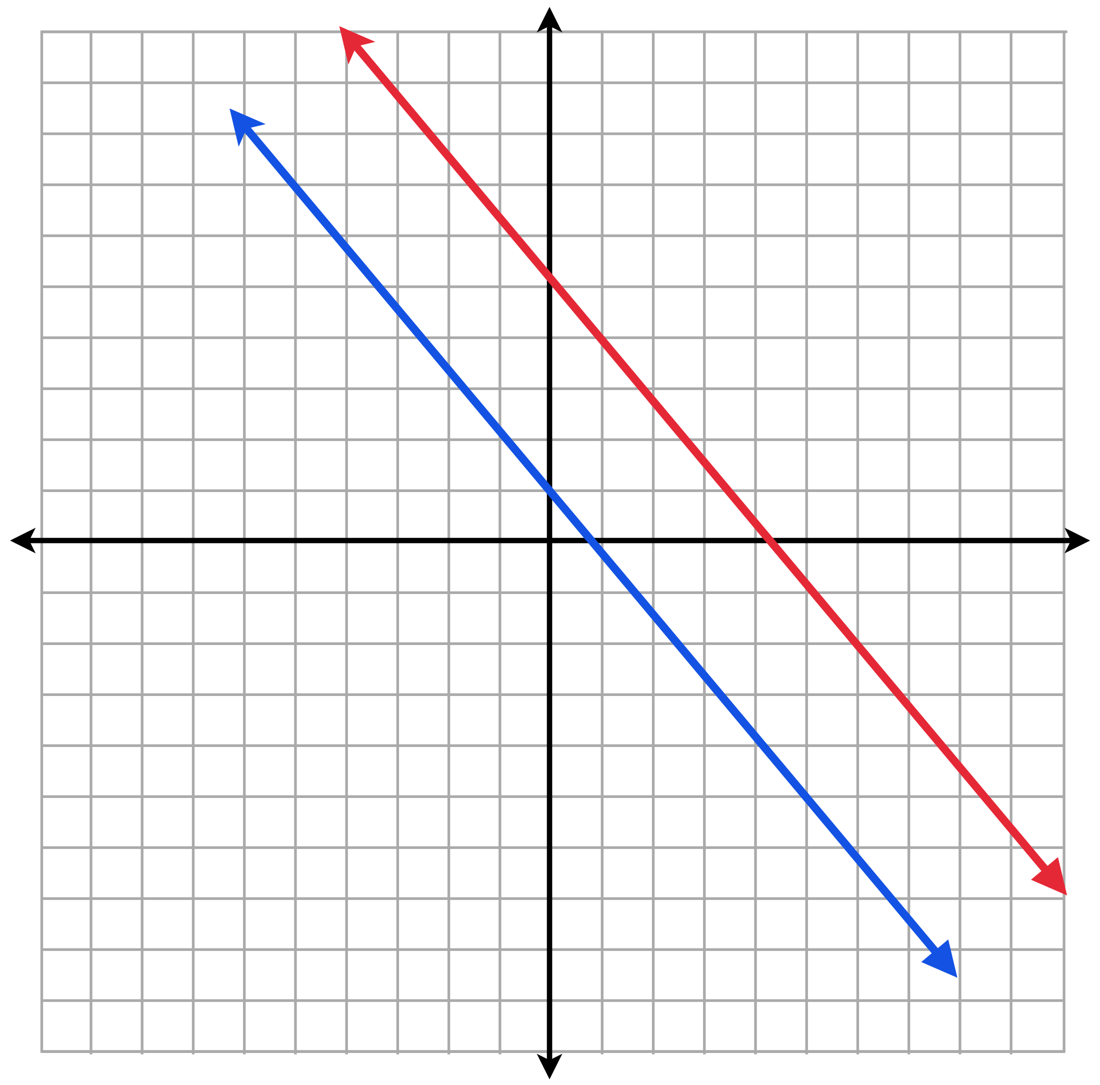
The solution of the system is **(-2, 5)**

