Chapter 8

8.1 Matrices and Systems of Equations

Homework

Read Sec 8.1

Do p582 19, 21, 23, 27, 37, 39, 59, 61, 73

Objectives

Write the augmented matrix for a linear system.

Perform matrix row operations.

Use matrices and Gaussian elimination to solve systems.

Use matrices and Gauss-Jordan elimination to solve systems.

Matrices

- Matrices are used to display information and to solve systems of linear equations. A matrix (plural: matrices) is a rectangular array. The information is arranged between brackets in **rows and columns**. The values in the matrix are called elements of the matrix.

- Element row1 , column 1
- Element row2 , column 1

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

- Element row1 , column 2
- Element row2 , column 2

- The elements a_{rc} list the row first and column second.

Matrices

- Matrices are identified in order of rows and columns. Rows are listed first, Columns noted second.

$$\begin{bmatrix} a_{11} & a_{12} \end{bmatrix}$$

- 1×2 matrix

$$\begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}$$

- 2×1 matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

- 3×3 matrix

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

- $m \times n$ matrix

- A square matrix has an equal number of rows and columns. An $m \times m$ matrix has order m.

Systems of Equations

- The first step in solving a system of linear equations using matrices is to write the matrix of the coefficients and constants from the system of equations.

- An **augmented matrix** has a vertical bar separating the columns of the matrix into two groups. The coefficients of each variable are placed to the left of a vertical line and the constants are placed to the right. **If any variable is missing, its coefficient is 0.**

$$\begin{cases} 3x + y + 2z = 31 \\ x + y + 2z = 19 \\ x + 3y + 2z = 25 \end{cases} \qquad \begin{bmatrix} 3 & 1 & 2 \\ 1 & 1 & 2 \\ 1 & 3 & 2 \end{bmatrix} \qquad \left[\begin{array}{ccc|c} 3 & 1 & 2 & 31 \\ 1 & 1 & 2 & 19 \\ 1 & 3 & 2 & 25 \end{array}\right]$$

- System of Equations
- Coefficient Matrix
- Augmented Matrix

Row Echelon

- A reduced matrix

$$\begin{cases} x + 2y - 5z = -19 \\ y + 3z = 9 \\ z = 4 \end{cases} \qquad \left[\begin{array}{ccc|c} 1 & 2 & -5 & -19 \\ 0 & 1 & 3 & 9 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

- I think you can see where this is heading.

- A matrix with 1's shown on the main diagonal and 0's below the 1's is said to be in **row-echelon form**.

- You will use **row operations** on the augmented matrix to produce a matrix in **row-echelon form**.

Row Operations

- The row operations that result in the matrix to be used to solve a system of equations are as follows:

1. The rows may be interchanged, exactly as we did with the system of equations.

2. A row (it's elements) may be multiplied by a nonzero number. Once again this is comparable to multiplying both sides of an equation.

3. The elements of any row (or the product from the previous multiplication) may be added to the corresponding elements in any other row. This matches the adding the equation from a system to eliminate a variable.

- Two matrices are row equivalent if one matrix can be obtained from the other matrix by a sequence of row operations.

Notation

- To facilitate explanation, and to help you keep your work organized, the matrix row operations can be noted symbolically as follows:

  1. Rows interchanged, Row i and Row j interchanged: $R_i \leftrightarrow R_j$

  2. Multiply each element in row j by k: kR_j

  3. Add the elements in row i to the corresponding elements in row j: $R_i + R_j$

  4. Add k times the elements in row i to the corresponding elements in row j: $kR_i + R_j$

Row Operations

1. Rows interchanged, Row i and Row j interchanged: $R_1 \leftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right] \quad R_1 \leftrightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & 6 & -3 & 7 \\ 4 & 12 & -20 & 8 \\ -3 & -2 & 1 & -9 \end{array}\right]$$

