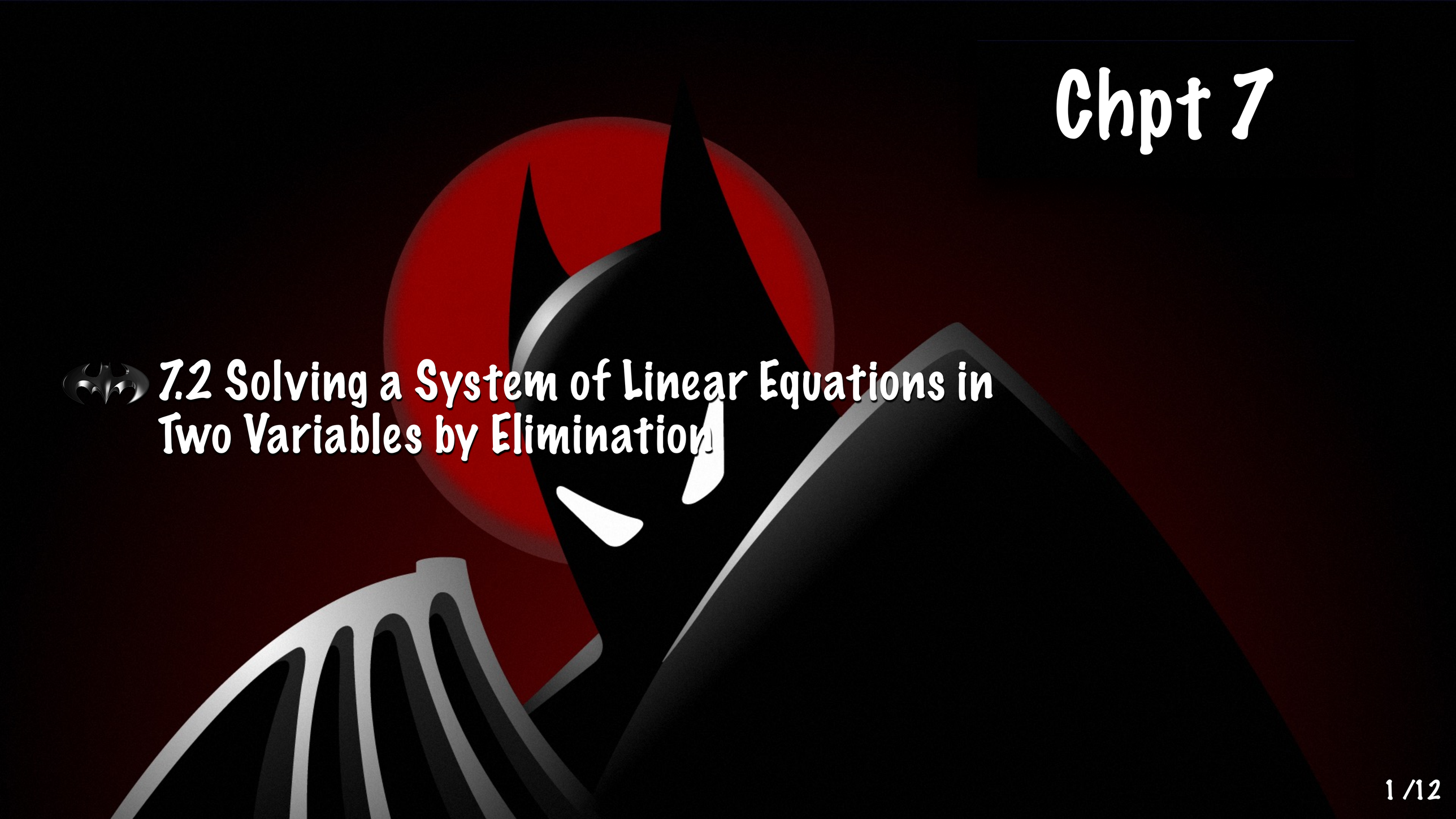
