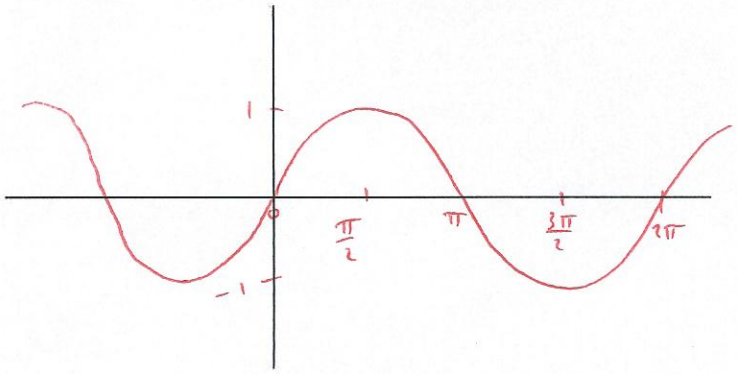
Trigonometric Functions Study Guide

SINE FUNCTIONS

Standard Form: $y = a\sin(bx - c) + d$

Parent Function $y = \sin x$



Amplitude: $|a|$

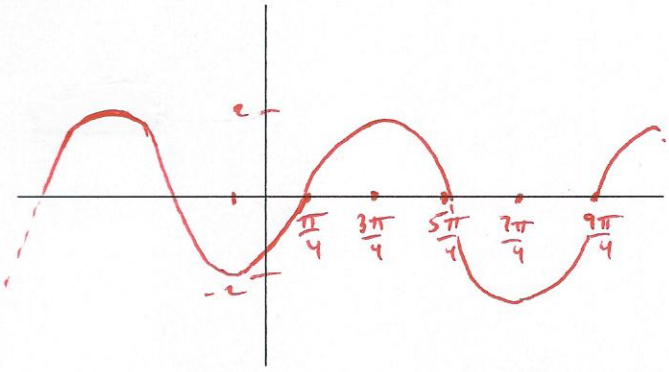
Period: $\frac{2\pi}{|b|}$

Phase Shift: $-\frac{c}{b}$

Initial Interval: $0 \le bx - c \le 2\pi$

Practice Problems

$$y = 2\sin\left(x - \frac{\pi}{4}\right)$$



Amplitude: 2

Period: 2π

Phase Shift: $\pi/4$

Initial Interval: $\frac{\pi}{4} \le x \le \frac{9\pi}{4}$

II

$0 \le x - \frac{\pi}{4} \le 2\pi$

$+\frac{\pi}{4}$ $+\frac{\pi}{4}$ $+\frac{\pi}{4}$

$\frac{\pi}{4} \le x \le \frac{9\pi}{4}$

$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$ add

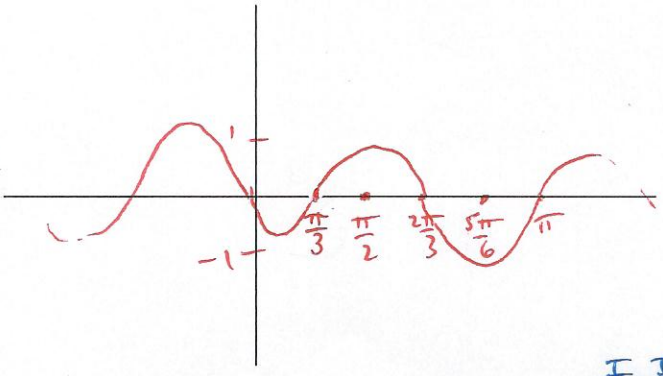
$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$

$\frac{3\pi}{4} + \frac{2\pi}{4} = \frac{5\pi}{4}$

$\frac{5\pi}{4} + \frac{2\pi}{4} = \frac{7\pi}{4}$

$\frac{7\pi}{4} + \frac{2\pi}{4} = \frac{9\pi}{4}$

$$y = \sin(3x - \pi)$$



Amplitude: 1

Period: $\frac{2\pi}{3}$

Phase Shift: $\frac{\pi}{3}$

Initial Interval: $\frac{\pi}{3} \le x \le \pi$

II

$0 \le 3x - \pi \le 2\pi$

$+\pi$ $+\pi$ $+\pi$

$\frac{\pi}{3} \le \frac{3x}{3} \le \frac{3\pi}{3}$

$\frac{\pi}{3} \le x \le \pi$

$\frac{\text{Period}}{4}$ $\frac{2\pi}{3} \div 4 = \frac{2\pi}{3} \cdot \frac{1}{4} = \boxed{\frac{\pi}{6}}$ Add

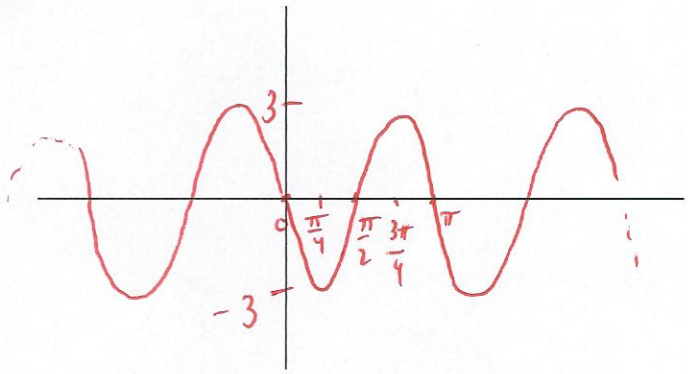
$\frac{2\pi}{6} + \frac{\pi}{6} = \frac{\pi}{2}$

$\frac{3\pi}{6} + \frac{\pi}{6} = \frac{2\pi}{3}$

$\frac{4\pi}{6} + \frac{\pi}{6} = \frac{5\pi}{6}$

$\frac{5\pi}{6} + \frac{\pi}{6} = \pi$

$y = -3\sin(2x)$



Amplitude: 3

Period: $\frac{2\pi}{2} = \pi$

Phase Shift: none

Initial Interval: $0 \le x \le \pi$

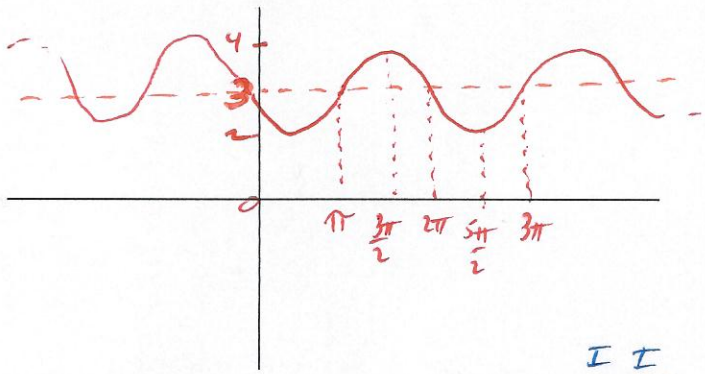
period / 4 $\boxed{\frac{\pi}{4}}$ add

$0 + \frac{\pi}{4} = \frac{\pi}{4}$

$\frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$

$\frac{2\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{4}$

$y = \sin(x - \pi) + 3$



Amplitude: 1

Period: 2π

Phase Shift: π

Initial Interval: $\pi \le x \le 3\pi$

I. I.

$0 \le x - \pi \le 2\pi$

$+\pi \quad +\pi \quad +\pi$

$\pi \le x \le 3\pi$

period / 4 $\frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$ add

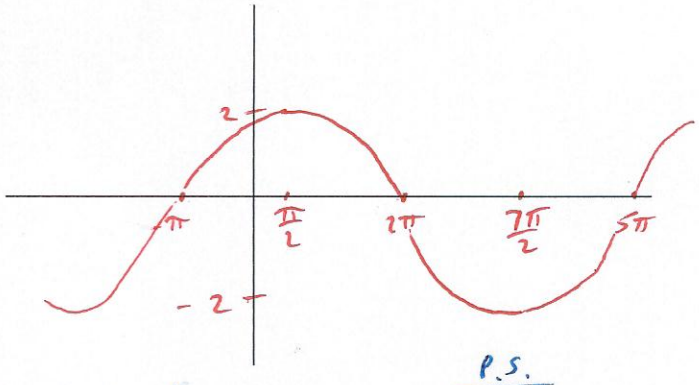
$\pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$\frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$\frac{4\pi}{2} + \frac{\pi}{2} = \frac{5\pi}{2}$