2. Multiply each element in row 1 by 1/4: $\frac{1}{4}R_1$

$$\left[\begin{array}{ccc|c} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right] \quad \frac{1}{4}R_1 \quad \left[\begin{array}{ccc|c} \frac{1}{4}\cdot 4 & \frac{1}{4}\cdot 12 & \frac{1}{4}\cdot -20 & \frac{1}{4}\cdot 8 \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 3 & -5 & 2 \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right]$$

Row Operations

3. Add 3 times the elements in row 2 to the corresponding elements in row 3: $3R_2 + R_3$

$$\left[\begin{array}{ccc|c} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ -3 & -2 & 1 & -9 \end{array}\right] \quad 3R_2 + R_3 \quad \left[\begin{array}{ccc|c} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ 3\cdot 1 + -3 & 3\cdot 6 + -2 & 3\cdot -3 + 1 & 3\cdot 7 + -9 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 4 & 12 & -20 & 8 \\ 1 & 6 & -3 & 7 \\ 0 & 16 & -8 & 12 \end{array}\right]$$

Solving with Gaussian Elimination

- Solving 3 x 3 systems with Gaussian Elimination is accomplished by:

1. Write the augmented matrix for the system.

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c1 \\ a_{21} & a_{22} & a_{23} & c2 \\ a_{31} & a_{32} & a_{33} & c3 \end{array}\right]$$

2. Use matrix row operations to simplify that matrix to a row-equivalent matrix in row echelon form with 1s down the main diagonal from 1,1 to 3,3 and 0s below the diagonal.

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c1 \\ a_{21} & a_{22} & a_{23} & c2 \\ a_{31} & a_{32} & a_{33} & c3 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & * & * & * \\ a_{21} & a_{22} & a_{23} & c2 \\ a_{31} & a_{32} & a_{33} & c3 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & * & * & * \\ 0 & * & * & * \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & * & * & * \end{array}\right]$$

- Get $a_{11} = 1$
- Using the first row, gets 0 in first column
- Get 1 for element 2,2

Solving with Gaussian Elimination

- Solving 3 x 3 systems with Gaussian Elimination is accomplished by:

$$\left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & * & * & * \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & * & * \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{array}\right]$$

- Using the 1 in 2,2, get element 2,3 = 0
- Get element 3,3 = 1

3. Once the matrix is in row echelon form, convert it back into a system of equations in row echelon form, then solve for the three variables.

Example

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

1. Write the augmented matrix for the system. $\left[\begin{array}{ccc|c} 2 & 1 & 2 & 18 \\ 1 & -1 & 2 & 9 \\ 1 & 2 & -1 & 6 \end{array}\right]$

2. Use matrix row operations to simplify that matrix to a row-equivalent matrix in row echelon form with 1s down the main diagonal from 1,1 to 3,3 and 0s below the diagonal.

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 18 \\ 1 & -1 & 2 & 9 \\ 1 & 2 & -1 & 6 \end{array}\right] \quad R_1 \leftrightarrow R_2 \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 2 & 1 & 2 & 18 \\ 1 & 2 & -1 & 6 \end{array}\right] \quad -2R_1 + R_2 \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

Example

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 18 \\ 1 & -1 & 2 & 9 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 1 & 2 & -1 & 6 \end{array}\right] \quad -1R_1 + R_3 \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 0 & 3 & -3 & -3 \end{array}\right] \quad R_2 \longleftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -3 & -3 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

$$\frac{1}{3}R_2 \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 3 & -2 & 0 \end{array}\right] \quad -3R_2 + R_3 \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

- Ta Da
- Row Echelon Form

Example

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$ $\left[\begin{array}{ccc|c} 2 & 1 & 2 & 18 \\ 1 & -1 & 2 & 9 \\ 1 & 2 & -1 & 6 \end{array}\right]$

3. Once the matrix is in row echelon form, convert it back into a system of equations in row echelon form, then solve for the three variables.