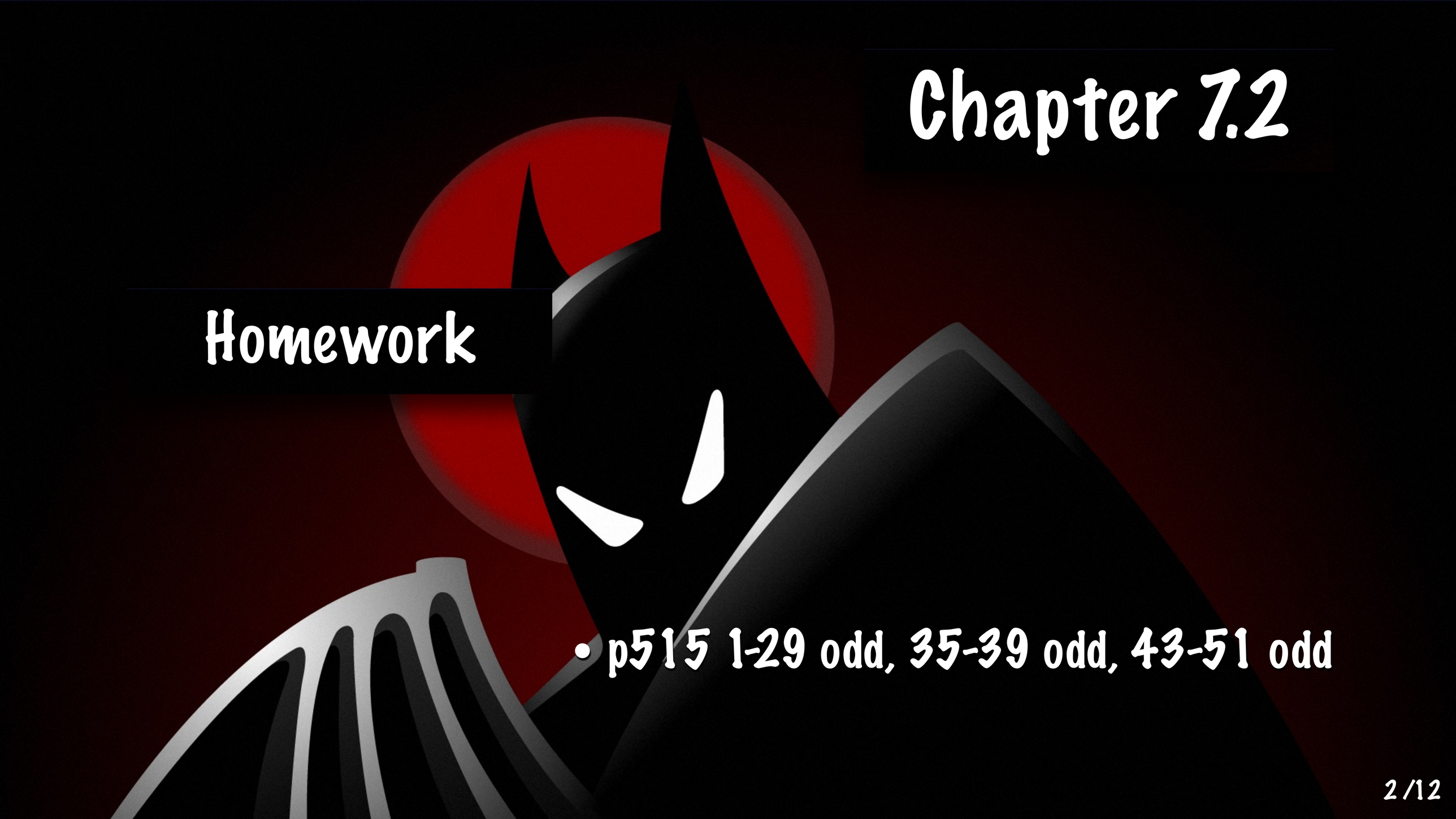
Chpt 7