The Number of Solutions to a System of Two Linear Equations

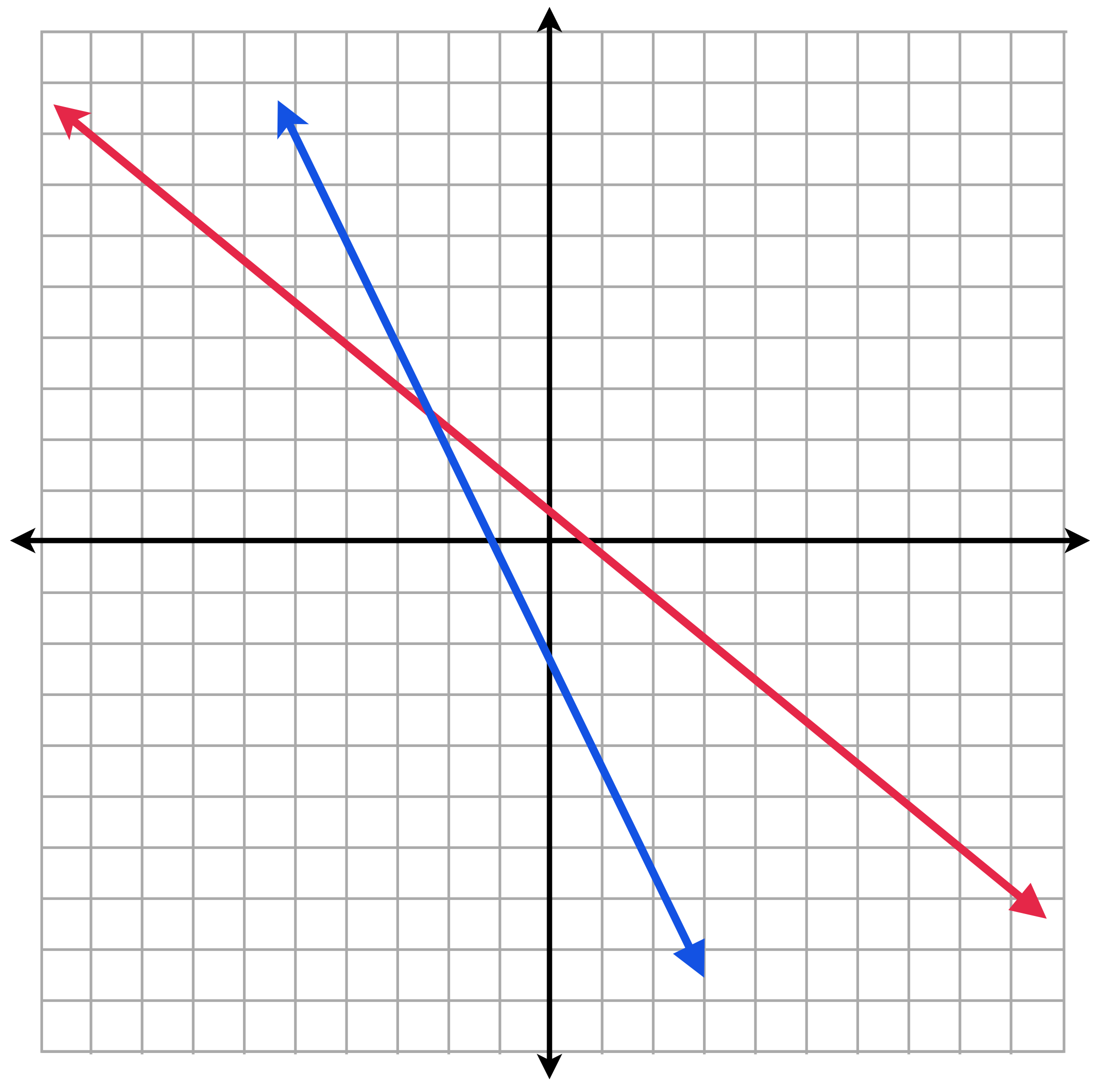
7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

- Keep in mind that a solution to a system of equations satisfies **every** equation in the system. It is very possible that there is no solution to the system.
- The number of solutions to a system of two equations in two variables is either 0, 1, or infinitely many.