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right] \qquad \begin{cases} x - y + 2z = 9 \\ y - z = -1 \\ z = 3 \end{cases} \qquad \begin{aligned} y - z &= -1 \\ y - 3 &= -1 \\ y &= 2 \end{aligned} \qquad \begin{aligned} x - y + 2z &= 9 \\ x - 2 + 2(3) &= 9 \\ x &= 5 \end{aligned}$$

- The solution to the system is (5, 2, 3)

Example 2

- Use matrices to solve the system: $\begin{cases} x + y + 2z = 3 \\ 3x + 4y + 4z = 9 \\ 5x + 2y + 15z = 13 \end{cases}$

1. Write the augmented matrix for the system. $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 3 & 4 & 4 & 9 \\ 5 & 2 & 15 & 13 \end{array}\right]$

2. Use matrix row operations to simplify that matrix to a row-equivalent matrix in row echelon form with 1s down the main diagonal from 1,1 to 3,3 and 0s below the diagonal.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 3 & 4 & 4 & 9 \\ 5 & 2 & 15 & 13 \end{array}\right] \quad -3R_1 + R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 5 & 2 & 15 & 13 \end{array}\right] \quad -5R_1 + R_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & -3 & 5 & -2 \end{array}\right]$$

Example 2

- Use matrices to solve the system: $\begin{cases} x + y + 2z = 3 \\ 3x + 4y + 4z = 9 \\ 5x + 2y + 15z = 13 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 3 & 4 & 4 & 9 \\ 5 & 2 & 15 & 13 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & -3 & 5 & -2 \end{array}\right] \quad 3R_2 + R_3 \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & -1 & -2 \end{array}\right] \quad -1R_3 \left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{cases} x + y + 2z = 3 \\ y - 2z = 0 \\ z = 2 \end{cases}$$

$$y - 2z = 0$$
$$y - 2(2) = 0$$
$$y = 4$$

$$x + y + 2z = 3$$
$$x + 4 + 2(2) = 3$$
$$x = -5$$

- The solution to the system is (-5, 4, 2)

Row Echelon Form

- The last matrix is in **row-echelon form.** The characteristics that define a matrix in row-echelon form are:

1. All rows consisting entirely of zeros occur at the bottom of the matrix

2. The first nonzero element of any row is 1, called a leading 1.

3. For two successive nonzero rows, the leading 1 in the upper row is farther to the left than the leading 1 in the lower row. Soon we will add another condition to find the **reduced row-echelon form**.

4. Every column with a leading 1 has zeros everywhere else in that column.

Example 3

- Use matrices to solve the system: $\begin{cases} x + 2y + 3z = 4 \\ 3x + 7y + 11z = 15 \\ -2x - 2y - z = 0 \end{cases}$

1. Write the augmented matrix for the system. $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & 7 & 11 & 15 \\ -2 & -2 & -1 & 0 \end{array}\right]$

2. Use matrix row operations to simplify that matrix to a row-equivalent matrix in row echelon form with 1s down the main diagonal from 1,1 to 3,3 and 0s below the diagonal.

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & 7 & 11 & 15 \\ -2 & -2 & -1 & 0 \end{array}\right] \quad -3R_1 + R_2 \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ -2 & -2 & -1 & 0 \end{array}\right] \quad 2R_1 + R_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 8 \end{array}\right]$$

Example 3

- Use matrices to solve the system: $\begin{cases} x+2y+3z=4 \\ 3x+7y+11z=15 \\ -2x-2y-z=0 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 3 & 7 & 11 & 15 \\ -2 & -2 & -1 & 0 \end{array}\right]$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 8 \end{array}\right] \quad -2R_2 + R_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\begin{cases} x+2y+3z=4 \\ \quad y+2z=3 \\ \quad\quad z=2 \end{cases}$$

$$y+2z=3$$
$$y+2(2)=3$$
$$y=-1$$

$$x+2y+3z=4$$
$$x+2(-1)+3(2)=4$$
$$x=0$$

- The solution to the system is (0, -1, 2)

Reduced Row-Echelon Form

- We can expand our method of using matrices to solve systems by using what is called **Gauss-Jordan elimination**. **Gauss-Jordan elimination** is taking the augmented matrix of the system of equations and transforming it into **reduced row-echelon form**. The last column of the augmented reduced matrix provides the values of the variables.