7.2 Solving a System of Linear Equations in Two Variables by Elimination



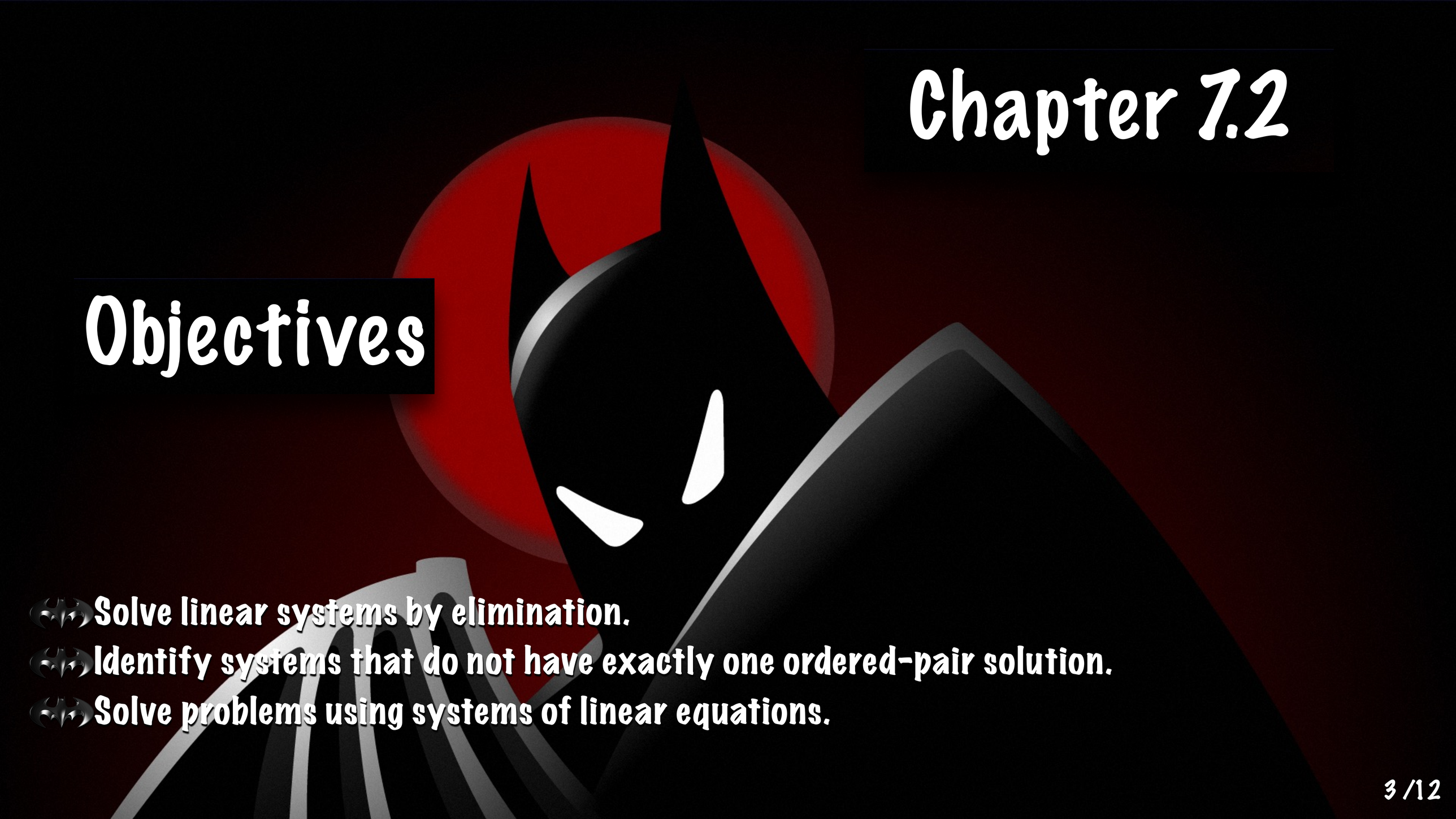
Chapter 7.2



Homework

- p515 1-29 odd, 35-39 odd, 43-51 odd

Chapter 7.2



Objectives

- Solve linear systems by elimination.
- Identify systems that do not have exactly one ordered-pair solution.
- Solve problems using systems of linear equations.

