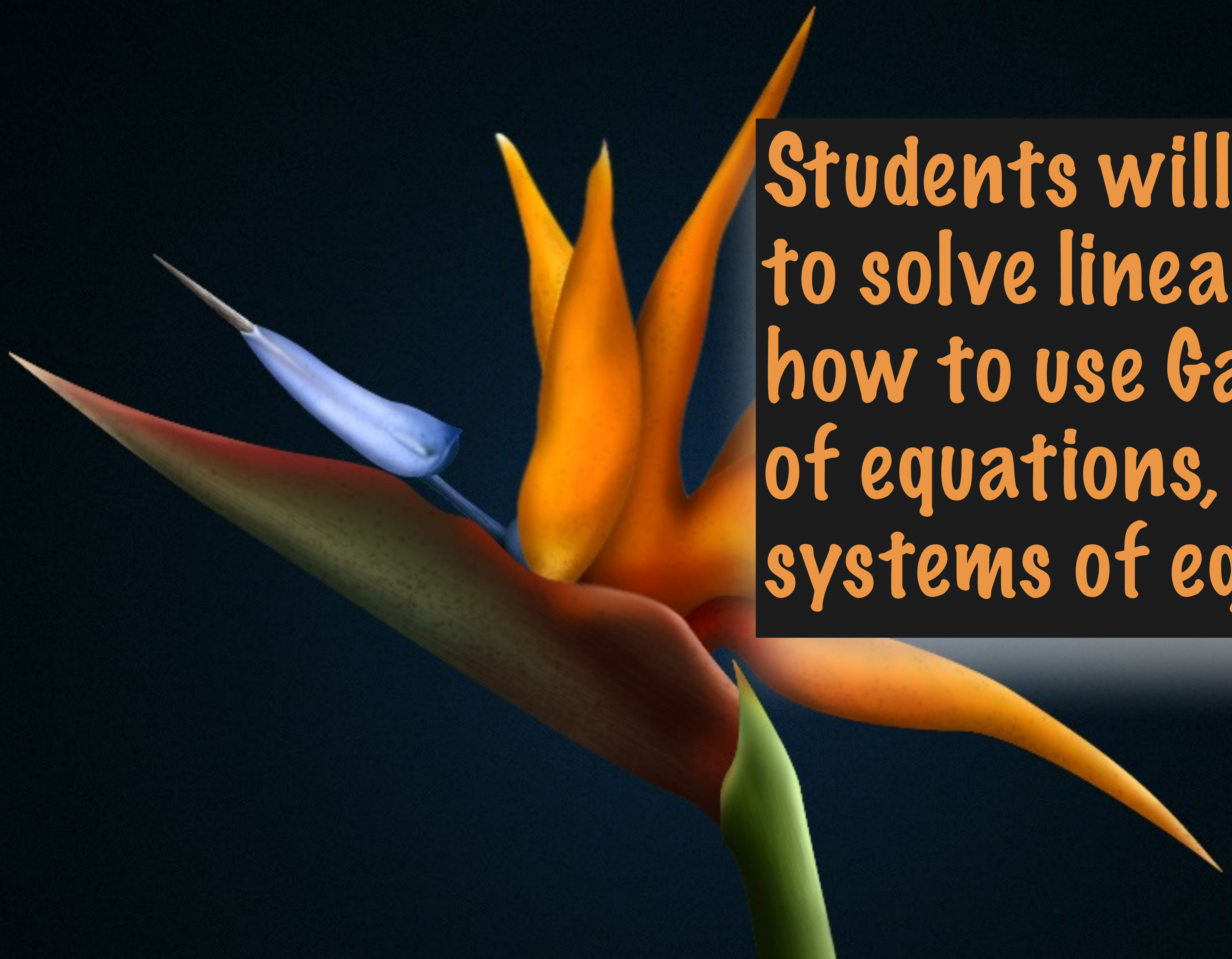
Chpt 7-3

7.3 Multivariable Linear Systems

7.3 Homework

7.3 p527 5-45 odd, 51, 53, 55, 67, 69, 71

7.3 Objectives



Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Linear Systems in 3 Variables

- There are several terms in the objective with which you may be unfamiliar.
  - Back-substitution
  - Row-echelon form
  - Gaussian elimination
  - Non-square systems
- We will define these terms throughout this presentation.
- We will also restrict ourselves to systems of 3 equations in 3 variables.
- In general, an equation of the form Ax + By + Cz = D where A, B, C, D are real numbers such that A, B, and C are not **all** 0, is a **linear equation in three variables, x, y, and z**.
- A solution of a system of linear equations in three variables is an **ordered triple** of real numbers that satisfies all equations of the system. The solution set of the system is the set of all its solutions.

