Chapter 7

7.5 SYSTEMS OF INEQUALITIES

Chapter 7

HOMEWORK

READ SEC 7.5
COMPLETE READING NOTES
DO P549 1-71 ODD

Chapter 7

OBJECTIVES

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Use mathematical models involving linear inequalities.
- Graph a system of inequalities.

Inequalities in Two Variables and Their Solutions

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Equations in the form $Ax + By = C$ are straight lines when graphed. If we change the symbol = to >, <, ≥. ≤ we obtain a **linear inequality in two variables**.

A **solution of an inequality in two variables**, x and y, is an **ordered pair** of real numbers with the following property: When the x-coordinate is substituted for x and the y-coordinate is substituted for y in the inequality, we obtain a true statement. Each ordered-pair solution is said to **satisfy** the inequality.

The **graph of an inequality in two variables** is the set of all points whose coordinates satisfy the inequality.

Graphing an Inequality in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

To graph an inequality in two variables follow the following procedure:

1. Replace the inequality symbol with an equal sign and graph the corresponding equation.

   1a. If the inequality includes = (≤ or ≥) draw a solid line.

   1b. If the inequality does not includes = (< or >) draw a dashed line.

2. Select a point from one area of the drawn equation and substitute into the original inequality.

3. If the point works, shade that area of the shape. If the point does not work, shade the opposite area.

Graphing an Inequality in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Graphing Inequalities

Remember!!!

Three Steps to graphing inequalities

1. Graph the equation
2. Solid or Dashed Line
3. Shade one area of the graph

Graphing an Inequality in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Graph the inequality **x – y < 2**.

