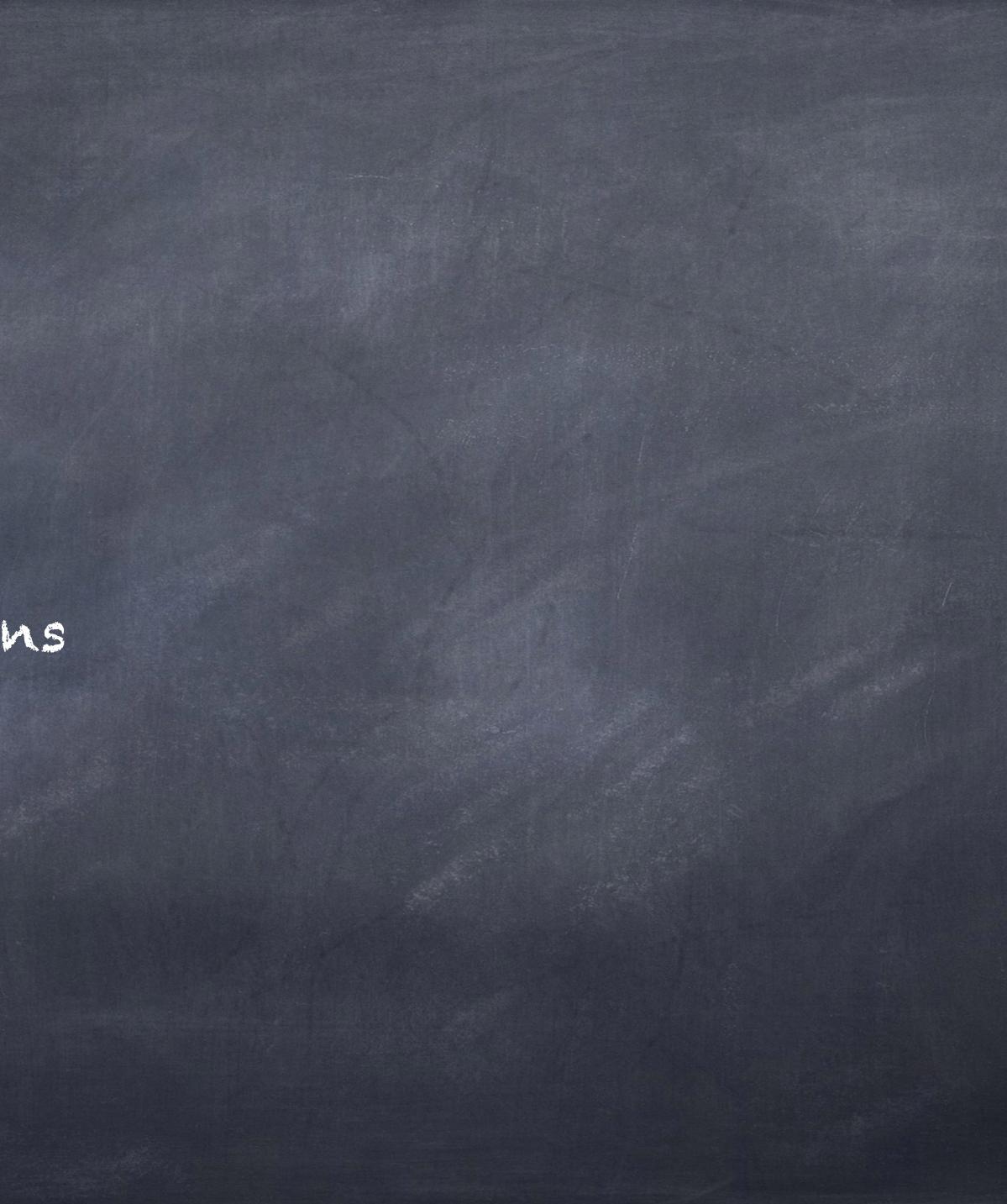
# CAR PET ST

#### 0 8.2 Matrix Operations





#### Chapter 232

#### HOMEWORK

o Read Sec 8.2 Complete Reading Notes
Do p597 1-69 every other odd 0 8.2 WS 0 8.2 WS 2



### CAQ PLET E

# O D C C C C C S

Use matrix notation.
Understand what is meant by equal matrices.
Add and subtract matrices.
Perform scalar multiplication.
Multiply matrices.
Model applied situations with matrix operations.



#### Matrix Notations

- different ways:
- A capital letter, such as A, B, or C, can denote a matrix.
- to denote a matrix.
- element in the ith row and jth column.

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

An array of numbers, arranged in rows and columns and placed in brackets, is called a matrix. We can represent a matrix in two

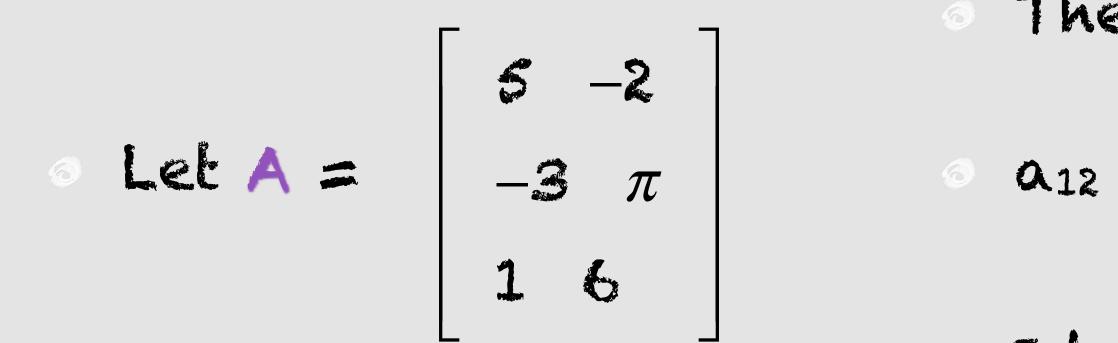
A lowercase letter enclosed in brackets, such as [a; ] can be used

