Chapter 9

Sequences and Series

★ 9.2 Arithmetic Sequences and Partial Sums

Chapter 9-2

Homework

- Read Sec 9.2 Complete Notes
- Do p659 1-79 every other odd

Chapter 9

Objectives

- ★ Find the common difference of an arithmetic sequence.
- ★ Write terms of an arithmetic sequence.
- ★ Use the formula for the general term of an arithmetic sequence
- ★ Use the formula for the sum of the first n terms of an arithmetic sequence.

Recursive Formula

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

An **arithmetic sequence** is a sequence in which each term after the first differs from the preceding term by a constant amount.

The difference (**d**) between consecutive terms is called the **common difference** of the sequence.

A **recursive formula** is a rule in which **one or** more **previous terms are used to generate the next term**.

The **recursive** form of an **arithmetic sequence**: $a_n = a_{n-1} + d$

Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

The **recursive** form of an **arithmetic sequence**: $a_n = a_{n-1} + d$

		+1	+1	+1	+1	+1	+1	+1	
Term number	n	1	2	3	4	5	6	7	8	Domain
Term value	a_n	1	4	7	10	13	16	19	22	Range
		+3	+3	+3	+3	+3	+3	+3	

The **common difference** is 3, this is an **arithmetic sequence**.

Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Definition of Arithmetic Sequence

A sequence is **arithmetic** if the differences between consecutive terms are the same. So, the sequence

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots$$

is arithmetic if there is a number d such that

$$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \cdots = d.$$

The number d is the **common difference** of the arithmetic sequence.

Using Recursive Form to Find Terms of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Find the first six terms of the **arithmetic sequence** in which $a_1 = 100$ and $a_n = a_{n-1} - 30$.

$$a_1 = 100$$

$$a_2 = a_1 - 30 = 100 - 30 = 70$$

$$a_3 = a_2 - 30 = 70 - 30 = 40$$

$$a_4 = a_3 - 30 = 40 - 30 = 10$$

$$a_5 = a_4 - 30 = 10 - 30 = -20$$

$$a_6 = a_5 - 30 = -20 - 30 = -50$$

Note that each term can be found by adding -30 to the previous term.

The first six terms of the sequence are 100, 70, 40, 10, -20, -50.