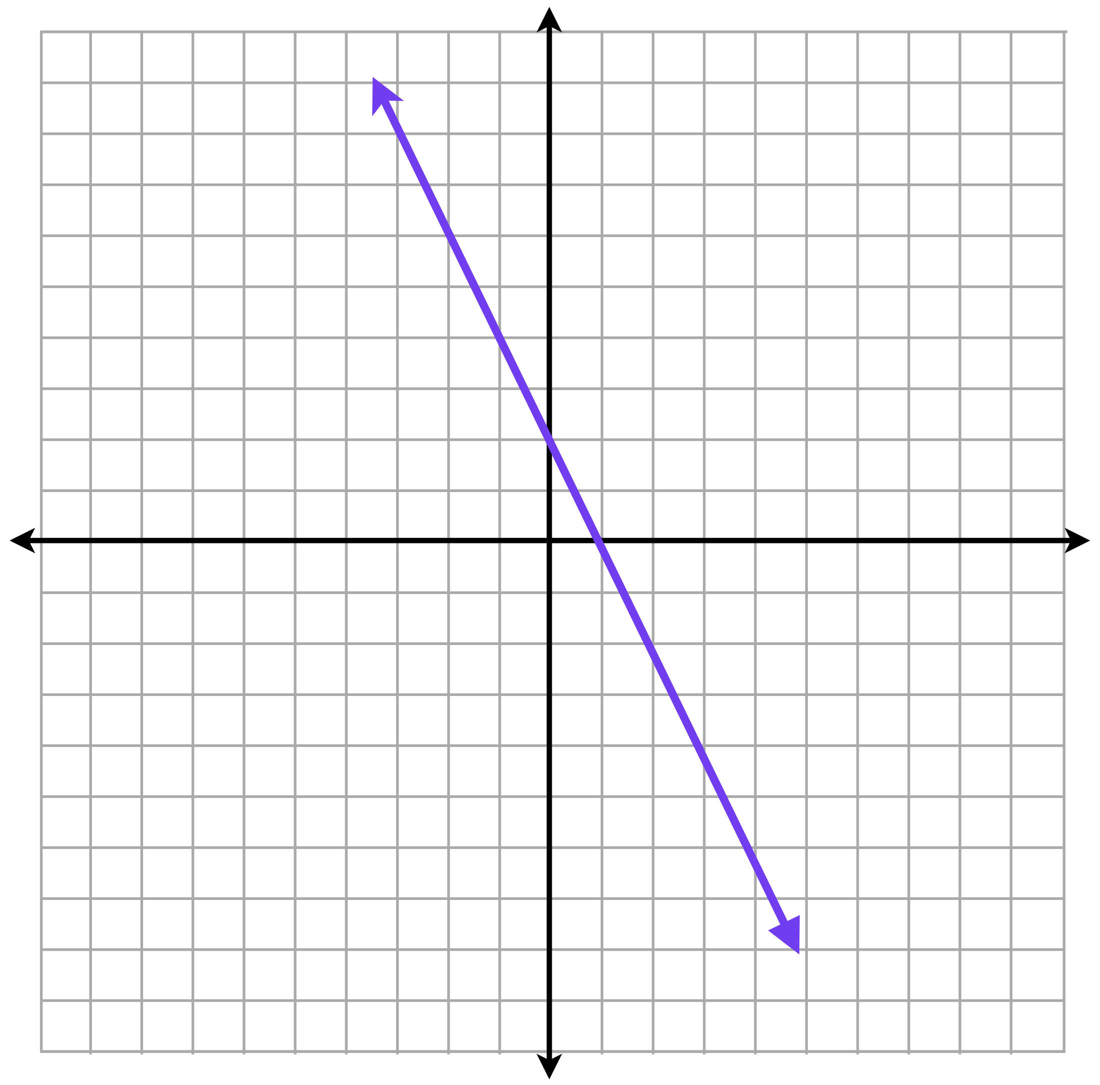
0 solutions



1 solution



Infinite solutions



A System with No Solution

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

Solve the system: $\begin{cases} 5x - 2y = 4 \\ -10x + 4y = 7 \end{cases}$