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Linear Systems in 3 Variables

- Show that the ordered triple (–1, –4, 5) is a solution of the system:

$$\begin{cases} x - 2y + 3z = 22 \\ 2x - 3y - z = 5 \\ 3x + y - 5z = -32 \end{cases}$$

$$x - 2y + 3z = 22$$
$$-1 - 2(-4) + 3(5) = 22$$
$$-1 + 8 + 15 = 22$$
$$22 = 22$$

$$2x - 3y - z = 5$$
$$2(-1) - 3(-4) - 5 = 22$$
$$-2 + 12 - 5 = 5$$
$$5 = 5$$

$$3x + y - 5z = -32$$
$$3(-1) + (-4) - 5(5) = -32$$
$$-3 - 4 - 25 = -32$$
$$-32 = -32$$

- Since the point satisfies all three equations it is a solution to the system.

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Row Echelon Form

- To find solutions to a system of linear equations in three variables we will use elimination. Our goal is to transform the system of equations into an equivalent system of equations with which back-substitution can be used.
  - Two systems are equivalent systems if they have the same solution set.
  - The converted system of a simplified system of 3 equations is referred to as the **row echelon form**.
  - Solving three-variable, three-equation linear systems is no more difficult than solving a two variable, two equation system. Solving a three equation, three variable system seems more difficult because there are more computations creating more chances for making a mistake. You will need to be very neat and organized with your work, and you should plan to use lots of scratch paper. The method for solving these systems is an extension of the two-variable solving by elimination method, so make sure you know the elimination method well and can use it correctly.

- Though the method of solution is based on elimination, trying to do actual addition of equations tends to get very confusing, so there is a systematic method for solving the three-or-more-variables systems. This method is called "Gaussian elimination" (with the equations ending up in what is called "**row echelon form**").

- Essentially, putting a system into **row echelon form** means changing the system into an equivalent system, through elimination, such that each succeeding equation loses a variable. In **reduced row echelon form** the leading coefficient in each equation is 1.

  - Here is an example of a system in reduced row echelon form.

$$\begin{cases} x - 3y + 2z = 1 \\ \quad\; y - 3z = -7 \\ \qquad\quad z = 3 \end{cases}$$

  - It is unimportant what we call these, what is important is the process of getting to the row echelon form.

Row Echelon Form

- In our preceding example $\begin{cases} x-2y+3z=22 \\ 2x-3y-z=5 \\ 3x+y-5z=-32 \end{cases}$ we confirmed the solution is (-1, -4, 5)

- The system $\begin{cases} x-2y+3z=22 \\ \quad y-7z=-39 \\ \quad\quad z=5 \end{cases}$ has the same solution of (-1, -4, 5)

- This second form is our **row echelon form**. Our goal is to change the initial system of equations into the second system of equation using elimination.

Slide 13

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Gaussian Elimination

- To change a system of equations into **row echelon form** follow the following procedure.
  - Rearrange the equations into what you think might me a more convenient form. Typically we try to get the leading coefficient of the first equation to be 1.
  - Multiply by a constant and add two equations to eliminate a variable as you did with two equation system. We are eliminating one variable. The variable we eliminated will often be the leading variable (x), but it can be any variable.
  - Replace one of the equations usually the 2nd with the new simplified equation.
  - Repeat with the remaining two original equations. Replacing one (usually equation 3) with the simplified equation.
  - Combine the 2 simplified equations to eliminate a second variable. Replace one of the simplified equations with the simplified equation with only one variable.
  - Voila! You have a simplified row echelons form of the system.

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Example

- Let us start easy with a system of two variables.

$$\begin{cases} 2x - 3y = 6 \\ x - 2y = 1 \end{cases}$$

- Rearrange the equations to get the first equation with a leading coefficient of 1.

$$\begin{cases} x - 2y = 1 \\ 2x - 3y = 6 \end{cases}$$

- Multiply the first equation by -2

$$\begin{cases} -2x + 4y = -2 \\ 2x - 3y = 6 \end{cases}$$

- Add the two equations thereby eliminating the variable x.

$$y = 4$$

- Replace the second equation with the reduced equation to create the row-echelon form of the system.

$$\begin{cases} x - 2y = 1 \\ y = 4 \end{cases}$$

- We back substitute 4 for y in the preceding (back) equation.

$$x - 2(4) = 1$$

$$x - 8 = 1$$

$$x = 9$$

- The solution to the system is **(9, 4)**.

- Always check your results.

Example

- We can now "back-substitute" the 4 for y in the first equation to find x.

$$\begin{cases} 2x - 3y = 6 \\ x - 2y = 1 \end{cases} \quad \begin{cases} x - 2y = 1 \\ 2x - 3y = 6 \end{cases} \quad \begin{cases} -2x + 4y = -2 \\ 2x - 3y = 6 \end{cases} \quad y = 4 \quad \begin{cases} x - 2y = 1 \\ y = 4 \end{cases}$$

  - That simply means we substitute 4 for y in the preceding (back) equation.

$$x - 2(4) = 1$$
$$x - 8 = 1$$
$$x = 9$$

  - The solution to the system is **(9, 4)**.