Solving Linear Systems by Elimination

Objective: Use method of elimination to solve systems of linear equations in two variables.

1. Re-write both equations in **standard form**, if necessary.

2. Multiply one or both equations by a value that obtains **opposite coefficients** for either variable.

3. Add the new equations so that **one variable is eliminated**, resulting in an equation with a single variable.

4. Solve the resulting equation for the remaining variable.

5. Substitute the value of the variable into either of the original equations and solve for the second variable.

6. Check your results.

7. Write your solution as an ordered pair.

Objective: Use method of elimination to solve systems of linear equations in two variables.

Method of Elimination

To use the **method of elimination** to solve a system of two linear equations in x and y, perform the following steps.

1. *Obtain coefficients* for x (or y) that differ only in sign by multiplying all terms of one or both equations by suitably chosen constants.
2. *Add* the equations to eliminate one variable, and solve the resulting equation.
3. *Back-substitute* the value obtained in Step 2 into either of the original equations and solve for the other variable.
4. *Check* your solution in both of the original equations.

Solving Linear Systems by Elimination

Objective: Use method of elimination to solve systems of linear equations in two variables.

Solve by elimination $\begin{cases} 4x+5y=3 \\ 2x-3y=7 \end{cases}$

1. $\begin{cases} 4x+5y=3 \\ 2x-3y=7 \end{cases}$

2. $\begin{cases} 4x+5y=3 \\ -2(2x-3y=7) \end{cases}$ $\quad \begin{cases} 4x+5y=3 \\ -4x+6y=-14 \end{cases}$

3. $0x+11y=-11$ $\qquad 11y=-11$

4. $y=-1$

5. $4x+5(-1)=3$

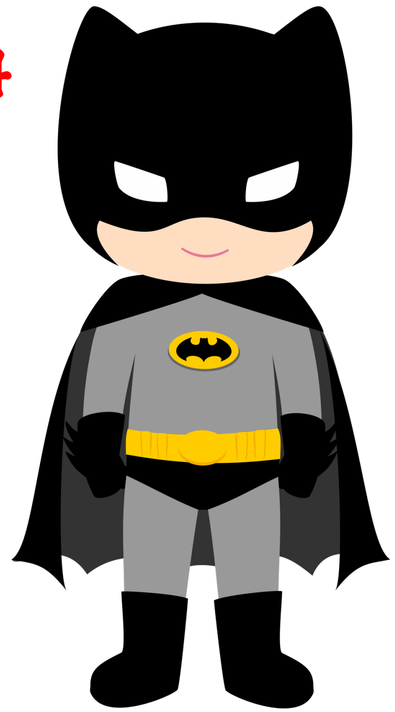
$4x=8$

$x=2$

5. $2x-3(-1)=7$

$2x=4$

$x=2$



6. Check your results.

7. The solution is (2, -1)

Solving Linear Systems by Elimination

Objective: Use method of elimination to solve systems of linear equations in two variables.

Solve by elimination $\begin{cases} 2x-3y=-15 \\ 2y=-5x+10 \end{cases}$

1. $\begin{cases} 2x-3y=-15 \\ 5x+2y=10 \end{cases}$

2. $\begin{cases} 2(2x-3y)=2(-15) \\ 3(5x+2y)=3(10) \end{cases}$ $\begin{cases} 4x-6y=-30 \\ 15x+6y=30 \end{cases}$

3. $19x+0y=0$ $\quad 19x=0$

4. $x=0$

5. $2y=-5(0)+10$
   $2y=10$
   $y=5$

6. Check your results.

7. The solution is $(0, 5)$

Word Problems

Objective: Use method of elimination to solve systems of linear equations in two variables.