1. Replace the inequality symbol with an equal sign and graph the corresponding equation.

x – y = 2

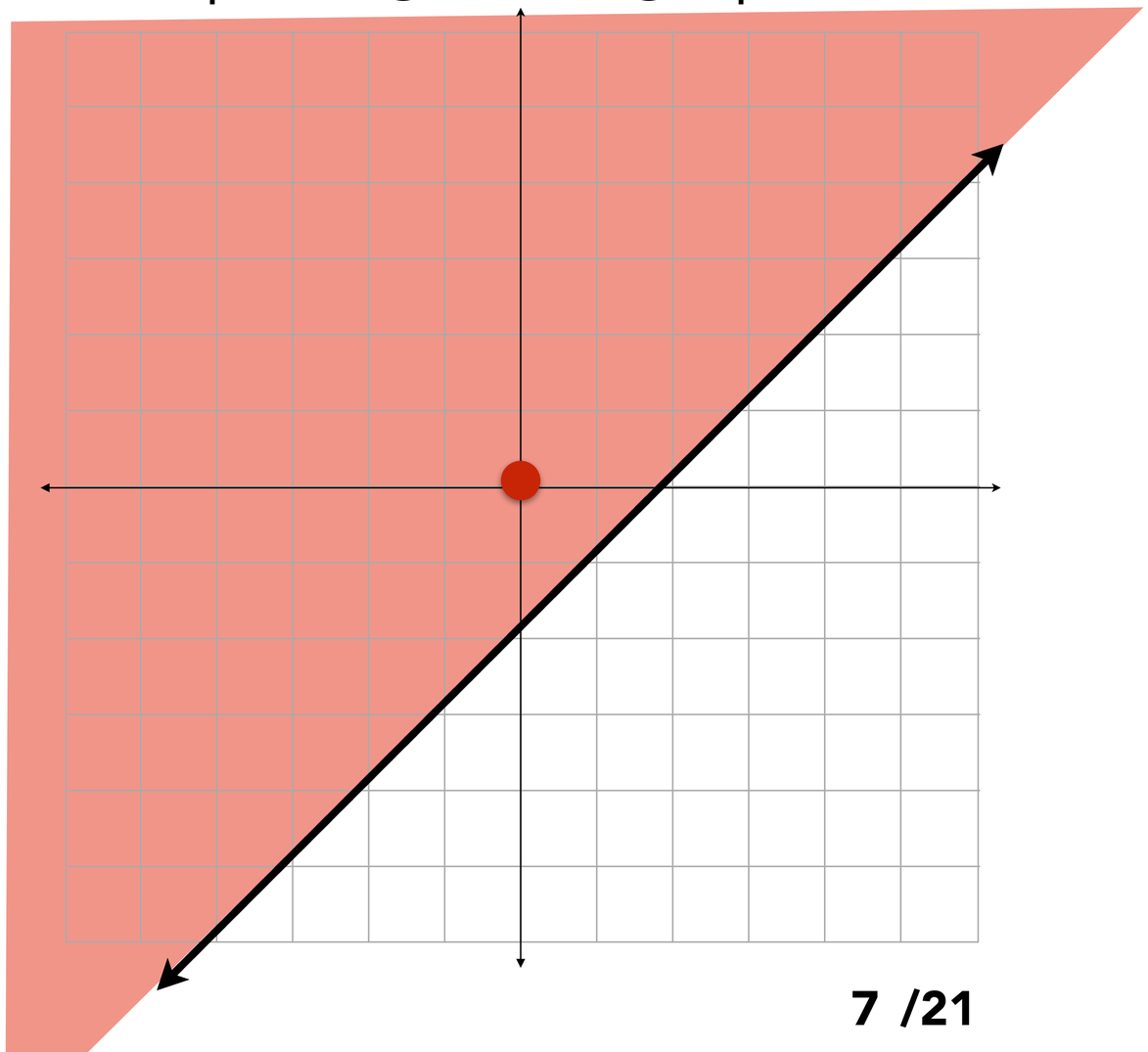
2. Solid or Dashed

3. Test a point and shade.

x – y < 2

0 – 0 < 2 Yep

Shade the side with the point



Graphing an Inequality in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Graph the inequality **$4x - 2y \geq 8$**.