  - Always check your results.

Example

- Let us go back to our original system from slide 4: $\begin{cases} x-2y+3z=22 \\ 2x-3y-z=5 \\ 3x+y-5z=-32 \end{cases}$

  - The first equation has a leading coefficient of 1. I like the arrangement. I will number the equations as we go along to help keep us organized.

  - Multiply by equation 1 by -2, add to equation 2, then replace equation 2 with the result.

$$\begin{cases} x-2y+3z=22 & (1) \\ 2x-3y-z=5 & (2) \\ 3x+y-5z=-32 & (3) \end{cases}$$

$$\begin{array}{rl} -2x+4y-6z=-44 & (1)\cdot(-2) \\ 2x-3y-z=5 & (2) \\ \hline y-7z=-39 & (4) \end{array}$$

$$\begin{cases} x-2y+3z=22 & (1) \\ y-7z=-39 & (4) \\ 3x+y-5z=-32 & (3) \end{cases}$$

Example

- Now repeat with equations 1 and 3, multiplying equation 1 by -3. See why we like the first equation to have a leading coefficient of 1?

$$\begin{cases} x - 2y + 3z = 22 & (1) \\ y - 7z = -39 & (4) \\ 3x + y - 5z = -32 & (3) \end{cases}$$

$$\begin{array}{rl} -3x + 6y - 9z = -66 & (1)\cdot(-3) \\ 3x + y - 5z = -32 & (3) \\ \hline 7y - 14z = -98 & (5) \end{array}$$

- Replace the original equation 3 with the simplified equation 5.

$$\begin{cases} x - 2y + 3z = 22 & (1) \\ y - 7z = -39 & (4) \\ 7y - 14z = -98 & (5) \end{cases}$$

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Example

- We repeat the process with equations 2 and 3. Multiply equation 2 by -7, add the new equations, thereby eliminating y, and replace equation 3 with the result.

$$\begin{cases} x-2y+3z=22 & (1) \\ y-7z=-39 & (4) \\ 7y-14z=-98 & (5) \end{cases}$$

$$\begin{array}{rl} -7y+49z=273 & (4)\cdot(-7) \\ 7y-14z=-98 & (5) \\ \hline 35z=175 & \\ z=5 & (6) \end{array}$$

$$\begin{cases} x-2y+3z=22 & (1) \\ y-7z=-39 & (4) \\ z=5 & (6) \end{cases}$$

- Reduced Row Echelon Form

- The rest should be obvious. What we do next is the back-substitution.

$$y-7z=-39$$
$$y-7(5)=-39$$
$$y=-4$$

$$x-2y+3z=22$$
$$x-2(-4)+3(5)=22$$
$$x=-1$$

- The solution is (-1, -4, 5)

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Gaussian Elimination

- To review the procedure for simplifying a system into **row echelon form.**
  - Rearrange the equations into what you think might me a more convenient form. Typically we try to get the leading coefficient of the first equation to be 1.
  - Multiply by a constant and add two equations to eliminate a variable as you did with two equation system. We are eliminating one variable. The variable we eliminated will often be the leading variable (x), but it can be any variable.
  - Replace one of the equations usually the 2nd with the new simplified equation.
  - Repeat with the remaining two original equations. Replacing one (usually equation 3) with the simplified equation.
  - Combine the 2 simplified equations to eliminate a second variable. Replace one of the simplified equations with the simplified equation with only one variable.
  - Voila! You have a simplified row echelons form of the system.

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Gaussian Elimination

Operations That Produce Equivalent Systems

Each of the following **row operations** on a system of linear equations produces an *equivalent* system of linear equations.

1. Interchange two equations.

2. Multiply one of the equations by a nonzero constant.

3. Add a multiple of one of the equations to another equation to replace the latter equation.

STUDY TIP

Arithmetic errors are often made when performing elementary row operations. You should note the operation performed in each step so that you can go back and check your work.

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Example: Solving a System in Three Variables