1. Write the augmented matrix for the system.
2. Using row operations, simplify the matrix to a row equivalent matrix in reduced row-echelon form, with 1s down the main diagonal from 1,1 to 3,3, and with 0s **above and below** the 1s.
   - 2a. Make element 1,1 = 1
   - 2b. Using row 1, get 0s in all of column 1.
   - 2c. Make element 2,2 = 1
   - 2d. Using row 2, get 0s in all of column 2.
   - 2e. Make element 3,3 = 1
   - 2f. Using row 3, get 0s in all of column 3.
3. Once you have 1s in the main diagonal and 0s above and below, the last column contains the values for the variables. Write them as an ordered triple. There is no need for back substitution.

Row-Echelon Form and Reduced Row-Echelon Form

A matrix in **row-echelon form** has the following properties.

1. Any rows consisting entirely of zeros occur at the bottom of the matrix.
2. For each row that does not consist entirely of zeros, the first nonzero entry is 1 (called a **leading 1**).
3. For two successive (nonzero) rows, the leading 1 in the higher row is farther to the left than the leading 1 in the lower row.

A matrix in *row-echelon form* is in **reduced row-echelon form** if every column that has a leading 1 has zeros in every position above and below its leading 1.

Example

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

- We previously solved this by finding the row-echelon form: $\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$

$R_2 + R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$ $-1R_3 + R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$ $R_3 + R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$

- The solution to the system is (5, 2, 3)

Caution

- When attempting to use Gauss Jordan Elimination by finding the reduced Row-Echelon form of the augmented matrix work **column by column**.

- Get the first column the way you want it, then move to the second column, and so on. Be careful not to work in a random order. That will lead to counterproductive work and a lot of wasted time.

Example

- Solve $\begin{cases} 2x + 4y = 6 \\ 3x + 7y = 5 \end{cases}$ $\left[\begin{array}{cc|c} 2 & 4 & 6 \\ 3 & 7 & 5 \end{array}\right]$ $\frac{1}{2}R_1$ $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 3 & 7 & 5 \end{array}\right]$

$-3R_1 + R_2$ $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & -4 \end{array}\right]$ $-2R_2 + R_1$ $\left[\begin{array}{cc|c} 1 & 0 & 11 \\ 0 & 1 & -4 \end{array}\right]$

- The solution to the system is (11, -4)

Example

Solve the system $\begin{cases} -x+y+2z=7 \\ 2x+3y+z=1 \\ -3x-4y+z=4 \end{cases}$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 7 \\ 2 & 3 & 1 & 1 \\ -3 & -4 & 1 & 4 \end{array}\right]$$

$-1R_1$ $\left[\begin{array}{ccc|c} 1 & -1 & -2 & -7 \\ 2 & 3 & 1 & 1 \\ -3 & -4 & 1 & 4 \end{array}\right]$

$-2R_1 + R_2$ $\left[\begin{array}{ccc|c} 1 & -1 & -2 & -7 \\ 0 & 5 & 5 & 15 \\ -3 & -4 & 1 & 4 \end{array}\right]$

$3R_1 + R_3$ $\left[\begin{array}{ccc|c} 1 & -1 & -2 & -7 \\ 0 & 5 & 5 & 15 \\ 0 & -7 & -5 & -17 \end{array}\right]$

$\frac{1}{5}R_2$ $\left[\begin{array}{ccc|c} 1 & -1 & -2 & -7 \\ 0 & 1 & 1 & 3 \\ 0 & -7 & -5 & -17 \end{array}\right]$

$7R_2 + R_3$ $\left[\begin{array}{ccc|c} 1 & -1 & -2 & -7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right]$