1. Replace the inequality symbol with an equal sign and graph the corresponding equation.

   $4x - 2y = 8$

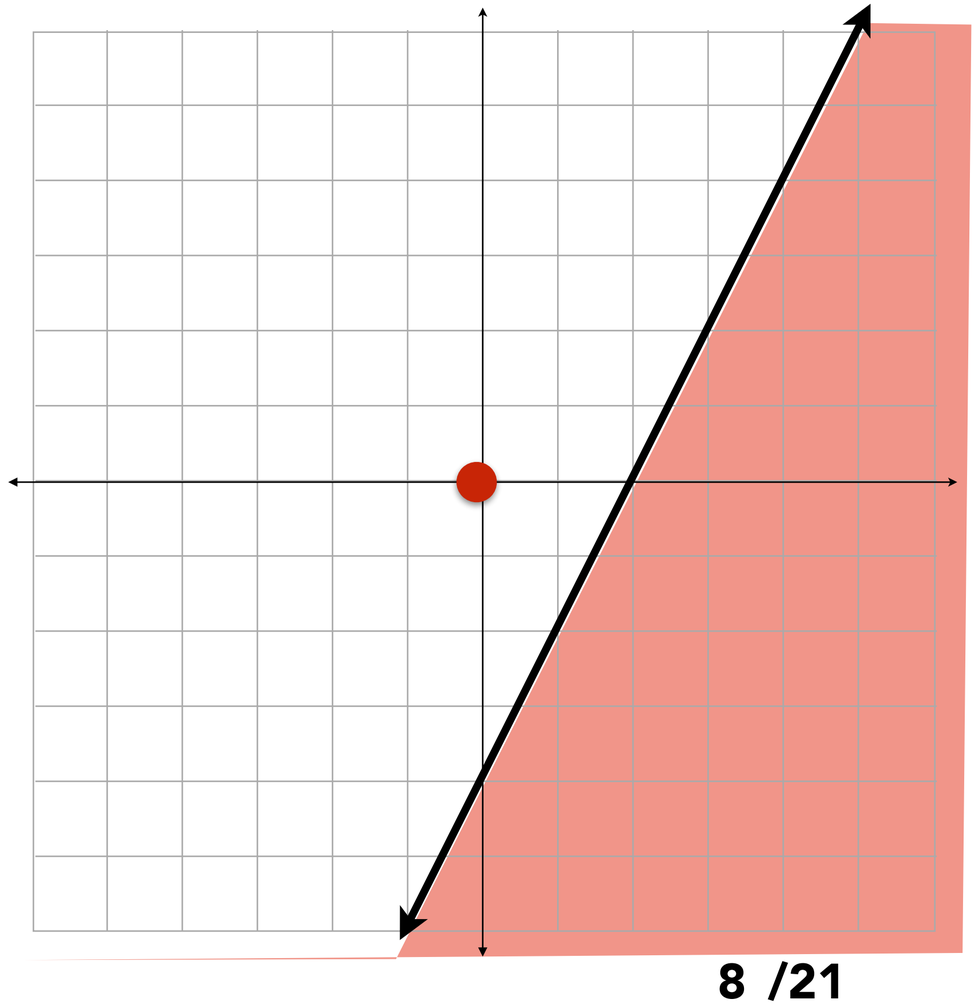
2. Solid or Dashed

3. Test a point and shade.

   $4x - 2y \geq 8$

   $0 - 0 \geq 8$ Nope

Shade the side opposite the point



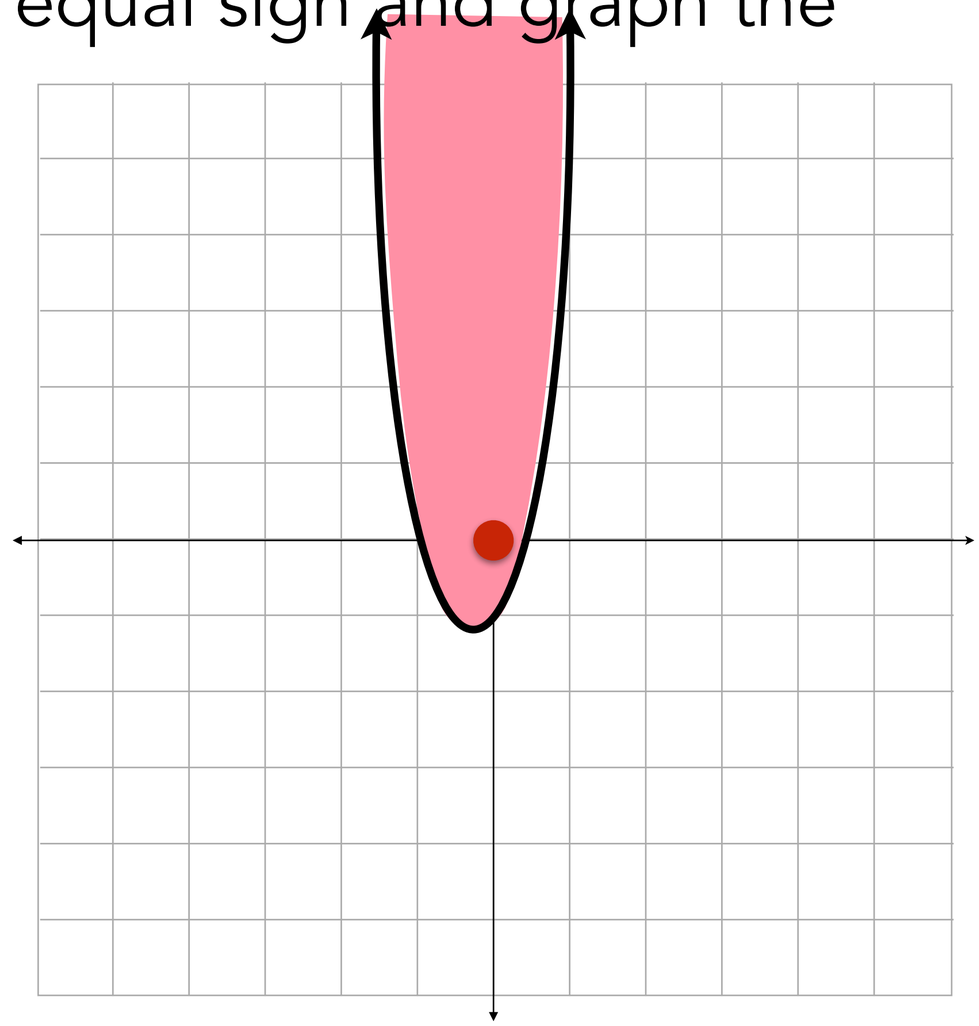
Graphing an Inequality in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Graph the inequality $\mathbf{y \geq 2x^2 + x - 1}$.

1. Replace the inequality symbol with an equal sign and graph the corresponding equation.

$$\mathbf{y = 2x^2 + x - 1}$$



2. Solid or Dashed

3. Test a point and shade.

$$\mathbf{y \geq 2x^2 + x - 1}$$

$$\mathbf{0 \geq 2(0)^2 + 0 - 1}$$ Yep

Shade the region with the point

Graphing an Inequality in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Graph the inequality $(x + 1)^2 + (y - 2)^2 > 16$.