Solve the system

$$\begin{cases} x-3y+2z=1 & (1) \\ 2x-5y+z=-5 & (2) \\ 3x+y-2z=-1 & (3) \end{cases}$$

$$\begin{array}{rl} -2x+6y-4z=-2 & (1)\cdot(-2) \\ 2x-5y+\ \ z=-5 & (2) \\ \hline y-3z=-7 & (4) \end{array}$$

$$\begin{cases} x-3y+2z=1 & (1) \\ y-3z=-7 & (4) \\ 3x+y-2z=-1 & (3) \end{cases}$$

$$\begin{array}{rl} -3x+9y-6z=-3 & (1)\cdot(-3) \\ 3x+\ \ y-2z=-1 & (3) \\ \hline 10y-8z=-4 & (5) \end{array}$$

$$\begin{cases} x-3y+2z=1 & (1) \\ y-3z=-7 & (4) \\ 10y-8z=-4 & (5) \end{cases}$$

$$\begin{array}{rl} -10y+30z=70 & (4)\cdot(-10) \\ 10y-\ \ 8z=-4 & (5) \\ \hline 22z=66 & \\ z=3 & (6) \end{array}$$

$$\begin{cases} x-3y+2z=1 & (1) \\ y-3z=-7 & (4) \\ z=3 & (6) \end{cases}$$

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Example

Solve the system

$$\begin{cases} x - 3y + 2z = 1 \\ 2x - 5y + z = -5 \\ 3x + y - 2z = -1 \end{cases} \qquad \begin{cases} x - 3y + 2z = 1 \\ y - 3z = -7 \\ z = 3 \end{cases}$$

Back substitution

$$y - 3z = -7$$
$$y - 3(3) = -7$$
$$y = 2$$

$$x - 3y + 2z = 1$$
$$x - 3(2) + 2(3) = 1$$
$$x = 1$$

The solution **(1, 2, 3)**

Check the solution by substituting back into all 3 equations.

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Example: Solving a System in Three Variables

- Solve the system $\begin{cases} x+3y-2z=1 \\ -x-2y+6z=-2 \\ 3x+11y+2z=6 \end{cases}$

$$\begin{array}{r} x+3y-2z=1 \\ -x-2y+6z=-2 \\ \hline y+4z=-1 \end{array}$$

$$\begin{cases} x+3y-2z=1 \\ y+4z=-1 \\ 3x+11y+2z=6 \end{cases}$$

$$\begin{array}{r} -3x-9y+6z=-3 \\ 3x+11y+2z=6 \\ \hline 2y+8z=3 \end{array}$$

$$\begin{cases} x-3y+2z=1 \\ y+4z=-1 \\ 2y+8z=3 \end{cases}$$

$$\begin{array}{r} -2y-8z=2 \\ 2y+8z=3 \\ \hline 0z=5 \end{array}$$

- We know what that means.

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Example: Solving a System in Three Variables

- Solve the system $\begin{cases} 2x+3y+4z=5 \\ x+y+z=2 \\ x-z=1 \end{cases}$

- Rearrange to get the first equation with a leading coefficient of 1.

$$\begin{cases} x+y+z=2 \\ 2x+3y+4z=5 \\ x-z=1 \end{cases}$$

$$\begin{array}{r} -2x-2y-2z=-4 \\ 2x+3y+4z=5 \\ \hline y+2z=1 \end{array}$$

$$\begin{cases} x+y+z=2 \\ y+2z=1 \\ x-z=1 \end{cases}$$

$$\begin{array}{r} -x-y-z=-2 \\ x\quad\;\; -z=1 \\ \hline -y-2z=-1 \end{array}$$

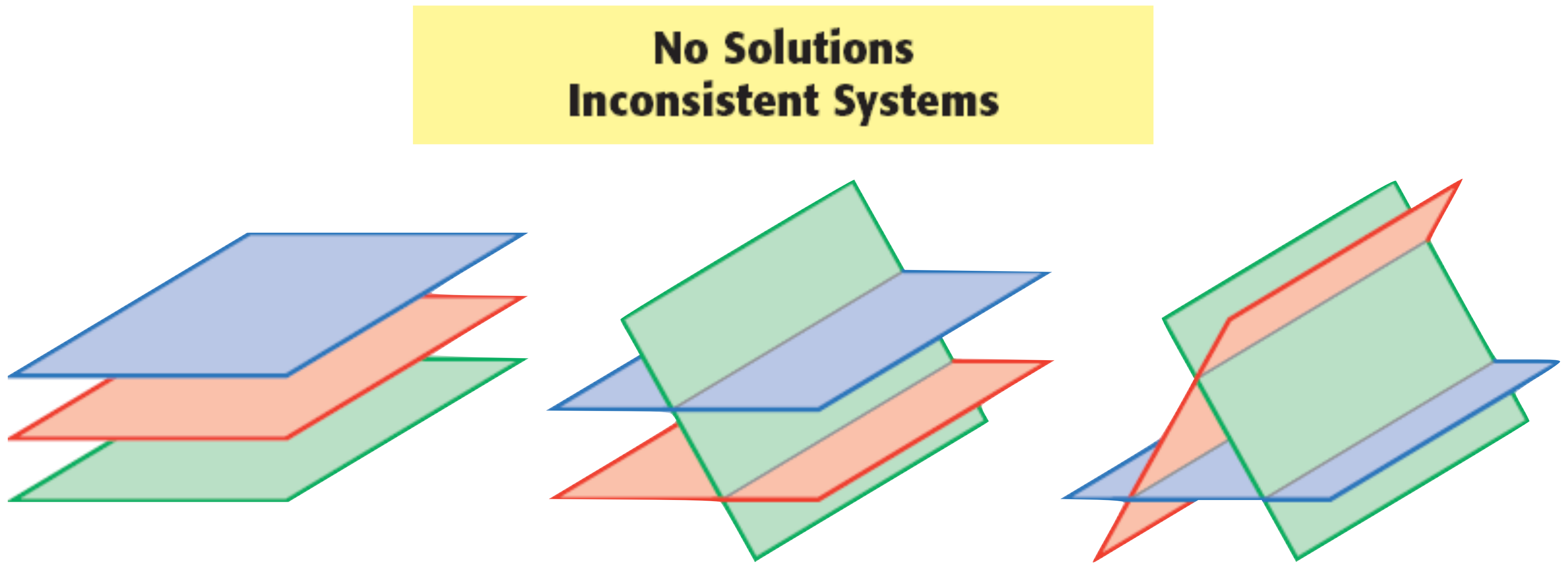
$$\begin{cases} x+y+z=2 \\ y+2z=1 \\ -y-2z=-1 \end{cases}$$