$\frac{5\pi}{2} + \frac{\pi}{2} = 3\pi$

$y = 2\sin\left(\frac{x}{3} + \frac{\pi}{3}\right)$



Amplitude: 2

Period: $2\pi \div \frac{1}{3} = 6\pi$

Phase Shift: $-\pi$

Initial Interval: $-\pi \le x \le 5\pi$

P.S.

$\frac{x}{3} + \frac{\pi}{3} = 0$

$3 \cdot \frac{x}{3} = -\frac{\pi}{3} \cdot 3$

$x = -\pi$

I. I.

$0 \le \frac{x}{3} + \frac{\pi}{3} \le 2\pi$

$-\frac{\pi}{3} \qquad -\frac{\pi}{3} \qquad -\frac{\pi}{3}$

$3 \cdot \left(-\frac{\pi}{3}\right) \le 3\left(\frac{x}{3}\right) \le \left(\frac{5\pi}{3}\right) \cdot 3$

$-\pi \le x \le 5\pi$

period / 4 $\frac{6\pi}{4} = \boxed{\frac{3\pi}{2}}$ add

$-\frac{2\pi}{2} + \frac{3\pi}{2} = \frac{\pi}{2}$

$\frac{\pi}{2} + \frac{3\pi}{2} = 2\pi$

$\frac{4\pi}{2} + \frac{3\pi}{2} = \frac{7\pi}{2}$

$\frac{7\pi}{2} + \frac{3\pi}{2} = 5\pi$

COSINE FUNCTIONS

Standard Form: $y = a\cos(bx - c) + d$

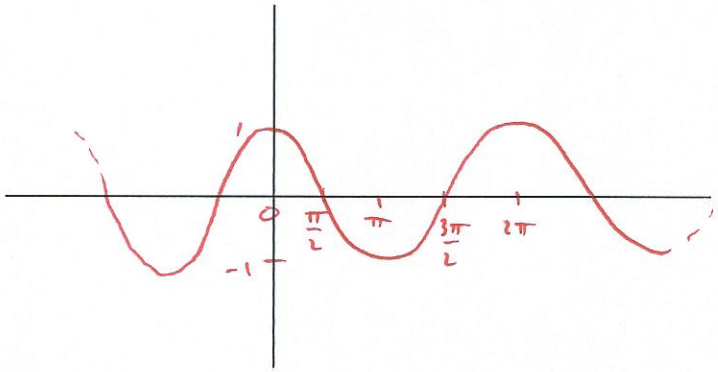
Amplitude: $|a|$

Period: $\frac{2\pi}{|b|}$

Phase Shift: $-\frac{c}{b}$

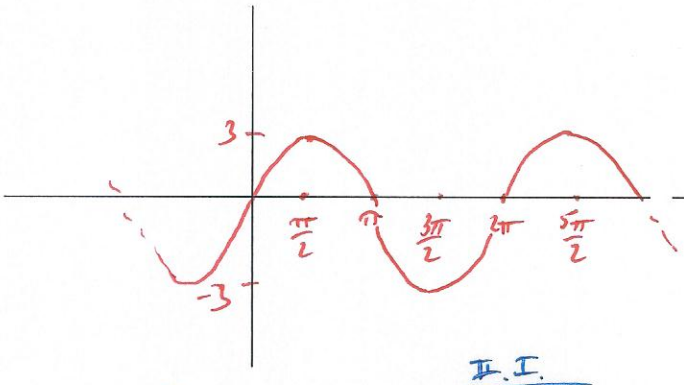
Initial Interval: $0 \le bx + c \le 2\pi$

Parent Function $y = \cos x$



Practice Problems

$y = 3\cos\left(x - \frac{\pi}{2}\right)$



Amplitude: 3

Period: 2π

Phase Shift: $\pi/2$

Initial Interval: $\frac{\pi}{2} \le x \le \frac{5\pi}{2}$

I. I.

$0 \le x - \frac{\pi}{2} \le 2\pi$

$+\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2}$

$\frac{\pi}{2} \le x \le \frac{5\pi}{2}$

Period / 4 $= \frac{2\pi}{4} = \frac{\pi}{2}$ Add

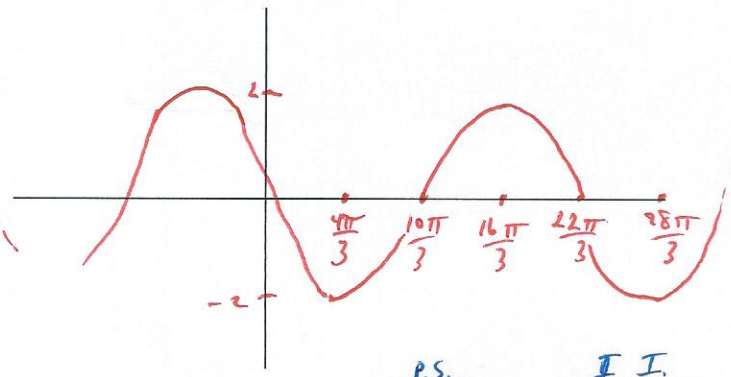
$\frac{\pi}{2} + \frac{\pi}{2} = \pi$

$\frac{2\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{2}$

$\frac{3\pi}{2} + \frac{\pi}{2} = 2\pi$

$\frac{4\pi}{2} + \frac{\pi}{2} = \frac{5\pi}{2}$

$y = -2\cos\left(\frac{x}{4} - \frac{\pi}{3}\right)$



Amplitude: 2

Period: $2\pi \div \frac{1}{4} = 8\pi$

Phase Shift: $\frac{4\pi}{3}$

Initial Interval: $\frac{4\pi}{3} \le x \le \frac{28\pi}{3}$

P.S.