A general element in matrix A is denoted by any. This refers to the



#### Matrix Notations

- A matrix of order m x n has m rows and n columns.
  - If m = h, a matrix has the same number of rows as columns and is called a square matrix.



Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

- The order of A is ...  $3 \times 2$
- $a_{12}$  is ... -2
  - Identify as1 1



#### Matrix Equality

- - i.e. if A = B, and A is  $m \ge n$ , then B is  $m \ge n$ ; and  $a_{ij} = b_{ij}$  for all i = 1, 2, ... and all j = 1, 2, ...

• Let 
$$A = \begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix}$$
 and  $B$ 

If A = B, then a = -1, b = -2, c = 8, d = 5.

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

Two matrices A and B are equal (A = B) if and only if they have the same order and corresponding elements are equal.

 $3 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ 



#### Matrix Addition

- Let  $A = [a_{ij}]$  and  $B = [b_{ij}]$  and both are  $m \ge n$  matrices.
  - Then  $A + B = [a_{ij} + b_{ij}].$ 
    - in the corresponding positions.
  - And  $A B = [a_{ij} + -b_{ij}].$
  - You cannot add matrices with differing dimensions.

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

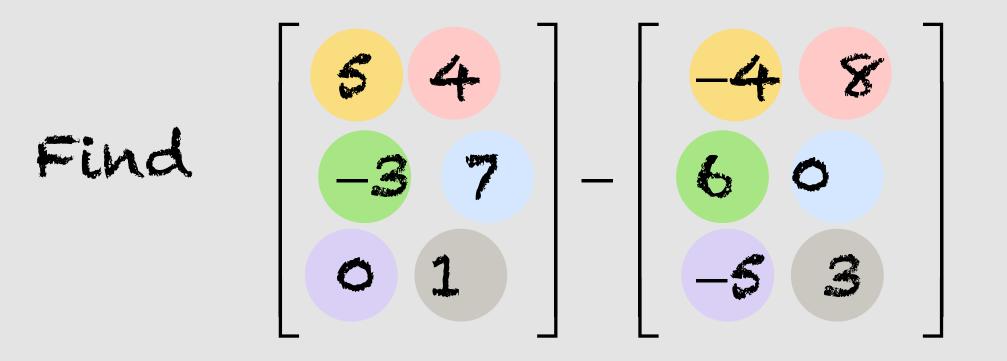
Matrices of the same order are added by adding the elements

Matrices of the same order are subtracted by adding the opposite of elements in the corresponding positions.



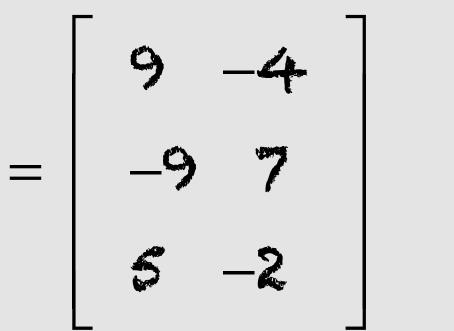


#### Adding Matrices



$$= \begin{bmatrix} 5 - -4 & 4 - 8 \\ -3 - 6 & 7 - 0 \\ 0 - -5 & 1 - 3 \end{bmatrix} = \begin{bmatrix} 9 \\ -5 \\ 5 \end{bmatrix}$$

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.





# Matrix Addition Properties

- Let A, B, and C be m x n matrices.
  - Commutative Property of matrix Addition. 1. A + B = B + A2. (A + B) + C = A + (B + C) Associative Property of matrix Addition.

  - 3. A + 0 = 0 + A = AMatrix Additive Identity
  - Matrix Additive Inverse 4. A + -A = -A + A = 0



Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

#### Let 0 be an $m \times n$ zero matrix. (a; j = 0, for every element)



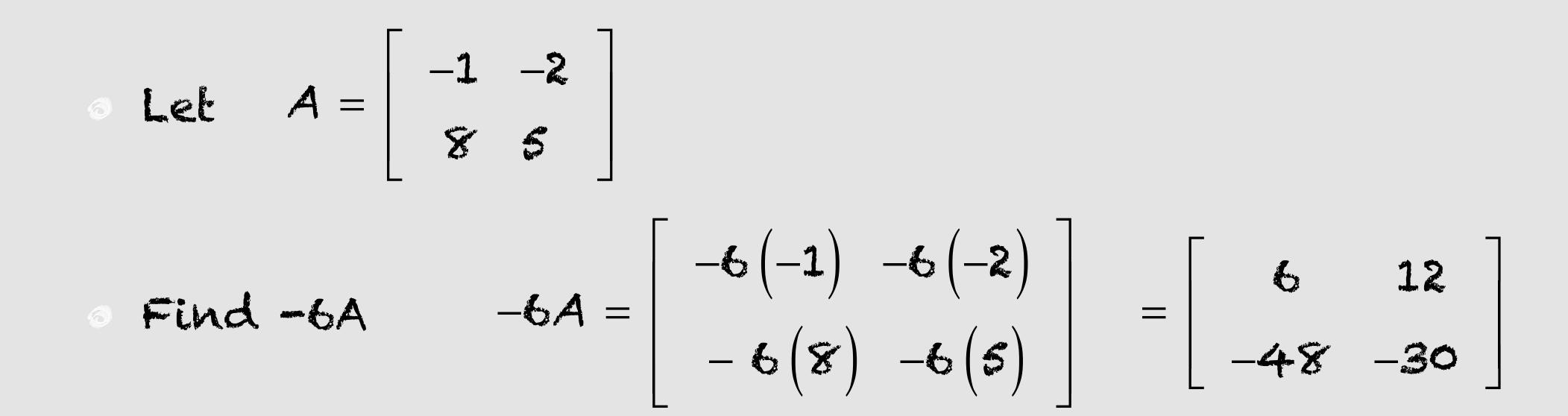






#### Matrix Scalar Multiplication

If  $A = [a_{ij}]$  is a matrix of order  $m \ge n$  and c is a scalar, then the matrix cA is an  $m \ge n$  matrix found by  $cA = [ca_{ij}]$ 





Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

The product of a matrix with a scalar is found by multiplying each element of A by the real number c. cA is a scalar multiple of A.

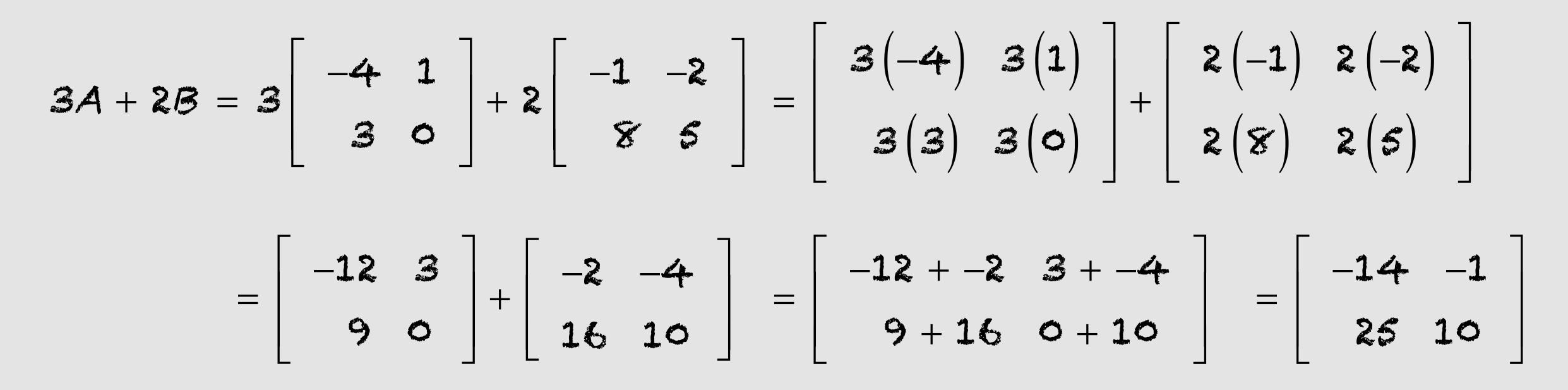




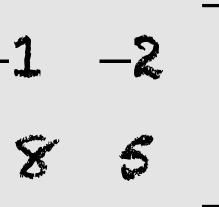


Let  $A = \begin{bmatrix} -4 & 1 \\ 3 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} -1 & -2 \\ 8 & 5 \end{bmatrix}$ 

#### Find 3A + 2B



Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.







# Properties of Scalar Multiplication

- Let A and B be m x n matrices, c and d are scalars. Then:
  - 1. (cd)A = c(dA)
  - 2. 1A = A

- 3. c(A + B) = cA + cB
- 4. (c + d)A = cA + dA

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

Associative Property of scalar multiplication.

Scalar Multiplicative Identity

Distributive Property

Distributive Property







#### Matrix Multiplication

- second matrix.

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

For the product of two matrices to be defined, the number of columns of the first matrix must equal the number of rows of the

The product of an  $m \ge n$  matrix, A, and and  $n \ge p$  matrix, B, is an  $m \ge p$  matrix AB, whose elements are found by:

The element in the ith row and jth column of AB is found by multiplying each element in the ith row of A by the corresponding element in the jth column of B and adding the products.