$R_2 + R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 2 & 4 \end{array}\right]$

$\frac{1}{2}R_3$ $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$-1R_3 + R_2$ $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$

$R_3 + R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$

The solution to the system is (-2, 1, 2)

Example

Solve the system $\begin{cases} x + 2y + 3z = 1 \\ x + 3y + z = -1 \\ 3x + 7y + 7z = 6 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 1 & 3 & 1 & -1 \\ 3 & 7 & 7 & 6 \end{array}\right]$

$-1R_1 + R_2$ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & -2 & -2 \\ 3 & 7 & 7 & 6 \end{array}\right]$ $-3R_1 + R_3$ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & -2 & -2 \\ 0 & 1 & -2 & 3 \end{array}\right]$ $-1R_2 + R_3$ $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 0 & 5 \end{array}\right]$

Oops, no solution

Example

Solve the system $\begin{cases} x+2y-3z=2 \\ 2x+5y-6z=1 \end{cases}$ $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 2 \\ 2 & 5 & -6 & 1 \end{array}\right]$

$-2R_1 + R_2$ $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 2 \\ 0 & 1 & 0 & -3 \end{array}\right]$ $-2R_2 + R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & -3 \end{array}\right]$

- Add a row $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & a \end{array}\right]$

$3R_3 + R_1$ $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3a+8 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & a \end{array}\right]$

- The solutions to the system are of the form: $(3a+8, -3, a)$

TI-84

- All we have done with matrices can be done on the TI-84. Keep in mind that you must demonstrate your ability to manipulate the matrices without the calculator. You are welcome to check your results with the calculator, but you must show every step as if you had **done the entire process by hand without the calculator**.

- To solve a system of equations using the TI-84 graphing calculator, we must enter the system of equations as an augmented matrix into the calculator. We will use the calculator to convert the matrix to reduced row-echelon form.

- To enter an augmented matrix, the system must first be in standard form. **Missing coefficients must be entered as 0s**.

- To enter the coefficients into a matrix press the **2nd** key and then the **x^{-1}** (MATRIX) key. This will show the matrix menu of the calculator. Simply choose a matrix to use.

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$ $\left[\begin{array}{ccc|c} 2 & 1 & 2 & 18 \\ 1 & -1 & 2 & 9 \\ 1 & 2 & -1 & 6 \end{array}\right]$

MATRIX

2nd x^{-1} ➢ EDIT ⋎ 1:[A] ENTER Enter Dimensions 3 ➢ 4 ENTER

MATRIX[A] 3 x 4

- Enter the coefficients into Augmented Matrix A

2	1	2	18
1	-1	2	9
1	2	-1	6

ENTER after each entry

QUIT

2nd MODE To get back to home screen.

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$ $\left[\begin{array}{ccc|c} 2 & 1 & 2 & 18 \\ 1 & -1 & 2 & 9 \\ 1 & 2 & -1 & 6 \end{array}\right]$

- Row operations are found under Matrix - Math

MATRIX
2nd x^{-1} ➢ MATH

$R_1 \leftrightarrow R_2$

MATRIX
2nd x^{-1} ➢ MATH ⋎ C:rowSwap(ENTER

MATRIX
2nd x^{-1} 1:[A] ENTER , 1 , 2) ENTER

$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 2 & 1 & 2 & 18 \\ 1 & 2 & -1 & 6 \end{array}\right]$

- Store the result in Matrix B

MATRIX
STO> 2nd x^{-1} ⋎ 2:[B] ENTER

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 2 & 1 & 2 & 18 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

$-2R_1 + R_2$

MATRIX

2nd x⁻¹ ➢ MATH ∀ F:*row+(ENTER

MATRIX

(-) 2 , 2nd x⁻¹ 2:[B] ENTER , 1 , 2) ENTER

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

- Store the result in Matrix C

MATRIX

STO> 2nd x⁻¹ ∀ 3:[C] ENTER

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

$-1R_1 + R_3$

MATRIX

2nd x⁻¹ ➢ MATH ⋎ F:*row+(ENTER

MATRIX

(-) 1 , 2nd x⁻¹ 3:[C] ENTER , 1 , 3) ENTER

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 0 & 3 & -3 & -3 \end{array}\right]$$