$\frac{x}{4} - \frac{\pi}{3} = 0$

$4\left(\frac{x}{4}\right) = 4\left(\frac{\pi}{3}\right)$

$x = \frac{4\pi}{3}$

I. I.

$0 \le \frac{x}{4} - \frac{\pi}{3} \le 2\pi$

$+\frac{\pi}{3} \quad +\frac{\pi}{3} \quad +\frac{\pi}{3}$

$\frac{\pi}{3} \le \frac{x}{4} \le \frac{7\pi}{3}$

$\frac{4\pi}{3} \le x \le \frac{28\pi}{3}$

Period / 4 $= \frac{8\pi}{4} = 2\pi$ Add

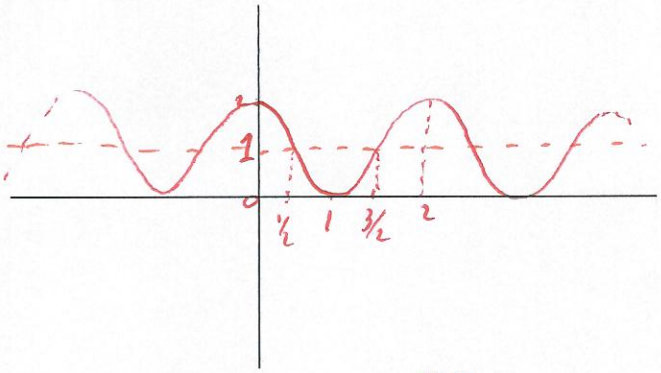
$\frac{4\pi}{3} + \frac{6\pi}{3} = \frac{10\pi}{3}$

$\frac{10\pi}{3} + \frac{6\pi}{3} = \frac{16\pi}{3}$

$\frac{16\pi}{3} + \frac{6\pi}{3} = \frac{22\pi}{3}$

$\frac{22\pi}{3} + \frac{6\pi}{3} = \frac{28\pi}{3}$

$y = \cos(\pi x) + 1$ ← vertical shift +1



Amplitude: 1

Period: $\frac{2\pi}{\pi} = 2$

Phase Shift: none

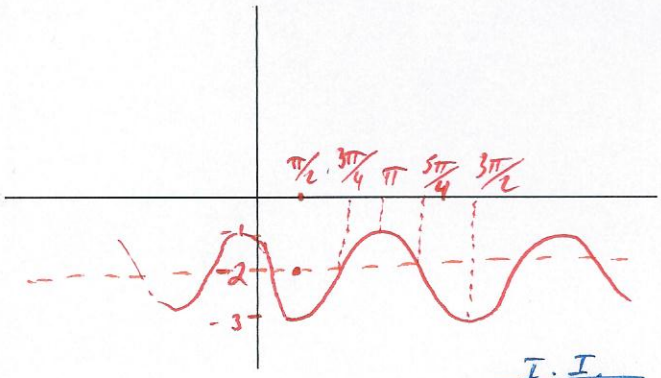
Initial Interval: $0 \le x \le 2$

I. I.

$0 \le \pi x \le 2\pi$

$0 \le x \le 2$

$y = -\cos(2x - \pi) - 2$ ← vertical shift −2



Amplitude: 1

Period: $\frac{2\pi}{2} = \pi$

Phase Shift: $\pi/2$

Initial Interval: $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

I. I.

$0 \le 2x - \pi \le 2\pi$

$+\pi \quad +\pi \quad +\pi$

$\frac{\pi}{2} \le \frac{2\pi}{2} \le \frac{3\pi}{2}$

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

period / 4 = $\frac{\pi}{4}$ add

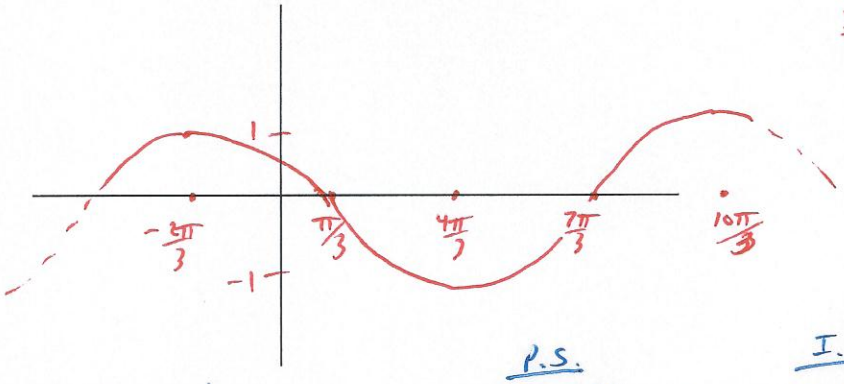
$\frac{2\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{4}$

$\frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$\frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$

$\frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

$y = \cos\left(\frac{x}{2} + \frac{\pi}{3}\right)$



Amplitude: 1

Period: $2\pi \div \frac{1}{2} = 4\pi$

Phase Shift: $-\frac{2\pi}{3}$

Initial Interval: $-\frac{2\pi}{3} \le x \le \frac{10\pi}{3}$

P.S.

$\frac{x}{2} + \frac{\pi}{3} = 0$

$2\left(\frac{x}{2}\right) = 2\left(-\frac{\pi}{3}\right)$

$x = -\frac{2\pi}{3}$

I. I.

$0 \le \frac{x}{2} + \frac{\pi}{3} \le 2\pi$

$-\frac{\pi}{3} \quad -\frac{\pi}{3} \quad -\frac{\pi}{3}$

$2\left(-\frac{\pi}{3}\right) \le 2\left(\frac{x}{2}\right) \le 2\left(\frac{5\pi}{3}\right)$