1. Replace the inequality symbol with an equal sign and graph the corresponding equation.

   $(x + 1)^2 + (y - 2)^2 = 16$

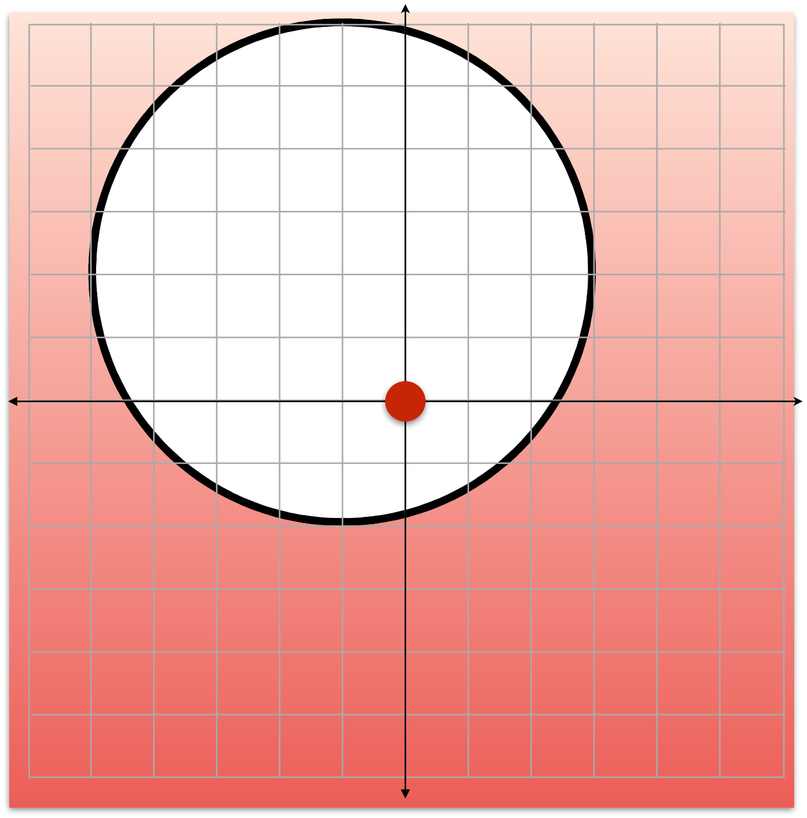
2. Solid or Dashed

3. Test a point and shade.

   $(x + 1)^2 + (y - 2)^2 > 16$

   $(0 + 1)^2 + (0 - 2)^2 > 16$ Nope

   Shade the region without the point



Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

To graph a system of inequalities, we graph each inequality in the system. The **solution** of the system is the set of all points in the **region where all the shaded regions overlap**. The points at which boundary lines intersect are called the **vertices** of the solution region.

Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

The latest guidelines, which apply to both men and woman, give healthy weight ranges, rather than specific weights, for your height. If x represents height in inches, and y represents weight, in pounds, the healthy weight region can be modeled by the following system of linear inequalities:

$$\begin{cases} 4.9x - y \geq 165 \\ 3.7x - y \leq 125 \end{cases}$$

Is a person that is 5’6” and 130 lbs in the healthy region?

Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Is a person that is 5’6” and 130 lbs in the healthy region?

$$\begin{cases} 4.9x - y \geq 165 \\ 3.7x - y \leq 125 \end{cases}$$

$4.9x - y \geq 165$ $\quad 4.9(66) - 130 \geq 165$

$193.4 \geq 165$

$3.7x - y \leq 125$ $\quad 3.7(66) - 130 \leq 125$

$114.2 \leq 125$

A person that is 5’6” and 130 lbs is in the healthy region.

Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

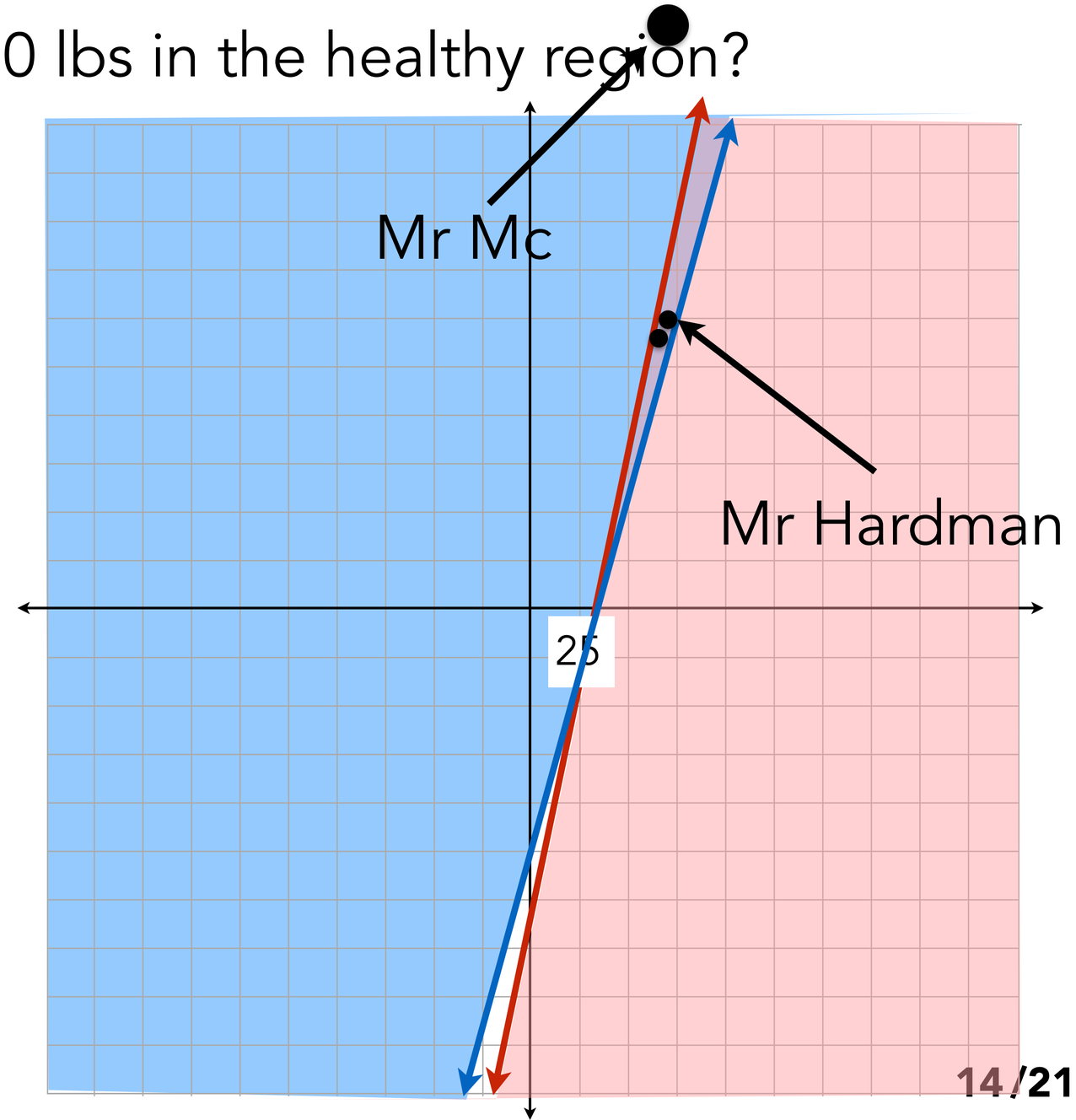
Is a person that is 5'6" and 130 lbs in the healthy region?