Arithmetic Sequence

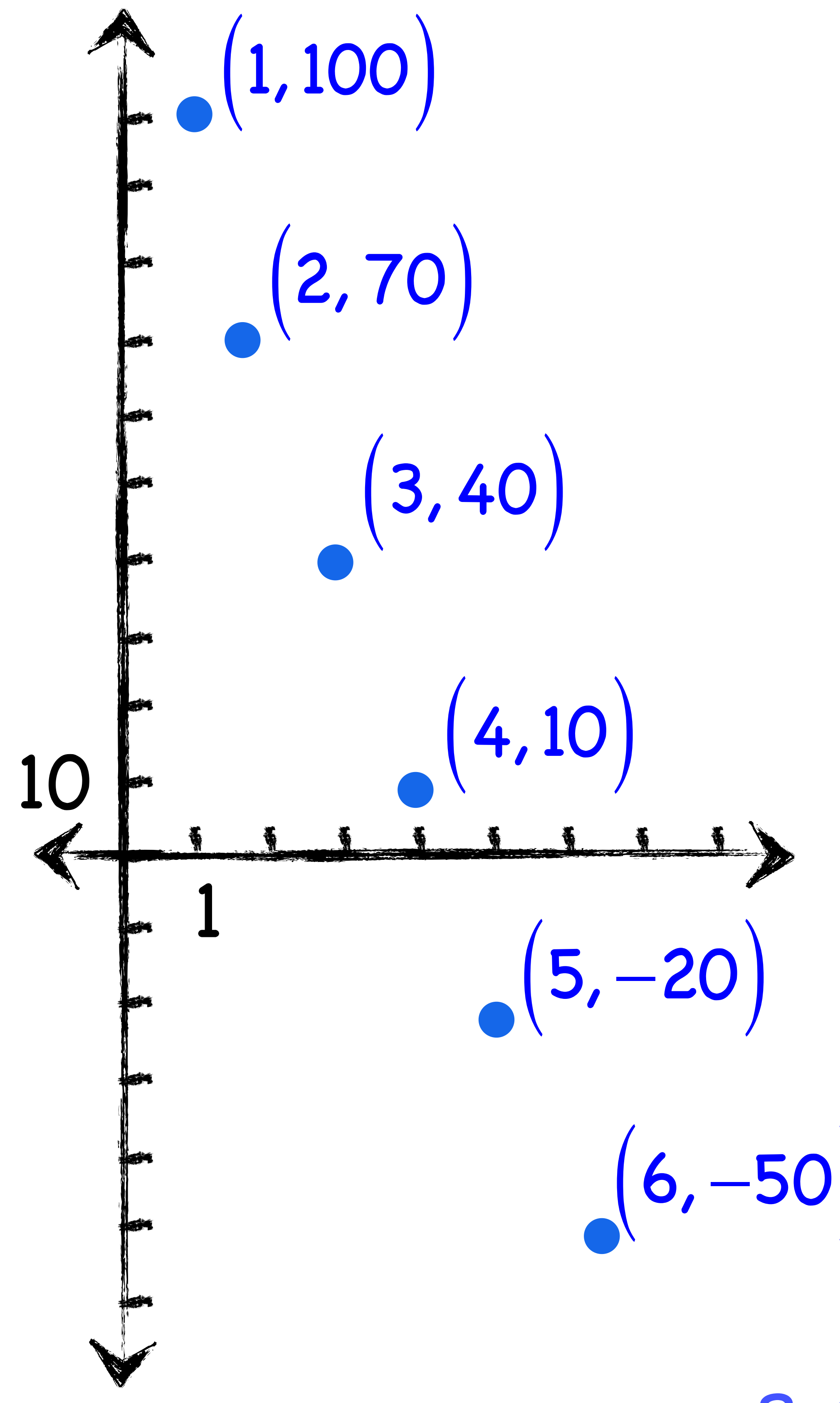
Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

The first six terms of the sequence are 100, 70, 40, 10, -20, -50.

If we graph those terms with the appropriate index as points, (n, a_n)

(1, 100) (2, 70) (3, 40) (4, 10), (5, -20) (6, -50)

The graph of each **arithmetic sequence** forms a set of discrete points lying on a straight line. An **arithmetic sequence** is a linear function whose domain is the set of positive integers (Natural Numbers).



Note that there is no line. The domain does not include non-integers.

Definition of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

If the first term of an **arithmetic sequence** is a_1, each term after the first is found by adding **d**, the common difference to the previous term.

100, 70, 40, 10, -20, -50

$$a_1 = 100$$

$$a_2 = a_1 + d$$

$$a_3 = a_2 + d = (a_1 + d) + d = a_1 + 2d$$

$$a_4 = a_3 + d = (a_1 + 2d) + d = a_1 + 3d$$

$$a_5 = a_4 + d = (a_1 + 3d) + d = a_1 + 4d$$

$$a_2 = 100 + 1(-30) = 70$$

$$a_3 = 100 + 2(-30) = 40$$

$$a_4 = 100 + 3(-30) = 10$$

$$a_5 = 100 + 4(-30) = -20$$

This suggests an **explicit formula** for finding a specific term of an **arithmetic sequence**.

$$a_n = a_1 + d(n-1)$$

Explicit Formula

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

We can find a specific term by using the **explicit formula** for determining the **n**th term of an arithmetic sequence with first term a_1 and common difference **d**.

$$a_n = a_1 + d(n-1)$$

The **explicit formula** defines the value of a term in a sequence by the position of the term in the sequence.

You can also start with the 0th term, a_0, the term prior to the first term of the sequence if there was one.

$$a_n = a_0 + nd$$

nth Term of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

The *n*th Term of an Arithmetic Sequence

The *n*th term of an arithmetic sequence has the form

$$a_n = dn + c$$ Linear form

where d is the common difference between consecutive terms of the sequence and $c = a_1 - d$. A graphical representation of this definition is shown in Figure 9.3. Substituting $a_1 - d$ for c in $a_n = dn + c$ yields an alternative *recursion* form for the *n*th term of an arithmetic sequence.

$$a_n = a_1 + (n - 1)\,d$$ Alternative form

Example: nth Term of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Find the ninth term of the arithmetic sequence whose first term is 6 and whose common difference is −5.