$-\frac{2\pi}{3} \le x \le \frac{10\pi}{3}$

$6\pi/3$

period / 4 = $\frac{4\pi}{4} = \pi$ add

$-\frac{2\pi}{3} + \frac{3\pi}{3} = \frac{\pi}{3}$

$\frac{\pi}{3} + \frac{3\pi}{3} = \frac{4\pi}{3}$

$\frac{4\pi}{3} + \frac{3\pi}{3} = \frac{7\pi}{3}$

$\frac{7\pi}{3} + \frac{3\pi}{3} = \frac{10\pi}{3}$

COSECANT FUNCTIONS

Standard Form: $y = a\csc(bx-c)+d$ → 1st graph $y = a\sin(bx-c)+d$

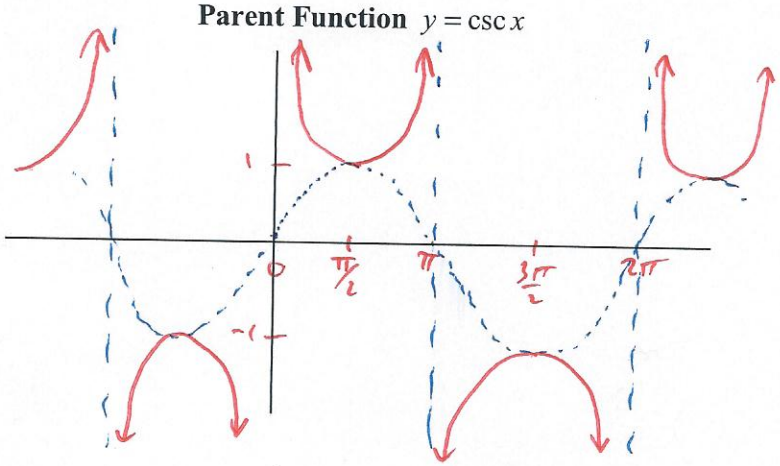
Amplitude: $|a|$

Period: $\frac{2\pi}{|b|}$

Phase Shift: $-\frac{c}{b}$

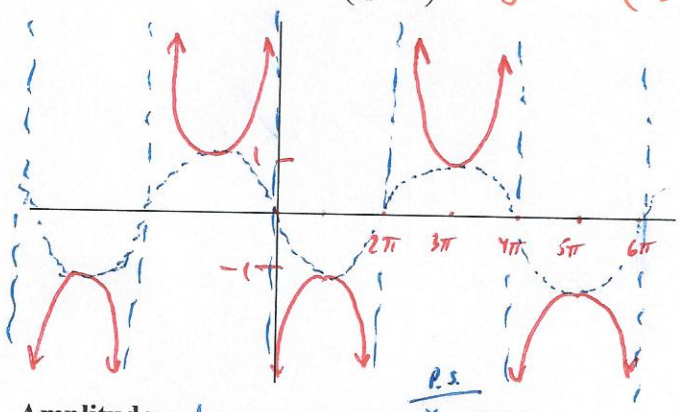
Initial Interval: $0 \le bx-c \le 2\pi$

Parent Function $y = \csc x$



Practice Problems

$y = \csc\left(\frac{x}{2}-\pi\right)$ $y = \sin\left(\frac{x}{2}-\pi\right)$



Amplitude: 1

Period: $2\pi \div \frac{1}{2} = 4\pi$ $2\left(\frac{x}{2}\right) = 2\pi$

Phase Shift: 2π $x = 2\pi$

Initial Interval: $2\pi \le x \le 6\pi$

P.S. $\frac{x}{2}-\pi = 0$

I.I. $0 \le \frac{x}{2}-\pi \le 2\pi$

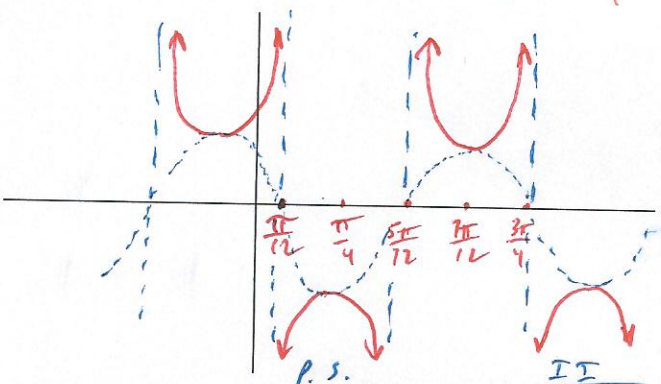
$+\pi \quad +\pi \quad +\pi$

$2(\pi) \le 2\left(\frac{x}{2}\right) \le 2(3\pi)$

$2\pi \le x \le 6\pi$

$\frac{\text{Period}}{4} = \frac{4\pi}{4} = \boxed{\pi}$ Add

$y = -\csc\left(3x-\frac{\pi}{4}\right)$ $y = -\sin\left(3x-\frac{\pi}{4}\right)$



Amplitude: 1

Period: $\frac{2\pi}{3}$

Phase Shift: $\pi/12$

Initial Interval: $\pi/12 \le x \le \frac{3\pi}{4}$

P.S. $3x-\frac{\pi}{4} = 0$

$\frac{3x}{3} = \frac{\pi}{4}\cdot\frac{1}{3}$

$x = \frac{\pi}{12}$

I.I. $0 \le 3x-\frac{\pi}{4} \le 2\pi$

$+\frac{\pi}{4} \quad +\frac{\pi}{4} \quad +\frac{\pi}{4}$

$\frac{1}{3}\left(\frac{\pi}{4}\right) \le \frac{3x}{3} \le \frac{1}{3}\left(\frac{9\pi}{4}\right)$

$\frac{\pi}{12} \le x \le \frac{3\pi}{4}$

$\frac{\text{Period}}{4} = \frac{2\pi}{3}\div 4 = \frac{2\pi}{3}\cdot\frac{1}{4} = \boxed{\frac{\pi}{6}}$ Add

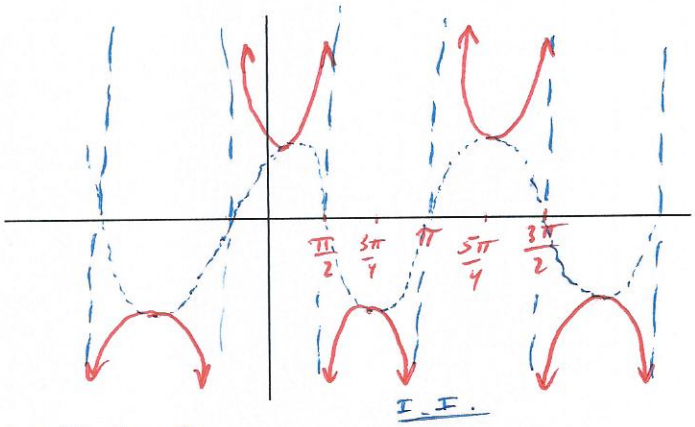
$\frac{\pi}{12}+\frac{2\pi}{12} = \boxed{\frac{\pi}{4}}$

$\frac{3\pi}{12}+\frac{2\pi}{12} = \boxed{\frac{5\pi}{12}}$

$\frac{5\pi}{12}+\frac{2\pi}{12} = \boxed{\frac{7\pi}{12}}$

$\frac{7\pi}{12}+\frac{2\pi}{12} = \boxed{\frac{3\pi}{4}}$

$y = 2\csc(\pi - 2x)$ $y = -2\sin(2x - \pi)$



Amplitude: 2

Period: $\frac{2\pi}{2} = \pi$

Phase Shift: $\pi/2$

Initial Interval: $\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

I. I.

$0 \le 2x - \pi \le 2\pi$

$+\pi \quad +\pi \quad +\pi$

$\frac{\pi}{2} \le \frac{2x}{2} \le \frac{3\pi}{2}$

$\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

Period/4 = $\frac{\pi}{4}$ All

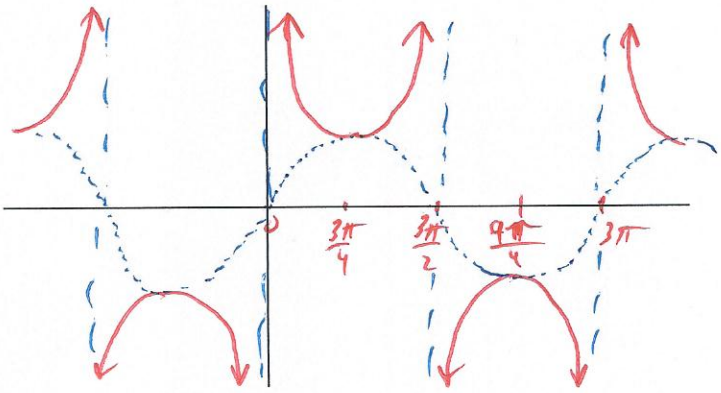
$\frac{2\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{4}$

$\frac{3\pi}{4} + \frac{\pi}{4} = \pi$

$\frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$

$\frac{5\pi}{4} + \frac{\pi}{4} = \frac{3\pi}{2}$

$y = \csc\left(\frac{2x}{3}\right)$ $y = \sin\left(\frac{2x}{3}\right)$



Amplitude: 1

Period: $2\pi \div \frac{2}{3} = 2\pi \cdot \frac{3}{2} = 3\pi$

Phase Shift: none

Initial Interval: $0 \le x \le 3\pi$

Period/4 = $\frac{3\pi}{4}$ All

$0 + \frac{3\pi}{4} = \frac{3\pi}{4}$

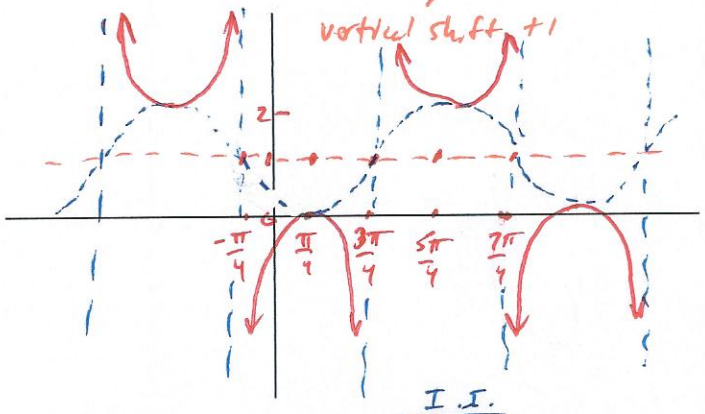
$\frac{3\pi}{4} + \frac{3\pi}{4} = \frac{3\pi}{2}$