(66, 130)

$$\begin{cases} 4.9x - y \geq 165 \\ 3.7x - y \leq 125 \end{cases}$$

$$\begin{cases} 4.9x - 165 = y \\ 3.7x - 125 = y \end{cases}$$

$$\begin{cases} 4.9(0) - 165 \geq 0 \\ 3.7(0) - 125 \leq 0 \end{cases}$$



Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

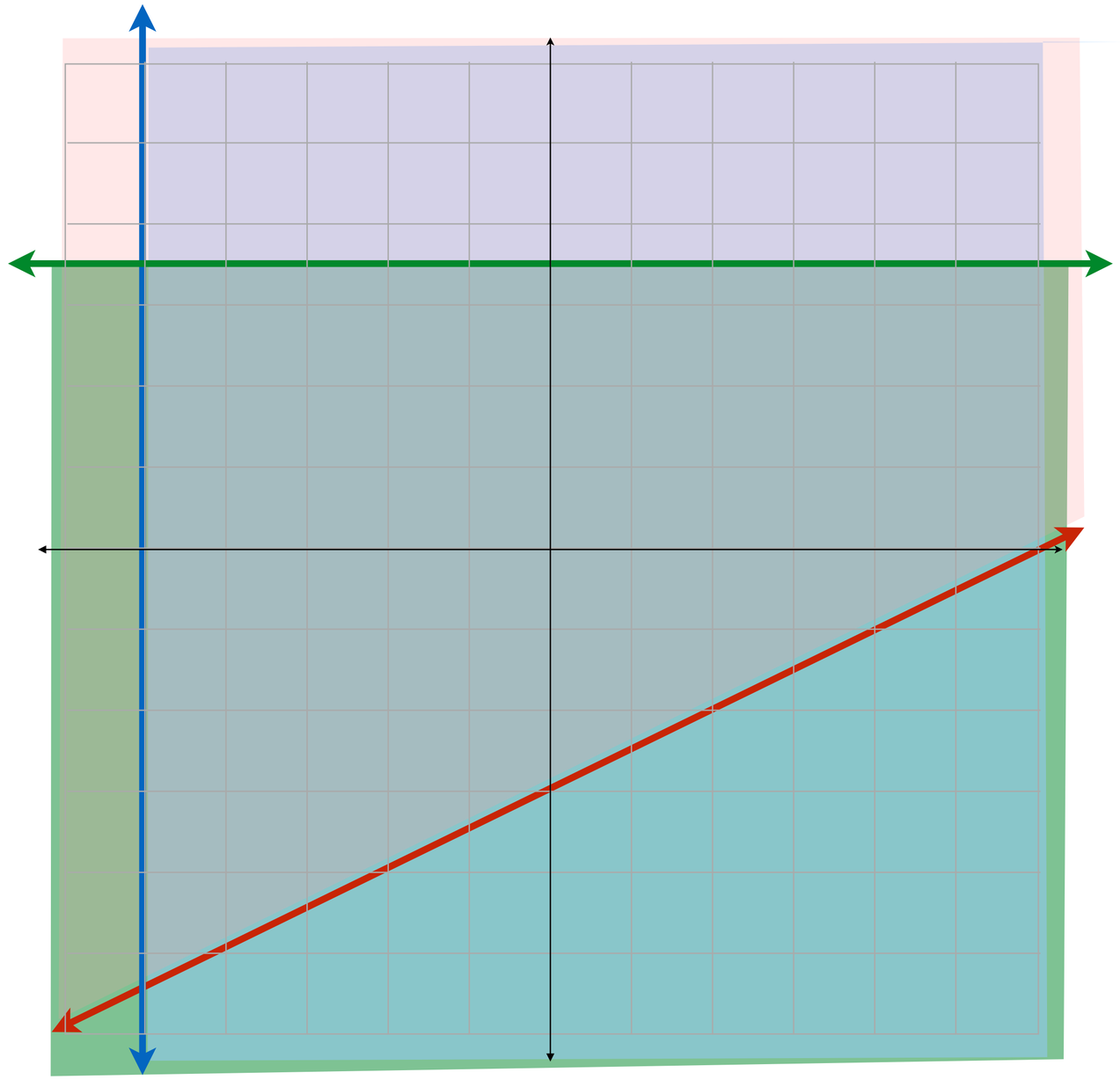