To find the ninth term, a_9, we replace n in the formula with 9, a_1 with 6, and d with −5.

$$a_n = a_1 + d(n-1)$$

$$n = 9,\ a_1 = 6,\ d = -5$$

$$a_9 = 6 + -5(9-1)$$

$$a_9 = 6 + -40 = -34$$

The ninth term is −34.

You can also start with the 0th term

$$a_n = a_0 + nd$$

$$a_0 = 6 - -5 = 11$$

$$a_9 = 11 + 9(-5) = -34$$

Example: nth Term of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Find the nth term of the arithmetic sequence whose fifth term is 19 and whose ninth term is 27.

$$a_n = a_1 + d(n-1)$$

Method 1 Use the formula and a system of equations to find a_1 and d.

$$a_5 = a_1 + d(5-1)$$
$$a_1 = a_5 - 4d$$
$$a_1 = 19 - 4d$$

$$a_9 = a_1 + d(9-1)$$
$$a_1 = a_9 - 8d$$
$$a_1 = 27 - 8d$$

$$19 - 4d = 27 - 8d$$
$$4d = 8$$
$$d = 2$$

$$a_1 = 19 - 4(2) = 11$$
$$a_1 = 27 - 8(2) = 11$$

$$a_n = 11 + 2(n-1)$$
$$a_n = 2n + 9$$

9 would be the term before a_1 = 11

Example: nth Term of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Find the nth term of the arithmetic sequence whose fifth term is 19 and whose ninth term is 27.

$$a_n = a_1 + d(n-1)$$

Method 2

We use the formula replacing a_1 and a_n with a_5 and a_9. Finding d, then using the formula again to find a_1.

$$a_9 = a_5 + d(9-5)$$

$$27 = 19 + 4d$$

$$d = 2$$

$$a_9 = a_1 + 2(9-1)$$

$$a_1 = 27 - 2(8)$$

$$a_1 = 11$$

$$a_n = 11 + 2(n-1)$$

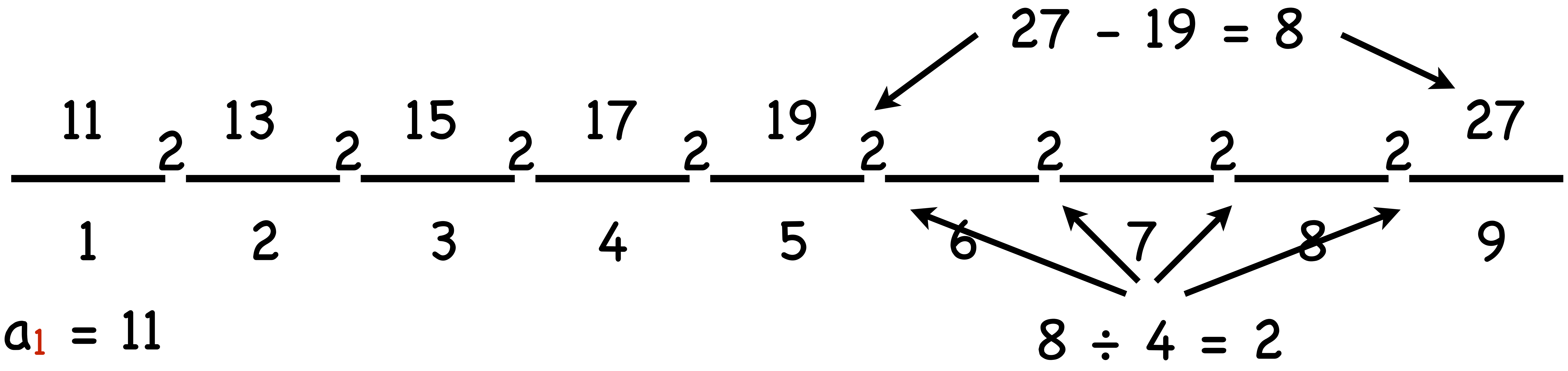
$$a_n = 2n + 9$$

Example: nth Term of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Find the nth term of the arithmetic sequence whose fifth term is 19 and whose ninth term is 27.