$\frac{6\pi}{4} + \frac{3\pi}{4} = \frac{9\pi}{4}$

$\frac{9\pi}{4} + \frac{3\pi}{4} = 3\pi$

$y = -\csc\left(x + \frac{\pi}{4}\right) + 1$ $y = -\sin\left(x + \frac{\pi}{4}\right) + 1$

vertical shift +1



Amplitude: 1

Period: 2π

Phase Shift: $-\pi/4$

Initial Interval: $-\frac{\pi}{4} \le x \le \frac{7\pi}{4}$

I. I.

$0 \le x + \frac{\pi}{4} \le 2\pi$

$-\frac{\pi}{4} \quad -\frac{\pi}{4} \quad -\frac{\pi}{4}$

$-\frac{\pi}{4} \le x \le \frac{7\pi}{4}$

Period/4 = $\frac{2\pi}{4} = \frac{\pi}{2}$ All

$-\frac{\pi}{4} + \frac{2\pi}{4} = \frac{\pi}{4}$

$\frac{\pi}{4} + \frac{2\pi}{4} = \frac{3\pi}{4}$

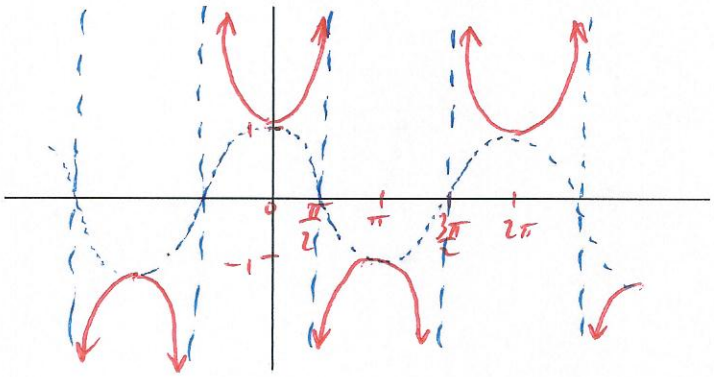
$\frac{3\pi}{4} + \frac{2\pi}{4} = \frac{5\pi}{4}$

$\frac{5\pi}{4} + \frac{2\pi}{4} = \frac{7\pi}{4}$

SECANT FUNCTIONS

Standard Form: $y = a\sec(bx-c)+d$ → 1st graph $y = a\cos(bx-c)+d$

Parent Function $y = \sec x$



Amplitude: $|a|$

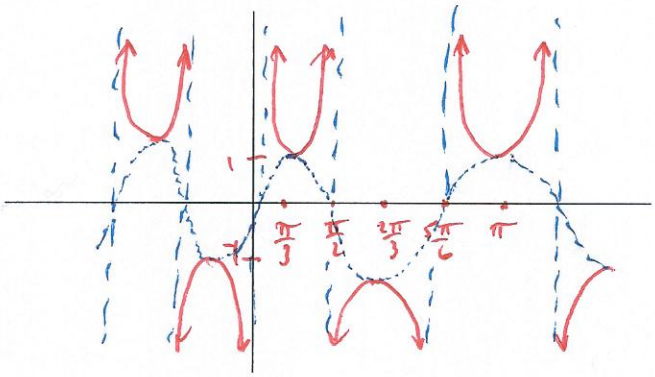
Period: $\frac{2\pi}{|b|}$

Phase Shift: $-\frac{c}{b}$

Initial Interval: $0 \le bx-c \le 2\pi$

Practice Problems

$y = \sec(3x-\pi)$ $y = \cos(3x-\pi)$



Amplitude: 1

Period: $\frac{2\pi}{3}$

Phase Shift: $\frac{\pi}{3}$

Initial Interval: $\frac{\pi}{3} \le x \le \pi$

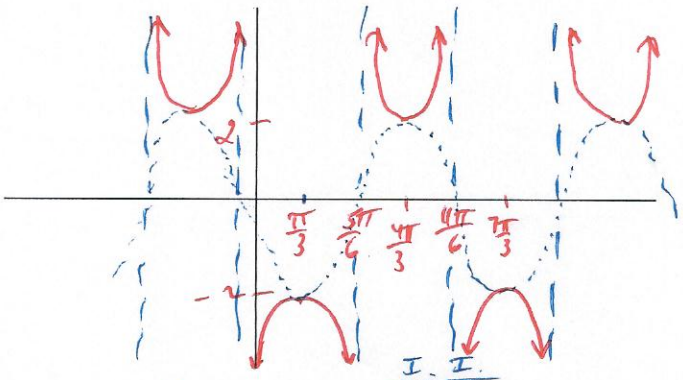
$0 \le 3x-\pi \le 2\pi$

$+\pi \quad +\pi \quad +\pi$

$\frac{\pi}{3} \le \frac{3x}{3} \le \frac{3\pi}{3}$

$\frac{\pi}{3} \le x \le \pi$

$y = -2\sec\left(x-\frac{\pi}{3}\right)$ $y = -2\cos\left(x-\frac{\pi}{3}\right)$



I. I.

Amplitude: 2

Period: 2π

Phase Shift: $\pi/3$

Initial Interval: $\frac{\pi}{3} \le x \le \frac{7\pi}{3}$

$0 \le x-\frac{\pi}{3} \le 2\pi$

$+\frac{\pi}{3} \quad +\frac{\pi}{3} \quad +\frac{\pi}{3}$

$\frac{6\pi}{3}$

$\frac{\pi}{3} \le x \le \frac{7\pi}{3}$

$\frac{\text{Period}}{4} = \frac{2\pi}{3}\cdot\frac{1}{2} = \boxed{\frac{\pi}{6}}$ Add

$\frac{2\pi}{6} + \frac{\pi}{6} = \boxed{\frac{\pi}{2}}$

$\frac{3\pi}{6} + \frac{\pi}{6} = \boxed{2\pi/3}$

$\frac{4\pi}{6} + \frac{\pi}{6} = \boxed{5\pi/6}$

$\frac{5\pi}{6} + \frac{\pi}{6} = \boxed{\pi}$

$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$ Add

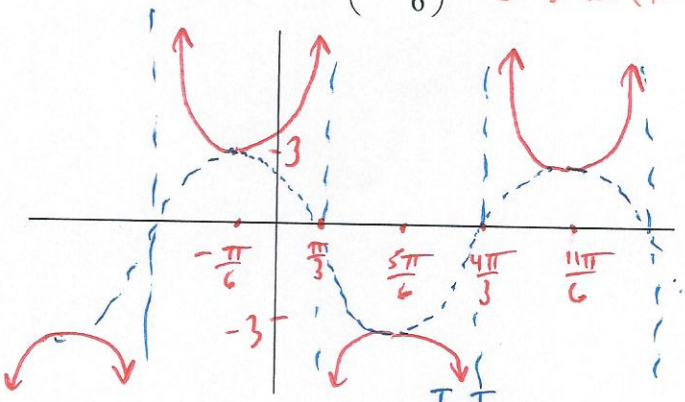
$\frac{\pi}{3} + \frac{\pi}{2} = \frac{2\pi}{6} + \frac{3\pi}{6} = \boxed{\frac{5\pi}{6}}$

$\frac{5\pi}{6} + \frac{3\pi}{6} = \frac{8\pi}{6} = \boxed{\frac{4\pi}{3}}$

$\frac{8\pi}{6} + \frac{3\pi}{6} = \boxed{\frac{11\pi}{6}}$

$\frac{11\pi}{6} + \frac{3\pi}{6} = \frac{14\pi}{6} = \boxed{\frac{7\pi}{3}}$

$y = 3\sec\left(x + \frac{\pi}{6}\right)$ $y = 3\cos\left(x + \frac{\pi}{6}\right)$



Amplitude: 3

Period: 2π

Phase Shift: $-\pi/6$

Initial Interval: $-\frac{\pi}{6} \le x \le \frac{11\pi}{6}$

I.I.