Graph the system of inequalities

$$\begin{cases} x - 2y \le 6 \\ x \ge -5 \\ y \le 4 \end{cases}$$

$x - 2y \le 6$

$x \ge -5$

$y \le 4$



Graphing a System of Inequalities in Two Variables

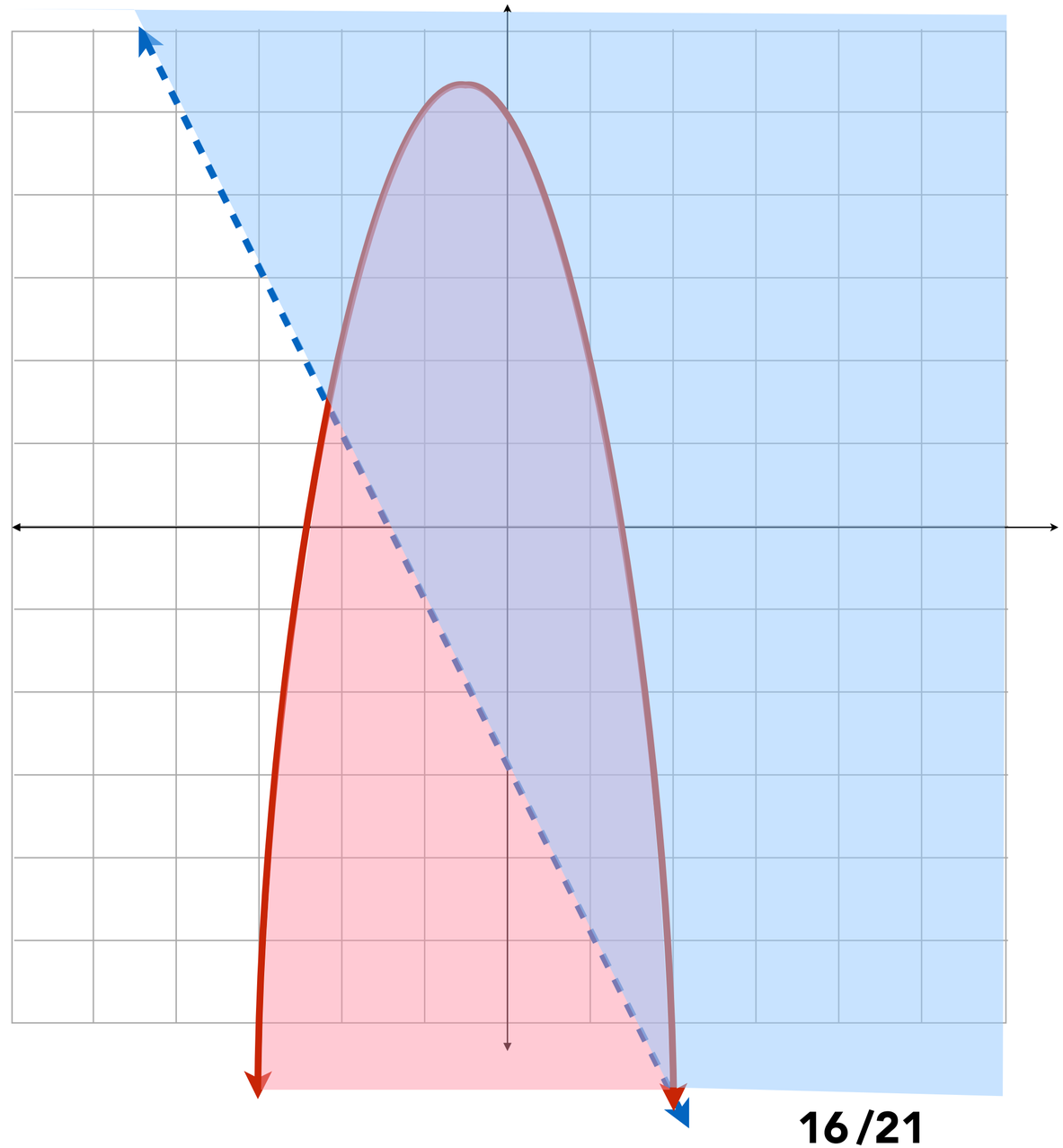
- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Graph the system of inequalities