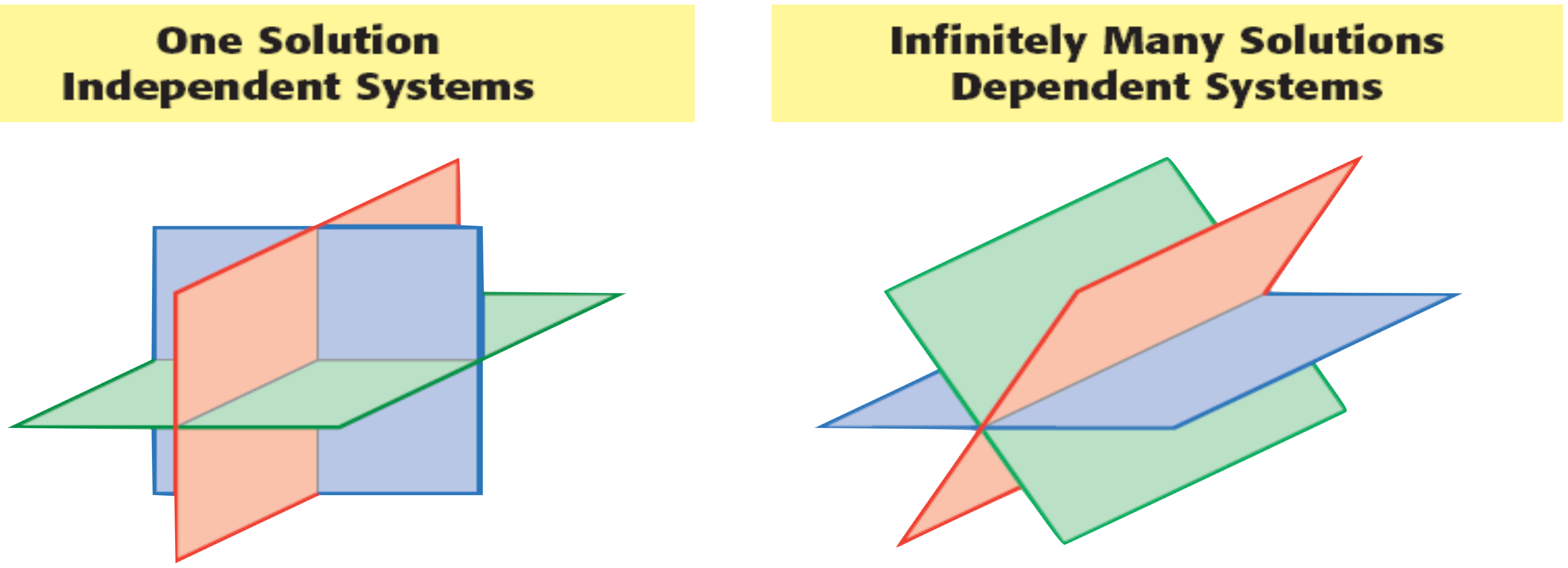
$$\begin{array}{r} y+2z=1 \\ -y-2z=-1 \\ \hline 0=0 \end{array}$$

You do not know what this means yet.

System of 3 variables

- The graph of a linear equation in three variables is a **plane**. When you graph a system of three linear equations in three dimensions, the result is three planes that may or may not intersect. The solution to the system is the set of points where all three planes intersect. These systems may have one, infinitely many, or no solution.





- **Independent system** A linear system is independent if none of the equations can be derived algebraically from the others. With equations in two variables that can be interpreted to mean the equations have different slopes and different y-intercepts. With equations in three variables that means the planes do not share a line.
- A **dependent system** is not independent. Two or more of the equations are equivalent equations.