$0 \le x + \frac{\pi}{6} \le 2\pi$

$-\frac{\pi}{6} \quad -\frac{\pi}{6} \quad -\frac{\pi}{6}$

$-\frac{\pi}{6} \le x \le \frac{11\pi}{6}$

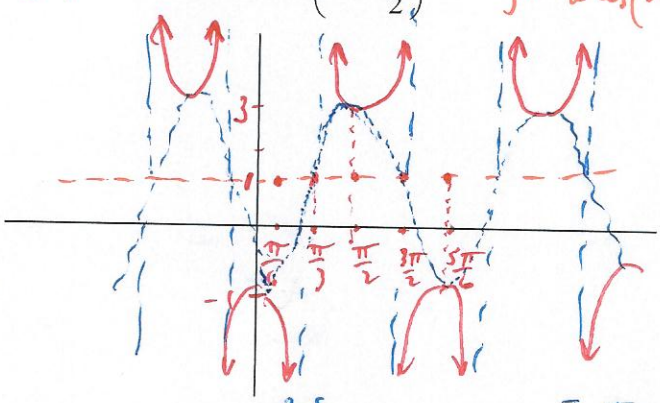
$\frac{\text{Period}}{4} = \frac{2\pi}{4} = \boxed{\frac{\pi}{2}}$ Add

$-\frac{\pi}{6} + \frac{3\pi}{6} = \boxed{\frac{\pi}{3}}$ $\quad \frac{5\pi}{6} + \frac{3\pi}{6} = \boxed{\frac{8\pi}{6}}$

$\frac{2\pi}{6} + \frac{3\pi}{6} = \boxed{\frac{5\pi}{6}}$ $\quad \frac{8\pi}{6} + \frac{3\pi}{6} = \boxed{\frac{11\pi}{6}}$

vertical shift +1

$y = -2\sec\left(3x - \frac{\pi}{2}\right) + 1$ $y = -2\cos\left(3x - \frac{\pi}{2}\right) + 1$



Amplitude: 2

Period: $\frac{2\pi}{3}$

Phase Shift: $\pi/6$

Initial Interval: $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$

P.S.

$3x - \frac{\pi}{2} = 0$

$\left(\frac{1}{3}\right)3x = \frac{\pi}{2}\left(\frac{1}{3}\right)$

$x = \frac{\pi}{6}$

I.I.

$0 \le 3x - \frac{\pi}{2} \le 2\pi$

$+\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2}$

$\left(\frac{1}{3}\right)\frac{\pi}{2} \le \frac{3x}{3} \le \frac{5\pi}{2}$

$\frac{\pi}{6} \le x \le \frac{5\pi}{6}$

$\frac{\text{Period}}{4} = \frac{2\pi}{3} \cdot \frac{1}{4} = \boxed{\frac{\pi}{6}}$ Add

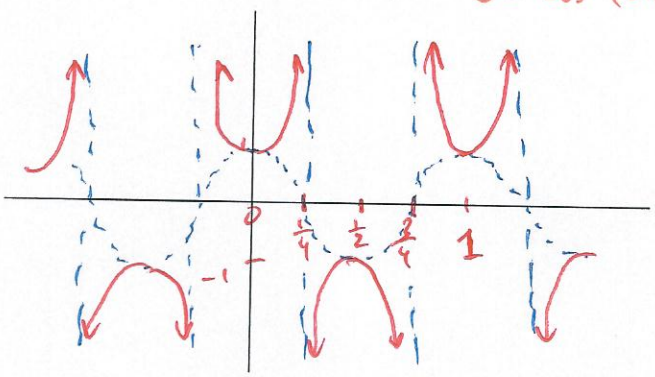
$\frac{\pi}{6} + \frac{\pi}{6} = \boxed{\frac{\pi}{3}}$

$\frac{2\pi}{6} + \frac{\pi}{6} = \boxed{\frac{\pi}{2}}$

$\frac{3\pi}{6} + \frac{\pi}{6} = \boxed{\frac{3\pi}{2}}$

$\frac{4\pi}{6} + \frac{\pi}{6} = \boxed{\frac{5\pi}{6}}$

$y = \sec(2\pi x)$ $y = \cos(2\pi x)$



Amplitude: 1

Period: $\frac{2\pi}{2\pi} = 1$

Phase Shift: none

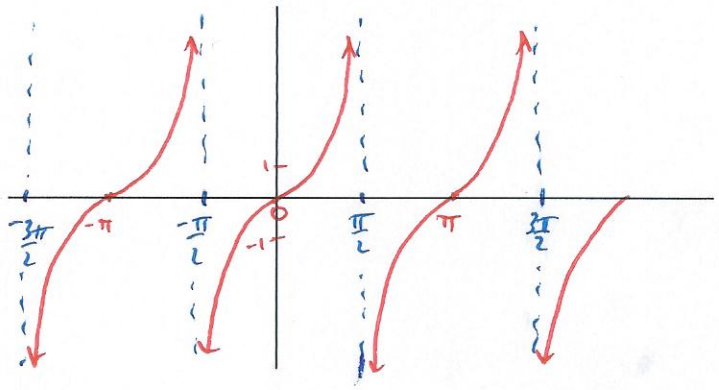
Initial Interval: $0 \le x \le 1$

$\frac{\text{Period}}{4} = \boxed{\frac{1}{4}}$

TANGENT FUNCTIONS

Standard Form: $y = a\tan(bx - c) + d$

Parent Function $y = \tan x$



Amplitude: $|a|$

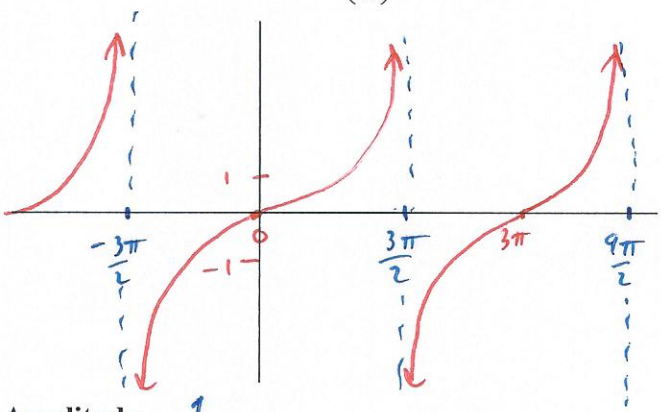
Period: $\frac{\pi}{|b|}$

Phase Shift:

Initial Interval: $-\frac{\pi}{2} < bx - c < \frac{\pi}{2}$

Practice Problems

$y = \tan\left(\frac{x}{3}\right)$



Amplitude: 1

Period: $\pi \div \frac{1}{3} = 3\pi$

Phase Shift:

Initial Interval: $-\frac{3\pi}{2} < x < \frac{3\pi}{2}$

$3\left(-\frac{\pi}{2}\right) < 3\left(\frac{x}{3}\right) < 3\left(\frac{\pi}{2}\right)$