Method 3



$a_1 = 11$

$$a_9 = a_5 + d(9 - 5)$$

$$27 = 19 + d(4)$$

$$d = 2$$

$$a_n = a_1 + d(n - 1)$$

$$a_n = 11 + 2(n - 1) = 2n + 9$$

Example

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Find the missing terms in the arithmetic sequence **17, ___, ___, ___, -7, …**

Step 1 Find the common difference (note the numbers are decreasing).

There are a couple of ways to approach this problem.

$a_n = a_1 + (n - 1)d$

$a_5 = a_1 + (5 - 1)d$

$-7 = 17 + 4d$

$-24 = 4d$

$-6 = d$

There are 4 differences between 17 and -7

The total difference is -7 - 17 = -24

-24/4 = -6

17, 11, 5, -1, -7, …

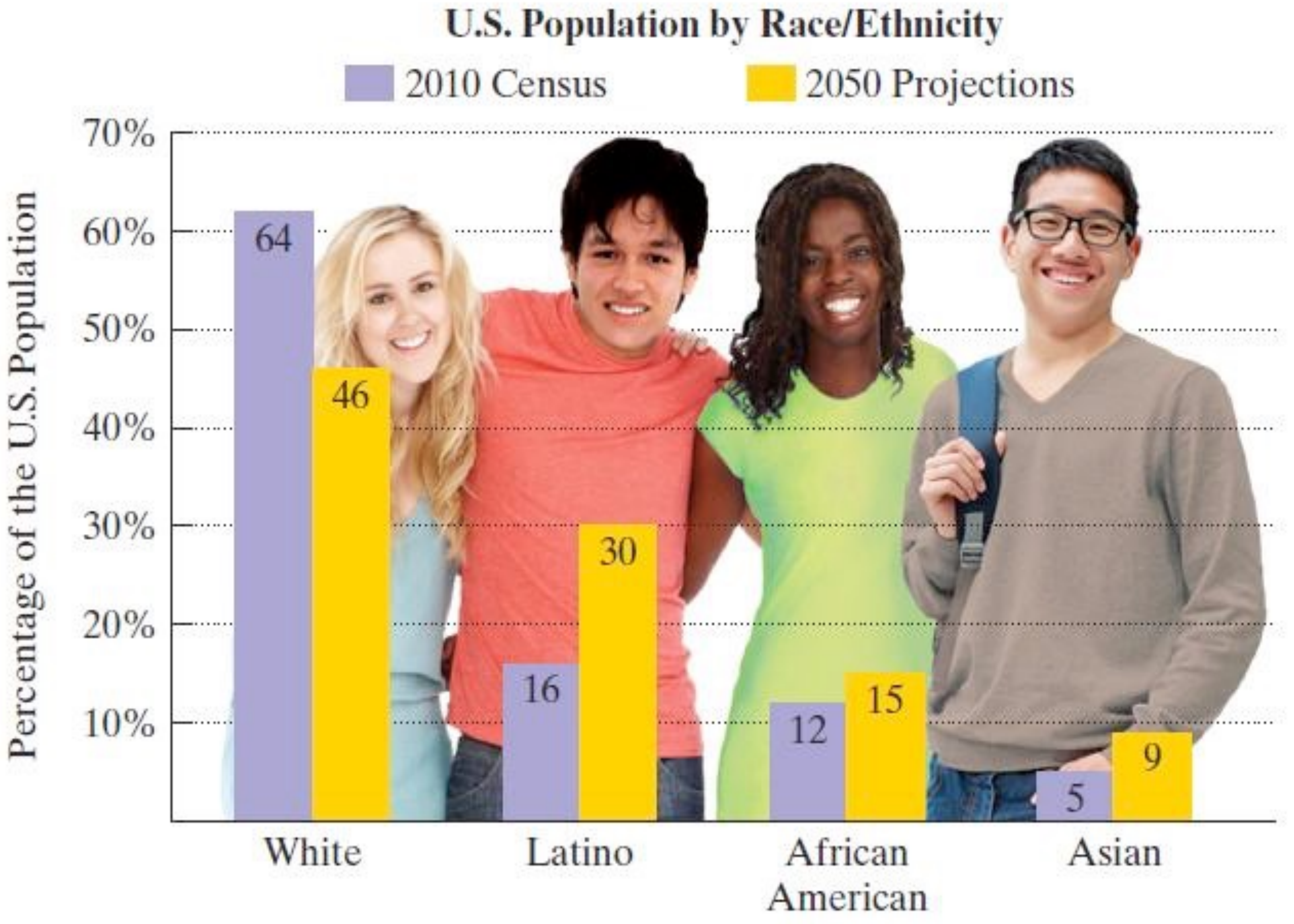
Modeling Changes in the U.S. Population

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