1. $5x - 2y = 4$

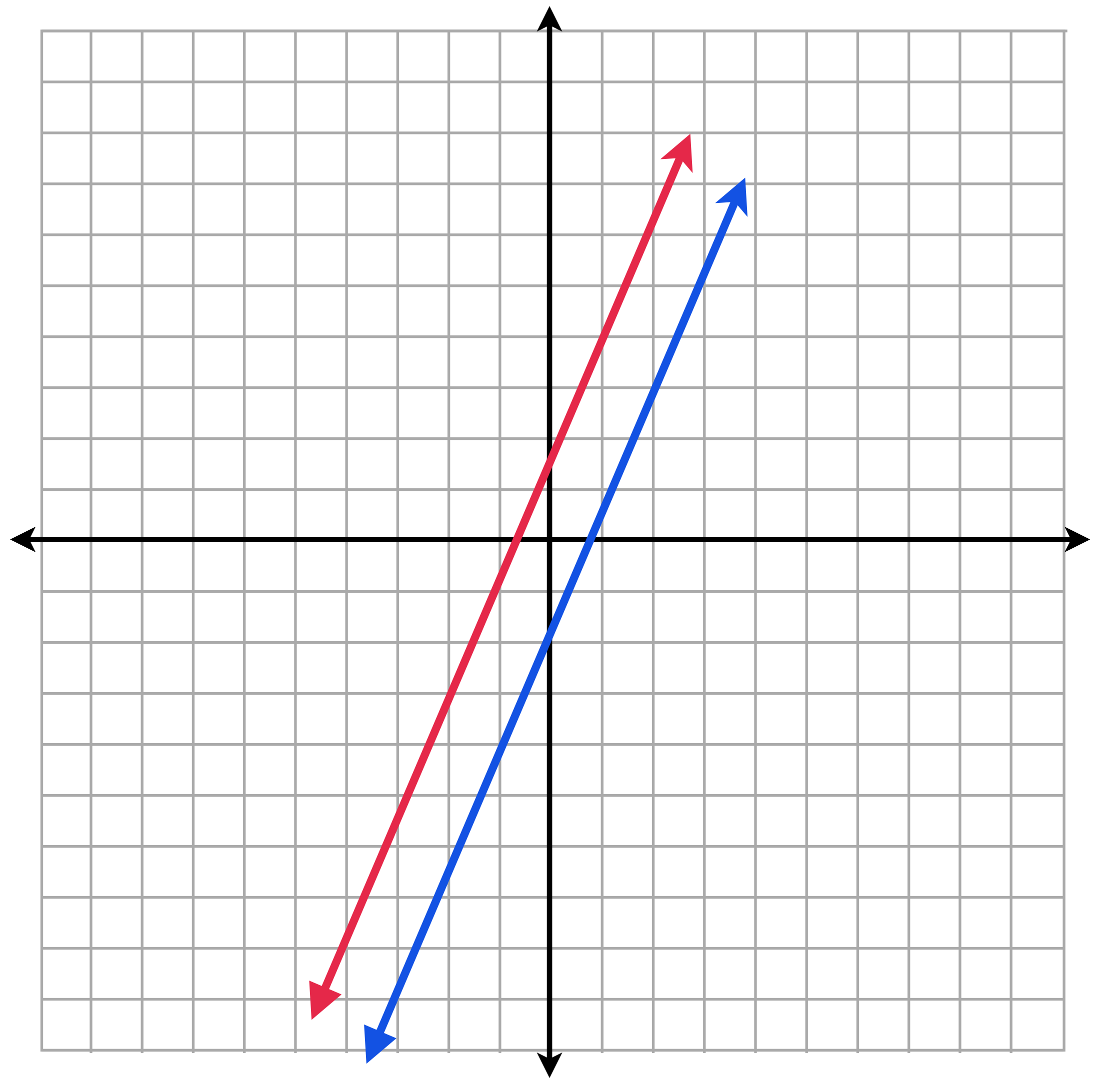
$y = \dfrac{5}{2}x - 2$

2. $-10x + 4y = 7$

$-10x + 4\left(\dfrac{5}{2}x - 2\right) = 7$

3. $-10x + 10x - 8 = 7$

$-8 = 7$



Obviously, this cannot happen.