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Example

- Solve the system

$$\begin{cases} 3x+2y+z=8 \\ 2x-3y+2z=-16 \\ x+4y-z=20 \end{cases}$$

- Rearrange

$$\begin{cases} x+4y-z=20 \\ 3x+2y+z=8 \\ 2x-3y+2z=-16 \end{cases}$$

$$\begin{array}{r} -3x-12y+3z=-60 \\ 3x+2y+z=8 \\ \hline -10y+4z=-52 \end{array}$$

$$\begin{cases} x+4y-z=20 \\ -10y+4z=-52 \\ 2x-3y+2z=-16 \end{cases}$$

$$\begin{array}{r} -2x-8y+2z=-40 \\ 2x-3y+2z=-16 \\ \hline -11y+4z=-56 \end{array}$$

$$\begin{cases} x+4y-z=20 \\ -10y+4z=-52 \\ -11y+4z=-56 \end{cases}$$

$$\begin{array}{r} -110y+44z=-572 \\ 110y-40z=560 \\ \hline 4z=-12 \\ z=-3 \end{array}$$

$$\begin{cases} x+4y-z=20 \\ -10y+4z=-52 \\ z=-3 \end{cases}$$

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Example

- Solve the system $\begin{cases} 3x+2y+z=8 \\ 2x-3y+2z=-16 \\ x+4y-z=20 \end{cases}$

$$\begin{cases} x+4y-z=20 \\ -10y+4z=-52 \\ z=-3 \end{cases}$$

$$-10y+4z=-52$$
$$-10y+4(-3)=-52$$
$$-10y-12=-52$$
$$-10y=-40$$
$$y=4$$

$$x+4y-z=20$$
$$x+4(4)--3=20$$
$$x+16+3=20$$
$$x+19=20$$
$$x=1$$

- The solution

(1, 4, -3)

- Check the solution by substituting back into all 3 equations.

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Example

Solve the system
$$\begin{cases} 5x-2y-3z=-7 \\ 2x-3y+z=-16 \\ 3x+4y-2z=7 \end{cases}$$

Rearrange
$$\begin{cases} 2x-3y+z=-16 & (1) \\ 5x-2y-3z=-7 & (2) \\ 3x+4y-2z=7 & (3) \end{cases}$$

$$\begin{aligned} -10x+15y-5z&=80 && (1)\cdot(-5) \\ 10x-4y-6z&=-14 && (2)\cdot(2) \\ \hline 11y-11z&=66 && (4) \end{aligned}$$

$$\begin{aligned} -6x+9y-3z&=48 && (1)\cdot(-3) \\ 6x+8y-4z&=14 && (3)\cdot(2) \\ \hline 17y-7z&=62 && (5) \end{aligned}$$

$$\begin{cases} 2x-3y+z=-16 & (1) \\ 11y-11z=66 & (4) \\ 17y-7z=62 & (5) \end{cases}$$

$$\begin{aligned} 187y-187z&=1122 && (4)\cdot(17) \\ -187y+77z&=-682 && (5)\cdot(-11) \\ \hline -110z&=440 \\ z&=-4 && (6) \end{aligned}$$

$$\begin{cases} 2x-3y+z=-16 & (1) \\ 11y-11z=66 & (4) \\ z=-4 & (6) \end{cases}$$

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Example

- Solve the system $\begin{cases} 5x-2y-3z=-7 \\ 2x-3y+z=-16 \\ 3x+4y-2z=7 \end{cases}$

$$\begin{cases} 2x-3y+z=-16 \\ 11y-11z=66 \\ z=-4 \end{cases}$$

$$\begin{aligned} 11y-11z&=66 \\ 11y-11(-4)&=66 \\ 11y&=22 \\ y&=2 \end{aligned}$$

$$\begin{aligned} 2x-3y+z&=-16 \\ 2x-3(2)+(-4)&=-16 \\ 2x-6-4&=-16 \\ 2x&=-6 \\ x&=-3 \end{aligned}$$

- The solution

(-3, 2, -4)

- Check the solution by substituting back into all 3 equations.