- When solving word problems I require students to use a 7(8) step process.

1. Write (summarize) what is given
   1a. When appropriate, draw a picture of the situation.
2. Write the question. You cannot answer a question until you know what it is.
3. For unknown values necessary for finding the solution assign a variable.
4. Write a verbal (word) model of how you plan to solve the problem.
5. Translate the verbal model into an algebraic equation.
6. Solve the algebraic equation.
7. Write a complete sentence stating your solution.

Application

Objective: Use method of elimination to solve systems of linear equations in two variables.

A man in a boat can row 8 miles downstream in 1 hour. He can row 6 miles upstream in 3 hours. How fast can the man row in still water, and what is the rate of the current?

1. 8 miles down, 1 hour
   6 miles up, 3 hours

2. rowing speed in still water?
   rate of current?

3. r = rowing speed,
   c = current speed

4. rowing speed + current speed = speed down
   rowing speed - current speed = speed up distance = rate x time

5. $\begin{cases} r + c = \frac{8}{1} \\ r - c = \frac{6}{3} \end{cases}$ $\begin{cases} r + c = 8 \\ r - c = 2 \end{cases}$

$5 + c = 8$

$c = 3$

The rowing speed is 5 mph and the current speed is 3 mph.

Slope

Objective: Use method of elimination to solve systems of linear equations in two variables.

- Remember, you can graph the equations to find the solution graphically.

STUDY TIP

A comparison of the slopes of two lines gives useful information about the number of solutions of the corresponding system of equations. To solve a system of equations graphically, it helps to begin by writing the equations in slope-intercept form. Try doing this for the systems in Example 4.

Graphical Interpretations of Solutions

For a system of two linear equations in two variables, the number of solutions is one of the following.

Number of Solutions	*Graphical Interpretation*	*Slopes of Lines*
1. Exactly one solution	The two lines intersect at one point.	The slopes of the two lines are not equal.
2. Infinitely many solutions	The two lines coincide (are identical).	The slopes of the two lines are equal.
3. No solution	The two lines are parallel.	The slopes of the two lines are equal.

Solving Linear Systems by Elimination

Objective: Use method of elimination to solve systems of linear equations in two variables.

Solve by elimination $\begin{cases} x+3y=5 \\ -2x-6y=1 \end{cases}$

1. $\begin{cases} x+3y=5 \\ -2x-6y=1 \end{cases}$

2. $\begin{cases} 2(x+3y)=2(5) \\ -2x-6y=1 \end{cases}$ $\quad \begin{cases} 2x+6y=10 \\ -2x-6y=1 \end{cases}$ $\quad \begin{cases} y=-\frac{1}{3}x+\frac{5}{3} \\ y=-\frac{1}{3}x-\frac{1}{3} \end{cases}$

3. $0x+0y=11 \quad 0=11$

4. Obviously, no solution.

Equal slopes, different intercepts, parallel lines, no solution.

Solving Linear Systems by Elimination

Objective: Use method of elimination to solve systems of linear equations in two variables.

Solve by elimination

$$\begin{cases} \frac{1}{4}x - \frac{1}{2}y = 1 \\ -x + 2y = -4 \end{cases}$$

1. $\begin{cases} x - 2y = 4 \\ -x + 2y = -4 \end{cases}$

2. $\begin{cases} x - 2y = 4 \\ -x + 2y = -4 \end{cases}$

$$\begin{cases} y = \frac{1}{2}x - 2 \\ y = \frac{1}{2}x - 2 \end{cases}$$

3. $0x + 0y = 0 \qquad 0 = 0$

Equal slopes, equal intercepts, same line, infinite solution.

4. Infinite solutions