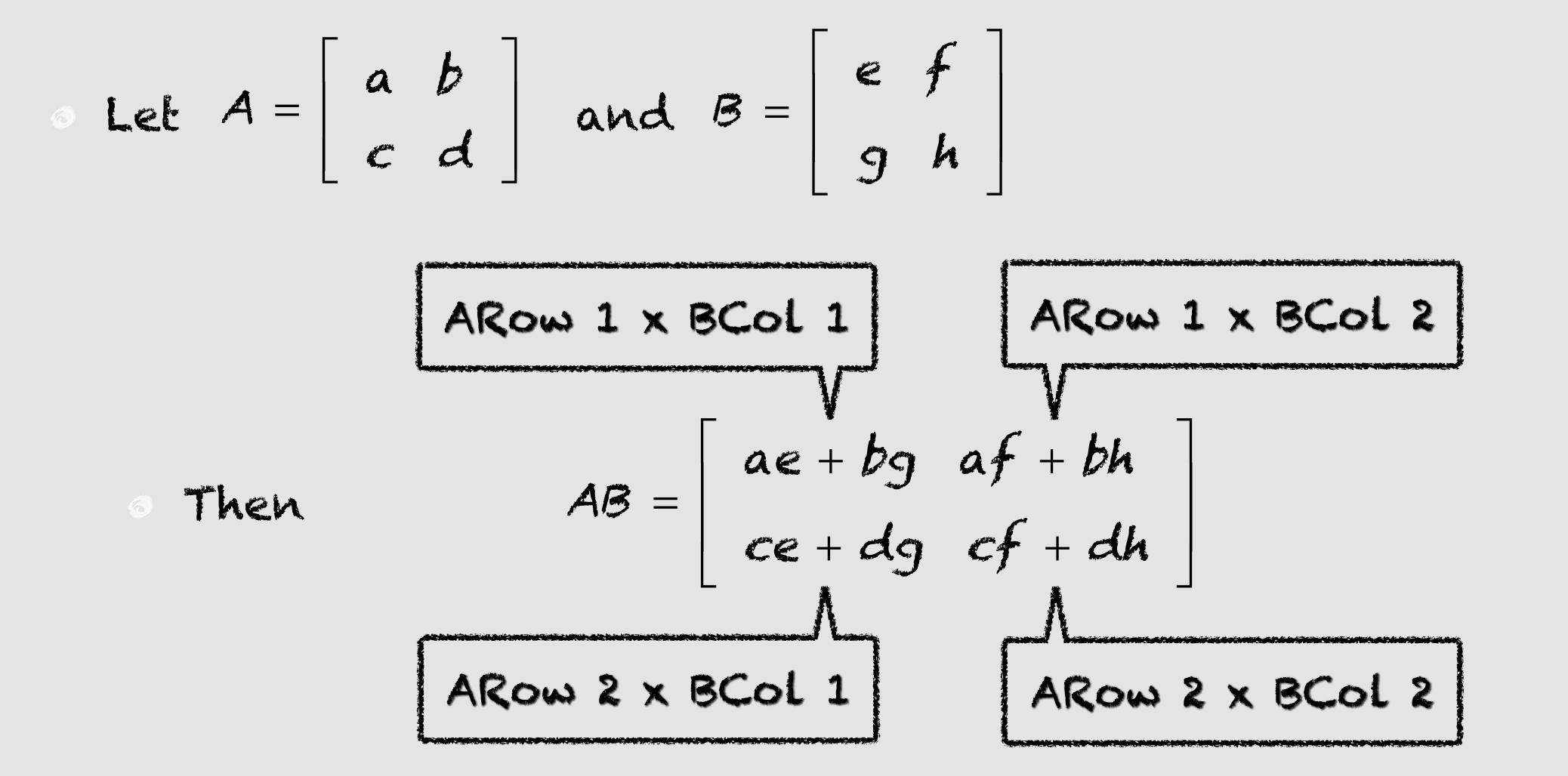








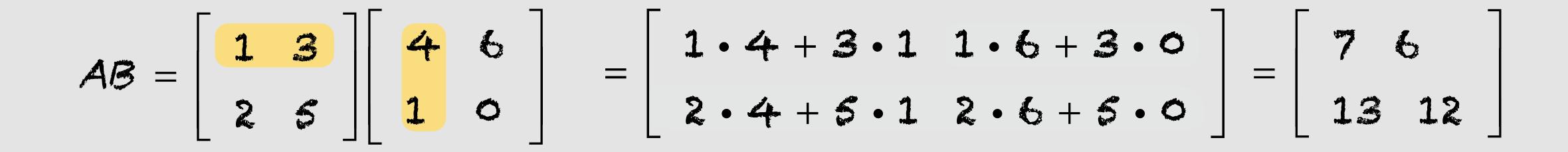
#### Matrix Multiplication



Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.



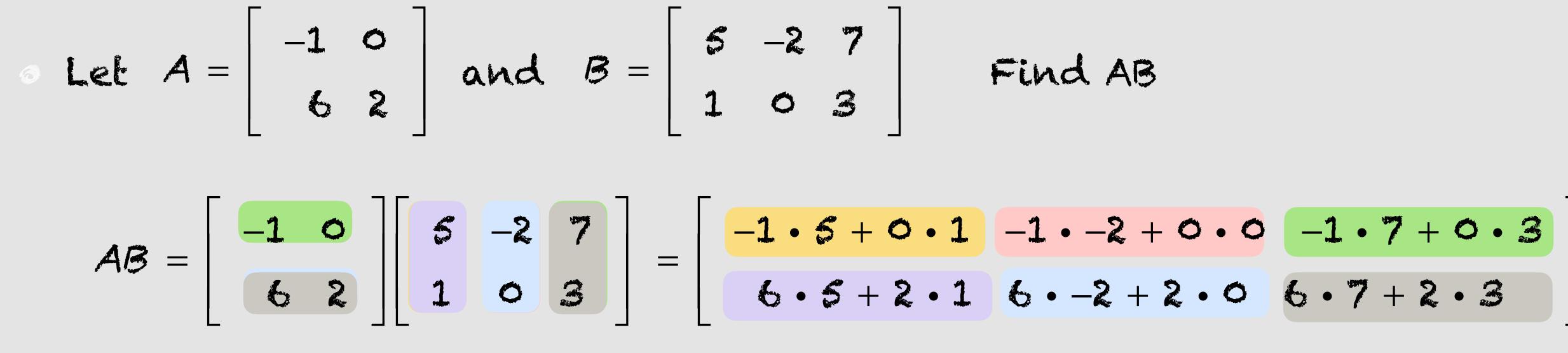
• Let 
$$A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$$
 and  $B = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ 



Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

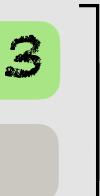






Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

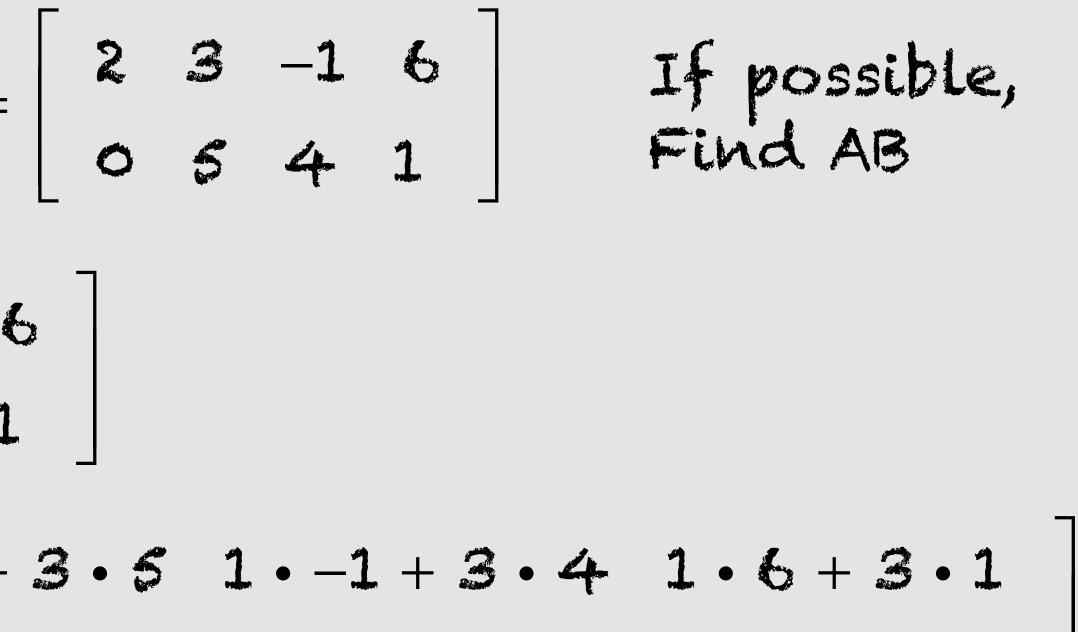






• Let 
$$A = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$$
 and  $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix}$ 
$$= \begin{bmatrix} 1 \cdot 2 + 3 \cdot 0 & 1 \cdot 3 + \\ 0 \cdot 2 + 2 \cdot 0 & 0 \cdot 3 + \\ 0 \cdot 2 + 2 \cdot 0 & 0 \cdot 3 + \\ = \begin{bmatrix} 2 & 18 & 11 & 9 \\ 0 & 10 & 8 & 2 \end{bmatrix}$$

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.



 $2 \cdot 5 \quad 0 \cdot -1 + 2 \cdot 4 \quad 0 \cdot 6 + 2 \cdot 1$ 



• Let 
$$A = \begin{bmatrix} 2 & 3 & -1 & 6 \\ 0 & 5 & 4 & 1 \end{bmatrix}$$
 and

- For the product of two matrices to be defined, the number of columns of the first matrix must equal the number of rows of the second matrix.
- Since that is not the case in this example, multiplication is not possible. The product is undefined.