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Example

Solve the system $\begin{cases} -x + y + 2z = 7 \\ 2x + 3y + z = 1 \\ -3x - 4y + z = 4 \end{cases}$

Slight Change $\begin{cases} x - y - 2z = -7 & (1) \\ 2x + 3y + z = 1 & (2) \\ -3x - 4y + z = 4 & (3) \end{cases}$

$$\begin{array}{rl} -2x + 2y + 4z = 14 & (1) \cdot (-3) \\ 2x + 3y + z = 1 & (2) \\ \hline 5y + 5z = 15 & (4) \end{array}$$

$$\begin{array}{rl} 3x - 3y - 6z = -21 & (1) \cdot (3) \\ -3x - 4y + z = 4 & (3) \\ \hline -7y - 5z = -17 & (5) \end{array}$$

$$\begin{cases} x - y - 2z = -7 & (1) \\ 5y + 5z = 15 & (4) \\ -7y - 5z = -17 & (5) \end{cases}$$

$$\begin{array}{rl} 35y + 35z = 105 & (4) \cdot (7) \\ -35y - 25z = -85 & (5) \cdot (5) \\ \hline 10z = 20 & \\ z = 2 & (6) \end{array}$$

$$\begin{cases} x - y - 2z = -7 & (1) \\ 5y + 5z = 15 & (4) \\ z = 2 & (6) \end{cases}$$

$$5y + 5z = 15$$
$$5y + 5(2) = 15$$
$$y = 1$$

$$x - y - 2z = -7$$
$$x - 1 - 2(2) = -7$$
$$x = -2$$

(–2, 1, 2)

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Public Service Announcement

- You may have noticed (I hope) and remember that it is not necessary to eliminate x first, then y second, solving initially for z. In fact, in some of the examples it would have been simpler to eliminate other variables first. The Gaussian elimination method which results in the row echelon form focuses on the columns of the system. The purpose of focusing on the columns will become clear in the next section involving using matrices to solve systems of equations.

- With a **non-square system** of equations (the number of equations does not equal the number of variables), we will never have exactly one solution. The solution will involve relationships between the variables.

STUDY TIP

When comparing descriptions of an infinite solution set, keep in mind that there is more than one way to describe the set.

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Non-square Systems

- With a **non-square system** of equations (the number of equations does not equal the number of variables), we will never have exactly one solution. The solution will involve relationships between the variables.
  - For example, we might write our solution as (a, a+3, a-2)
    - This describes a relationship between the 3 coordinates.
    - However, the same relationship could be written (a-3, a, a-5) or (a+2, a+5, a)
      - Those are 3 different descriptions of the same solution set.

STUDY TIP

When comparing descriptions of an infinite solution set, keep in mind that there is more than one way to describe the set.

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Dependent System

- Solve the system $\begin{cases} 2x+3y+4z=5 \\ x+y+z=2 \\ x-z=1 \end{cases}$

- Rearrange $\begin{cases} x+y+z=2 \\ 2x+3y+4z=5 \\ x-z=1 \end{cases}$

$$\begin{array}{r} -2x-2y-2z=-4 \\ 2x+3y+4z=5 \\ \hline y+2z=1 \end{array}$$

$$\begin{cases} x+y+z=2 \\ y+2z=1 \\ x-z=1 \end{cases}$$

$$\begin{array}{r} -x-y-z=-2 \\ x\qquad -z=1 \\ \hline -y-2z=-1 \end{array}$$

$$\begin{cases} x+y+z=2 \\ y+2z=1 \\ -y-2z=-1 \end{cases}$$

$$\begin{array}{r} y+2z=1 \\ -y-2z=-1 \\ \hline 0=0 \end{array}$$

Let z = a

$$y+2a=1$$
$$y=1-2a$$

$$x+y+z=2$$
$$x+1-2a+a=2$$
$$x=1+a$$

$$(1+a,\ 1-2a,\ a)$$

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Non-square Systems

Solve the system $\begin{cases} x-4y+2z=5 \\ 3x+5y-2z=1 \end{cases}$

$$\begin{array}{r} -3x+12y-6z=-15 \\ 3x+5y-2z=1 \\ \hline 17y-8z=-14 \end{array}$$