$$\begin{cases} y \le 5 - x - x^2 \\ y > -2x - 3 \end{cases}$$

$y \le 5 - x - x^2$

$y > -2x - 3$



Remember

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Remember!!!!!

To determine which direction to shade, **test a point**.
Often it is easy to use the origin as a test point.

Test a Point

I do not care what anyone else has told you, **test a point**! You will save yourself a lot of heartache and anguish if you test a point rather than use some shortcut you have been told about the direction of the inequality, or any other nonsense.

Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Graph the system of inequalities

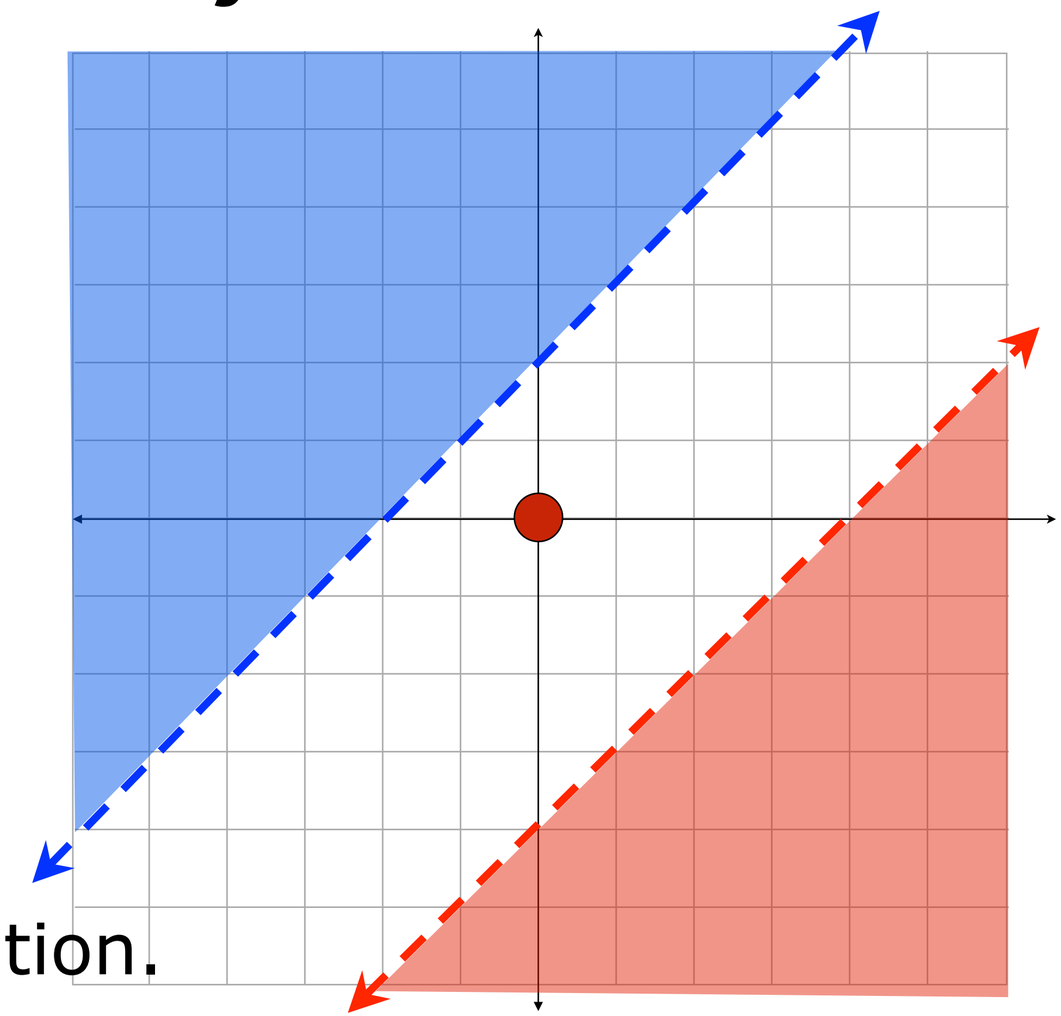
$$\begin{cases} x - y < -2 \\ 2x - 2y > 8 \end{cases}$$

$x - y < -2$

graph $x - y = -2$, dashed, shade above.