The data in the graph show that in 2010, 16% of the U.S. population was Latino. On average, this is projected to increase by approximately 0.35% per year.

Our data is percentage and since the growth is a constant percentage growth, slope, this is a linear growth or arithmetic sequence.

Write a formula for the nth term of the arithmetic sequence that describes the percentage of the U.S. population that will be Latino n years after 2009.



Modeling Changes in the U.S. Population

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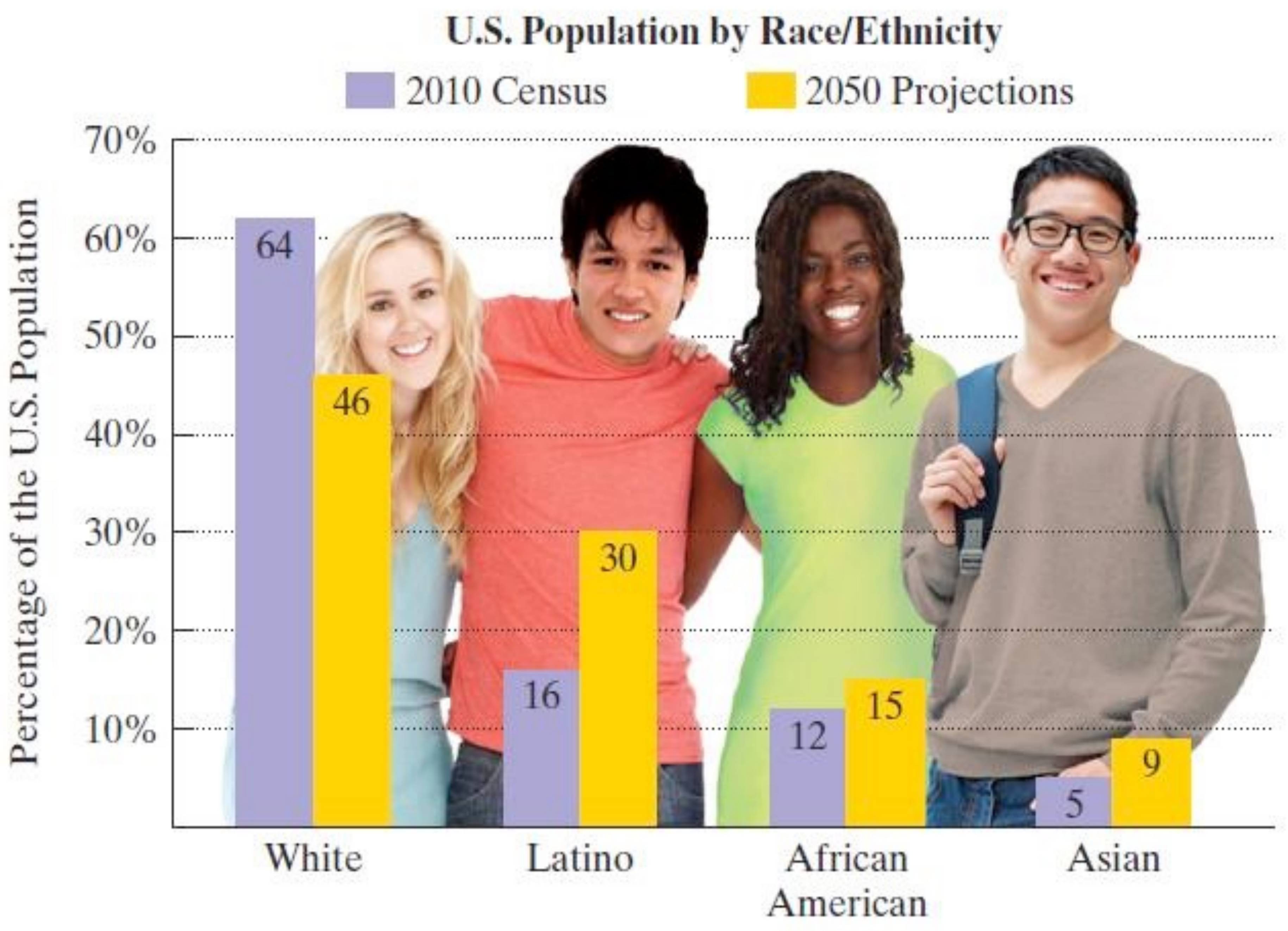
The data in the graph show that in 2010, 16% of the U.S. population was Latino. On average, this is projected to increase by approximately 0.35% per year.