$$\begin{cases} x-4y+2z=5 \\ 17y-8z=-14 \\ z=a \end{cases}$$

$$17y-8a=-14$$

$$17y=8a-14$$

$$y=\frac{8}{17}a-\frac{14}{17}$$

$$x-4y+2z=5$$

$$x-4\left(\frac{8}{17}a-\frac{14}{17}\right)+2a=5$$

$$x-\frac{32}{17}a+\frac{56}{17}+\frac{34}{17}a=5$$

$$x=-\frac{2}{17}a+\frac{29}{17}$$

The solutions

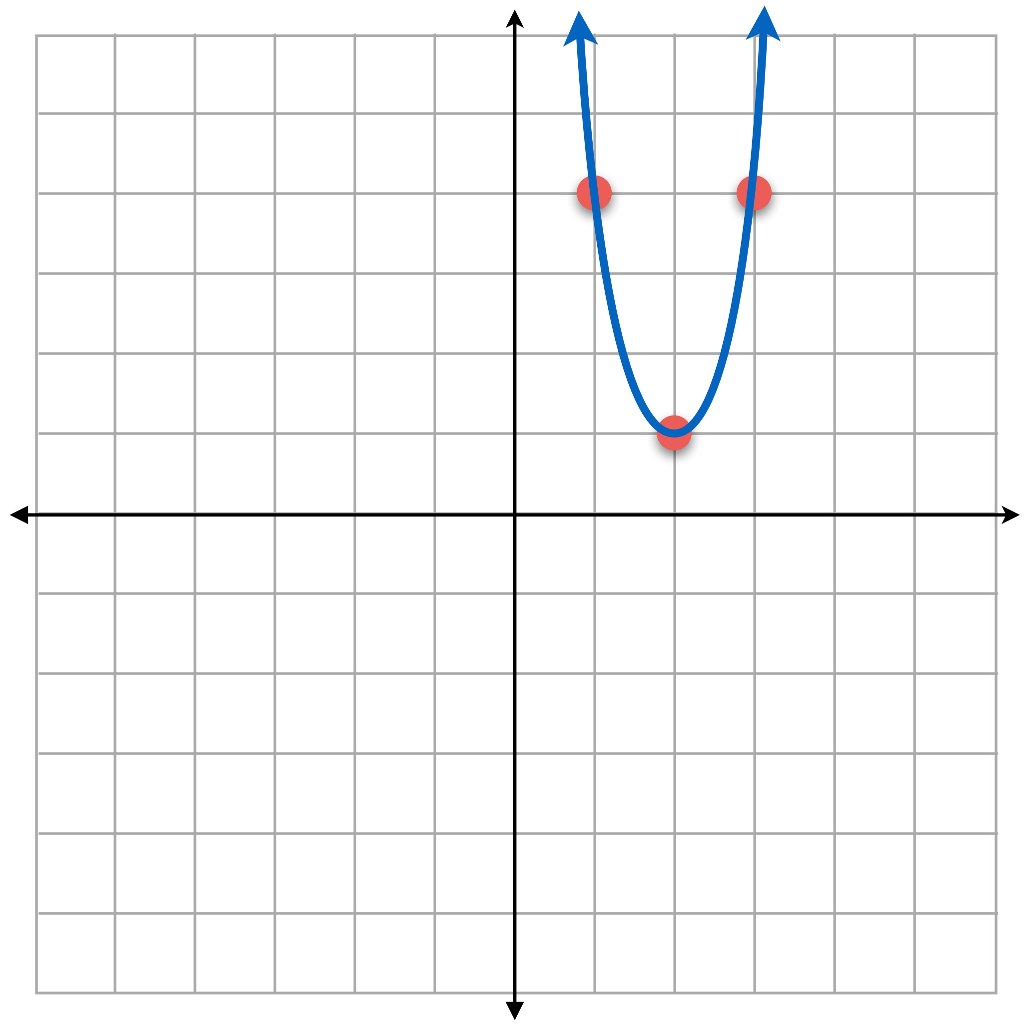
$$\left(-\frac{2}{17}a+\frac{29}{17},\frac{8}{17}a-\frac{14}{17},a\right)$$

$$\left(-\frac{2}{17}z+\frac{29}{17},\frac{8}{17}z-\frac{14}{17},z\right)$$

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Application

- Find the quadratic function **$y = ax^2 + bx + c$** whose graph passes through the points (1, 4), (2, 1), and (3, 4).



  - Begin by substituting each ordered pair into the equation $y = ax^2 + bx + c$

$$4 = a(1)^2 + b(1) + c \qquad 1 = a(2)^2 + b(2) + c \qquad 4 = a(3)^2 + b(3) + c$$

$$a + b + c = 4 \qquad 4a + 2b + c = 1 \qquad 9a + 3b + c = 4$$

- To find a, b, and c, we form a system with these equations and solve the system.

Students will know how to use back-substitution to solve linear systems in row-echelon form, how to use Gaussian elimination to solve systems of equations, and how to solve nonsquare systems of equations.

Application

$$a+b+c=4$$
$$4a+2b+c=1$$
$$9a+3b+c=4$$

$$\begin{array}{r} -4a-4b-4c=-16 \\ 4a+2b+c=1 \\ \hline -2b-3c=-15 \end{array}$$

$$\begin{array}{r} -9a-9b-9c=-36 \\ 9a+3b+c=4 \\ \hline -6b-8c=-32 \end{array}$$

$$\begin{cases} a+b+c=4 \\ -2b-3c=-15 \\ -6b-8c=-32 \end{cases}$$

$$\begin{array}{r} 6b+9c=45 \\ -6b-8c=-32 \\ \hline c=13 \end{array}$$

$$\begin{cases} a+b+c=4 \\ -2b-3c=-15 \\ c=13 \end{cases}$$

$$-2b-3(13)=-15$$
$$-2b-39=-15$$
$$b=-12$$

$$a+b+c=4$$
$$a-12+13=4$$
$$a=3$$

The equation for the quadratic function whose graph passes through the points (1, 4), (2, 1), and (3, 4) is

$$y=3x^2-12x+13$$