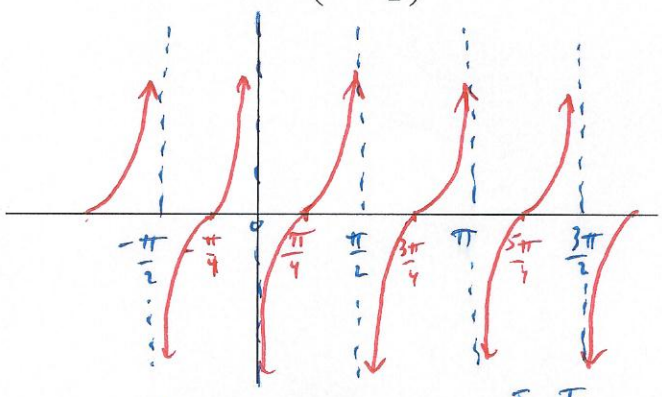
$-\frac{3\pi}{2} < x < \frac{3\pi}{2}$

$\frac{3\pi}{2}$ all

$\frac{3\pi}{2} + \frac{3\pi}{2} = \frac{6\pi}{2}$

$\frac{3\pi}{2} + \frac{6\pi}{2} = \frac{9\pi}{2}$

$y = \tan\left(2x - \frac{\pi}{2}\right)$



Amplitude: 1

Period: $\frac{\pi}{2}$

Phase Shift:

Initial Interval: $0 < x < \frac{\pi}{2}$

I. I.

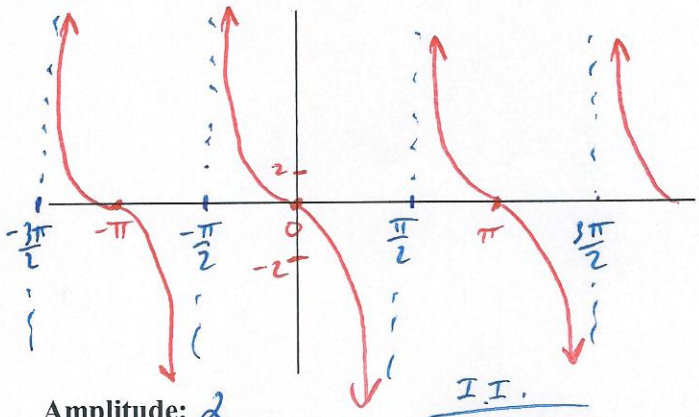
$-\frac{\pi}{2} < 2x - \frac{\pi}{2} < \frac{\pi}{2}$

$+\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2}$

$\frac{0}{2} < \frac{2x}{2} < \frac{\pi}{2}$

$0 < x < \frac{\pi}{2}$

$y = -2\tan(x+\pi)$



Amplitude: 2

Period: π

Phase Shift: $-$

Initial Interval: $-\frac{3\pi}{2} < x < -\frac{\pi}{2}$

I. I.

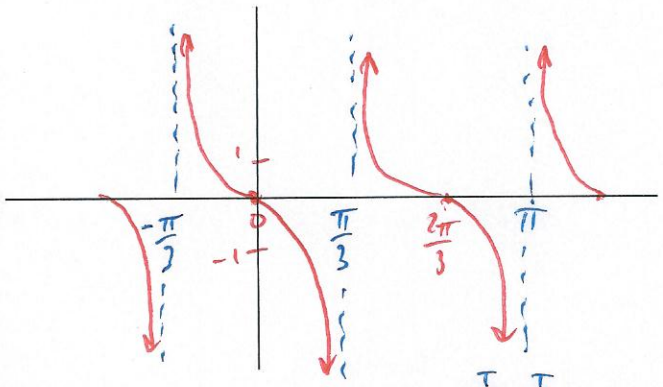
$-\frac{\pi}{2} < x+\pi < \frac{\pi}{2}$

$-\pi \quad -\pi \quad -\pi$

$-\frac{3\pi}{2} < x < -\frac{\pi}{2}$

$\frac{\text{period}}{2} = \boxed{\frac{\pi}{2}}$ Add $-\frac{3\pi}{2} + \frac{\pi}{2} = -\frac{2\pi}{2}$

$y = -\tan\left(\frac{3x}{2}\right)$



Amplitude: 1

Period: $\pi \div \frac{3}{2} = \frac{2\pi}{3}$

Phase Shift: $-$

Initial Interval: $-\frac{\pi}{3} < x < \frac{\pi}{3}$

I. I.

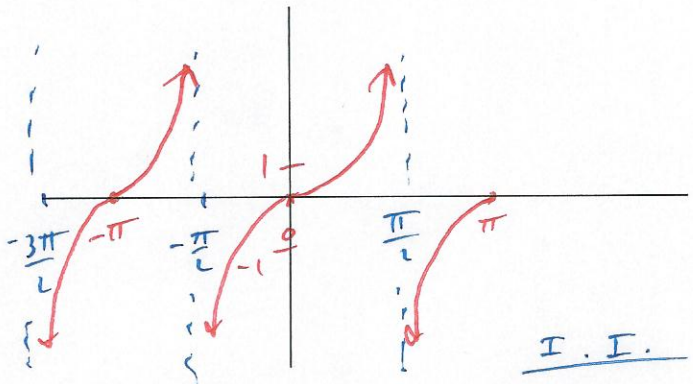
$\left(\frac{2}{3}\right)\left(-\frac{\pi}{2}\right) < \left(\frac{3x}{2}\right)\left(\frac{2}{3}\right) < \frac{\pi}{2}\left(\frac{2}{3}\right)$

$-\frac{\pi}{3} < x < \frac{\pi}{3}$

$\frac{\text{Period}}{2} = \frac{2\pi}{3} \cdot \frac{1}{2} = \boxed{\frac{\pi}{3}}$ $\frac{\pi}{3} + \frac{2\pi}{3} = \pi$