There is no solution to the system $\varnothing$

A System with Infinitely Many Solutions

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

Solve the system: $\begin{cases} x = 4y - 8 \\ 5x - 20y = -40 \end{cases}$

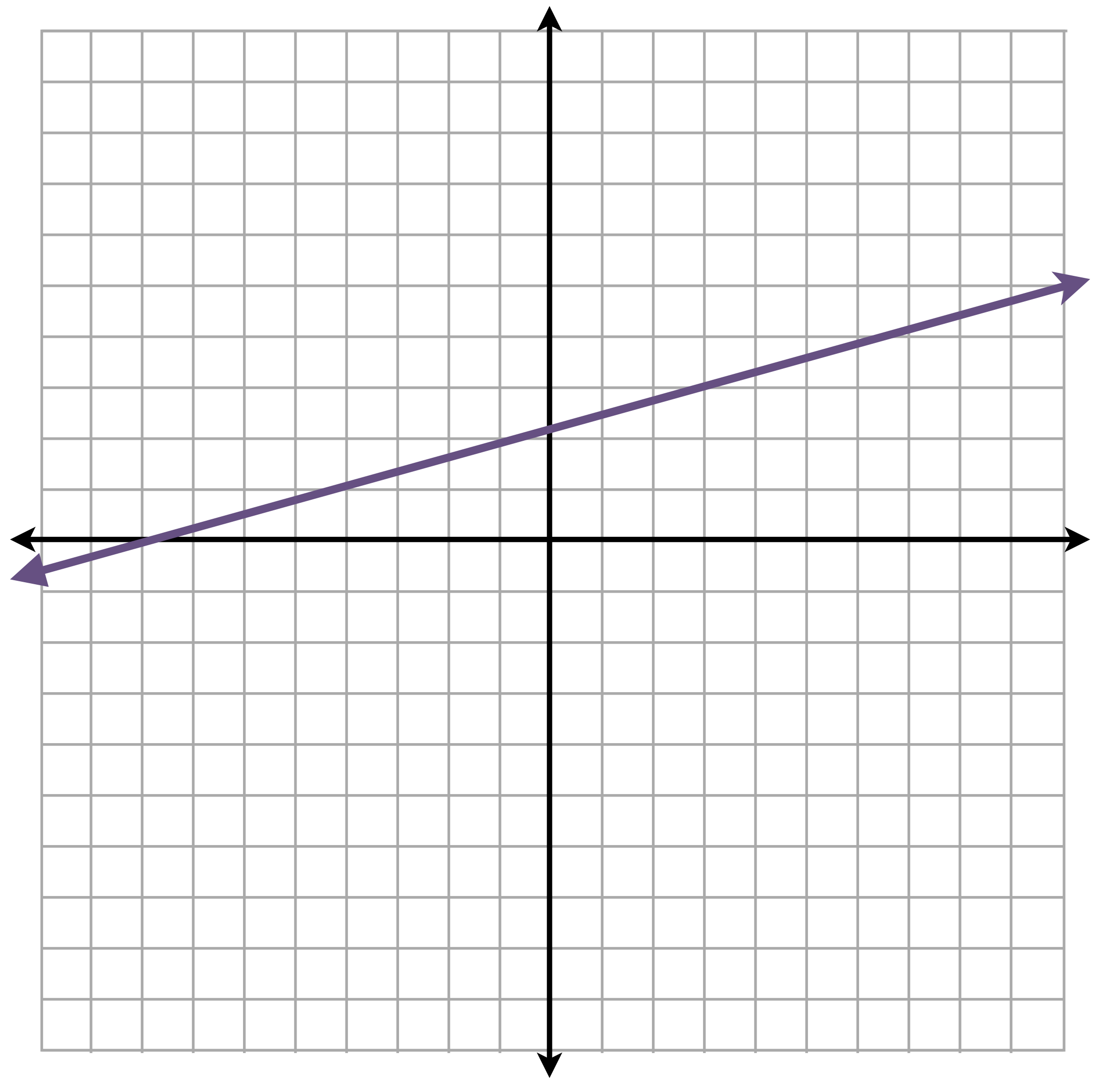
1. $x = 4y - 8$

2. $5x - 20y = -40$

$$5(4y - 8) - 20y = -40$$

3. $20y - 40 - 20y = -40$

$$-40 = -40$$



Obviously, this will always happen no matter the value of y.

There are infinitely many solutions to the system.

Example: Finding a Break-Even Point

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

- A company that manufactures running shoes has a fixed cost of \$300,000. Additionally, it costs \$30 to produce each pair of shoes. They are sold at \$80 per pair.

a. Write the cost function, C, of producing x pairs of running shoes.

$$C(x) = 30x + 300{,}000$$

b. Write the income function, S, from the sale of x pairs of running shoes.

$$S(x) = 80x$$

Finding a Break-Even Point

- A company that manufactures running shoes has a fixed cost of $300,000. Additionally, it costs $30 to produce each pair of shoes. They are sold at $80 per pair.

c. Determine the break-even point for sales.

- The break even point is where sales = costs or C(x) = S(x).

$$C(x) = 30x + 300,000$$
$$S(x) = 80x$$

$$30x + 300,000 = 80x$$
$$300,000 = 50x$$
$$x = 6000$$

$$S(x) = 80x$$
$$S(x) = 80(6000)$$
$$S(x) = 480,000$$

The break even point is at (6000, 480,000)

Income of $480,000 on sales of 6000 units.

Non-Linear Systems

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

We solved linear systems by substitution. Non-linear systems can also be solved by substitution.