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

 $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$ If possible, Find AB

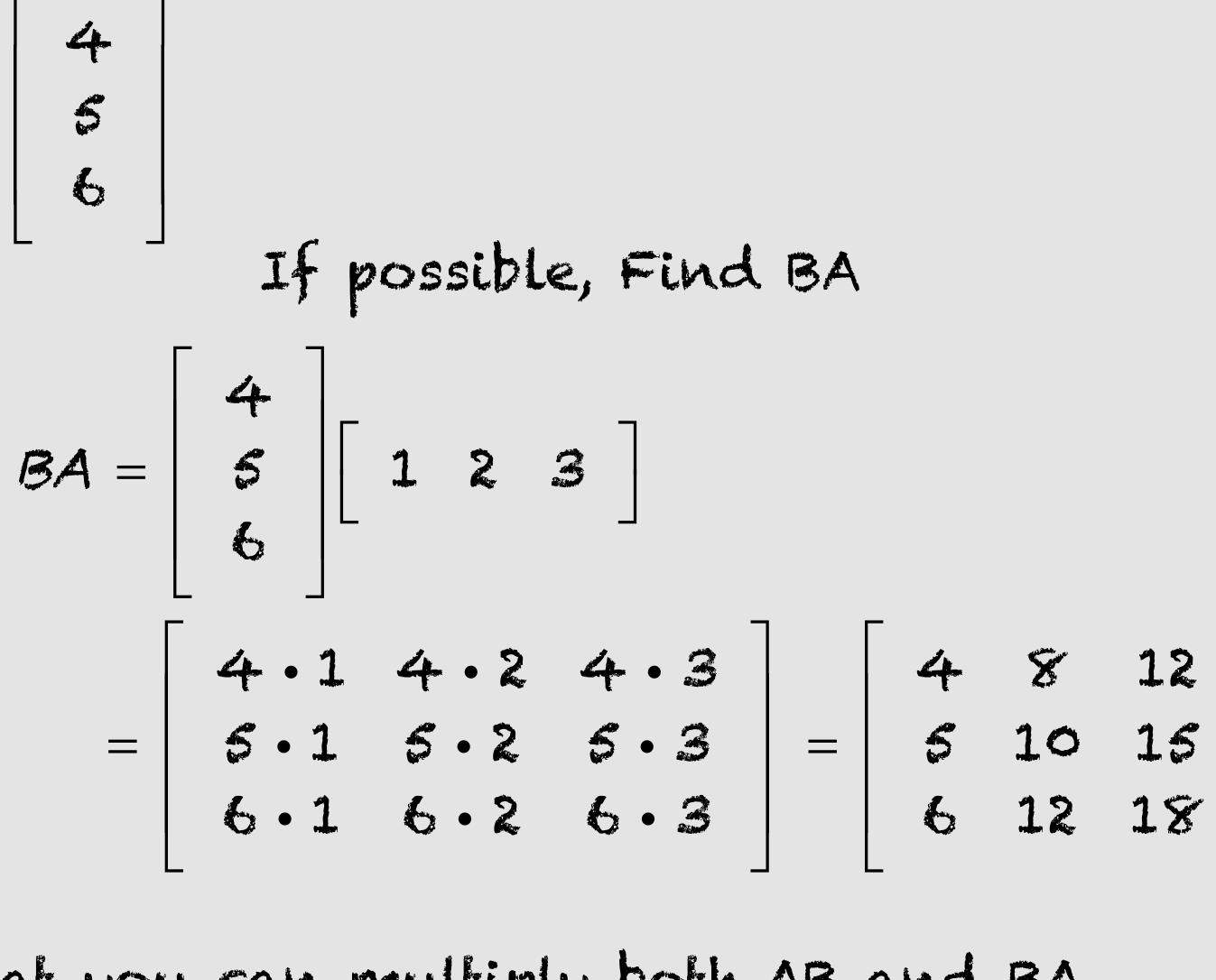
You will note that matrix multiplication is not commutative.



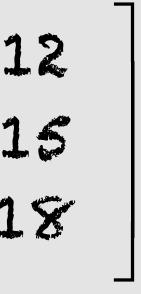
Let  $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ If possible, Find AB  $AB = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$  $= \begin{bmatrix} 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 \end{bmatrix}$ = [32]

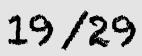
It will not often happen that you can multiply both AB and BA

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.









Let  $A = \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$  $AB = \begin{bmatrix} 1 & -1 & 4 \\ 4 & -1 & 3 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 4 \\ 1 & -1 & 3 \end{bmatrix}$  $2 \cdot 1 + 0 \cdot 1 + -2 \cdot 1$   $2 \cdot 1 + 0 \cdot 2 + -2 \cdot -1$   $2 \cdot 0 + 0 \cdot 4 + -2 \cdot 3$ 

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

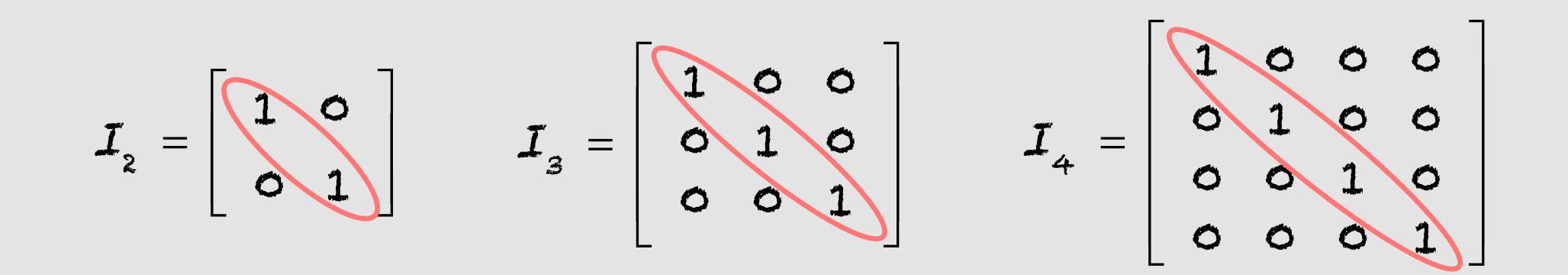
If possible, Find AB

- $1 \cdot 1 + -1 \cdot 1 + 4 \cdot 1$   $1 \cdot 1 + -1 \cdot 2 + 4 \cdot -1$   $1 \cdot 0 + -1 \cdot 4 + 4 \cdot 3$  $= | 4 \cdot 1 + -1 \cdot 1 + 3 \cdot 1 + 4 \cdot 1 + -1 \cdot 2 + 3 \cdot -1 + 4 \cdot 0 + -1 \cdot 4 + 3 \cdot 3$