MATRIX

- Store the result in Matrix D STO> 2nd x⁻¹ ⋎ 4:[D] ENTER

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -2 & 0 \\ 0 & 3 & -3 & -3 \end{array}\right]$$

$R_2 \longleftrightarrow R_3$

MATRIX
2nd x⁻¹ ➢ MATH ∨ C:rowSwap(ENTER

MATRIX
2nd x⁻¹ 4:[D] ENTER , 2 , 3) ENTER

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -3 & -3 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

- Store the result in Matrix E

MATRIX
STO> 2nd x⁻¹ ∨ 5:[E] ENTER

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 3 & -3 & -3 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

$\frac{1}{3}R_2$

MATRIX

2nd x⁻¹ ➢ MATH ⋎ E:*row(ENTER

MATRIX

1 ÷ 3 , 2nd x⁻¹ 5:[E] ENTER , 2) ENTER

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

- Store the result in Matrix F

MATRIX

STO> 2nd x⁻¹ ⋎ 6:[F] ENTER

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 3 & -2 & 0 \end{array}\right]$$

$-3R_2 + R_3$

MATRIX

2nd x⁻¹ ➢ MATH ∨ F:*row+(ENTER

MATRIX

(-) 3 , 2nd x⁻¹ 6:[F] ENTER , 2 , 3) ENTER

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

- Store the result in Matrix G

MATRIX

STO> 2nd x⁻¹ ∨ 7:[G] ENTER

- At this point you can convert the matrix back into a row-echelon system of equations and solve algebraically. OR ...

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 9 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

- We can continue on to the reduced row-echelon form of the matrix.

$R_2 + R_1$

MATRIX

2nd x^{-1} ➢ MATH ∨ D:row+(ENTER

MATRIX

2nd x^{-1} 7:[G] ENTER , 2 , 1) ENTER

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

- Store the result in Matrix H

MATRIX

STO> 2nd x^{-1} ∨ 8:[H] ENTER

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 8 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$-1R_3 + R_1$

MATRIX

2nd x⁻¹ ➢ MATH ∀ F:*row+(ENTER

MATRIX

(-) 1 , 2nd x⁻¹ 8:[H] ENTER , 3 , 1) ENTER

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

- Store the result in Matrix I

MATRIX

STO> 2nd x⁻¹ ∀ 9:[I] ENTER

TI-84 Row Operations

- Use matrices to solve the system: $\begin{cases} 2x + y + 2z = 18 \\ x - y + 2z = 9 \\ x + 2y - z = 6 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R_3 + R_2$

MATRIX

2nd x^{-1} ➢ MATH ⩔ D:row+(ENTER

MATRIX

2nd x^{-1} 9:[I] ENTER , 3 , 2) ENTER

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

- You have your solution (5,2,3)

TI-84 Reduced Row-Echelon

- I am certain you had so much fun doing all the row operations for the previous steps on the TI that I almost hate to spoil all the fun showing you how to get to the reduced row-echelon form without all the intermediate steps.

Remember

- You must show all the steps on any assessment. Going directly to the reduced row-echelon form of the matrix will lose you several points on any problem involving matrices.

TI-84

- Use matrices to solve the system: $\begin{cases} 2x + \ \ y + 2z = 18 \\ \ \ x - \ \ y + 2z = 9 \\ \ \ x + 2y - \ \ z = 6 \end{cases}$

- Enter the coefficients into Matrix A

(You should still have Matrix A)

MATRIX
2nd x^{-1} ➢ MATH ⋎ B:rref(ENTER

MATRIX
2nd x^{-1} 1:[A])

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

- You have your solution (5,2,3)

- What? You expected Minions?