$$a_n = a_1 + d(n-1)$$

$a_1 = 16$, $d = .35$

$$a_n = 16 + .35(n-1)$$

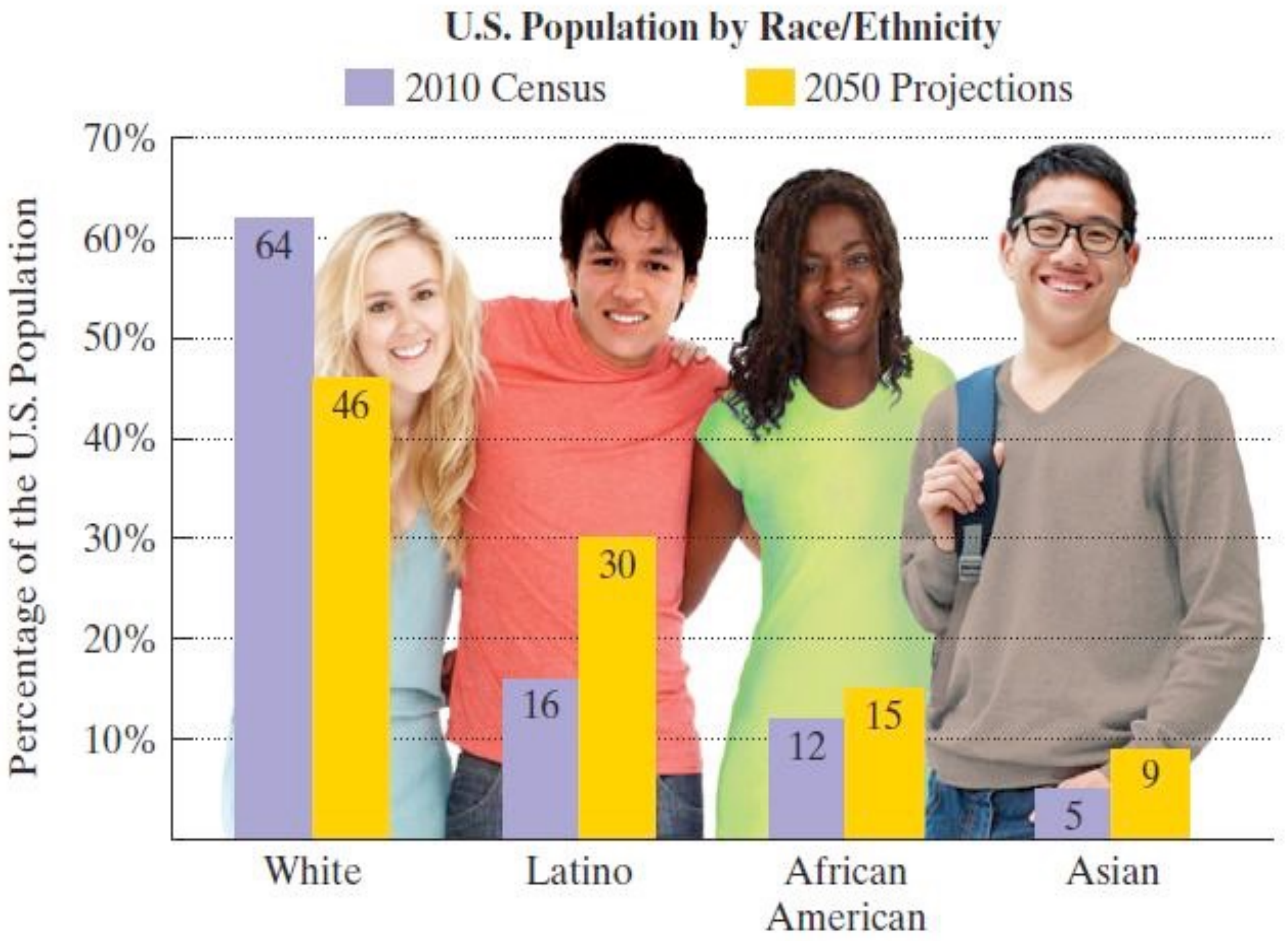
$$= 16 + .35n - .35 = 15.65 + .35n$$



The formula for the percentage of the U.S. population that will be Latino n years after 2009 is 15.65 + .35n.

Modeling Changes in the U.S. Population

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.



The formula for the percentage of the U.S. population that will be Latino n years after 2009 is 15.65 + .35n.

What percentage of the U.S. population is projected to be Latino in 2050?

$$a_n = 15.65 + .35n \qquad n = 2050 - 2009 = 41$$

$$a_{41} = 15.65 + .35(41) = 30$$

30% of the U.S. population is projected to be Latino in 2050.

The Sum of the First n Terms of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

A series is the sum of the terms of a sequence. The partial sum, S_n, is the sum of the first n terms of an arithmetic sequence

Let us take the series	$S_7 =$	$2 + 4 + 6 + 8 + 10 + 12 + 14$
Reverse the order	$S_7 =$	$14 + 12 + 10 + 8 + 6 + 4 + 2$
Add the two	$2S_7 =$	$16 + 16 + 16 + 16 + 16 + 16 + 16$

We have 7 16s or 7 (14+2)s

Adding them; $2S_7 = 7(14+2)$

Solving $$S_7 = \frac{7(2+14)}{2} = \frac{n(a_1 + a_n)}{2}$$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

a_1 is the first term, a_n is the nth term.

The Sum of the First n Terms of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Find the sum of the first 15 terms of the arithmetic sequence: 3, 6, 9, 12, …