## Identily Matrix

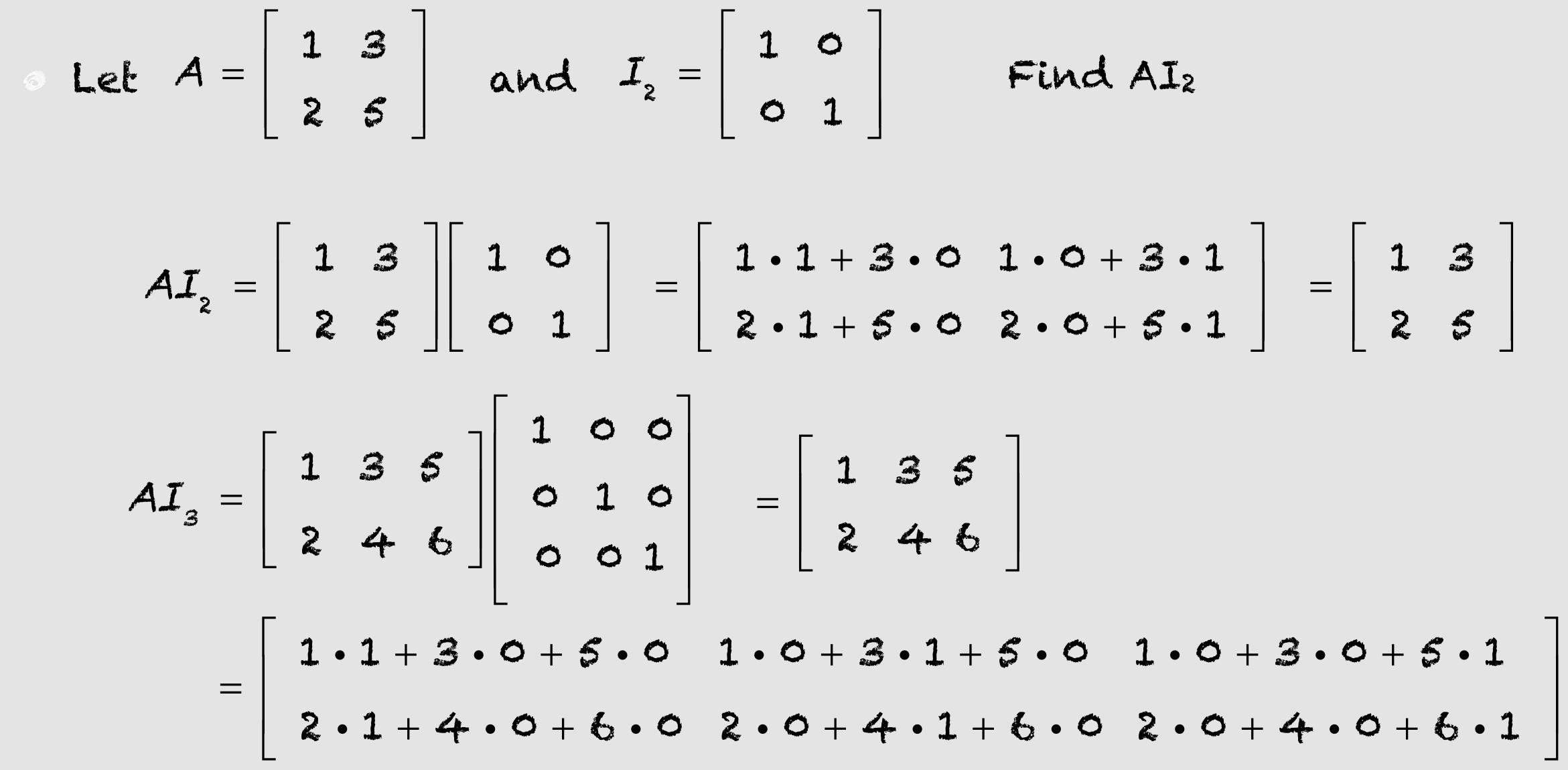
The identity matrix is an n x n matrix with elements of the main diagonal equal to 1 and all other elements equal 0.



Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.



### Identity Matrix



Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.



#### Properties of Matrix Multiplication

Let A, B, and C be matrices, and c is a scalar. Then:

- 1.  $I_m A = A I_n = A$ Matrix Multiplicative Identity A = m x n Associative Property of matrix multiplication.
- 2. (AB)C = A(BC)
- 3. A(B + C) = AB + ACDistributive Property of matrix multiplication

5. c(AB) = (cA)B

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

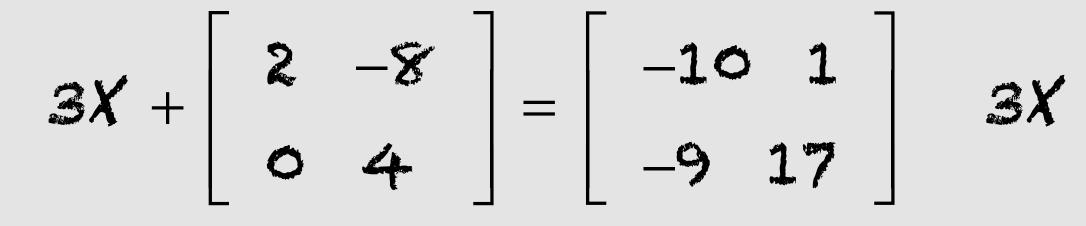
- 4. (A + B)C = AC + BC Distributive Property of matrix multiplication
  - Associative Property of scalar multiplication