$2x - 2y > 8$

graph the $y = x - 4$, dashed, shade below.



No overlapping region, thus no solution.

Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

Lauren wants to paint a maximum of 70 plates for an art show. It costs her $50 plus $2 per plate to produce red plates and $3 per plate to produce gold plates. She can spend no more than $215. Write and graph a system of inequalities that can be used to determine the number of each plate that Lauren can make.

1. No more than 70 plates, fixed cost $50, $2/red, $3/gold, limit $215

2. How many plates for no more than $215?

3. Let *r* represent the number of red plates, and let *g* represent the number of gold plates.

Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

4. Verbal Models

red plates + # gold plates ≤ total plates

$$x + y \leq 70.$$

red plates • cost + # gold plates • cost + fixed cost ≤ total cost

$$2x + 3y + 50 \leq 215$$

The number of plates cannot be negative

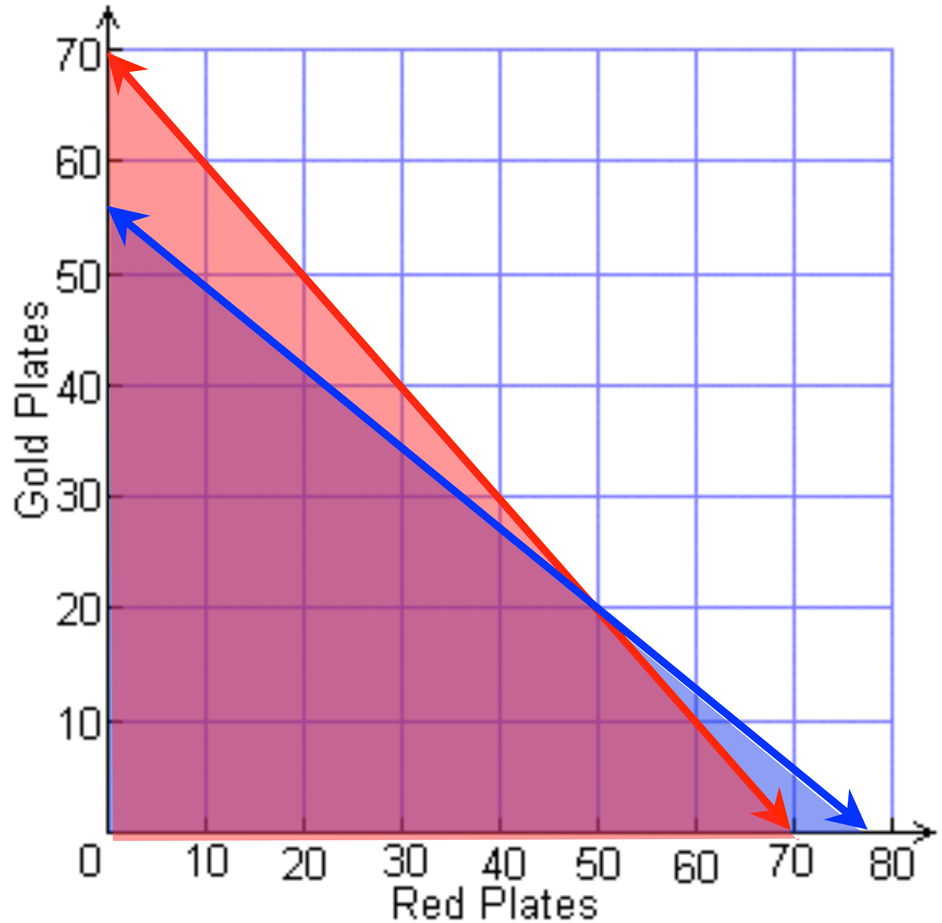
$$x \geq 0 \quad \text{and} \quad y \geq 0$$

Graphing a System of Inequalities in Two Variables

- Graph a linear inequality in two variables.
- Graph a nonlinear inequality in two variables.
- Graph a system of inequalities.

The system of inequalities is

$$\begin{cases} x + y \le 70. \\ 2x + 3y + 50 \le 215 \\ x \ge 0 \\ y \ge 0 \end{cases}$$



Graph line $x + y = 70$, solid, and shade below.

Graph the line $2x + 3y \le 165$, solid and shade below.

The overlapping region is the solution region.