Solve the system: $\begin{cases} x^2+y^2=5 \\ x+y=1 \end{cases}$

1.
$$x+y=1$$
$$y=-x+1$$

2.
$$x^2+y^2=5$$
$$x^2+(-x+1)^2=5$$

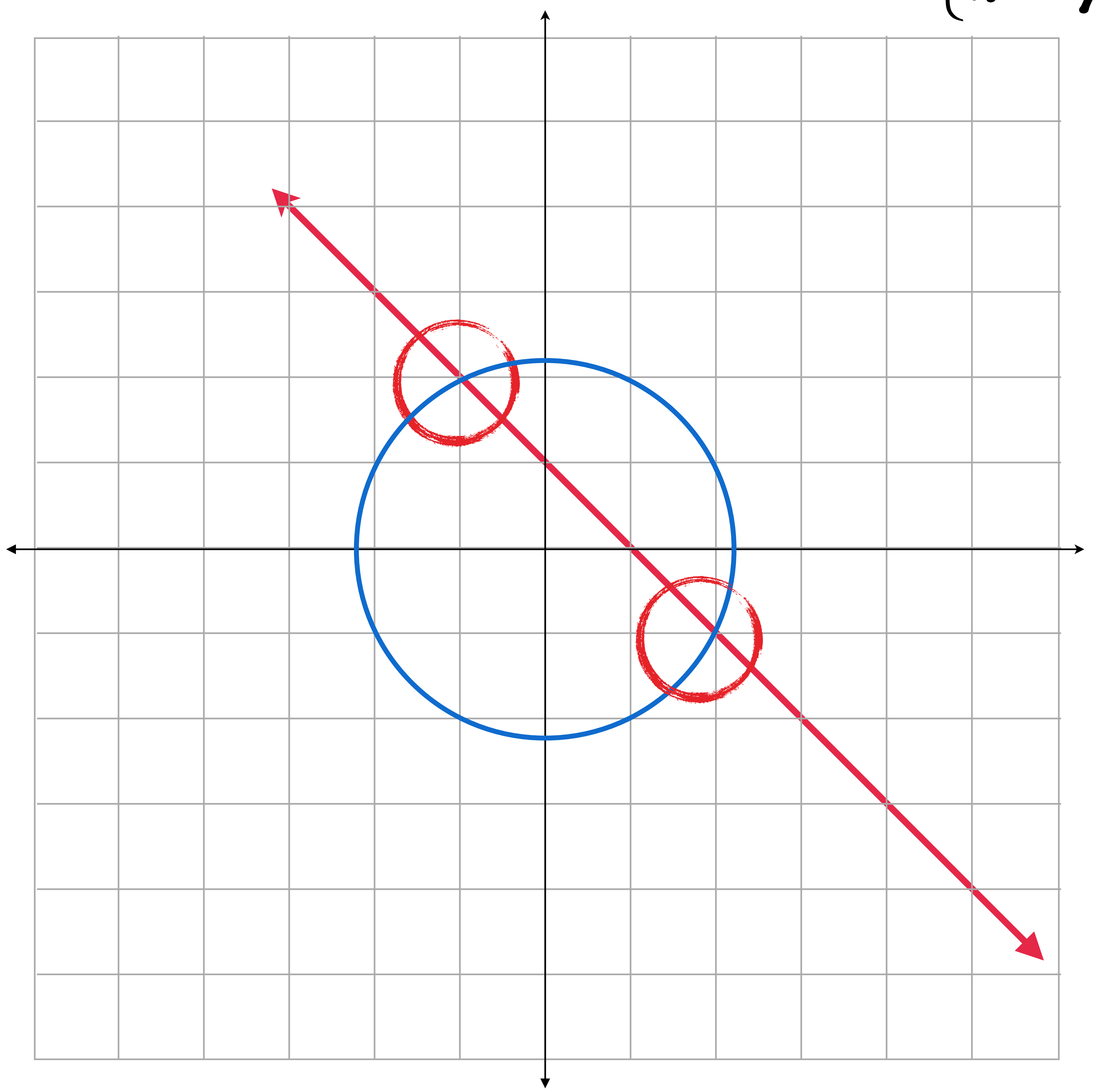
3.
$$x^2+x^2-2x+1=5$$
$$2x^2-2x+1=5$$
$$2x^2-2x-4=0$$
$$x^2-x-2=0$$
$$(x+1)(x-2)=0$$
$$x=-1 \text{ or } 2$$