# Solving a Matrix Equation

Solve for X in the matrix equation 3X + A = B, where

$$A = \begin{bmatrix} 2 & -8 \\ 0 & 4 \end{bmatrix} \quad B = \begin{bmatrix} -10 & 1 \\ -9 & 17 \end{bmatrix}$$



 $X = \frac{1}{3} \begin{bmatrix} -12 & 9 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ -3 & \frac{13}{3} \end{bmatrix}$ 3



Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

# $3X + \begin{bmatrix} 2 & -8 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -10 & 1 \\ -9 & 17 \end{bmatrix} \quad 3X = \begin{bmatrix} -10 & 1 \\ -9 & 17 \end{bmatrix} - \begin{bmatrix} 2 & -8 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -12 & 9 \\ -9 & 13 \end{bmatrix}$

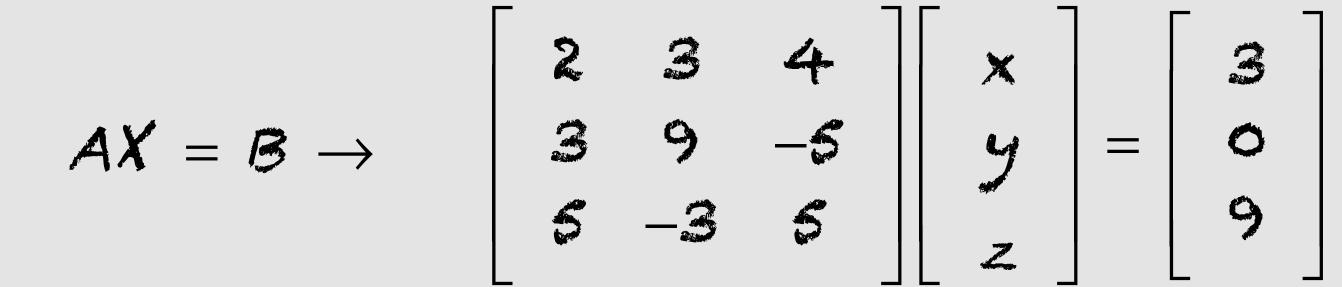


# System of Equations

- Write the following system of equations as a matrix equation. 2x + 3y + 4z = 3 $\begin{cases} 3x + 9y - 5z = 0 \end{cases}$ 5x - 3y + 5z = 9
- matrix, X = solution matrix, and B = constant matrix.

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

Let's define this as a matrix equation AX = B, with A = coefficient





# System of Equations

#### Write the following system of equations as a matrix equation. $\begin{cases} 2x + 3y + 4z = 3 \\ 3x + 9y - 5z = 0 \end{cases}$ 2 3 5 5x - 3y + 5z = 9

You can now solve the system by using Gauss-Jordan. Create augmented matrix [AB] Gauss-Jordan gives matrix [I3|X]

Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.

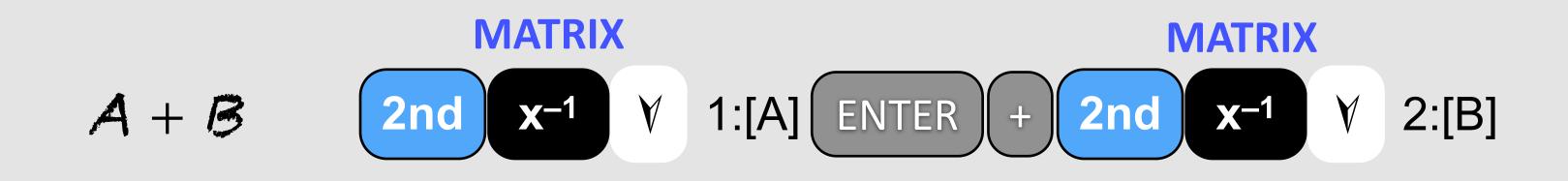
	2	4	×		3
3	9	-5	9	=	0
\$	-3	5	Z		9

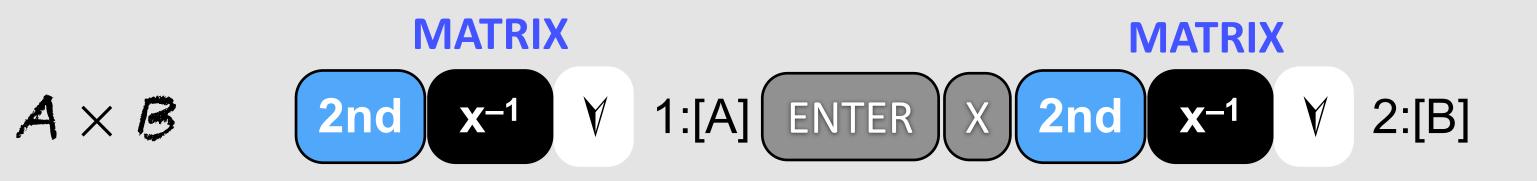
1	0	0	1.33696
0	1	0	28261
0	0	1	.29348





- From the previous section you should be able to perform elementary row operations on matrices.
  - You can also perform matrix operations; add, subtract, and multiply simply by selecting the matrices and performing the operations as you would with any values.





Objectives: Add and subtract matrices, multiply matrices by real numbers, and multiply two matrices.