$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

$y = \tan(x+\pi)$



Amplitude: 1

Period: π

Phase Shift: $-$

Initial Interval: $-\frac{3\pi}{2} < x < -\frac{\pi}{2}$

I. I.

$-\frac{\pi}{2} < x+\pi < \frac{\pi}{2}$

$-\pi \quad -\pi \quad -\pi$

$-\frac{3\pi}{2} < x < -\frac{\pi}{2}$

COTANGENT FUNCTIONS

Standard Form: $y = a\cot(bx - c) + d$

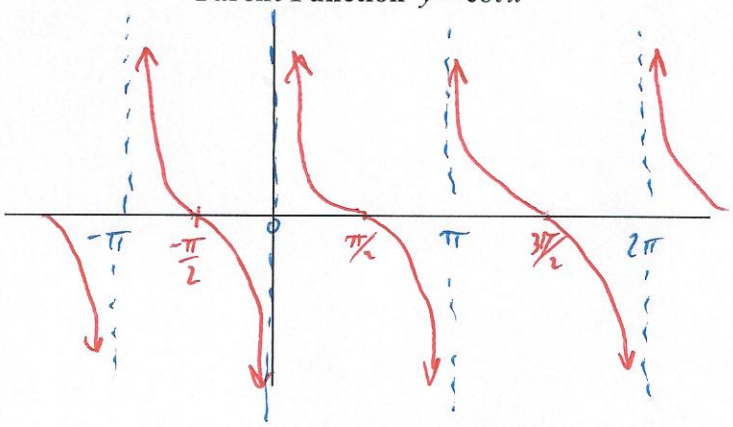
Amplitude: $|a|$

Period: $\frac{\pi}{|b|}$

Phase Shift: —

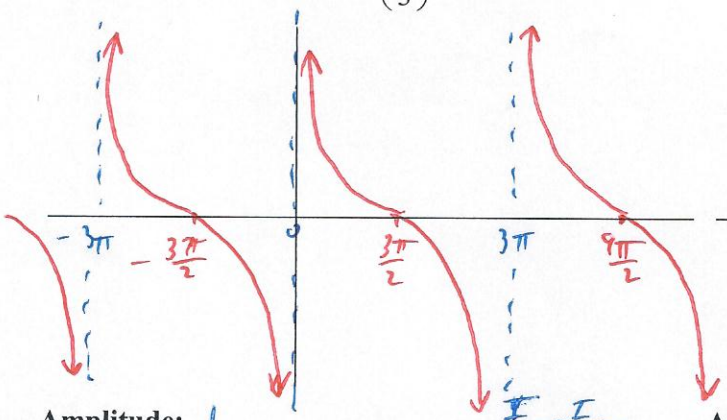
Initial Interval: $0 < bx - c < \pi$

Parent Function $y = \cot x$



Practice Problems

$y = \cot\left(\frac{x}{3}\right)$



Amplitude: 1

Period: $\pi \div \frac{1}{3} = 3\pi$

Phase Shift: —

Initial Interval: $0 < x < 3\pi$

$0 < \frac{x}{3} < \pi$

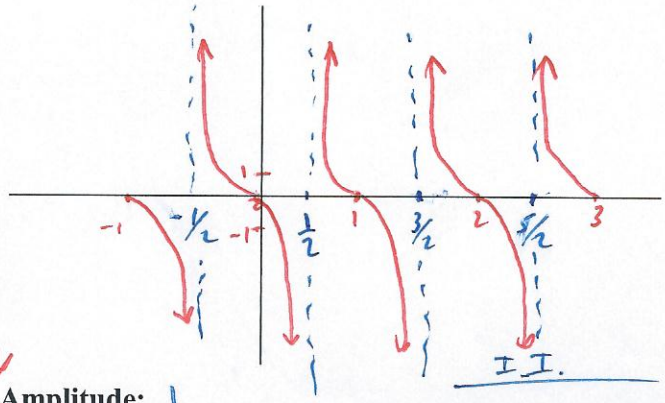
$0 < x < 3\pi$

$\frac{\text{period}}{2} = \frac{3\pi}{2}$ add

$3\pi + \frac{3\pi}{2} = \frac{9\pi}{2}$

$\frac{6\pi}{2} + \frac{3\pi}{2}$

$y = \cot\left(\pi x - \frac{\pi}{2}\right)$



Amplitude: 1

Period: $\frac{\pi}{\pi} = 1$

Phase Shift: —

Initial Interval: $\frac{1}{2} < x < \frac{3}{2}$

$0 < \pi x - \frac{\pi}{2} < \pi$

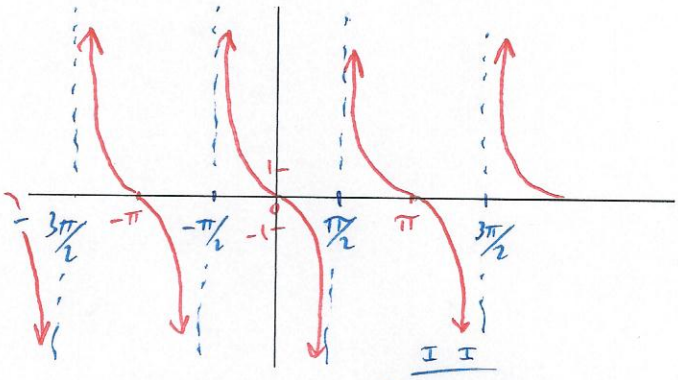
$+\frac{\pi}{2}$ $+\frac{\pi}{2}$ $+\frac{\pi}{2}$

$\frac{1}{\pi}\left(\frac{\pi}{2}\right) < \frac{\pi x}{\pi} < \frac{3\pi}{2}\left(\frac{1}{\pi}\right)$

$\frac{\text{period}}{2} = \frac{1}{2}$ add

$\frac{1}{2} < x < \frac{3}{2}$

$$y = \cot\left(x - \frac{\pi}{2}\right)$$



Amplitude: 1

Period: π

Phase Shift: $-$

Initial Interval: $\frac{\pi}{2} < x < \frac{3\pi}{2}$

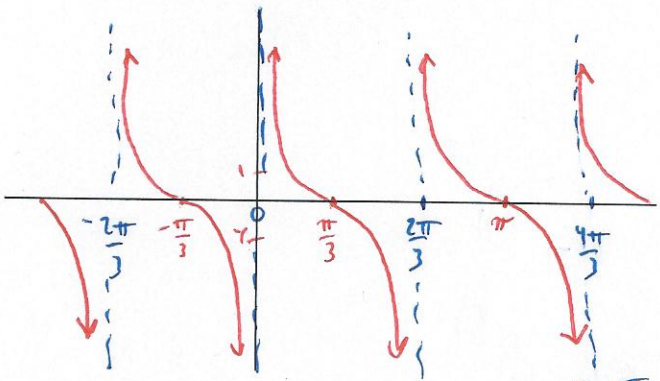
I I

$0 < x - \frac{\pi}{2} < \pi$

$+\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2}$

$\frac{\pi}{2} < x < \frac{3\pi}{2}$

$$y = \cot\left(\frac{3x}{2} - \pi\right)$$



Amplitude: 1

Period: $\pi \div \frac{3}{2} = \frac{2\pi}{3}$

Phase Shift: $-$

Initial Interval: $\frac{2\pi}{3} < x < \frac{4\pi}{3}$

I I

$0 < \frac{3x}{2} - \pi < \pi$

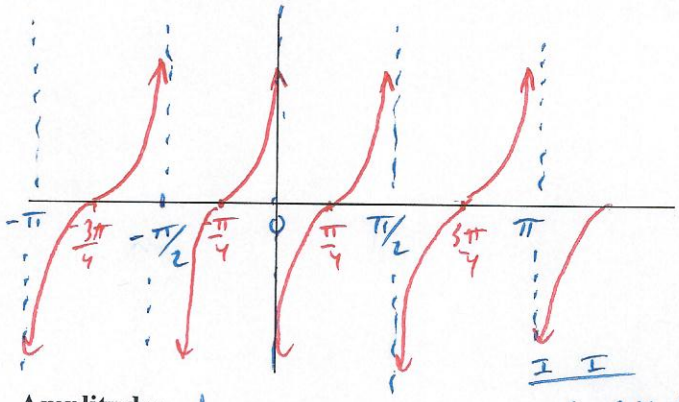
$+\pi \quad +\pi \quad +\pi$

$\frac{2}{3}(\pi) < \frac{2}{3}\left(\frac{3x}{2}\right) < \frac{2}{3}(2\pi)$

$\frac{2\pi}{3} \div 2$ Add

$\frac{2\pi}{3} \cdot \frac{1}{2} = \boxed{\frac{\pi}{3}}$

$$y = -\cot(2x + \pi)$$



Amplitude: 1

Period: $\frac{\pi}{2}$

Phase Shift: $-$

Initial Interval: $-\frac{\pi}{2} < x < 0$

I I

$0 < 2x + \pi < \pi$

$-\pi \quad -\pi \quad -\pi$

$-\frac{\pi}{2} < \frac{2x}{2} < \frac{0}{2}$

$-\pi/2 < x < 0$

$\frac{\text{Period}}{2} = \frac{\pi}{2} \div 2 = \frac{\pi}{2} \cdot \frac{1}{2} = \boxed{\frac{\pi}{4}}$ Adj