$$S_n = \frac{n(a_1 + a_n)}{2}$$

We need the 15th term and the common difference.

$d = 3$ $a_{15} = a_1 + 3(15 - 1) = 3 + 3(14) = 45$

$$S_n = \frac{n(a_1 + a_n)}{2} = \frac{15(3+45)}{2} = \frac{15(48)}{2} = 15(24) = 360$$

The sum of the first 15 terms of the sequence is 360.

The Sum of the First n Terms of an Arithmetic Sequence

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Evaluate $\sum_{i=1}^{20} 7n + 1$

$$S_n = \frac{n(a_1 + a_n)}{2}$$

$$a_1 = 7(1) + 1 = 8$$

$$a_{20} = 7(20) + 1 = 141$$

$$S_{20} = \frac{20(8 + 141)}{2}$$

$$S_{20} = 10(149) = 1490$$

Save some money

Objectives: Recognize, write, and manipulate arithmetic sequences, and find the nth partial sum of an arithmetic sequence.

Suppose you foolishly put $100 under your mattress at the end of the month. You continue to be foolish and put money under your mattress each month but increasing the amount by $5 each time. How much money is under your mattress after one year?

$a_1 = 100 \qquad a_{12} = 100 + 5(11) = 155$

$$S_n = \frac{n(a_1 + a_n)}{2} \qquad S_{12} = \frac{12(100+155)}{2} \qquad S_{20} = 6(255) = \$1530$$

You would have $1530 after 12 months. Of course, had you put the money into a savings account at the bank you would have earned a little interest. And let's be honest, if you have money under your mattress, you would spend it.