

Transformations of Functions

We will now look at graphing a function without actually knowing the equation. Based on the graph of a function, it will be possible to shift, or transform the graph in any manner indicated.

For example, if given the picture of a graph and told “This is the graph of the function f .” Proceed to first identify the coordinates of any points that would aid in the graphing of the function’s transformations. These could be intercepts, vertices or any other coordinate that may help. These will serve as a guide for the graph of the function’s transformation.

To graph the function of $f(x+6)$, the function will need to shift to the left 6 spaces. To accomplish this, subtract 6 from all x values in the original function. The results will be the coordinates for the new graph. Likewise, to graph $f(x-4)$, this function will need to shift to the right 4 spaces, so add 4 to all x values.

In order to graph $f(x)+5$, the function will shift up 5 spaces, requiring that 5 be added to all y values. If asked to graph $f(x)-3$, the will function shift down 3 spaces, meaning subtract 3 from all y values.

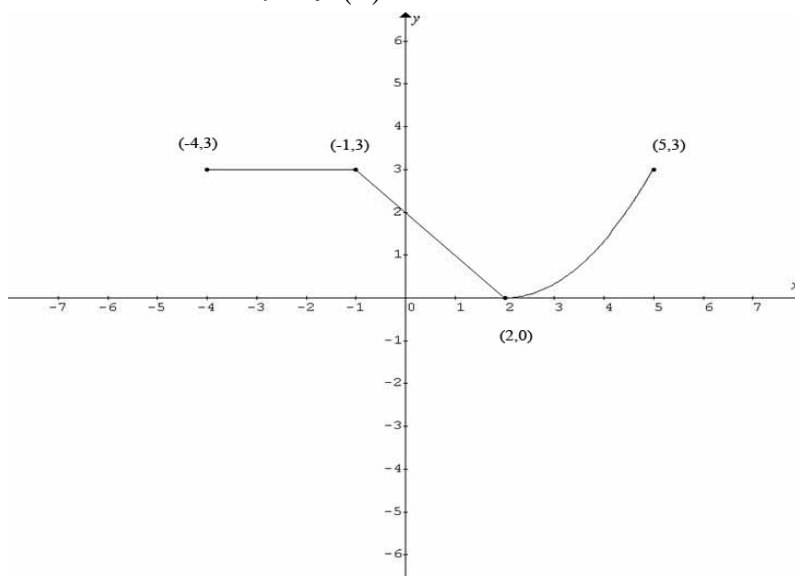
If the number is in the parenthesis, the function is shifting using P.L.N.R.. If the number is after the $f(x)$, simply shift as indicated, + says shift up, - says shift down.

Any number in front of the $f(x)$ will affect the scale of the function. This means it will affect the rate at which the function grows. When graphing, for example, $-f(x)$, change the sign of all y values on the graph of the function. This will cause the graph of the function to flip upside down. A number other than -1 can also be used. Lets say we need to graph $3f(x)$, this means the actual curve will increase 3 times as fast. It will therefore, be necessary to multiply all y values by 3. This will result in the coordinates for the new function. If the 3 were grouped with the x such as $f(3x)$, the horizontal change is the inverse of what it appears to be. So instead of multiplying x values by 3, divide by 3.

When graphing $f(-x)$, take the opposite of the x values of the function. This will cause the graph of the function to flip along a vertical axis.

Combinations of these rules will be encountered throughout your study of functions, for example, to shift right 3 and up 6. Just stick with the rules and the graph will be translated to its new location. If faced with a problem such as $2f(x)+3$, follow the order of operations. Multiply all y values by 2 first, then add 3 to each. Referring to the previous two topics, quadratic functions and absolute value functions, you will find references to these rules and examples throughout.

The following is the graph of the function $y = f(x)$. Use this to graph each function for letters A-D.



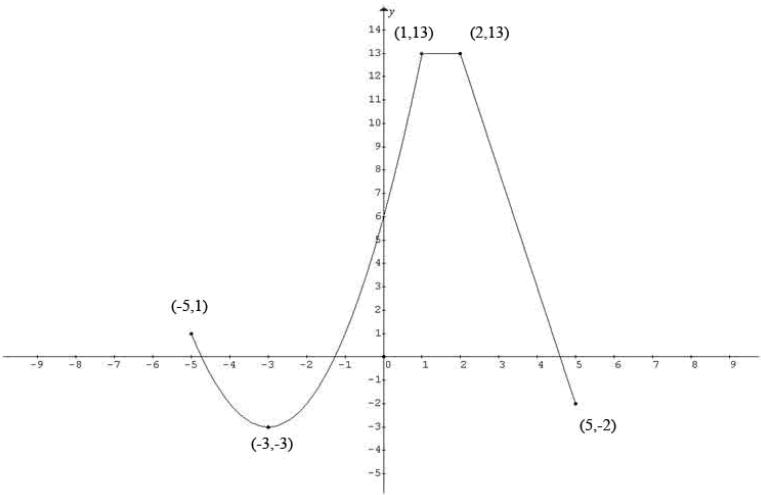
A) $f(x+2)$

B) $f(x)-3$

C) $-f(x)+1$

D) $f(-x)$

The following is the graph of the function $y = f(x)$. Use this to graph each function for letters E-H.



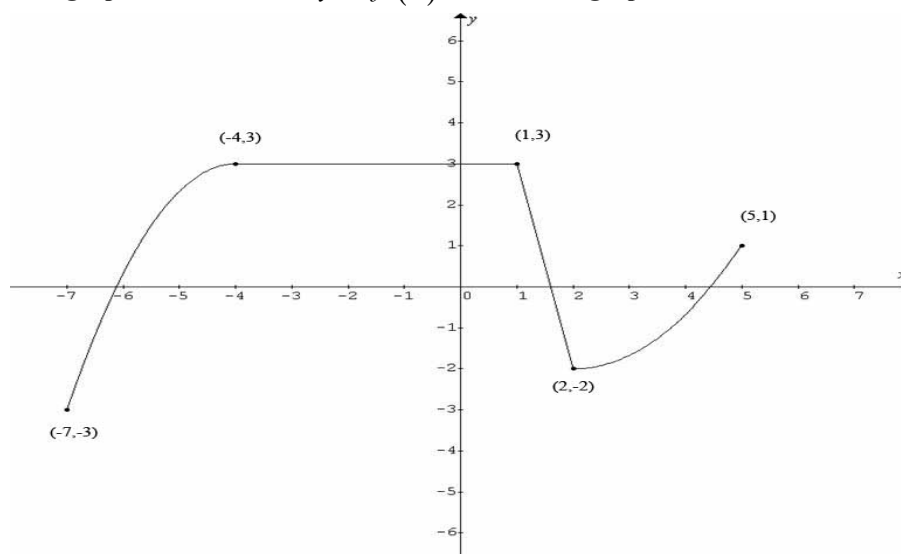
E) $f(x-1)$

F) $|f(x)|$

G) $f(x)-4$

H) $f(x+2)-3$

The following is the graph of the function $y = f(x)$. Use this to graph each function for letters I-L.



I) $f(-x)$

J) $f(x-2)-1$

K) $-f(x)+1$

L) $\frac{1}{3}f(x)$