4.
$$x+y=1 \qquad x+y=1$$
$$(-1)+y=1 \qquad 2+y=1$$
$$y=2 \qquad y=-1$$

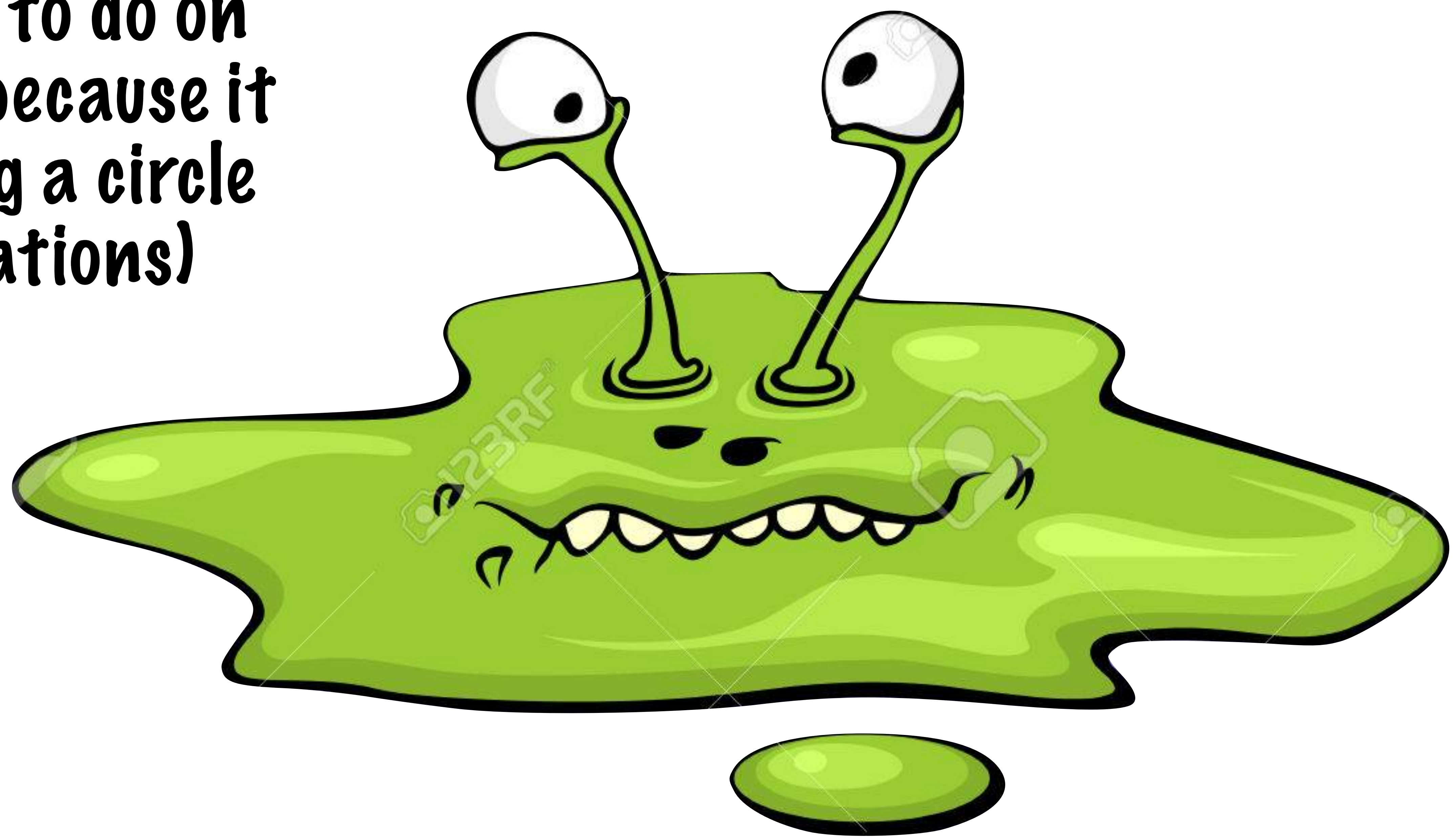
5. **Check** the solutions.

The solutions to the system are **(-1, 2)** and **(2, -1)**

Solve the system graphically: $\begin{cases} x^2 + y^2 = 5 \\ x + y = 1 \end{cases}$



This one is tough to do on your calculator because it requires graphing a circle (parametric equations)



Non-Linear Systems

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

Solve the system: $\begin{cases} x^2 - y = -6 \\ x - 4y = 8 \end{cases}$

1. $x - 4y = 8$

$x = 4y + 8$

2. $x^2 - y = -6$

$(4y+8)^2 - y = -6$

3. $16y^2 + 64y + 64 - y = -6$

$16y^2 + 63y + 70 = 0$

$$y = \frac{-63 \pm \sqrt{63^2 - 4(16)(70)}}{2(16)}$$

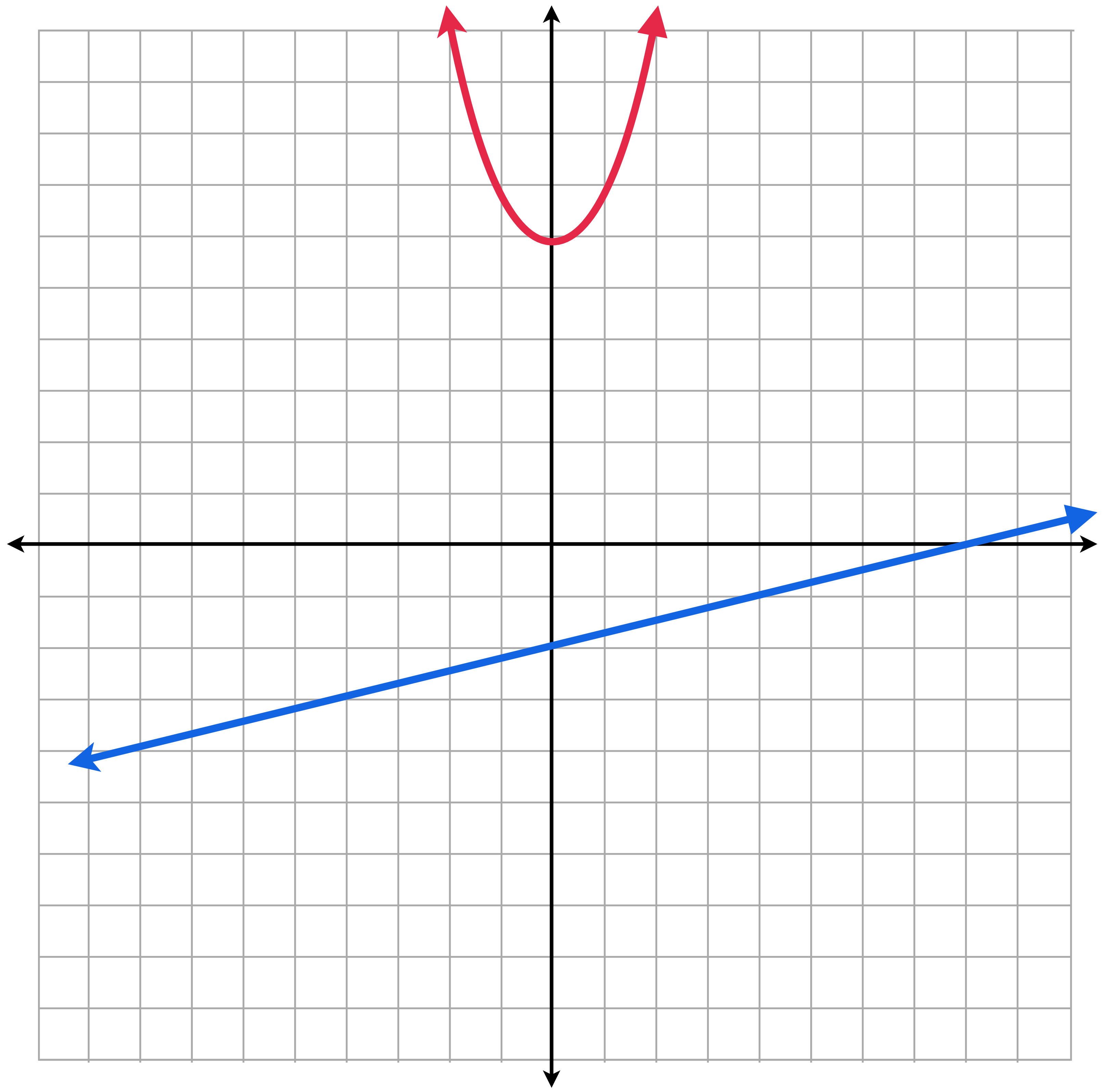
$$y = \frac{-63 \pm \sqrt{-511}}{32}$$

There are no real solutions.

Non-linear Systems

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

Solve the system graphically: $\begin{cases} x^2 - y = -6 \\ x - 4y = 8 \end{cases}$



This system can be easily seen on your calculator but you may have to resize the window.

Non-Linear Systems

7.1 Objective: Use substitution and graphing to solve systems of linear and nonlinear equations

Solve the system: $\begin{cases} y = e^x \\ x + y = 1 \end{cases}$

1. $y = e^x$

2. $x + y = 1$

   $x + e^x = 1$

3. $x + e^x = 1$

   $e^x = 1 - x$

   $\ln(1 - x) = x$

   $\frac{\ln 1}{\ln x} = x$

   $\frac{0}{\ln x} = x$

   $x = 0$

4. $x + y = 1$

   $(0) + y = 1$

   $y = 1$

5. **Check** the solutions.

   The solution to the system is **(0, 1)**

Solve the system graphically: $\begin{cases} y = e^x \\ x + y = 1 \end{cases}$